# To Integrate or Not To Integrate? The Role of Country Size<sup>1</sup>

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#### Abstract

Public goods are an important aspect to consider when countries integrated economically. Using a Hotelling style model with elements from Alesina and Spolaore (2003), we analyse the effects of economic integration between two countries, the only difference between them being size. In the first part of the paper we consider the sharing of fixed costs ad the effect on transport costs when countries integrate. We find that if the two countries' capitals are retained, the smaller country benefits. The smaller country will vote for integration and the larger country will not. When we consider only allowing 1 capital in the integrated country, we find that retaining the capital of the larger country is the most popular choice. Since integration will not be voted for when the capitals are retained in the first part, in the second part of the paper we try to find cases to the contrary by including infrastructure choice. Here, we look at the benefits from choosing a level of infrastructure to reduce transport costs. The country can either choose the infrastructure independently or as an integrated country. Overall benefit may increase with integration in this case. We also find that, when the size difference between the countries is not too big or when the social planner is choosing the level of infrastructure, integration may be preferred by voters in both countries.

# 1 Introduction

Countries integrate and form unions for both economical and political reasons, an example is the union of east and west Germany. Not all unions results in new countries, we can have unions where each member country retains its sovereignty. One such union of recent interest is the European Union (EU). The decision to integrate can be analysed from many perspectives. From the public good perspective, this means that after integration, the countries can share the benefits and costs of their public goods.

The European Constitution is an international treaty intended to create a constitution for the EU. Thus, the constitution can be seen as a huge step towards European integration. While in the process of ratification by its member states, the constitution was rejected by the French and the Dutch in mid 2005. Soon after, Luxembourg voted "yes" on the constitution. Polls in the United Kingdom had shown that more voters are against the constitution than for it. It would seem that a country's relative size would determine if they would prefer integration.

To model this sort of behaviour, an obvious choice would be Hotelling or location models. In Alesina and Spolaore (2003)'s size of nations model, formation of countries are examined with a Hotelling type setup. The world lies on a straight line and the number and size of the nations formed is based different factors such as fixed costs and transport costs. In their model, Alesina and Spolaore find the equilibrium and efficient number of nations. It was established that the public good (i.e. the capital) will be optimally located in the centre of the country. Our paper adopts this concept as a major assumption. Most of the initial framework in our paper is based on the size of nations model, see appendix for more details.

Following Alesina and Spolaore (2003), regionalism is explored in Goyal and Staal (2004) Regional preferences were defined in terms of size, location and diversity. This will in term define the number of resultant countries. Alesina and Spolaore extend their paper in Alesina, Spolaore and Wacziarg (2000), the relationship between openness in trade and the equilibrium size and number of nations were explored.

The second model in this paper looks at infrastructure investment; we borrow in part from Ghosh and Meagher (2005), where infrastructure investments are treated as a device to reduce the magnitude of transport cost in a Hotelling style model. We can thus explore if it is more efficient to make infrastructure investments as a singular country or as an integrated one.

The papers mentioned are location models which explores the creation of nations and their resultant size and numbers. One paper which explore what happens when countries which are already formed opens up trade between them is Tharakan and Thisse (2002). Tharakan and Thisse looks at how benefits, from openness to trade, differ between countries when they have different sizes. Country size was found to play an important part in benefits from trade.

Since the world is formed into nations only once and is left to evolve over time, it makes more sense to look at integration of countries already formed than to look at formation of these countries. In our model, we do exactly this and examine what happens when the countries move towards some form of economic integration from a public good perspective. To this purpose, we use a framework with 2 countries of unequal size with their populations distributed uniformly; a geographically smaller country would also have a smaller population.

In this paper, we look at two main model, one where two countries of different size integrate, share the use of their public good and costs. another where two countries of different sizes integrate and make infrastructure investment decisions together. A country's gain would depend largely on the size of their direct neighbours. Therefore, we not only need to look at different sizes but specifically adjacent countries with different sizes. This framework will help us examine how country size affects the decision to integrate. We can investigate who are the winners and losers, if integration will occur and under what circumstances it will occur.

# 2 Fixed costs and transport costs

In this model, it is assumed that the two countries each incur a fixed cost (of maintaining a capital) and individuals incur a transport cost (of being away from the capital). We analyse the changes to costs when a country goes from non-integrated to integrated. When integrated, the fixed costs are shared between the two countries.

## 2.1 Basic Setup (Base Case)

This setup is modified from the basic utility functions found in Alesina and Spolaore (2003).

#### Definitions

 $s_j = \text{size of country j}$ 

 $l_i$  = individual i's distance from the public good

 $\alpha = \text{cost}$  of being away from the public good (as per distance from the public good)

k = cost of the public good per country or cost of maintaining a capital (assumed to be the same for every country)

Costs faced by individual i =

$$\frac{k}{s_j} + \alpha l_i \tag{1}$$

Costs faced by country j =

$$k + \alpha \int l_i \tag{2}$$

We assume 2 countries of different sizes, the smaller country will be indexed by s (C<sub>s</sub>) and the larger country by L (C<sub>s</sub>). s<sub>s</sub>=1 for C<sub>s</sub> and s<sub>L</sub> = S for C<sub>L</sub>. We can normalize the size of C<sub>s</sub> to 1 without loss of generality. S becomes a size dissimilarity index. We locate the 2 countries side by side on a linear model with a uniform population distribution. Following Alesina and Spolaore (2003), the capital of each country will be located in its centre. Individual I's transport cost ( $\alpha l_i$ ) is increasing with the distance from the capital. The individual located at the edge of the country will face the highest transport cost of ( $\alpha s_j/2$ ), see Figure 1. This will be our base case for comparison purposes.



Figure 1: Fixed Costs and Transport Costs (Base Case)

Total costs of  $C_s = \frac{1}{4}\alpha + k$ Total costs of  $C_L = \frac{1}{4}\alpha S^2 + k$  Total costs of world =  $\alpha \left(\frac{1}{4}S^2 + \frac{1}{4}\right) + 2k$ 

Integration between the 2 countries can happen in a number of ways. When the countries form unions, such as the EU, it is logical to assume that the capitals of these countries would not be changed. This is analysed in the first case where both capitals are retained after integration. When the countries form a new country like Germany, we can assume that eventually there will be only one capital. We examine this scenario with three cases, when either one of the capitals is retained and when a new capital is created. We will explore these alternatives in the following sections.

## 2.2 Retain the original capitals

**Proposition 1** When the 2 capitals are retained, integration is better for overall welfare, but politically, it will never happen.

*Proof:* If the 2 original capitals are retained, individuals, in  $C_L$ , who were too far from their capital may now find  $C_s$ 's capital nearer and use it instead. The population between the 2 capitals will split into half and use their nearest capital. There is a reduction in transport costs faced by  $C_L$  as observed in Figure 2, (changes in transport costs shown by the dotted lines). The combined country will now share the fixed costs of 2k.



Figure 2: Fixed Costs and Transport costs (Retain Capitals)

We notice that  $C_s$  make a gain on the reduction in fixed costs.  $C_L$  on the other hand makes a loss on the increase in fixed costs and a gain from transport cost reductions.

Total costs of  $C_s = \frac{1}{4}\alpha + \frac{2k}{S+1}$ Total costs of  $C_L = \alpha \left(\frac{1+S}{4}\right)^2 + \frac{1}{8}\alpha S^2 - \frac{1}{8}\alpha + \frac{2Sk}{S+1}$  Total costs of world  $=\frac{1}{8}\alpha + \alpha \left(\frac{1+S}{4}\right)^2 + \frac{1}{8}\alpha S^2 + 2k$ 

When we look at increase in welfare when integrated, we find that welfare in  $C_s$  is better, welfare in  $C_L$  is better if  $k > \frac{\alpha(1-S^2)}{16}$  and overall welfare is better. When we look at wheter a country's citizens would vote for integration, we find that  $C_s$  will always vote for integration and  $C_L$  will never vote for integration. Even though overall welfare is better, integration will not be voted for by  $C_L$ . See appendix for more information.  $\Box$ 

## 2.3 One capital cases

**Proposition 2** When the 2 countries integrate and end up with only one capital, there exist cases where overall welfare is higher when integrated, but politically, integration will not be voted for.

*Proof:* See following three lemmas

#### 2.3.1 Retain capital s

In this scenario, capital s is retained, the new transport costs are as observed in Figure 3 (changes in transport costs shown by the dotted lines). The combined country will now share the fixed costs of k for one capital.

**Lemma 1** When the 2 countries integrate and retain the smaller country's capital, when  $\tilde{k}_{rs} > k > k_{rs}^*$ , overall welfare is higher when integrated, but politically, integration will not be voted for.

Total costs of  $C_s = \frac{1}{4}\alpha + \frac{k}{S+1}$ Total costs of  $C_L = \frac{1}{2}\alpha \left(S + \frac{1}{2}\right)^2 - \frac{1}{8}\alpha + \frac{Sk}{S+1}$ Total costs of world  $= \frac{1}{8}\alpha + \frac{1}{2}\alpha \left(S + \frac{1}{2}\right)^2 + k$ 

When we look at increase in welfare when integrated, we find that welfare in  $C_s$  is better, welfare in  $C_L$  is better if  $k > \frac{S\alpha(S+2)(S+1)}{4}$  and overall welfare is better if  $k > \frac{\alpha(2S+S^2)}{4}$ . The minimum k needed for overall welfare to be positive (when capital s is retained) is

$$k_{rs}^* = \frac{\alpha(2S+S^2)}{4}.$$
 (3)

 $k_{rs}^*$  is increasing with S.



Figure 3: Fixed costs and Transport Costs (Retain capital s)

When we look at wheter a country's citizens would vote for integration, we find that  $C_s$  will always vote for integration and  $C_L$  will only vote for integration if  $k > \frac{\alpha S(S+1)^2}{2}$ . Since, we need both countries to vote for integration, the minimum k needed for the vote to pass (when capital s is retained) is

$$\tilde{k}_{rs} = \frac{\alpha S(S+1)^2}{2}.$$
(4)

 $\tilde{k}_{rs}$  is increasing with S. Since,  $\tilde{k}_{rs} > k_{rs}^*$ , When  $\tilde{k}_{rs} > k > k_{rs}^*$ , overall welfare is positive and the vote will not pass. See appendix for more information.

#### 2.3.2 Retain capital L

In this scenario, capital L is retained, the new transport costs are as observed in Figure 4 (changes in transport costs shown by the dotted lines). The combined country will now share the fixed costs of k for one capital.

**Lemma 2** When the 2 countries integrate and retain the larger country's capital, when  $\tilde{k}_{rL} > k > k_{rL}^*$ , overall welfare is higher when integrated, but

politically, integration will not be voted for.



Figure 4: Fixed costs and Transport costs (Retain capital L)

Total costs of  $C_s = \frac{1}{2}\alpha \left(1 + \frac{S}{2}\right)^2 - \frac{1}{8}\alpha S^2 + \frac{k}{S+1}$ Total costs of  $C_L = \frac{1}{4}\alpha S^2 + \frac{Sk}{S+1}$ Total costs of world  $=\frac{1}{2}\alpha \left(1 + \frac{S}{2}\right)^2 + \frac{1}{8}\alpha S^2 + k$ 

When we look at increase in welfare when integrated, we find that welfare in  $C_s$  is better if  $k > \frac{\alpha(S+1)(2S+1)}{4S}$ , welfare in  $C_L$  is better and overall welfare is better if  $k > \frac{\alpha(2S+S^2)}{4}$ . The minimum k needed for overall welfare to be positive (when capital L is retained) is

$$k_{rL}^* = \alpha \left(\frac{1+2S}{4}\right). \tag{5}$$

 $k_{rL}^*$  is increasing with S.

When we look at wheter a country's citizens would vote for integration, we find that  $C_s$  will only vote for integration if  $k > \frac{\alpha(S+1)^2}{2S}$  and  $C_L$  will always vote for integration. Since, we need both countries to vote for integration, the minimum k needed for the vote to pass (when capital L is retained) is

$$\tilde{k}_{rL} = \frac{\alpha (S+1)^2}{2S}.$$
(6)

 $\tilde{k}_{rL}$  is increasing with S. Since,  $\tilde{k}_{rL} > k_{rL}^*$ , when  $\tilde{k}_{rL} > k > k_{rL}^*$ , overall welfare is positive and the vote will not pass. See appendix for more information.

#### 2.3.3 Set up a new capital

In this scenario, we allow for a new capital to be set up which would minimise overall transport costs. The location which minimises overall transport cost is in the middle of the overall length  $\left(\frac{1+S}{2}\right)$ . See Figure 5 for new transport costs (changes in transport costs shown by the dotted lines). The combined country will now share the fixed costs of k for one capital.

**Lemma 3** When the 2 countries integrate and set up a new capital in the middle (where transport costs are minimized), when  $\tilde{k}_{new} > k > k_{new}^*$ , overall welfare is higher when integrated, but politically, integration will not be voted for.



Figure 5: Fixed costs and Transport costs (New capital)

Total costs of  $C_s = \frac{\alpha}{2} \left(\frac{1+S}{2}\right)^2 - \frac{\alpha}{2} \left(\frac{S-1}{2}\right)^2 + \frac{k}{S+1}$ Total costs of  $C_L = \frac{\alpha}{2} \left(\frac{1+S}{2}\right)^2 + \frac{\alpha}{2} \left(\frac{S-1}{2}\right)^2 + \frac{Sk}{S+1}$ Total costs of world  $= \alpha \left(\frac{1+S}{2}\right)^2 + k$ 

When we look at increase in welfare when integrated, we find that welfare in  $C_s$  is better if  $k > \frac{\alpha(S+1)(2S-1)}{4S}$ , welfare in  $C_L$  is better if  $k > \alpha \frac{(1+S)}{4}$  and overall welfare is better if  $k > \frac{\alpha S}{2}$ . The minimum k needed for overall welfare to be positive (when new capital is chosen) is

$$k_{new}^* = \frac{\alpha S}{2}.\tag{7}$$

 $k_{new}^*$  is increasing with S.

When we look at whether a country's citizens would vote for integration, we find that  $C_s$  will only vote for integration if  $k > \alpha \left(\frac{1+S}{2}\right)$  and  $C_L$  will only vote for integration if  $k > \frac{\alpha(S+S^2)}{2}$ . Since, we need both countries to vote for integration, the minimum k needed for the vote to pass (new capital is chosen) is

$$\tilde{k}_{new} = \frac{\alpha(S+S^2)}{2}.$$
(8)

as  $\frac{\alpha(S+S^2)}{2} > \alpha\left(\frac{1+S}{2}\right)$ .  $\tilde{k}_{new}$  is increasing with S. Since,  $\tilde{k}_{new} > k^*_{new}$ , when  $\tilde{k}_{new} > k > k^*_{new}$ , overall welfare is positive and the vote will not pass. See appendix for more information.

#### 2.3.4 Comparing the different levels of k

**Proposition 3** Out of the cases where 1 capital is left, retaining the capital of the larger country would be the most popular choice. This is because lower parameter restrictions are needed to require voters to vote for integration.

*Proof:* When we rank the different levels of k, we find the following (smallest on the left, largest on the right): For 1.618>S>2.2143



Figure 6: Comparing k, For 1.618>S>2.2143

For S < 1.618 or S > 2.2143

Since all the k values are increasing with S, the lower the k, the less restrictions there are on the parameters of S. Among the three levels of k required for voting to work, the lowest is  $\tilde{k}_{rL}$ , this means that this method



Figure 7: Comparing k, For S<1.618 or S>2.2143

of integration is most likely to be approved by the countries as it requires the lowest level of parameterization. We also find that although relocating to a new capital requires the lowest level of k  $(k_{new}^*)$  to create an overall welfare increase, it requires the second highest level of k  $(\tilde{k}_{new})$  for voting to be approved than the case where we retain the larger capital.

### 2.4 Discussion

The first case where both capitals are retained is closer to the case where countries form unions but do not form a whole new country. The two countries will have an overall increase in welfare if they integrate. Therefore, from the social welfare perspective it is efficient to do so. This outcome would be possible when the decision to integrate is made by social planners. If the decision to integrate is made by voters, the two countries will never merge. This will result in an overall economic inefficiency.

The case, where only 1 capital will be left, fits the story of countries merging to make a new country. In this case, choosing to retain the capital of the larger country is the most probable way of ensuring voter's approval. The case of creating a new capital is the most socially efficient, but it has a lower chance of being voted through as S gets smaller. It is also possible for welfare to increase after integration and not get voted for. This makes voting somewhat inefficient as a decision making mechanism.

# 3 Infrastructure Investment

In the previous sections we found that politically, integration is possible with one capital. In the case where both capitals are retained, integration will not be voted through. This does not explain real life cases where economic unions do exist. In this section, we add infrastructure choice, to the case where both capitals are retained. The basic setup is again modified from the basic utility functions found in Alesina and Spolaore (2003). The idea of infrastructure is borrowed from Ghosh and Meagher (2005).

Infrastructure is a public good which can improve access to other public goods; better roads would reduce the time it takes to get to the capital. Governments often have to make choices on how much infrastructure to invest in. This section analyses the choice of a level of infrastructure which will reduce a country's transport cost ( $\alpha$ ). When the countries are not integrated, they make separate choices of infrastructure. When they are integrated, they choose a level of infrastructure together. The same basic structure is used from the previous model, instead of looking at the sharing of fixed costs; we now look at the benefits from making an infrastructure investment. We now find cases where it is possible to retain both capitals and have both countries vote for integration.

#### Definitions

 $I_j$  = level of infrastructure chosen by country j, will directly reduce an individual's transport costs by  $I_j$ 

 $\beta I s_j$  = total variable costs of infrastructure per country (variable on  $s_j$ )  $\gamma I^2$  = fixed costs of infrastructure per country

#### Individual i's benefits =

$$I_j l_i - \beta I - \frac{\gamma I^2}{s_j} \tag{9}$$

#### Country j's benefits from choosing $I_i$ level of infrastructure =

$$I \int l_i - \beta I s_j - \gamma I^2 \tag{10}$$

The effect a level I of infrastructure will have on the countries if they are not integrated is shown in Figure 8 . I effectively lowers transport costs for all individuals.

 $\mathbf{C}_s$ 's benefits from choosing  $\mathbf{I}_s$  level of infrastructure =  $I_s(\frac{1}{2})^2 - \beta I_s - \gamma I_s^2$ <br/> $\mathbf{C}_L$ 's benefits from choosing  $\mathbf{I}_L$  level of infrastructure =<br/> $I_L(\frac{S}{2})^2 - \beta I_LS - \gamma I_L^2$ <br/>World benefit =  $I_s(\frac{1}{2})^2 - \beta I_s - \gamma I_s^2 + I_L(\frac{S}{2})^2 - \beta I_LS - \gamma I_L^2$ 

When the countries are integrated, they share a common level of I which is different from their original Is, some individuals in  $C_L$  also benefit by going to  $C_s$ 's capital. This is shown by the dotted lines in Figure 9.



Figure 8: Effect of infrastructure on costs (not integrated)



Figure 9: Effect of infrastructure on costs (integrated)

If the countries choose to integrate, their benefits become as follows:

$$C_s$$
's benefit from choosing I level of infrastructure  
=  $I \sum_{i=0}^{1} l_i - \beta I - \frac{\gamma I^2}{(1+S)} = I \left(\frac{1}{2}\right)^2 - \beta I - \frac{\gamma I^2}{(1+S)}$ 

 $C_L$ 's benefit from choosing I level of infrastructure

$$= I\sum_{i=1}^{L} l_i - \beta IS - \frac{\gamma I^2 S}{(1+S)} = I\left(\left(\frac{1+S}{4}\right)^2 + \frac{1}{2}\left(\frac{S}{2}\right)^2 - \frac{1}{2}\left(\frac{1}{2}\right)^2\right) - \beta IS - \frac{\gamma I^2 S}{(1+S)}$$

World benefit from choosing I level of infrastructure

$$= I \sum_{i=0}^{1} l_i + I \sum_{i=1}^{L} l_i - \beta I (1+S) - \gamma I^2 = I \left( \frac{1}{2} \left( \frac{1}{2} \right)^2 + \left( \frac{1+S}{4} \right)^2 + \frac{1}{2} \left( \frac{S}{2} \right)^2 \right) - \beta I (1+S) - \gamma I^2$$

The level of infrastructure chosen will depend on how it is chosen. We consider 2 alternatives, a voting outcome (where voters vote on a level of infrastructure) and a social optimum outcome (where a social planner chooses the level of infrastructure).

### 3.1 Voting Outcome

When considering the voting outcome for a choice of I, we take the median voter's choice. To find his choice, we need to find  $l_m$  (the distance the median voter is away from the capital. The median voter's choice becomes a maximization of  $I_j l_m - \beta I_j^2$ . See Appendix for details on finding the location of the median voter.

When not integrated, the median voter is located in the middle of the country,  $l_m = s_j/4$ . In C<sub>s</sub>,  $l_s = 1/4$ . In C<sub>L</sub>,  $l_L = S/4$ . When integrated, there are 2 possible cases for  $l_m$  (The distance the median voter is away from his nearest capital.) When  $S \leq 3$ ,  $l_m = (1 + S)/8$ . When S > 3,  $l_m = S/6$ .

#### 3.1.1 Voting outcome when the countries remain separate

The infrastructure level chosen by  $C_s$  via voting is =

$$I_s = \begin{cases} \frac{1}{2\gamma} \left(\frac{1}{4} - \beta\right) & if \quad \beta < \frac{1}{4} \\ 0 & if \quad \beta \ge \frac{1}{4} \end{cases}$$
(11)

There is an upper limit on  $\beta =$ 

$$\beta_s = \frac{1}{4} \tag{12}$$

, as the I chosen cannot be negative.

The infrastructure level chosen by  $C_L$  via voting is =

$$I_L = \begin{cases} \frac{1}{2\gamma} \left(\frac{S^2}{4} - S\beta\right) & if \quad \beta < \frac{S}{4} \\ 0 & if \quad \beta \ge \frac{S}{4} \end{cases}$$
(13)

There is an upper limit on  $\beta =$ 

$$\beta_L = \frac{S}{4} \tag{14}$$

, as the I chosen cannot be negative.

#### 3.1.2 Voting outcome when the countries integrate

Since the median voter can have 2 locations given the size of the country, we need to consider the I chosen for these 2 cases.

In Voting Case 1, the infrastructure level chosen when  $S \leq 3 =$ 

$$I_{i(v1)} = \begin{cases} \frac{S+1}{16\gamma} (S+1-8\beta) & if \quad \beta < \frac{S+1}{8} \\ 0 & if \quad \beta \ge \frac{S+1}{8} \end{cases}$$
(15)

There is an upper limit on  $\beta =$ 

$$\beta_{i(v1)} = \frac{S+1}{8} \tag{16}$$

, as the I chosen cannot be negative.

In Voting Case 2, the infrastructure level chosen when  $S \leq 3 =$ 

$$I_{i(v2)} = \begin{cases} \frac{1}{12\gamma} (S^2 + S + \beta(6S - 6)) & if \quad \beta < \frac{S^2 + S}{6 - 6S} \\ 0 & if \quad \beta \ge \frac{S^2 + S}{6 - 6S} \end{cases}$$
(17)

There is an upper limit on  $\beta =$ 

$$\beta_{i(v2)} = \frac{S^2 + S}{6 - 6S} \tag{18}$$

, as the I chosen cannot be negative.  $\beta_{i(v2)} < 0$  when S>1, since  $\beta > 0$ ,  $I_{i(v2)}$  will always be zero.

## 3.2 Social Optimum

The social planner will choose the level of I by maximizing total benefits.

#### 3.2.1 When not integrated,

When not integrated, we find that the social planner will choose the same levels of I as the median voter. Therefore, the Is and the upper limits on  $\beta s$  are the same as the voting outcome when not integrated.

#### 3.2.2 When integrated,

The infrastructure level chosen by  $C_L$ 's social planner is =

$$I_{i(so)} = \begin{cases} \frac{1}{32\gamma} (3S^2 + 2S + 3 - \beta(6S + 16)) & if \quad \beta < \frac{3S^2 + 2S + 3}{16 + 16S} \\ 0 & if \quad \beta \ge \frac{3S^2 + 2S + 3}{16 + 16S} \end{cases}$$
(19)

There is an upper limit on  $\beta =$ 

$$\beta_{i(so)} = \frac{3S^2 + 2S + 3}{16 + 16S} \tag{20}$$

, as the I chosen cannot be negative.

Since the upper limits on  $\beta$  determine the level of Is chosen, the relationship between the betas will determine what levels of Is will be chosen by each country when not integrated and by the new country when integrated.

For voting outcome 1  $\beta_L > \beta_{i(v1)} > \beta_s$ For voting outcome 2  $\beta_L > \beta_s > 0 > \beta_{i(v2)}$ For social optimum  $\beta_L > \beta_{i(so)} > \beta_s$ 

There are ranges of  $\beta$  where a country may choose a level of I when not integrated and none when integrated, and vice versa. See appendix for more details.

## 3.3 Changes to benefits from integration

**Proposition 4** Overall welfare is better with integration when  $\beta_s > \beta$  and with S upper bounded by  $\overline{S}$ .

A country will benefit from integration if the change in benefit from integration is positive:

(Benefit from a level of infrastructure chosen as an integrated country) - (Benefit from a level of infrastructure chosen as a separate country) > 0

For 
$$C_s$$
,  $\left( (I_i) \left( \frac{1}{4} \right) - \beta (I_i) - \frac{\gamma (I_i)^2}{1+S} \right) - \left( (I_s) \left( \frac{1}{4} \right) - \beta (I_s) - \gamma (I_s)^2 \right) > 0$   
For  $C_L$ ,  $\left( (I_i) \left( \left( \frac{1+S}{4} \right)^2 + \frac{1}{8}S^2 - \frac{1}{8} \right) - \beta (I_i)S - \frac{\gamma S (I_i)^2}{1+S} \right)$ 

$$-\left(\left(I_L\right)\left(\frac{S^2}{4}\right) - \beta\left(I_L\right)S - \gamma\left(\frac{1}{2\gamma}\left(I_L\right)^2\right) > 0$$

In voting case 2, where  $I_{i(v2)} = 0$ , we find that no country will benefit from integration. In both voting case 1 and the social optimum, we find that both countries will benefit from integration, when  $\beta_s > \beta$ . Also, only  $C_L$  will benefit from integration when  $\beta_i > \beta > \beta_s$ . For the cases where,  $\beta_L > \beta > \beta_{i(v1)}$  or  $\beta > \beta_L$ , we find that no country's will benefit from integration. In cases where, welfare is positive, we find that S will be upper bounded by  $\overline{S}$ . See appendix for detail calculations.

# 3.4 Will the majority in each country vote for integration?

**Proposition 5** Majorities in both countries will prefer integration when  $\beta_s > \beta$  and with S upper bounded by  $\overline{S}$ .

A country will vote for integration if the median voter finds it beneficial to do so. Therefore, we need to look at the change in benefits of the median voter if integration is chosen. The changes in benefits for the median voter are:  $(I_i l_m - \beta I_i - (\frac{\gamma I_i^2}{1+S}) - (I_n l_m - \beta I_n - (\frac{\gamma I_n^2}{size of country})))$ . Where  $I_i$  is the I chosen if integrated and  $I_n$  is the I chosen if not integrated. The median voter in  $C_s$  is located at (1/4) and in  $C_L$  is located in (S/4).

In voting case 2, we find that there are no cases where a country would prefer integration. In voting case 1 and the social optimum, we find that the only case where both countries might prefer integration is when  $\beta_s > \beta$ . Since we need both countries to vote for integration for it to pass,  $\beta_s > \beta$ , is the only case where integration will pass via a vote. For integration to pass when  $\beta_s > \beta$ , we find that S will be upper bounded by  $\overline{S}$ . See appendix for detail calculations.

#### 3.5 Discussion

We can see that  $\gamma$ , the fixed component of infrastructure costs, only affects the change in benefits in terms of its magnitude. The lower fixed costs are, the more likely that a benefit can result from choosing a level of I as an integrated country.  $\beta$ , the variable component, will affect the positivity or negativity of the change in benefits. In this second model, there can be benefits in both countries when  $\beta_s > \beta$ . Thus, it would be socially beneficial under these circumstances to integrate, voters will also be likely to vote for integration in this case. Integration will not be voted when the difference in size is too big and the variable costs of integration too high. If we look at the welfare levels of these outcomes, we find that overall welfare are non positive. Therefore, voting is efficient here as integration is voted through when overall welfare is better and vice versa.

# 4 Conclusions

If countries are to share the fixed costs of maintaining their capitals, the large country would not vote for integration. This is despite of the fact that overall welfare will increase as a result. This seems to fit with the EU scenario, larger countries had rejected the constitution and smaller ones such as Luxembourg have voted it through. This suggests that individual voting may not be the best method when making integration decisions. Therefore, it may be better that such decision be left to a central governing body which can unbiasedly analyze the overall welfare effects of integration.

If we think of the countries integrating to form a new country, it is reasonable for the new country to have one capital. If this is the case, retaining the capital of the larger country is the most popular choice and would be most likely to be voted through. We have seen this historically with Germany where Berlin was retained and Bonn (West Germany's old capital) was moved to Berlin.

When the countries are to choose the same level of infrastructure if they choose to integrate, it is now possible to retain both capitals and have voter prefer integration, when the size difference is small or when the social planner is the one to choose the level of infrastructure. When the size difference between the countries is too big, and the level of infrastructure is decided upon via voting, then no country will prefer to integrate. This gives insight to reasons why countries may prefer to unionize (eg. the EU), as such union are seen as not only as a means to share the fixed costs of the public good but also as a means to cooperate in future changes to public goods (via improvements in infrastructure).

# Appendix

# A Alesina and Spolaore's Size of Nations Model

The basic setup of this paper's model is adapted from Alesina and Spolaore's size of nations model. Presented here, is a simplified version of parts of the model used in this paper. The world population has a mass of one are distributed uniformly on a straight line. An individual's utility on the line is given as:

 $u_i = y - t_i + g - al_i$ 

y is income,  $t_i$  are taxes paid by the individual, g is the gains from the public good, a is the transport cost and  $l_i$  is the distance the individual is away from the public good.

Total taxes must cover the cost of the public good,  $\int t_i di = k + \gamma s$ , where s is the size of the country. The sum of everybody's utility is given as  $\int u_i di = y - (kN + \gamma + a \int l_i di)$  where N is the number of countries in the world. The social planner chooses the optimum number of nations by maximizing total utility. The optimal number of nations is given as  $N^* = \sqrt{a/4k}$ . Since The size of the world is 1, the size of each nation is  $1/N^*$ 

# B Fixed costs and transport cost: Increase in welfare when integrated

## **B.1** Retain the original capitals

Total costs are lower for  $C_s$  if:  $\left(\frac{1}{4}\alpha + k\right) - \left(\frac{1}{4}\alpha + \frac{2k}{S+1}\right) = k\frac{S-1}{S+1} > 0$ , always true, therefore, welfare in  $C_s$  is better in this case

Total costs are lower for  $C_L$  if:  $\left(\frac{1}{4}\alpha S^2 + k\right) - \left(\alpha \left(\frac{1+S}{4}\right)^2 + \frac{1}{8}\alpha S^2 - \frac{1}{8}\alpha + \frac{2Sk}{S+1}\right)$   $= \frac{1}{16} \left(S-1\right) \frac{-16k-\alpha+S^2\alpha}{S+1} > 0 \text{ or } k > \frac{\alpha(1-S^2)}{16}$ therefore, welfare in  $C_L$  will be better if  $k > \frac{\alpha(1-S^2)}{16}$ 

Total costs are lower for the world if:  $\left(\alpha\left(\frac{1}{4}S^2 + \frac{1}{4}\right) + 2k\right) - \left(\frac{1}{8}\alpha + \alpha\left(\frac{1+S}{4}\right)^2 + \frac{1}{8}\alpha S^2 + 2k\right)$  $= \frac{1}{16}\alpha\left(S-1\right)^2 > 0$ , always true, therefore, overall welfare is always better.

# B.2 Retain Capital S

Total costs are lower for  $C_s$  if:  $\left(\frac{1}{4}\alpha + k\right) - \left(\frac{1}{4}\alpha + \frac{k}{S+1}\right) = S\frac{k}{S+1} > 0$ , always true, therefore, welfare in  $C_s$  is better in this case

Total costs are lower for  $C_L$  if:  $\left(\frac{1}{4}\alpha S^2 + k\right) - \left(\frac{1}{2}\alpha \left(S + \frac{1}{2}\right)^2 - \frac{1}{8}\alpha + \frac{Sk}{S+1}\right) = -\frac{1}{4}\frac{-4k+2S\alpha+3S^2\alpha+S^3\alpha}{S+1}$   $= -\frac{1}{4}\frac{-4k+2S\alpha+3S^2\alpha+S^3\alpha}{S+1} > 0 \text{ or } k > \frac{S\alpha(S+2)(S+1)}{4}$ therefore, welfare in  $C_L$  will be better if  $k > \frac{S\alpha(S+2)(S+1)}{4}$ 

Total costs are lower for the world if:  $\left(\alpha \left(\frac{1}{4}S^2 + \frac{1}{4}\right) + 2k\right) - \left(\frac{1}{8}\alpha + \frac{1}{2}\alpha \left(S + \frac{1}{2}\right)^2 + k\right) = -\frac{1}{4}\left(-4k + 2S\alpha + S^2\alpha\right)$   $= -\frac{1}{4}\left(-4k + 2S\alpha + S^2\alpha\right) > 0 \text{ or } k > \frac{\alpha(2S+S^2)}{4}, \text{ therefore, overall welfare will be better if } k > \frac{\alpha(2S+S^2)}{4}$ 

The minimum k needed for overall welfare to be positive (when capital s is retained) is  $k_{rs}^* = \frac{\alpha(2S+S^2)}{4}$ .  $k_{rs}^*$  is increasing with S.

# B.3 Retain Capital L

Total costs are lower for  $C_s$  if:  $\left(\frac{1}{4}\alpha + k\right) - \left(\frac{1}{2}\alpha \left(1 + \frac{S}{2}\right)^2 - \frac{1}{8}\alpha S^2 + \frac{k}{S+1}\right)$   $= -\frac{1}{4}\frac{\alpha + 3S\alpha + 2S^2\alpha - 4Sk}{S+1} > 0 \text{ or } \frac{\alpha(S+1)(2S+1)}{4S} < k,$ therefore, welfare in  $C_s$  will be better if  $k > \frac{\alpha(S+1)(2S+1)}{4S}$ 

Total costs are lower for  $C_L$  if:  $\left(\frac{1}{4}\alpha S^2 + k\right) - \left(\frac{1}{4}\alpha S^2 + \frac{Sk}{S+1}\right) = \frac{k}{S+1}$  $= \frac{k}{S+1} > 0$  always true, therefore, welfare in  $C_L$  is better in this case

Total costs are lower for the world if:  $\left(\alpha\left(\frac{1}{4}S^2+\frac{1}{4}\right)+2k\right)-\left(\frac{1}{2}\alpha\left(1+\frac{S}{2}\right)^2+\frac{1}{8}\alpha S^2+k\right)$  $=-\frac{1}{4}\left(-4k+\alpha+2S\alpha\right)>0 \text{ or } k>\alpha\left(\frac{1+2S}{4}\right), \text{ therefore, overall welfare will be better if } k>\alpha\left(\frac{1+2S}{4}\right)$ 

The minimum k needed for overall welfare to be positive (when capital L is retained) is  $k_{rL}^* = \alpha \left(\frac{1+2S}{4}\right)$ .  $k_{rL}^*$  is increasing with S.

#### B.4 Set up a new capital

Total costs are lower for  $C_s$  if:  $\left(\frac{1}{4}\alpha + k\right) - \left(\frac{\alpha}{2}\left(\frac{1+S}{2}\right)^2 - \frac{\alpha}{2}\left(\frac{S-1}{2}\right)^2 + \frac{k}{S+1}\right)$   $= -\frac{1}{4}\frac{-\alpha+S\alpha+2S^2\alpha-4Sk}{S+1} > 0 \text{ or } k > \frac{\alpha(S+1)(2S-1)}{4S},$ therefore, welfare in  $C_s$  will be better if  $k > \frac{\alpha(S+1)(2S+1)}{4S}$ 

Total costs are lower for  $C_L$  if:  $\left(\frac{1}{4}\alpha S^2 + k\right) - \left(\frac{\alpha}{2}\left(\frac{1+S}{2}\right)^2 + \frac{\alpha}{2}\left(\frac{S-1}{2}\right)^2 + \frac{Sk}{S+1}\right)$   $= -\frac{1}{4}\frac{-4k+\alpha+S\alpha}{S+1} > 0 \text{ or } \alpha\frac{(1+S)}{4} < k,$ therefore, welfare in  $C_L$  will be better if  $k > \alpha\frac{(1+S)}{4}$ 

Total costs are lower for the world if:  $\left(\alpha\left(\frac{1}{4}S^2 + \frac{1}{4}\right) + 2k\right) - \left(\alpha\left(\frac{1+S}{2}\right)^2 + k\right)$  $= -\frac{1}{2}\left(-2k + S\alpha\right) > 0 \text{ or } k > \frac{\alpha S}{2}, \text{ therefore, overall welfare will be better if } k > \frac{\alpha S}{2}.$ 

The minimum k needed for overall welfare to be positive (when new capital is chosen) is  $k_{new}^* = \frac{\alpha S}{2}$ .  $k_{new}^*$  is increasing with S.

# C Fixed costs and transport cost: Will the majority in each country vote for integration?

Since transport costs are decreasing the nearer the individual is to the capital, to find if the country will vote for integration or not, we need only determine if the individual at the capital will vote for or against integration. This works because if the individual at the capital does not benefit from the merge, at least half of the country does not benefit either.

# C.1 Retain the original capitals

#### Country s

a) Individual in the capital of  $C_s$ 's increase in transport cost: 0 - 0 = 0b) Individual in the capital of  $C_s$ 's savings in fixed  $cost: k - \frac{2k}{1+S} = k\frac{S-1}{S+1}$  $C_s$  will vote for it if: b>a:  $k\frac{S-1}{S+1} > 0$ , always true, therefore, C<sub>s</sub> will always vote for integration

#### Country L

a) Individual in the capital of  $C_L$ 's increase in transport cost: 0 - 0 = 0b) Individual in the capital of  $C_L$ 's savings in fixed  $cost: \frac{k}{S} - \frac{2k}{1+S} = -k\frac{S-1}{S(S+1)}$  $C_L$  will vote for it if:b>a:  $-k\frac{S-1}{S(S+1)} > 0$ , always not true, therefore,  $C_L$  will never vote for integration

#### Will the vote not pass even though welfare is better?

Yes, overall welfare is better but  $C_L$  will not vote for it.

## C.2 Retain Capital S

#### Country s

a) Individual in the capital of  $C_s$ 's increase in transport cost:0 - 0 = 0b) Individual in the capital of  $C_s$ 's savings in fixed  $cost:k - \frac{k}{1+S} = S\frac{k}{S+1}$  $C_s$  will vote for it if: b>a:

 $S_{\frac{k}{S+1}} > 0$ , always true, therefore,  $C_s$  will always vote for integration

#### Country L

a) Individual in the capital of  $C_L$ 's increase in transport  $\cot(\alpha(\frac{S+1}{2}) - 0 = \frac{1}{2}\alpha(S+1)$ b) Individual in the capital of  $C_L$ 's savings in fixed  $\cot(\frac{k}{S}) - \frac{k}{1+S} = \frac{k}{S(S+1)}$  $C_L$  will vote for it if:b>a:  $\frac{k}{S(S+1)} - \frac{1}{2}\alpha(S+1) = -\frac{1}{2}\frac{-2k+S\alpha+2S^2\alpha+S^3\alpha}{S(S+1)} > 0$ or  $k > \frac{\alpha S(S+1)^2}{2}$ 

 $C_L$  will only vote for integration if  $k > \frac{\alpha S(S+1)^2}{2}$ .

#### Will the vote not pass even though welfare is better?

Yes, because (the level of k required for  $C_L$  to vote for integration is greater than the level of k required to have overall better welfare),  $\frac{\alpha S(S+1)^2}{2} > \frac{\alpha(2S+S^2)}{4}$ , therefore, there are points where overall welfare may be positive and yet  $C_L$  will not vote for it.

Since, we need both countries to vote for integration, the minimum k needed for the vote to pass (when capital s is retained) is  $\tilde{k}_{rs} = \frac{\alpha S(S+1)^2}{2}$ .  $\tilde{k}_{rs}$  is increasing with S.  $\tilde{k}_{rs} > k_{rs}^*$ , there exists points where overall welfare is positive and the vote will not pass.

# C.3 Retain Capital L

#### Country s

a) Individual in the capital of  $C_s$ 's increase in transport  $\cot(\alpha(\frac{1+S}{2}) - 0 = \frac{1}{2}\alpha(S+1)$ b) Individual in the capital of  $C_s$ 's savings in fixed  $\cot(k) - \frac{k}{1+S} = S\frac{k}{S+1}$  $C_s$  will vote for it if: b>a:  $S\frac{k}{S+1} - \frac{1}{2}\alpha(S+1) = -\frac{1}{2}\frac{\alpha+2S\alpha+S^2\alpha-2Sk}{S+1} > 0$ or  $k > \frac{\alpha(S+1)^2}{2S}$  $C_s$  will only vote for integration if  $k > \frac{\alpha(S+1)^2}{2S}$ 

#### Country L

a) Individual in the capital of  $C_L$ 's increase in transport cost: 0 - 0 = 0b) Individual in the capital of  $C_L$ 's savings in fixed cost:  $\frac{k}{S} - \frac{k}{1+S} = \frac{k}{S(S+1)}$  $C_L$  will vote for it if: b>a:

 $\frac{k}{S(S+1)} > 0$ , always true, C<sub>L</sub> will always vote for integration.

#### Will the vote not pass even though welfare is better?

Yes, because (the level of k required for  $C_s$  to vote for integration is greater than the level of k required to have overall better welfare),  $\frac{\alpha(S+1)^2}{2S} > \alpha \left(\frac{1+2S}{4}\right)$ , therefore, there are points where overall welfare may be positive and yet  $C_s$ will not vote for it.

Since, we need both countries to vote for integration, the minimum k needed for the vote to pass (when capital L is retained) is  $\tilde{k}_{rL} = \frac{\alpha(S+1)^2}{2S}$ .  $\tilde{k}_{rL}$  is increasing with S.  $\tilde{k}_{rL} > k_{rL}^*$ , there exists points where overall welfare is positive and the vote will not pass.

#### C.4 Set up a new capital

#### Country s

a) Individual in the capital of  $C_s$ 's increase in transport cost:  $\alpha(\frac{1+S}{2}-\frac{1}{2})-0 = \frac{1}{2}S\alpha$ b) Individual in the capital of  $C_s$ 's savings in fixed cost:  $k - \frac{k}{1+S} = S\frac{k}{S+1}$  $C_s$  will vote for it if: b>a:  $S\frac{k}{S+1} - \frac{1}{2}S\alpha$ or  $-\frac{1}{2}S\frac{-2k+\alpha+S\alpha}{S+1} > 0$  or  $k > \alpha(\frac{1+S}{2})$  $C_s$  will only vote for integration if  $k > \alpha(\frac{1+S}{2})$ 

Country L

a) Individual in the capital of  $C_L$ 's increase in transport cost:  $\alpha(\frac{2+S}{2}-\frac{1+S}{2}) =$  $\frac{1}{2}\alpha$ 

b) Individual in the capital of  $C_L$ 's savings in fixed  $\operatorname{cost}: \frac{k}{S} - \frac{k}{1+S} = \frac{k}{S(S+1)}$ 
$$\begin{split} & \frac{k}{S(S+1)} - \frac{1}{2}\alpha \\ & \text{or } -\frac{1}{2} \frac{-2k + S\alpha + S^2\alpha}{S(S+1)} > 0 \text{ or } k > \frac{\alpha(S+S^2)}{2}, \\ & \text{C}_s \text{ will only vote for integration if } k > \frac{\alpha(S+S^2)}{2} \end{split}$$

#### Will the vote not pass even though welfare is better?

Yes,  $\cos$  (the level of k required for  $C_s$  to vote for integration is greater than the level of k required to have overall better welfare),  $\frac{\alpha(S+S^2)}{2} > \frac{\alpha S}{2}$ , there are points where overall welfare is positive where  $C_s$  and/or  $C_L$  will not vote for it. Since, we need both countries to vote for integration, the minimum k needed for the vote to pass (new capital is chosen) is  $\tilde{k}_{new} = \frac{\alpha(S+S^2)}{2}$ , as  $\frac{\alpha(S+S^2)}{2} > \alpha\left(\frac{1+S}{2}\right)$ .  $\tilde{k}_{new}$  is increasing with S.  $\tilde{k}_{new} > k_{new}^*$ , there exists points where overall welfare is positive and the vote will not pass.

#### Infrastructure investment: Finding the me-D dian voter

If the two countries choose not to integrate, individuals at the borders will face the highest transport costs, see Figure 10.



Figure 10: Transport costs when not integrated

When the 2 countries merge, transport costs will change for individuals in country 2 who are near the border of country 1. Figure 11 illustrates this.



Figure 11: Transport costs when integrated

To determine the voting equilibrium when voters vote on a level of I to reduce their transport costs, we need to find the person with the median transport costs (the median voter).  $l_m$  = the distance the median voter is away from the capital.

### D.1 Location of the median voter when not integrated

This is quite straight forward when the two countries do not integrate. The transport costs a country will face, when not integrated, is illustrated by Figure 12. The distribution of costs can be represented by Figure 13.

We need only look at the distribution of transport costs; the median voter is located where the area of the distribution is half of the size of the country. The median voter is located at  $s_j/4$  from the capital,  $l_m = s_j/4$ .



Figure 12: Transport cost of country j when not integrated



Figure 13: Distribution of transport costs when not integrated

#### D.2 Location of the median voter when integrated

The median voter is harder to find if the 2 countries choose to integrate. We find that they now face asymmetric transport costs. With the assumption of linear transport costs, we can identify three symmetric groups of areas, a, b and c, see Figure 14.



Figure 14: Transport costs when integrated (Classified into three groups)

The distribution of transport costs can be represented by Figure 15.

The median voter is located where the area of the distribution is half of the size of the two countries. Since the total length of the countries is 1+S, this area needs to be (1+S)/2.

 $l_m$  is the distance the median voter is from his nearest capital. The median voter will be between 0 and 1/2, when  $4l_m = (s_1+s_2)/2$ , and be between 1/2 and (1+S)/4, when  $4s_1/2 + 3l_m = (s_1+s_2)/2$ .

This is falls in area A with  $l_m = (1+S)/8$  (when  $S \leq 3$ ) or in area B where  $l_m = (S/6)$  (when S > 3). The median voter will not be located in area C as (area A + area B) > (1+S)/2.



Figure 15: Distribution of transport costs when integrated

# E Infrastructure investment: Relationship of the upper limits on $\beta$

Since the upper limits on  $\beta$  determine the level of Is chosen, the relationship between the betas will determine what levels of Is will be chosen by each country when not integrated and by the new country when integrated. There are ranges of  $\blacksquare$  where a country may choose a level of I when not integrated and none when integrated, and vice versa.

#### Voting Case 1

 $\begin{array}{l} \beta_L > \beta_{i(v1)} > \beta_s \\ \text{When } \beta_s > \beta, \text{ all the Is are non zero} \\ \text{When } \beta_{i(v1)} > \beta > \beta_s, \ I_s = 0 \\ \text{When } \beta_L > \beta > \beta_{i(v1)}, \ I_s = 0, \ I_{i(v1)} = 0 \\ \text{When } \beta > \beta_L, \text{ all the Is are zero} \end{array}$ 

#### Voting Case 2

 $\begin{array}{l} \beta_L > \beta_s > 0 > \beta_{i(v2)} \\ \text{When } \beta_s > \beta, \ I_{i(v2)} = 0 \\ \text{When } \beta_L > \beta > \beta_s, \ I_s = 0, I_{i(v2)} = 0 \\ \text{When } \beta > \beta_L, \ \text{all the Is are zero} \end{array}$ 

#### Social Optimum

 $\beta_L > \beta_{i(so)} > \beta_s$ When  $\beta_s > \beta$ , all the Is are non zero When  $\beta_{i(so)} > \beta > \beta_s$ ,  $I_s = 0$ When  $\beta_L > \beta > \beta_{i(so)}$ ,  $I_s = 0, I_{i(so)} = 0$ When  $\beta > \beta_L$ , all the Is are zero

# F Infrastructure investment: Changes to benefits from integration (Detailed calculations)

For benefits to be greater when integrated, benefit from integration needs to be positive.

For C<sub>s</sub>, benefit from integration =  $\left((Ii)\left(\frac{1}{4}\right) - \beta\left(Ii\right) - \frac{\gamma(Ii)^2}{1+S}\right) - \left((Is)\left(\frac{1}{4}\right) - \beta\left(Is\right) - \gamma\left(Is\right)^2\right)$ 

For C<sub>L</sub>, benefit from integration =  

$$\left((Ii)\left(\left(\frac{1+S}{4}\right)^2 + \frac{1}{8}S^2 - \frac{1}{8}\right) - \beta(Ii)S - \frac{\gamma S(Ii)^2}{1+S}\right) - \left((IL)\left(\frac{S^2}{4}\right) - \beta(IL)S - \gamma\left(\frac{1}{2\gamma}(IL)\right)^2\right)$$

For the world, benefit from integration =  $C_s$ 's benefit from integration and  $C_L$ 's benefit from integration.

# F.1 Voting Case 1

When 
$$\beta < \beta_s$$
  
 $C_s$ 's benefit from integration =  
 $\left(\left(\frac{S+1}{16\gamma}(S+1-8\beta)\right)\left(\frac{1}{4}\right) - \beta\left(\frac{S+1}{16\gamma}(S+1-8\beta)\right) - \frac{\gamma\left(\frac{S+1}{16\gamma}(S+1-8\beta)\right)^2}{1+S}\right)$   
 $-\left(\left(\frac{1}{2\gamma}\left(\frac{1}{4}-\beta\right)\right)\left(\frac{1}{4}\right) - \beta\left(\frac{1}{2\gamma}\left(\frac{1}{4}-\beta\right)\right) - \gamma\left(\frac{1}{2\gamma}\left(\frac{1}{4}-\beta\right)\right)^2\right)$   
 $= \frac{1}{256\gamma}\left(-S^3 + S^2 + 64S\beta^2 - 32S\beta + 5S - 1\right) > 0$ 

By solving numerically, there exists upper bound S, where benefit from integration is positive for  $\beta < \beta_s$  where  $S \leq \overline{S}$  and  $\overline{S}$  is decreasing in  $\beta$ .

$$C_L \text{'s benefit from integration} = \left(\frac{S+1}{16\gamma}(S+1-8\beta)\right) \left(\left(\frac{1+S}{4}\right)^2 + \frac{1}{8}S^2 - \frac{1}{8}\right)$$

$$\begin{aligned} &-\beta \left( \frac{S+1}{16\gamma} (S+1-8\beta) \right) S - \frac{\gamma S \left( \frac{S+1}{16\gamma} (S+1-8\beta) \right)^2}{1+S} \\ &- \left( \left( \frac{1}{2\gamma} (\frac{S^2}{4} - S\beta) \right) (\frac{S^2}{4}) - \beta \left( \frac{1}{2\gamma} (\frac{S^2}{4} - S\beta) \right) S - \gamma \left( \frac{1}{2\gamma} (\frac{S^2}{4} - S\beta) \right)^2 \right) \\ &= -\frac{1}{256\gamma} \left( 2S^4 - 8S^3\beta - 5S^3 + 40S^2\beta - 3S^2 - 64S\beta^2 + 8S\beta + S - 8\beta + 1 \right) > 0 \end{aligned}$$

By solving numerically, there upper bound exists  $\bar{S}$ , where benefit from integration is positive for  $\beta < \beta_s$  where  $S \leq \bar{S}$  and  $\bar{S}$  is decreasing in  $\beta$ .

World's benefit from integration =  

$$\frac{1}{256\gamma} \left( -S^3 + S^2 + 64S\beta^2 - 32S\beta + 5S - 1 \right) \\
- \frac{1}{256\gamma} \left( 2S^4 - 8S^3\beta - 5S^3 + 40S^2\beta - 3S^2 - 64S\beta^2 + 8S\beta + S - 8\beta + 1 \right) \\
= \frac{1}{128\gamma} \left( \begin{array}{c} -S^4 + 4S^3\beta + 2S^3 - 20S^2\beta + 2S^2 \\ + 64S\beta^2 - 20S\beta + 2S + 4\beta - 1 \end{array} \right) > 0 \\
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By solving numerically, there exists upper bound  $\overline{S}$ , where benefit from integration is positive for  $\beta < \beta_s$  where  $S \leq \overline{S}$  and  $\overline{S}$  is decreasing in  $\beta$ .

When 
$$\beta_{i(v1)} > \beta > \beta_s$$
,  $I_s = 0$   
 $C_s$ 's benefit from integration =  
 $\left(\left(\frac{S+1}{16\gamma}(S+1-8\beta)\right)\left(\frac{1}{4}\right) - \beta\left(\frac{S+1}{16\gamma}(S+1-8\beta)\right) - \frac{\gamma\left(\frac{S+1}{16\gamma}(S+1-8\beta)\right)^2}{1+S}\right)$   
 $= \frac{1}{256\gamma}\left(S+1\right)\left(-S^2+2S+64\beta^2-32\beta+3\right) > 0$ 

By solving numerically, we find that benefits from integration is negative for  $\beta_{i(v1)} > \beta > \beta_s$  and S  $\leq 3$ .

$$\begin{split} C_L's \text{ benefit from integration} &= \\ \left(\frac{S+1}{16\gamma}(S+1-8\beta)\right) \left(\left(\frac{1+S}{4}\right)^2 + \frac{1}{8}S^2 - \frac{1}{8}\right) \\ &-\beta \left(\frac{S+1}{16\gamma}(S+1-8\beta)\right)S - \frac{\gamma S\left(\frac{S+1}{16\gamma}(S+1-8\beta)\right)^2}{1+S} \\ &- \left(\left(\frac{1}{2\gamma}\left(\frac{S^2}{4} - S\beta\right)\right)\left(\frac{S^2}{4}\right) - \beta \left(\frac{1}{2\gamma}\left(\frac{S^2}{4} - S\beta\right)\right)S - \gamma \left(\frac{1}{2\gamma}\left(\frac{S^2}{4} - S\beta\right)\right)^2\right) \\ &= -\frac{1}{256\gamma}\left(2S^4 - 8S^3\beta - 5S^3 + 40S^2\beta - 3S^2 - 64S\beta^2 + 8S\beta + S - 8\beta + 1\right) > \\ 0 \end{split}$$

By solving numerically, there exists upper bound  $\overline{S}$ , where benefit from integration is positive for  $\beta_{i(v1)} > \beta > \beta_s$  where  $S \leq \overline{S}$  and  $\overline{S}$  is decreasing in  $\beta$ .

World's benefit from integration =  

$$\frac{1}{256\gamma} (S+1) (-S^2 + 2S + 64\beta^2 - 32\beta + 3) - \frac{1}{256\gamma} (2S^4 - 8S^3\beta - 5S^3 + 40S^2\beta - 3S^2 - 64S\beta^2 + 8S\beta + S - 8\beta + 1)$$

$$= \frac{1}{128\gamma} \left( \begin{array}{c} -S^4 + 4S^3\beta + 2S^3 - 20S^2\beta + 2S^2 \\ +64S\beta^2 - 20S\beta + 2S + 32\beta^2 - 12\beta + 1 \end{array} \right) > 0$$

By solving numerically, we find that benefits from integration is negative for  $\beta_{i(v1)} > \beta > \beta_s$  and S  $\leq 3$ .

When  $\beta_L > \beta > \beta_{i(v1)}, I_s = 0, I_{i(v1)} = 0$ 

All countries are no better off, they either lose benefits or are indifferent if they integrate.

When  $\beta > \beta_L$ ,  $I_s = 0$ ,  $I_{i(v1)} = 0$ ,  $I_L = 0$ All countries are no better off, they either lose benefits or are indifferent if they integrate.

# F.2 Voting Case 2

All countries are no better off, they either lose benefits or are indifferent if they integrate.

# F.3 Social Optimum

$$\begin{aligned} & \text{When } \beta < \beta_s \\ & \text{C}_s\text{'s benefit from integration} = \\ & \left(\frac{1}{32\gamma}(3S^2 + 2S + 3 - \beta(6S + 16))\right)\left(\frac{1}{4}\right) \\ & -\beta\left(\frac{1}{32\gamma}(3S^2 + 2S + 3 - \beta(6S + 16))\right) - \frac{\gamma\left(\frac{1}{32\gamma}(3S^2 + 2S + 3 - \beta(6S + 16))\right)^2}{1 + S} \\ & -\left(\left(\frac{1}{2\gamma}\left(\frac{1}{4} - \beta\right)\right)\left(\frac{1}{4}\right) - \beta\left(\frac{1}{2\gamma}\left(\frac{1}{4} - \beta\right)\right) - \gamma\left(\frac{1}{2\gamma}\left(\frac{1}{4} - \beta\right)\right)^2\right) \\ & = -\frac{1}{1024\gamma(S+1)} \left(\begin{array}{c} 9S^4 + 60S^3\beta - 12S^3 - 156S^2\beta^2 + 88S^2\beta \\ -18S^2 - 256S\beta^2 + 108S\beta - 12S + 1 \end{array}\right) > 0 \end{aligned}$$

By solving numerically, there exists upper bound  $\overline{S}$ , where benefit from integration is positive for  $\beta < \beta_s$  where  $S \leq \overline{S}$  and  $\overline{S}$  is decreasing in  $\beta$ .

$$\begin{split} C_L's \text{ benefit from integration} &= \\ \left(\frac{1}{32\gamma}(3S^2 + 2S + 3 - \beta(6S + 16))\right) \left(\left(\frac{1+S}{4}\right)^2 + \frac{1}{8}S^2 - \frac{1}{8}\right) \\ -\beta \left(\frac{1}{32\gamma}(3S^2 + 2S + 3 - \beta(6S + 16))\right)S - \frac{\gamma S\left(\frac{1}{32\gamma}(3S^2 + 2S + 3 - \beta(6S + 16))\right)^2}{1+S} \\ -\left(\left(\frac{1}{2\gamma}\left(\frac{S^2}{4} - S\beta\right)\right)\left(\frac{S^2}{4}\right) - \beta \left(\frac{1}{2\gamma}\left(\frac{S^2}{4} - S\beta\right)\right)S - \gamma \left(\frac{1}{2\gamma}\left(\frac{S^2}{4} - S\beta\right)\right)^2\right) \end{split}$$

$$=\frac{1}{1024\gamma(S+1)}\left(\begin{array}{c}-7S^5+32S^4\beta+14S^4-100S^3\beta^2-68S^3\beta+22S^3+256S^2\beta^2\\-232S^2\beta+16S^2+256S\beta^2-20S\beta-7S+32\beta-6\end{array}\right)>0$$

By solving numerically, there exists upper bound  $\bar{S}$  and lower bound  $\bar{S}$ , where benefit from integration is positive for  $\beta < \beta_s$  where  $\bar{S} \leq \bar{S}$ ,  $\bar{S}$  is decreasing in  $\beta$  and  $\bar{S}$  is increasing in  $\beta$ .

World's benefit from integration =  

$$-\frac{1}{1024\gamma(S+1)} \begin{pmatrix} 9S^4 + 60S^3\beta - 12S^3 - 156S^2\beta^2 + 88S^2\beta \\ -18S^2 - 256S\beta^2 + 108S\beta - 12S + 1 \end{pmatrix}$$

$$+\frac{1}{1024\gamma(S+1)} \begin{pmatrix} -7S^5 + 32S^4\beta + 14S^4 - 100S^3\beta^2 - 68S^3\beta + 22S^3 \\ +256S^2\beta^2 - 232S^2\beta + 16S^2 + 256S\beta^2 - 20S\beta - 7S + 32\beta - 6 \end{pmatrix}$$

$$= \frac{1}{1024\gamma} \begin{pmatrix} -7S^4 + 32S^3\beta + 12S^3 - 100S^2\beta^2 - 160S^2\beta \\ +22S^2 + 512S\beta^2 - 160S\beta + 12S + 32\beta - 7 \end{pmatrix} > 0$$

By solving numerically, there exists upper bound  $\overline{S}$ , where benefit from integration is positive for  $\beta < \beta_s$  where  $S \leq \overline{S}$  and  $\overline{S}$  is decreasing in  $\beta$ .

$$\begin{split} & \textbf{When } \beta_{i(so)} > \beta > \beta_s, \ I_s = 0 \\ & \textbf{C}_s\text{'s benefit from integration} = \\ & \left(\frac{1}{32\gamma}(3S^2 + 2S + 3 - \beta(6S + 16))\right)\left(\frac{1}{4}\right) \\ & -\beta\left(\frac{1}{32\gamma}(3S^2 + 2S + 3 - \beta(6S + 16))\right) - \frac{\gamma\left(\frac{1}{32\gamma}(3S^2 + 2S + 3 - \beta(6S + 16))\right)^2}{1+S} \\ & = \frac{1}{1024\gamma(S+1)} \left(\begin{array}{c} -9S^4 - 60S^3\beta + 12S^3 + 156S^2\beta^2 - 88S^2\beta + 18S^2 \\ +512S\beta^2 - 236S\beta + 28S + 256\beta^2 - 128\beta + 15 \end{array}\right) > 0 \\ & \textbf{By solving numerically, we find benefit from integration is negative for } \beta_{i(so)} > \\ & \beta > \beta_s \text{ where } S > 1. \end{split}$$

$$\begin{split} &C_L \text{'s benefit from integration} = \\ & \left(\frac{1}{32\gamma}(3S^2 + 2S + 3 - \beta(6S + 16))\right) \left(\left(\frac{1+S}{4}\right)^2 + \frac{1}{8}S^2 - \frac{1}{8}\right) \\ & -\beta \left(\frac{1}{32\gamma}(3S^2 + 2S + 3 - \beta(6S + 16))\right)S - \frac{\gamma S \left(\frac{1}{32\gamma}(3S^2 + 2S + 3 - \beta(6S + 16))\right)^2}{1+S} \\ & - \left(\left(\frac{1}{2\gamma}(\frac{S^2}{4} - S\beta)\right) \left(\frac{S^2}{4}\right) - \beta \left(\frac{1}{2\gamma}(\frac{S^2}{4} - S\beta)\right)S - \gamma \left(\frac{1}{2\gamma}(\frac{S^2}{4} - S\beta)\right)^2\right) \\ & = \frac{1}{1024\gamma(S+1)} \left(\begin{array}{c} -7S^5 + 32S^4\beta + 14S^4 - 100S^3\beta^2 - 68S^3\beta \\ +22S^3 + 256S^2\beta^2 - 232S^2\beta + 16S^2 + 256S\beta^2 - 20S\beta - 7S + 32\beta - 6\end{array}\right) \\ & > 0 \end{split}$$

By solving numerically, there exists upper bound  $\bar{S}$ , where benefit from integration is positive for  $\beta_{i(so)} > \beta > \beta_s$  where  $S \leq \bar{S}$  and  $\bar{S}$  is decreasing in  $\beta$ .

World's benefit from integration =

$$\begin{split} & \frac{1}{1024\gamma(S+1)} \left( \begin{array}{c} -9S^4 - 60S^3\beta + 12S^3 + 156S^2\beta^2 - 88S^2\beta + 18S^2 \\ +512S\beta^2 - 236S\beta + 28S + 256\beta^2 - 128\beta + 15 \end{array} \right) \\ & + \frac{1}{1024\gamma(S+1)} \left( \begin{array}{c} -7S^5 + 32S^4\beta + 14S^4 - 100S^3\beta^2 - 68S^3\beta + 22S^3 \\ +256S^2\beta^2 - 232S^2\beta + 16S^2 + 256S\beta^2 - 20S\beta - 7S + 32\beta - 6 \end{array} \right) \\ & = \frac{1}{1024\gamma} \left( \begin{array}{c} -7S^4 + 32S^3\beta + 12S^3 - 100S^2\beta^2 - 160S^2\beta \\ +22S^2 + 512S\beta^2 - 160S\beta + 12S + 256\beta^2 - 96\beta + 9 \end{array} \right) > 0 \\ & \text{By solving numerically, we find benefit from integration is negative for } \beta_{i(so)} > \\ & \beta > \beta_s \text{ where } S > 1. \end{split}$$

When  $\beta_L > \beta > \beta_{i(so)}$ ,  $I_s = 0$ ,  $I_{i(so)} = 0$ All countries are no better off, they either lose benefits or are indifferent if they integrate.

When  $\beta > \beta_L$ ,  $I_s = 0$ ,  $I_{i(so)} = 0$ ,  $I_L = 0$ All countries are no better off, they either lose benefits or are indifferent if they integrate.

# G Infrastructure investment: Will the majority in each country vote for integration? (Detailed Calculations)

A country will vote for integration if the median voter finds it beneficial to do so. Therefore, we need to look at the change in benefits of the median voter if integration is chosen. The changes in benefits for the median voter are:  $(I_i l_m - \beta I_i - (\frac{\gamma I_i^2}{1+S})) - (I_n l_m - \beta I_n - (\frac{\gamma I_n^2}{sizeof country}))$ . Where  $I_i$  is the I chosen if integrated and  $I_n$  is the I chosen if not integrated. The median voter in  $C_s$  is located at (1/4) and in  $C_L$  is located in (S/4).

## G.1 Voting Case 1

When  $\beta < \beta_s$ For  $\mathbf{C}_s$   $\mathbf{C}_s$  will vote for integration if change in benefits are positive:  $\left(-\frac{1}{256\gamma}\left(S^3 + 3S^2 - 64S\beta^2 + 3S - 3\right)\right) - \left(-\frac{1}{64\gamma}\left(2S - 8S\beta + S^2 - 1\right)\right)$  $= \frac{1}{256\gamma}\left(-S^3 + S^2 + 64S\beta^2 - 32S\beta + 5S - 1\right) > 0$ 

By solving numerically, there exists upper bound  $\overline{S}$ , where integration will be voted for, where  $\beta < \beta_s$ ,  $S \leq \overline{S}$  and  $\overline{S}$  is decreasing in  $\beta$ .

#### For $C_L$

 $C_L \text{ will vote for integration if change in benefits are positive:} \left(-\frac{1}{256\gamma}\left(-3S^3+3S^2+3S-64\beta^2+1\right)\right) - \left(\frac{1}{64}\frac{S}{\gamma}\left(S^2-2S+8\beta-1\right)\right) = \frac{1}{256\gamma}\left(-S^3+5S^2-32S\beta+S+64\beta^2-1\right) > 0$ or  $\left(-S^3+5S^2-32S\beta+S+64\beta^2-1\right) > 0$ By solving numerically, we find that integration will be voted for, for all S

By solving numerically, we find that integration will be voted for, for all S $\leq$ 3 where  $\beta < \beta_s$ .

When 
$$\beta_{i(v1)} > \beta > \beta_s$$
,  $I_s = 0$   
For  $\mathbf{C}_s$ 

C<sub>s</sub> will vote for integration if change in benefits are positive:  $\left(-\frac{1}{256\gamma}\left(S+1\right)\left(S^2+2S-64\beta^2+1\right)\right) - \left(-\frac{1}{64\gamma}\left(S+1\right)\left(S-8\beta+1\right)\right)$   $= \frac{1}{256\gamma}\left(S+1\right)\left(-S^2+2S+64\beta^2-32\beta+3\right) > 0$ By solving numerically, we find that integration will not be voted for, for

By solving numerically, we find that integration will not be voted for, for all  $S \leq 3$  where  $\beta_{i(v1)} > \beta > \beta_s$ .

#### For $C_L$

 $C_L \text{ will vote for integration if change in benefits are positive:} \left( -\frac{1}{256\gamma} \left( -3S^3 + 3S^2 + 3S - 64\beta^2 + 1 \right) \right) - \left( \frac{1}{64} \frac{S}{\gamma} \left( S^2 - 2S + 8\beta - 1 \right) \right)$  $= \frac{1}{256\gamma} \left( -S^3 + 5S^2 - 32S\beta + S + 64\beta^2 - 1 \right) > 0$ 

By solving numerically, there exists upper bound  $\overline{S}$ , where integration will be voted for, where  $\beta_{i(v1)} > \beta > \beta_s$ ,  $S \leq \overline{S}$  and  $\overline{S}$  is decreasing in  $\beta$ .

When 
$$\beta_L > \beta > \beta_{i(v1)}$$
,  $I_s = 0$ ,  $I_{i(v1)} = 0$   
For  $\mathbf{C}_s$ 

Since there is no infrastructure for  $C_s$  with or without integration, voters in  $C_s$  will be indifferent.

### For $C_L$

 $C_L$  will vote for integration if change in benefits are positive:  $\left(\frac{1}{64\gamma}\left(S^3 - 16S\beta^2\right)\right) - \left(\frac{1}{32}\frac{S^2}{\gamma}\left(S - 4\beta\right)\right)$   $= -\frac{1}{64}\frac{S}{\gamma}\left(S - 4\beta\right)^2 > 0$ This is strictly possible, therefore,  $C_{-}$  will not note for integration

This is strictly negative, therefore,  $C_L$  will not vote for integration.

### When $\beta > \beta_L$ , $I_s = 0$ , $I_{i(v1)} = 0$ , $I_L = 0$ For $C_s$

Since there is no infrastructure for  $C_s$  with or without integration, voters in  $C_s$  will be indifferent.

#### For $C_L$

Since there is no infrastructure for  $C_L$  with or without integration, voters in  $C_L$  will be indifferent.

# G.2 Voting Case 2

When  $\beta < \beta_s$ ,  $I_{i(v2)} = 0$ For  $\mathbf{C}_s$  $\mathbf{C}_s$  will vote for integration if change in benefits are positive:  $\left(-\frac{1}{64\gamma}\left(16\beta^2 - 1\right)\right) - \left(-\frac{1}{8\gamma}\left(\beta - \frac{1}{4}\right)\right)$  $= -\frac{1}{64\gamma}\left(4\beta - 1\right)^2 > 0$ 

This is strictly negative, therefore,  $C_s$  will not vote for integration.

#### For $C_L$

 $C_L$  will vote for integration if change in benefits are positive:  $\left(\frac{1}{64\gamma}\left(S^3 - 16S\beta^2\right)\right) - \left(\frac{1}{32}\frac{S^2}{\gamma}\left(S - 4\beta\right)\right)$   $= -\frac{1}{64}\frac{S}{\gamma}\left(S - 4\beta\right)^2 > 0$ This is strictly nogative, therefore,  $C_{\gamma}$  will not yote for integral

This is strictly negative, therefore,  $C_L$  will not vote for integration.

When  $\beta_L > \beta > \beta_s$ ,  $I_s = 0$ ,  $I_{i(v2)} = 0$ For  $\mathbf{C}_s$ 

Since there is no infrastructure for  $C_s$  with or without integration, voters in  $C_s$  will be indifferent.

For  $C_L$   $C_L$  will vote for integration if change in benefits are positive:  $\left(\frac{1}{64\gamma}\left(S^3 - 16S\beta^2\right)\right) - \left(\frac{1}{32}\frac{S^2}{\gamma}\left(S - 4\beta\right)\right)$  $= -\frac{1}{64}\frac{S}{\gamma}\left(S - 4\beta\right)^2 > 0$ 

This is strictly negative, therefore,  $C_L$  will not vote for integration.

When 
$$\beta > \beta_L$$
,  $I_s = 0$ ,  $I_{i(v1)} = 0$ ,  $I_L = 0$   
For  $\mathbf{C}_s$ 

Since there is no infrastructure for  $C_s$  with or without integration, voters in  $C_s$  will be indifferent.

For  $C_L$ 

Since there is no infrastructure for  $C_L$  with or without integration, voters in  $C_L$  will be indifferent.

#### **G.3** Social Optimum

When  $\beta < \beta_s$ For  $C_s$  $\mathbf{C}_s$  will vote for integration if change in benefits are positive:  $\left(\frac{1}{1024\gamma(S+1)}\left(-9S^4 - 12S^3 + 256S^2\beta^2 - 22S^2 + 256S\beta^2 + 4S + 7\right)\right)$  $= \frac{1}{1024\gamma(S+1)} \begin{pmatrix} -9S^4 + 12S^3 + 256S^2\beta^2 - 128S^2\beta \\ +18S^2 + 256S\beta^2 - 128S\beta + 12S - 1 \end{pmatrix} > 0$ By solving numerically, there exists upper bound  $\bar{S}$ , where integration will

be voted for, where  $\beta < \beta_s$ ,  $S \leq \overline{S}$  and  $\overline{S}$  is decreasing in  $\beta$ .

For 
$$C_L$$
  
 $C_L$  will vote for integration if change in benefits are positive:  
 $\left(\frac{1}{1024\gamma(S+1)}\left(7S^4 + 4S^3 - 22S^2 + 256S\beta^2 - 12S + 256\beta^2 - 9\right)\right)$   
 $-\left(\frac{1}{128}\frac{S}{\gamma}\left(S^2 - 2S + 16\beta - 3\right)\right)$   
 $=\frac{1}{1024\gamma(S+1)}\left(\begin{array}{c}-S^4 + 12S^3 - 128S^2\beta + 18S^2\\+256S\beta^2 - 128S\beta + 12S + 256\beta^2 - 9\end{array}\right) > 0$ 

By solving numerically, there exists upper bound S, where integration will be voted for, where  $\beta < \beta_s$ ,  $S \leq \overline{S}$  and  $\overline{S}$  is decreasing in  $\beta$ .

When 
$$\beta_{i(so)} > \beta > \beta_s$$
,  $I_s = 0$   
For  $\mathbf{C}_s$   
 $\mathbf{C}_s$  will vote for integration if change in benefits are positive:  
 $\left(-\frac{1}{1024\gamma(S+1)}\left(9S^4 + 12S^3 - 256S^2\beta^2 + 22S^2 - 512S\beta^2 + 12S - 12S^2\beta^2\right)\right)$ 

$$\begin{pmatrix} -\frac{1}{1024\gamma(S+1)} \left(9S^4 + 12S^3 - 256S^2\beta^2 + 22S^2 - 512S\beta^2 + 12S - 256\beta^2 + 9\right) \end{pmatrix} - \\ \begin{pmatrix} -\frac{1}{128\gamma} \left(2S - 16\beta - 16S\beta + 3S^2 + 3\right) \end{pmatrix} \\ = \frac{1}{1024\gamma(S+1)} \begin{pmatrix} -9S^4 + 12S^3 + 256S^2\beta^2 - 128S^2\beta + 18S^2 \\ +512S\beta^2 - 256S\beta + 28S + 256\beta^2 - 128\beta + 15 \end{pmatrix} > 0 \\ \text{This is strictly negative, therefore, C_s will not vote for integration.}$$

For  $C_L$  $C_L$  will vote for integration if change in benefits are positive:  $\left(\frac{1}{1024\gamma(S+1)}\left(7S^4 + 4S^3 - 22S^2 + 256S\beta^2 - 12S + 256\beta^2 - 9\right)\right)$ 

$$- \left(\frac{1}{128}\frac{S}{\gamma}\left(S^2 - 2S + 16\beta - 3\right)\right) \\ = \frac{1}{1024\gamma(S+1)} \left(\begin{array}{c} -S^4 + 12S^3 - 128S^2\beta + 18S^2\\ +256S\beta^2 - 128S\beta + 12S + 256\beta^2 - 9\\ - \end{array}\right) > 0$$

By solving numerically, there exists upper bound S, where integration will be voted for, where  $\beta < \beta_s$ ,  $S \leq \overline{S}$  and  $\overline{S}$  is decreasing in  $\beta$ .

When 
$$\beta_L > \beta > \beta_{i(so)}$$
,  $I_s = 0$ ,  $I_{i(so)} = 0$   
For  $\mathbf{C}_s$ 

Since there is no infrastructure for  $C_s$  with or without integration, voters in  $C_s$  will be indifferent.

#### For $C_L$

C<sub>L</sub> will vote for integration if change in benefits are positive:  $\begin{pmatrix} \frac{1}{64\gamma} \left(S^3 - 16S\beta^2\right) \right) - \left(\frac{1}{32} \frac{S^2}{\gamma} \left(S - 4\beta\right)\right)$   $= -\frac{1}{64} \frac{S}{\gamma} \left(S - 4\beta\right)^2 > 0$ This is strictly negative, therefore, C<sub>L</sub> will not vote for integration.

# When $\beta > \beta_L$ , $I_s = 0$ , $I_{i(so)} = 0$ , $I_L = 0$ For $C_s$

Since there is no infrastructure for  $C_s$  with or without integration, voters in  $C_s$  will be indifferent.

#### For $C_L$

Since there is no infrastructure for  $C_L$  with or without integration, voters in  $C_L$  will be indifferent.

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