Conditional OLS Minimum Variance Hedge Ratio

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Abstract

The paper proposes a new methodology to estimate time dependent minimum variance hedge ratios that capture the predictability of returns. Compared to the existing literature on hedging, the advantages of the so called conditional OLS hedge ratio are threefold. First, it recognizes the less than perfect correlation between spot and futures prices. Second, it captures the stochastic movements in hedge ratios arising from the time variation in expected returns. Third, it is easy to estimate and produces virtually instant estimates of the hedge ratio. These characteristics make the conditional OLS hedge ratio potentially superior to the traditional naïve, static OLS, and GARCH hedges as a tool for risk management.

These theoretical considerations motivated the empirical analysis of the article. The ability of the conditional OLS hedge ratio to minimize the risk of a hedged portfolio is compared to conventional static and dynamic approaches, such as the naïve hedge, the roll-over OLS hedge and the bivariate GARCH(1,1) error correction model. The paper analyzes a cross section of six currencies and concludes that modeling the stochastic movements in the hedge ratio via conditional OLS substantially reduces in-sample volatility. With the noticeable exception of British Pound, the conditional OLS hedge ratio also enhances out-of-sample hedging effectiveness. This suggests that the proposed methodology is a potentially superior way to manage foreign exchange risk.

Keywords: Minimum Variance Hedge Ratio, Conditional OLS, Hedging Effectiveness

1. Introduction

There is now mounting evidence to suggest that returns on risky assets time vary. This phenomenon seems to prevail across a wide cross section of assets and across both developed and emerging markets.[1] Most importantly, the evidence indicates that conditional asset pricing models with time varying risk and risk premia capture this predictability (Ferson and Harvey, 1993; Evans, 1994; Ferson and Korajczyk, 1995; Harvey, 1995; Miffre, 2001). It follows therefore that the predictable variations in returns are probably consistent with rational pricing in efficient markets and most likely result from variations in the returns required by investors over time to compensate them for risk and deferred consumption (see Fama, 1991, for an early review).

If the predictable variations in returns are rational, hedging strategies that ignore this predictability might lead to sub-optimal hedging decisions. The purpose of this paper is to propose a new methodology that takes the predictability of returns into account while estimating time dependent minimum variance hedge ratios. The approach modifies the traditional OLS hedge ratio to incorporate conditioning information. It relies on previous work on conditional asset pricing and on portfolio performance evaluation (Ferson and Warther, 1996; Ferson and Schadt, 1996).

This hedge ratio, referred to as the conditional OLS (ordinary least squares) hedge ratio, presents three advantages compared to the traditional naïve, static OLS and GARCH (generalized autoregressive conditional heteroscedasticity) hedge ratios

[1] Among others, the assets tested for time varying risk premia include long-term US corporate bonds (Chang and Huang, 1990), international equity indices (Ferson and Harvey, 1993; Harvey, 1995) and futures contracts (Bessembinder and Chan, 1992).

typically encountered in the literature on hedging. First, unlike the naïve hedge ratio, the conditional OLS hedge ratio recognizes the less than perfect correlation between spot and futures prices. Second, as opposed to the naïve and OLS hedge ratios, the conditional OLS hedge ratio is time dependent. It takes into account the stochastic movements in hedge ratios arising from the time variation in expected returns. Third, the conditional OLS hedge ratio is simple to estimate and does not suffer from the problems of convergence typically run into with the family of GARCH models. Like GARCH, it readjusts hedging positions as new information becomes available. However it does not require the tedious maximization of the GARCH log likelihood function and produces virtually instant estimates of the hedge ratio. For these reasons the investment community could welcome the conditional OLS hedge ratio as an alternative tool for risk management.

These theoretical considerations motivated the empirical part of the paper. The hypothesis tested here is that hedging effectiveness improves when one allows via conditional OLS for some stochastic movements in the minimum variance hedge ratio. The ability of the conditional OLS hedge ratio to minimize portfolio risk is compared to conventional static and dynamic approaches, such as the naïve hedge, the roll-over OLS hedge and the bivariate error correction model with a GARCH(1,1) error structure (Kroner and Sultan, 1993). The paper looks at the exchange rate between the US Dollar and the following six currencies: Australian Dollar (AD), British Pound (BP), Canadian Dollar (CD), Deutsche Mark (DM), Japanese Yen (JY), and Swiss Franc (SF). For each of the six currencies, modeling the stochastic movements in the hedge ratio via conditional OLS substantially reduces in-sample volatility. With the noticeable exception of BP, the conditional OLS hedge ratio also enhances out-of-sample hedging effectiveness. This suggests that the proposed methodology is a potentially superior way to manage foreign exchange risk. For BP however, adjusting the hedge ratio on a weekly basis, via either GARCH or conditional OLS, does not decrease out-of-sample volatility. For this currency, a naïve one-to-one hedge achieves the desired outcome better.

The remainder of the article is organized as follows. Section 2 reviews the literature on hedging. Section 3 presents the theory that underpins the conditional OLS minimum variance hedge ratio and describes the methodologies used to estimate competing hedge ratios. Section 4 introduces the data set. Section 5 presents the empirical results and section 6 concludes.

2. A Brief Review of the Literature on Hedging

Traditionally, three approaches have been proposed as a way to minimize the risk of a cash position. The first one, called the naïve or one-to-one hedge, assumes that the correlation between the spot and the futures is perfect and sets the hedge ratio equal to one over the period of the hedge. The problems with this hedge ratio are twofold. First, it fails to recognize that the correlation between spot and futures prices is less than perfect. Second, it fails to consider the stochastic nature of futures and spot prices and the resulting time variation in hedge ratios.

The second approach, the static OLS hedge, accurately recognizes that the correlation between futures and spot prices is less than perfect and estimates the hedge ratio as the

OLS coefficient of a regression of spot return on futures return (Ederington, 1979; Anderson and Danthine, 1981; Figlewski, 1984). However, the main shortcoming of this measure is that it imposes the restriction of a constant joint distribution of spot and futures price changes. As such it could lead to sub-optimal hedging decisions in periods of high basis volatility. The naïve and OLS approaches are static risk management strategies that involve a one-time decision about the best hedge and do not require any adjustment of the hedge ratio once this decision has been taken.

More recently, quite a large literature has been developed to overcome these problems. The articles that challenge the limitations of the naïve and OLS hedge ratios model the dynamics in the joint distribution of returns and the time variation in the optimal hedge ratio. The hedge ratio is then estimated using the family of GARCH models introduced by Engle (1982) and Bollerslev (1986). These papers study a variety of commodities (Baillie and Myers, 1991; Myers, 1991), Treasury securities (Cecchetti, Cumby, and Figlewski, 1988), foreign exchange instruments (Kroner and Sultan, 1991, 1993; Gagnon, Lypny, and McCurdy, 1998), and stock indices (Park and Switzer, 1995). The conclusions that emerge from these studies are that optimal hedge ratios are time dependent and that dynamic hedging reduces portfolio variance marginally better than static hedging.

3. Theoretical Discussion

3. 1. Conditional OLS Hedge Ratio

Consider a U.S. investor who is long foreign assets. He wants to minimize his exposure to exchange rate risk by going short b_t dollars of futures contracts. The time t return on his hedged portfolio p_t equals

$$p_t = s_t - b_t f_t$$

where s_t and f_t are the changes in the futures and spot prices respectively and b_t is the time t minimum variance hedge ratio.

The variance of the returns on his hedged portfolio, conditional upon a set of instruments Z available at time $t-1$, equals

$$s^2\left(p_t \middle| Z_{t-1}\right) = s^2\left(s_t \middle| Z_{t-1}\right) + \left(b_t \middle| Z_{t-1}\right)^2 s^2\left(f_t \middle| Z_{t-1}\right) - 2\left(b_t \middle| Z_{t-1}\right) s\left(s_t, f_t \middle| Z_{t-1}\right)$$

$s^2\left(. \middle| Z_{t-1}\right)$ and $s\left(.,. \middle| Z_{t-1}\right)$ denote variance and covariance conditional upon Z_{t-1}.

The minimum variance hedge ratio is traditionally defined as the value of $\left(b_t \middle| Z_{t-1}\right)$ that sets the first derivative of $s^2\left(p_t \middle| Z_{t-1}\right)$ with respect to $\left(b_t \middle| Z_{t-1}\right)$ equal to zero (see, for example, Figlewski, 1984, for an OLS estimation or Kroner and Sultan, 1991, for a GARCH estimation). It is the optimal proportion of the spot asset that should be hedged. It is equal to

$$\left(b_t \middle| Z_{t-1}\right) = \frac{s\left(s_t, f_t \middle| Z_{t-1}\right)}{s^2\left(f_t \middle| Z_{t-1}\right)}.$$

A traditional approach to measuring minimum variance hedge ratio is to regress the return on the spot asset onto the return on the futures contract, implicitly assuming b_t is constant.

$$s_t = a + bf_t + e_t \quad (1)$$

The slope coefficient is the OLS minimum variance hedge ratio. The hedger in this case makes no use of publicly available information while forming expectations of future hedge ratios.

Alternatively the hedger could model the time variation in the hedge ratio by assuming a linear relationship between b_t and a set of L mean zero instruments available at time $t-1$, z_{t-1}.[2] b_t then equals

$$(b_t | z_{t-1}) = b_0 + b_1 z_{t-1} \quad (2)$$

where b_0 measures the mean hedge ratio, b_1 is a L-vector of parameter estimates, $z_{t-1} = Z_{t-1} - E(Z)$ is a L-vector of normalized deviation from Z_{t-1}. $b_1 z_{t-1}$ captures the time variation in the hedge ratio. It measures the departure from the mean hedge ratio b_0. In this case the risk-minimizing hedge ratio is time-dependent and changes as new information arrives to the market. The time-varying hedge ratio will reduce to the static OLS hedge ratio if no information is conveyed to the market.

Substituting b in (1) by $(b_t | Z_{t-1})$ in (2) yields

$$s_t = a + b_0 f_t + b_1 f_t z_{t-1} + e_t \quad (3)$$

Equation (3) is a regression of the spot return onto a constant, the futures return, and the product of the futures return with the lagged information variables. The conditional model (3) adds a vector of L predictors $f_t z_{t-1}$ to the regression traditionally used to

[2] This assumption is in line with conventional research on conditional asset pricing models (see, among others, Harvey, 1989).

measure the static OLS hedge ratio. These regressors pick up the variations through time in the hedge ratios that are related to changing economic conditions. The resulting model is referred to as the conditional OLS model with constant a.[3]

We could equally well relax the restriction that a is constant. A linear relationship similar to (2) is used to model the time variation in a

$$\left(a_t \middle| Z_{t-1}\right) = a_0 + a_1 z_{t-1} \tag{4}$$

Substituting a in (3) by $\left(a_t \middle| Z_{t-1}\right)$ in (4) leads to the following regression

$$s_t = a_0 + a_1 z_{t-1} + b_0 f_t + b_1 f_t z_{t-1} + e_t \tag{5}$$

This model is referred to as the conditional OLS model with time-varying a.

Note also that the static OLS model (1) is nested into the conditional OLS models (3) and (5). In particular the conditional models can be assessed against the static model by testing the restrictions $b_1 = 0$ in (3) and $a_1 = b_1 = 0$ in (5).

The time t one-step ahead hedge ratios are estimated using the *ex-ante* model (3) over the in-sample period. This produces estimates of the coefficients b_0 and b_1 in (3) and hence an estimate of the conditional hedge ratio $\left(b_t \middle| Z_{t-1}\right)$ in (2). The sample is then

[3] Z_{t-1} is an approximation of Ω_{t-1}, the actual information set used by hedgers while estimating time varying hedge ratios. The omission of some instruments of Ω_{t-1} in Z_{t-1} could result in serial correlation and heteroscedasticity in the regression errors. To correct for this, the standard errors in (3) are adjusted for serial correlation and heteroscedasticity using Newey and West (1987) correction with Parzen weights. The bandwidth is set equal to one-third of the sample size (see Andrews, 1991).

rolled over to the next weekly observation and the model is re-estimated over the new sample to produce a new estimate of the conditional hedge ratio. Given the sample, this generates a time series of 180 rolling one-step ahead hedge ratios for each hedged portfolio over the out-of-sample period June, 25 1997 – November, 29 2000.[4] The assumption is therefore that hedgers reconsider their position on a weekly basis. They update the parameter estimates each Wednesday as new information becomes available.

3. 2. Alternative Hedge Ratios

The ability of the conditional OLS hedge ratio to minimize portfolio risk is compared to conventional static and dynamic approaches, such as the naïve hedge, the static OLS hedge and the bivariate error correction model with a GARCH(1,1) error structure (Kroner and Sultan, 1993). Each is defined in turn below.

The naïve approach simply sets the hedge ratio equal to one over the period of the hedge. It therefore assumes a perfect correlation between the futures and spot prices and no time variation in the hedge ratio. The static OLS hedge ratio is defined as b in (1). The bivariate GARCH error correction model captures the dynamics in the second moments of the distribution of returns via a GARCH(1,1) error structure and imposes the long-run cointegrating relationship between futures and spot returns via an error correction term $(\ln S_{t-1} - \ln F_{t-1})$ (see, Kroner and Sultan, 1993, for more details). The specification of the model is as follows:

$$s_t = a_S + b_S(\ln S_{t-1} - \ln F_{t-1}) + u_{St} \tag{6a}$$

$$f_t = a_F + b_F(\ln S_{t-1} - \ln F_{t-1}) + u_{Ft} \tag{6b}$$

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[4] The same procedure is applied to the conditional OLS model with time-varying a .

$$h_{St}^2 = c_S + a_S h_{St-1}^2 + b_S u_{St-1}^2 \quad (6c)$$

$$h_{Ft}^2 = c_F + a_F h_{Ft-1}^2 + b_F u_{Ft-1}^2 \quad (6d)$$

$$h_{SFt} = c_{SF} + a_{SF} h_{SF,t-1} + b_{SF} u_{St-1} u_{Ft-1} \quad (6e)$$

where $\frac{h_{SF,t}}{h_{Ft}^2}$ measures the GARCH(1,1) conditional hedge ratio.

4. Data

The data comprise weekly US spot and futures exchange rates between the US Dollar and the following currencies: Australian Dollar (AD), British Pound (BP), Canadian Dollar (CD), Deutsche Mark (DM), Japanese Yen (JY), Swiss Franc (SF). This cross section was chosen because it has been the subject of an important scrutiny in the literature (see, for example, Kroner and Sultan, 1991, 1993; Gagnon, Lypny, and McCurdy, 1998). To compile the time series of futures prices, the closing prices on the nearest maturity futures contract are collected, except in the maturity month when the prices on the second nearest futures are used (Bessembinder, 1992).[5] Returns are computed as 100 times the difference in the natural logarithms of the prices at the end of Wednesday.

The dataset spans the period July, 18 1990 – November 29, 2000. The sample is split into two sub-samples. The in-sample period covers the period July, 18 1990 – June, 18 1997, approximately two-third of the dataset, and is used for estimation. The out-of-

[5] Rolling the hedge forward avoids thin trading and expiration effects. However it introduces some uncertainty about the difference between the futures price of the contract being closed out and the futures price of the new contract that is entered into. At that specific time, hedgers face roll-over basis risk as well as hedge risk.

sample period, used for forecasting and hedging decisions, covers the remaining one-third. A total of 542 observations is considered for each series.

Table I presents the correlation between the changes in the spot and futures prices for each currency. It is clear from this table that the correlation between the spot and futures prices is less than perfect. The correlation is particularly low for Canadian Dollar (0.9639). At least for this asset, basis risk might be substantial and a naïve one-to-one hedge might not be optimal.

Insert Table I

The instruments used to capture the time variation in the spot and futures returns follow from the literatures on return predictability and conditional asset pricing (see, for example, Fama and French, 1989; Ferson and Harvey, 1993; Ferson and Schadt, 1996). These instruments include: the change in the Standard & Poor's composite price index, the dividend yield on the Standard & Poor's composite index, and the term structure of interest rates, defined as the difference in the yields on 30 year Treasury bond and 3 month Treasury bill. The instruments have a mean of zero so that the mean of the conditional hedge ratio equals the static OLS hedge ratio.

5. Empirical Results

5. 1. The statistical significance of conditioning information

Valid conclusions regarding the ability of the conditional OLS hedge to reduce portfolio volatility can only be drawn within a well-specified conditional model. To ensure that this condition is met, model (3) is estimated via OLS over the in-sample period and the hypothesis of a constant hedge ratio is tested using a c^2 test with L degrees of freedom. Table II reports probability values for the c^2 test of the marginal explanatory power of the conditioning information variables, along with the parameter estimates for b_1 in (3). In the left-hand columns, we also report estimates of the static OLS hedge ratios (equation (1)), thereby imposing the restriction $b_1 = 0$ in (3).

The information variables are jointly significant at standard levels of statistical significance. The c^2 test consistently indicates rejection of the hypothesis of a constant hedge ratio at 5 percent level with an average probability value across currencies of 0.01. Clearly, according to table II, it is important to allow the risk minimizing hedge ratio to be time-dependent instead of restricting it to be constant.

Insert Table II

In table III, time variations in both a and b are considered. The table reports estimates of a_1 and b_1 in (5) and also tests the hypotheses of constant a and b in (1). Although marginally for Australian Dollar (with a probability-value of 0.086), the results indicate that the hypothesis of a constant a is rejected. b also appears to be time dependent: the hedge ratios are indeed correlated with the public information variables. The hypothesis that the coefficients a_1 and b_1 are jointly zero produces probability values that never exceed 0.02 and average 0.002 across the six currencies. It follows from tables II and III that the conditional models (3) and (5) are correctly specified and can be used for risk management.

Insert Table III

The paper compares hedging effectiveness across a wide range of hedge ratios. In particular special attention is given to estimating the minimum variance hedge ratio via a bivariate error correction model with a GARCH(1,1) error structure (system 6). The parameter estimates are reported in table IV. The results indicate that the parameters a_i, b_i, c_i for $i = \{S, F, SF\}$ are statistically significant for most currencies, suggesting that the GARCH(1,1) error structure captures the dynamics in the second moments of the joint distribution of returns. Ultimately this implies that the conditional variances (h_{St}^2 and h_{Ft}^2) and covariances h_{SFt} are changing over time and therefore that the risk-minimizing hedge ratios are indeed time-varying. Similar results were reported in Kroner and Sultan (1991,1993) or Gagnon, Lypny, and McCurdy (1998) among others. As opposed to the evidence presented in these papers however, the parameter estimates for b_F and b_S are insignificant, suggesting that there is no major error correction mechanism operating between the spot and futures markets.

Insert Table IV

Figures 1 to 6 display plots of the estimated hedge ratios for each currency both over the in- and out-of-sample periods. It is clear from these graphs that the conditional hedge ratios are changing over time. It follows that restricting it to be constant might lead to hedging decision that are less than optimal.

Insert Figures 1 to 6

5. 2. Hedging Effectiveness

The ability of the conditional OLS hedge (with constant and time-varying a) to minimize portfolio risk is compared to conventional methods, such as the naïve hedge, the static OLS hedge and the GARCH(1,1) hedge. The ability of the competing alternatives to minimize portfolio volatility depends on how close the estimated hedge ratio is to the hedge ratio that would set the portfolio volatility equal to zero. The lower the variance of the hedged portfolio, the better the hedge ratio is at miminizing portfolio risk. For each of the respective hedge ratios the variance of the hedged portfolio is computed each week as

$$s^{2}\left(s_t - b_t f_t\right)$$

The in-sample results are summarized in Table V. Panel A reports the variance of the unhedged asset and the hedged portfolios under different estimates of the hedge ratio. Panel B presents measures of hedging effectiveness, estimated as the percentage reduction in variance compared to an unhedged position.

Insert Table V

The in-sample results in table V clearly indicate that a hedger who uses the conditional OLS hedge can substantially reduce the volatility of his portfolio. For example, the variance of the returns on the conditional hedge with constant a equals 0.02636 for CD. This compares favorably with the variance of the unhedged position (0.3525), of the naïve hedge (0.033), of the static OLS hedge (0.0274), and of the GARCH(1,1) hedge (0.0272). Looking at panel B, the conditional OLS hedge indeed eliminates 92.52 percent of the variability in CD price changes (against 90.63, 92.24, and 92.28 percents for the naïve, the static OLS, and the GARCH(1,1) hedges respectively). Note also that allowing for time variation in a does not improve hedging effectiveness. The results for the other currencies follow the same trend: hedging effectiveness is improved when one allows for time dependency in the hedge ratio via the conditional OLS model with constant a. Finally it is surprising to note that the results in Table V indicate that the OLS hedge in most instances performs better than the naïve and GARCH(1,1) hedges.

The magnitude of variance reduction varies across currencies. Hedging effectiveness ranges from 90.63 percent for CD (with the naïve hedge) to 98.20 percent for SF (with the conditional OLS hedges) over the in-sample period. As mentioned in table I, the low correlation between futures and spot price changes for CD also hints towards a low hedging performance of the naïve one-to-one hedge ratio for this currency.

How much variance reduction can be achieved if one uses historical hedge ratios to determine appropriate hedging strategies for future time periods?[6] Table VI reports the results. Panel A reports the variance of the unhedged asset and the hedged portfolios under different estimates of the hedge ratio. Hedging effectiveness is presented in Panel B.

Insert Table VI

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[6] Out-of-sample conditional OLS and GARCH(1,1) hedge ratios are estimated by using only information available at the time the hedge is placed. Roll-over estimates of the static OLS hedge ratios are used as a basis for risk management in the out-of-sample period.

With the exception of BP, the conditional OLS approach to risk management reduces out-of-sample volatility more substantially than the alternative competing approaches. For example the results for SF clearly indicate that out-of-sample hedging effectiveness is improved when one allows via conditional OLS for time variation in the estimated hedge ratio. For this currency the conditional OLS approach increases hedging effectiveness by 0.44, 0.2, and 0.28 percents compared to the naïve, roll-over OLS and GARCH(1,1) measures respectively. The conclusions are similar for AD, CD, DM and JY. For BP however, the costs involved in readjusting the futures position weekly might have to be weighted against the relatively marginal increase in hedging effectiveness obtained by weekly adjusting the hedge ratio. The simplicity of the naïve method and the very low transaction costs it involves might be more appealing to a hedger than an increase in hedging effectiveness of 0.01 percent obtained by rolling over the OLS estimate. For BP, the conditional OLS hedges perform worse than any alternative measures.

With therefore the noticeable exception of BP out-of-sample, the conditional OLS hedge with constant a improves hedging effectiveness more than any competing alternatives. In particular, it reduces the variance of the hedged portfolio at least as well as the conditional OLS hedge with time-varying a both in- and out-of-samples. Although unconditional a and conditional a are significantly different (table III), relying on unconditional a does not lead to sub-optimal hedging decisions (tables IV and V). Since conditioning a on past information does not improve hedging effectiveness, the remainder of the paper exclusively focuses on the conditional OLS model with constant a.

Finally the hypothesis that the conditional OLS hedge ratio substantially differs from its naïve, roll-over OLS and GARCH(1,1) counterparts is tested. A casual look at figures 1 to 6 already suggests that the correlation between the dynamic hedge ratios is less than perfect. This hypothesis is formally tested in table VII. Panel A reports correlation estimates between the conditional OLS hedge ratios and the competing alternatives for the six currencies. In panel B we formally test the hypothesis that the conditional OLS and the alternative hedge ratios are equal by regressing the difference on a constant and testing the statistical significance of the estimated coefficient. Looking first at panel A, the average correlation between the conditional OLS and roll-over OLS (GARCH(1,1)) hedge ratios equals 0.397 (0.156). As the competing hedge ratios seem to differ, the inferences drawn from one model might be at odds with the one advocated by a competing approach. The results reported in panel B confirm this view. For all currencies, the conditional OLS hedge ratio differs from the naïve hedge ratio. It is statistically different from the roll-over OLS estimate for five out of six currencies. It is statistically different from the GARCH(1,1) estimate for four out of six currencies. It follows that in most cases estimating time dependent hedge ratios via conditional OLS produces estimates of the hedge ratio that are significantly different from their naïve, roll-over OLS, and GARCH(1,1) counterparts.

Insert Table VII

6. Conclusions

This article proposes a new methodology to estimate minimum variance hedge ratios that relies on previous work on conditional asset pricing (Ferson and Warther, 1996;

Ferson and Schadt, 1996). The advantages of this approach are that (1) it recognizes the less than perfect correlation between spot and futures prices, (2) it captures the stochastic movements in hedge ratios arising from the time variation in expected returns and (3) it is easy to estimate and produces virtually instant estimates of the hedge ratio. These characteristics make the conditional OLS hedge ratio potentially superior to the traditional naïve, static OLS, and GARCH hedges as a tool for risk management.

These theoretical considerations motivated the empirical analysis of the article. The hypothesis tested here is that the variance of the hedged portfolio is minimized when one allows via conditional OLS for some stochastic movements in hedge ratios. The ability of the conditional OLS hedge ratio to minimize portfolio risk is compared to conventional static and dynamic approaches; such as the naïve hedge, the roll-over OLS hedge and the bivariate GARCH(1,1) model with an error correction term. Special attention is given to measuring hedging effectiveness in- and out-of-samples.

The article demonstrates that, at least in-sample, modeling the stochastic movements in the hedge ratio via conditional OLS substantially reduces portfolio volatility. With the noticeable exception of BP, the conditional OLS hedge ratio also enhances out-of-sample hedging effectiveness. For BP however, adjusting the hedge ratio on a weekly basis, via either GARCH or conditional OLS, does not decrease out-of-sample volatility. For this currency, a naïve one-to-one hedge achieves the desired outcome better.

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Table I. Correlation between Spot and Futures Price Changes

	Correlation
AD	0.9768
BP	0.9858
CD	0.9672
DM	0.9824
JY	0.9805
SF	0.9850

Table II: Statistical Significance of Conditioning Information: Conditional OLS Model with Constant *a*

The coefficients a and b are estimated from the following unconditional OLS regression:

$$s_t = a + bf_t + e_t \tag{2}$$

where s_t and f_t are the spot and futures returns respectively. For the conditional OLS model with constant a, the regression is as follows:

$$s_t = a + b_0 f_t + b_1 f_t z_{t-1} + e_t \tag{3}$$

or alternatively

$$s_t = a + b_0 f_t + b_{1,RM} f_t RM_{t-1} + b_{1,DY} f_t DY_{t-1} + b_{1,TS} f_t TS_{t-1} + e_t$$

z_{t-1} is a vector of mean zero pre-determined instruments available at time $t-1$, consisting of the lagged return on the Standard & Poor's composite index RM_{t-1}, the lag in the dividend yield on the Standard & Poor's composite index DY_{t-1}, and the lagged term structure of interest rates TS_{t-1}. a, b_0, b_1, $b_{1,RM}$, $b_{1,DY}$, and $b_{1,TS}$ are the estimated parameters. *p*-value is the probability value of a c^2 test for the significance of b_1. The figures in parenthesis are heteroscedasticity and serial correlation consistent *t*-ratios. $\overline{R}^2$ is the adjusted *R*-squared of the regression. The data are weekly and the model is estimated over the in-sample period: July, 25 1990 – June, 18 1997.

Table II - *Continued*

	Unconditional OLS (2)			Conditional OLS model with constant a (3)						
	a	b	$\overline{R}^2$	a	b_0	$b_{1,RM}$	$b_{1,DY}$	$b_{1,TS}$	p-value: $b_1 = 0$	$\overline{R}^2$
AD	-0.005	0.940	0.944	-0.006	0.953	-1.786	-3.548	0.002	0.04	0.944
	(-1.16)	(100.04)		(-0.46)	(48.16)	(-2.25)	(-1.41)	(0.01)		
BP	-0.003	0.968	0.973	-0.002	0.987	-0.291	-6.185	2.047	<0.01	0.974
	(-1.22)	(62.04)		(-0.15)	(105.57)	(-0.53)	(-3.18)	(2.86)		
CD	-0.010	0.883	0.922	-0.009	0.915	-1.403	-9.714	2.346	<0.01	0.925
	(-3.41)	(37.73)		(-1.11)	(37.41)	(-0.82)	(-3.06)	(2.10)		
DM	-0.002	0.995	0.969	-0.001	0.999	-1.042	-1.134	2.321	0.01	0.966
	(-0.56)	(101.62)		(-0.06)	(85.47)	(-2.38)	(-1.46)	(3.49)		
JY	0.000	0.956	0.965	-0.004	0.949	-0.517	0.035	2.661	<0.01	0.966
	(-0.09)	(82.00)		(-0.29)	(61.38)	(-1.46)	(0.05)	(4.27)		
SF	-0.003	0.990	0.982	-0.003	0.988	-0.605	-1.286	1.236	0.03	0.982
	(-1.32)	(125.90)		(-0.26)	(96.81)	(-2.01)	(-1.64)	(2.86)		

Table III: Statistical Significance of Conditioning Information: Conditional OLS Model with Time-Varying *a*

The conditional OLS regression with time-varying a is as follows:

$$s_t = a_0 + a_1 z_{t-1} + b_0 f_t + b_1 f_t z_{t-1} + e_t \tag{5}$$

or alternatively

$$s_t = a_0 + a_{1,RM} RM_{t-1} + a_{1,DY} DY_{t-1} + a_{1,TS} TS_{t-1} + b_0 f_t + b_{1,RM} f_t RM_{t-1} + b_{1,DY} f_t DY_{t-1} + b_{1,TS} f_t TS_{t-1} + e_t$$

where s_t and f_t are the spot and futures returns respectively, z_{t-1} is a vector of mean zero pre-determined instruments available at time $t-1$, consisting of the lagged return on the Standard & Poor's composite index RM_{t-1}, the lag in the dividend yield on the Standard & Poor's composite index DY_{t-1}, and the lagged term structure of interest rates TS_{t-1}. a_0, $a_{1,RM}$, $a_{1,DY}$, $a_{1,TS}$, b_0, b_1, $b_{1,RM}$, $b_{1,DY}$, and $b_{1,TS}$ are the estimated parameters. *p*-value is the probability value of a c^2 test for the significance of a_1 and/or b_1. The figures in parenthesis are heteroscedasticity and serial correlation consistent *t*-ratios. $\overline{R}^2$ is the adjusted *R*-squared of the regression. The data are weekly and the model is estimated over the in-sample period: July, 25 1990 – June, 18 1997.

Table III - *Continued*

	Conditional a					Conditional b						
	a_0	$a_{1,RM}$	$a_{1,DY}$	$a_{1,TS}$	p-value: $a_1 = 0$	b_0	$b_{1,RM}$	$b_{1,DY}$	$b_{1,TS}$	p-value: $b_1 = 0$	p-value: $a_1 = b_1 = 0$	$\overline{R}^2$
AD	-0.001	1.225	-0.735	-0.183	0.086	0.953	-1.855	-3.553	0.020	0.084	0.012	0.944
	(-0.19)	(2.09)	(-0.56)	(-0.52)		(99.64)	(-1.06)	(-1.10)	(0.03)			
BP	-0.003	0.830	0.963	-0.592	0.005	0.988	-0.286	-6.191	2.020	<0.001	<0.001	0.974
	(-0.84)	(2.34)	(1.42)	(-2.21)		(122.95)	(-0.55)	(-8.93)	(5.17)			
CD	-0.012	1.311	0.984	-0.303	0.001	0.920	-1.478	-10.207	2.200	0.001	<0.001	0.925
	(-4.01)	(2.60)	(2.45)	(-1.85)		(35.69)	(-0.81)	(-2.81)	(1.88)			
DM	-0.007	0.935	1.804	-0.477	<0.001	0.998	-0.819	-2.377	1.447	0.007	<0.001	0.969
	(-2.70)	(1.14)	(5.33)	(-2.92)		(123.41)	(-2.00)	(-2.29)	(2.69)			
JY	-0.017	-0.101	2.841	-0.112	<0.001	0.949	-0.481	-0.128	2.711	<0.001	<0.001	0.966
	(-4.40)	(-0.13)	(5.37)	(-0.45)		(123.60)	(-1.41)	(-0.14)	(4.37)			
SF	-0.009	0.506	1.496	-0.263	0.013	0.987	-0.539	-1.205	1.258	0.031	0.001	0.982
	(-4.07)	(1.11)	(3.12)	(-1.96)		(237.78)	(-1.82)	(-1.56)	(2.84)			

Table IV: Bivariate error correction model with a GARCH(1,1) error structure

The table reports parameter estimates from the following system of regressions:

$$
\begin{aligned}
s_t &= a_S + b_S\left(\ln S_{t-1} - \ln F_{t-1}\right) + u_{St} \\
f_t &= a_F + b_F\left(\ln S_{t-1} - \ln F_{t-1}\right) + u_{Ft} \\
h^2_{St} &= c_S + a_S h^2_{St-1} + b_S u^2_{St-1} \\
h^2_{Ft} &= c_F + a_F h^2_{Ft-1} + b_F u^2_{Ft-1} \\
h_{SFt} &= c_{SF} + a_{SF} h_{SF,t-1} + b_{SF} u_{St-1} u_{Ft-1}.
\end{aligned}
$$

where s_t and f_t are the spot and futures returns respectively, S_{t-1} and F_{t-1} are the spot and futures exchange rates at time $t-1$, $\ln$ is the natural logarithm operator, u_{St} and u_{Ft} are residuals, h^2_{St}, h^2_{Ft}, and h_{SFt} are conditional variances and covariance respectively, a_S, a_F, b_S, b_F, a_i, b_i, c_i for $i = \{S, F, SF\}$ are the estimated parameters. t-ratios in parenthesis. The data are weekly and the model is estimated over the in-sample period: July, 25 1990 – June, 18 1997.

Table IV - *Continued*

	AD	BP	CD	DM	JY	SF
α_S	0.0197	0.0671	-0.0248	-0.0427	0.1211	0.0040
	(0.32)	(1.11)	(-0.89)	(-0.47)	(1.55)	(0.05)
α_F	0.0100	0.0686	-0.0116	-0.0448	0.1173	0.0103
	(0.16)	(1.06)	(-0.40)	(-0.50)	(1.48)	(0.12)
β_S	-0.0860	0.0057	-0.0240	-0.0053	0.0364	-0.0044
	(-1.47)	(0.17)	(-0.62)	(-0.10)	(0.72)	(-0.08)
β_F	-0.0714	0.0247	-0.0310	-0.0127	0.0369	-0.0086
	(-1.22)	(0.69)	(-0.72)	(-0.25)	(0.71)	(-0.15)
c_S	1.0627	0.2506	0.0611	1.1759	3.1061	0.3235
	(13.06)	(23.36)	(2.74)	(6.67)	(7.75)	(4.66)
c_F	1.5264	2.2689	0.6577	0.8975	3.0136	0.2344
	(12.53)	(27.70)	(16.20)	(7.15)	(8.29)	(3.34)
c_{SF}	1.3025	0.7040	0.4917	1.0375	2.9909	0.2685
	(12.96)	(5.80)	(12.22)	(6.97)	(8.05)	(3.97)
a_S	0.0038	0.8862	0.0059	0.4383	-0.4413	0.0397
	(0.05)	(152.89)	(2.30)	(8.04)	(-3.30)	(3.40)
a_F	-0.3124	0.0266	0.0184	0.5705	-0.3522	0.0376
	(-3.79)	(0.84)	(3.24)	(14.24)	(-2.92)	(3.16)
a_{SF}	-0.1893	0.6898	0.0148	0.4980	-0.3839	0.0381
	(-2.54)	(13.07)	(4.60)	(10.74)	(-3.05)	(3.31)
b_S	0.0718	0.0034	0.8029	0.1367	0.0406	0.8459
	(3.64)	(5.37)	(10.84)	(8.73)	(3.58)	(31.08)
b_F	0.0626	0.0159	-0.7931	0.1012	0.0564	0.8809
	(3.18)	(12.26)	(-23.58)	(5.95)	(4.27)	(30.91)
b_{SF}	0.0621	0.0027	-0.5022	0.1192	0.0477	0.8669
	(3.22)	(6.00)	(-4.86)	(7.41)	(4.14)	(31.79)

Table V. Hedging Effectiveness: In-Sample Results (July, 25 1990 – June, 18 1997)

	AD	BP	CD	DM	JY	SF
Panel A: Variance						
No Hedge	1.13380	2.34798	0.35247	2.60277	2.18146	2.96649
Naïve One-to-One Hedge	0.06815	0.06693	0.03302	0.08150	0.07952	0.05432
Static OLS Hedge	0.06381	0.06438	0.02736	0.08143	0.07512	0.05402
GARCH (1,1) Hedge with an Error Correction Term	0.06480	0.07675	0.02720	0.08407	0.07573	0.05433
Conditional OLS Hedge with Constant Alpha	0.06274	0.06117	0.02636	0.08025	0.07319	0.05328
Conditional OLS Hedge with Time-Varying Alpha	0.06275	0.06117	0.02637	0.08025	0.07319	0.05328
Panel B: Hedging Effectiveness						
Naïve One-to-One Hedge	93.99%	97.15%	90.63%	96.87%	96.35%	98.17%
Static OLS Hedge	94.37%	97.26%	92.24%	96.87%	96.56%	98.18%
GARCH (1,1) Hedge with an Error Correction Term	94.28%	96.73%	92.28%	96.77%	96.53%	98.17%
Conditional OLS Hedge with Constant Alpha	94.47%	97.39%	92.52%	96.92%	96.64%	98.20%
Conditional OLS Hedge with Time-Varying Alpha	94.47%	97.39%	92.52%	96.92%	96.64%	98.20%

The variance of the portfolio returns is used as a measure of dispersion. Hedging effectiveness is computed as the percentage change in the variance of the hedged portfolio returns compared to the unhedged position.

Table VI. Hedging Effectiveness: Out-of-Sample Results (June, 25 1997 – November, 29 2000)

	AD	BP	CD	DM	JY	SF
Panel A: Variance						
No hedge	2.44216	1.01989	0.50052	2.00555	3.36907	2.09479
Naïve One-to-One Hedge	0.08890	0.03275	0.02251	0.09109	0.17062	0.13624
Roll-over OLS Hedge	0.08929	0.03267	0.02210	0.08942	0.14761	0.13115
GARCH (1,1) Hedge with an Error Correction Term	0.09254	0.03373	0.02225	0.10044	0.15902	0.13288
Conditional OLS Hedge with Constant Alpha	0.08803	0.03376	0.02180	0.08860	0.14735	0.12704
Conditional OLS Hedge with Time-Varying Alpha	0.08822	0.03379	0.02182	0.08867	0.14861	0.12717
Panel B: Hedging Effectiveness						
Naïve One-to-One Hedge	96.36%	96.79%	95.50%	95.46%	94.94%	93.50%
Roll-over OLS Hedge	96.34%	96.80%	95.58%	95.54%	95.62%	93.74%
GARCH (1,1) Hedge with an Error Correction Term	96.21%	96.69%	95.55%	94.99%	95.28%	93.66%
Conditional OLS Hedge with Constant Alpha	96.40%	96.69%	95.64%	95.58%	95.63%	93.94%
Conditional OLS Hedge with Time-Varying Alpha	96.39%	96.69%	95.64%	95.58%	95.59%	93.93%

The variance of the portfolio returns is used as a measure of dispersion. Hedging effectiveness is computed as the percentage change in the variance of the hedged portfolio returns compared to the unhedged position.

Table VII. The Conditional OLS Hedge Ratios with Constant *a* versus the Competing Hedge Ratios

Panel A: Correlation between the conditional OLS hedge ratio and the hedge ratio estimated via

	Roll-Over OLS	GARCH(1,1)
AD	0.3789	0.0876
BP	0.3569	0.1097
CD	0.4507	0.2766
DM	0.0443	0.1115
JY	0.4977	0.2088
SF	0.6534	0.1424
Mean	0.3970	0.1561

Panel B: Regression of the difference in hedge ratios on a constant

	Naïve (1)	Roll-over OLS (2)	GARCH(1,1) (3)
AD	-0.0474	0.0052	-0.0013
	(-29.60)	(3.64)	(-0.60)
BP	-0.0198	0.0066	0.0092
	(-14.53)	(5.31)	(5.57)
CD	-0.0962	0.0099	0.0008
	(-43.13)	(5.06)	(0.37)
DM	-0.0060	0.0002	0.0077
	(-6.13)	(0.16)	(4.15)
JY	-0.0491	-0.0033	-0.0085
	(-30.44)	(-2.20)	(-4.96)
SF	-0.0226	-0.0091	-0.0040
	(-19.63)	(-9.56)	(-2.94)

The table reports coefficient estimates for c in the following regressions:

(1) $HR_{Cond.OLS,t} - 1 = c + e_t$

(2) $HR_{Cond.OLS,t} - HR_{Roll-over\,OLS,t} = c + e_t$

(3) $HR_{Cond.OLS,t} - HR_{GARCH(1,1),t} = c + e_t$

$HR_{Cond.OLS,t}$ is the conditional OLS hedge ratio with constant a, $HR_{Roll-over\,OLS,t}$ is the roll-over OLS hedge ratio, $HR_{GARCH(1,1),t}$ is the GARCH(1,1) hedge ratio. *t*-ratios in parenthesis. The models are estimated over the whole sample (August, 01 1990 - November 29, 2000).

Figure 1: Australian Dollar Hedge Ratios

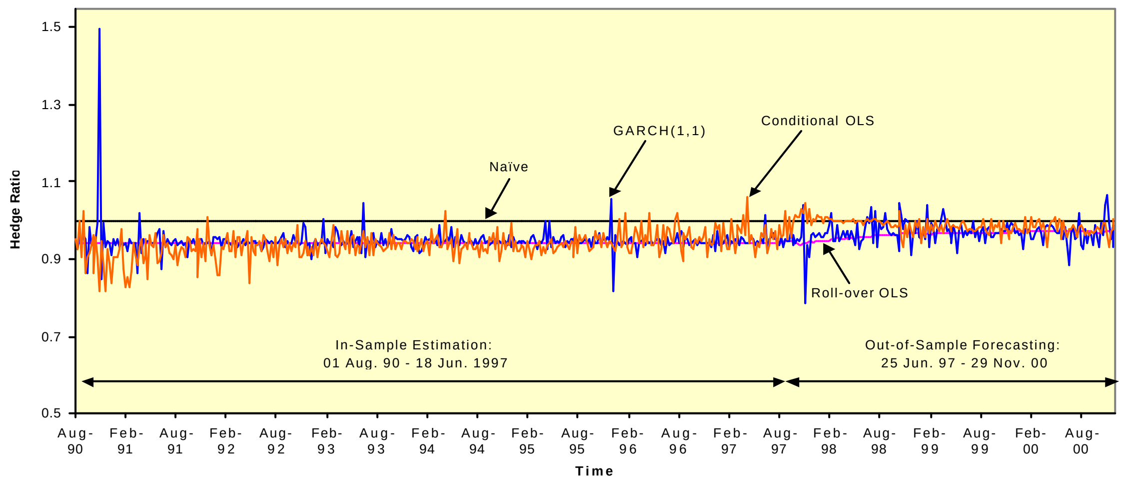

Figure 2: British Pound Hedge Ratios

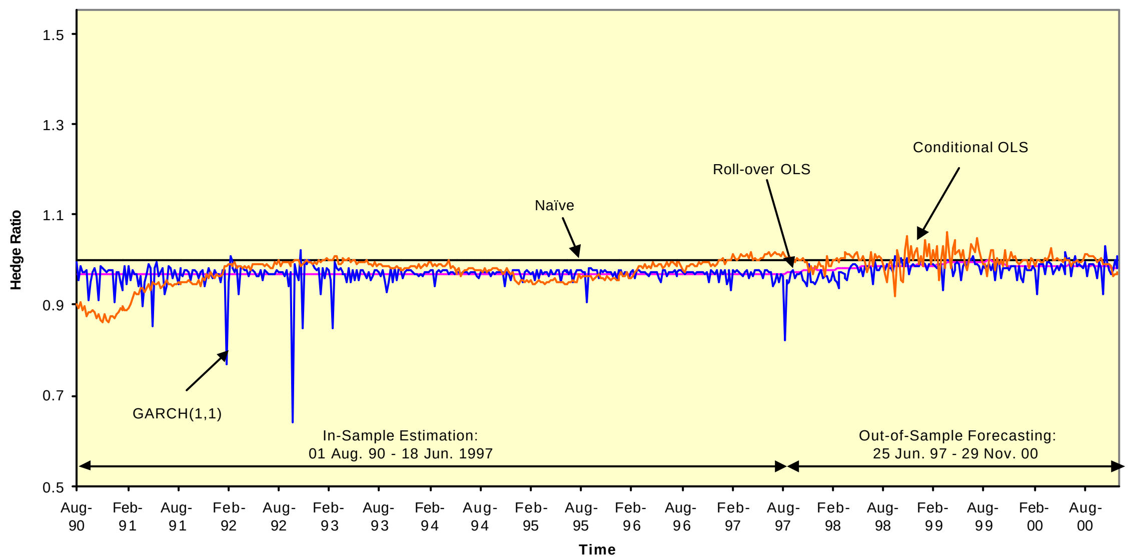

Figure 3: Canadian Dollar Hedge Ratio

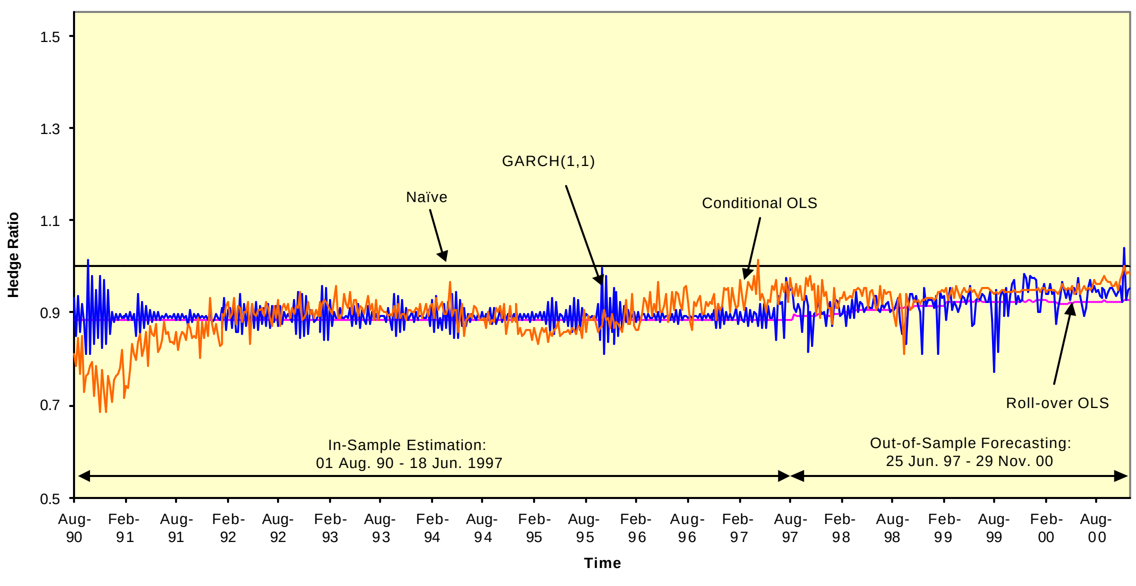

Figure 4: Deutsche Mark Hegde Ratios

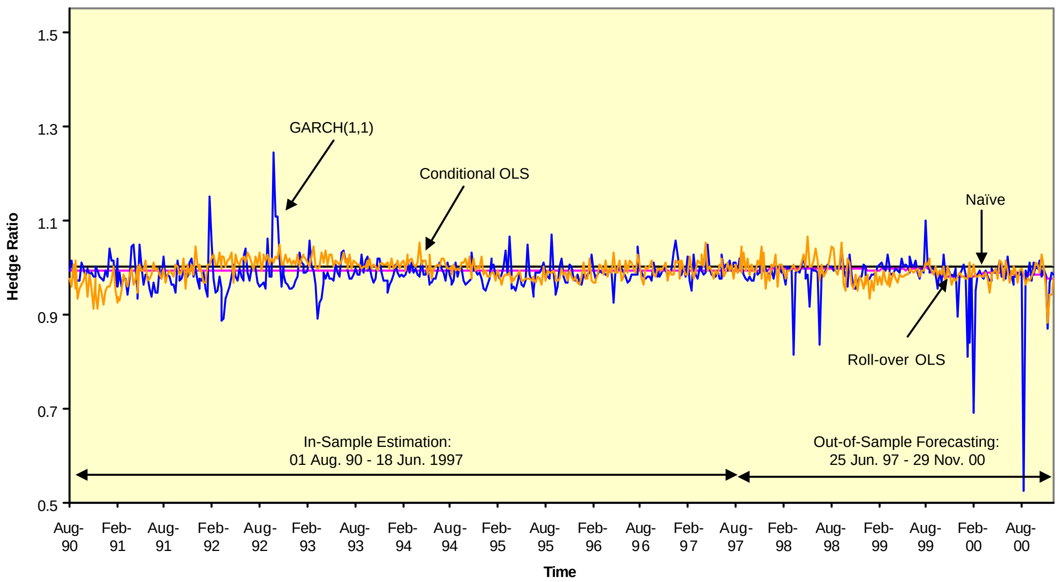

Figure 5: Japanese Yen Hedge Ratios

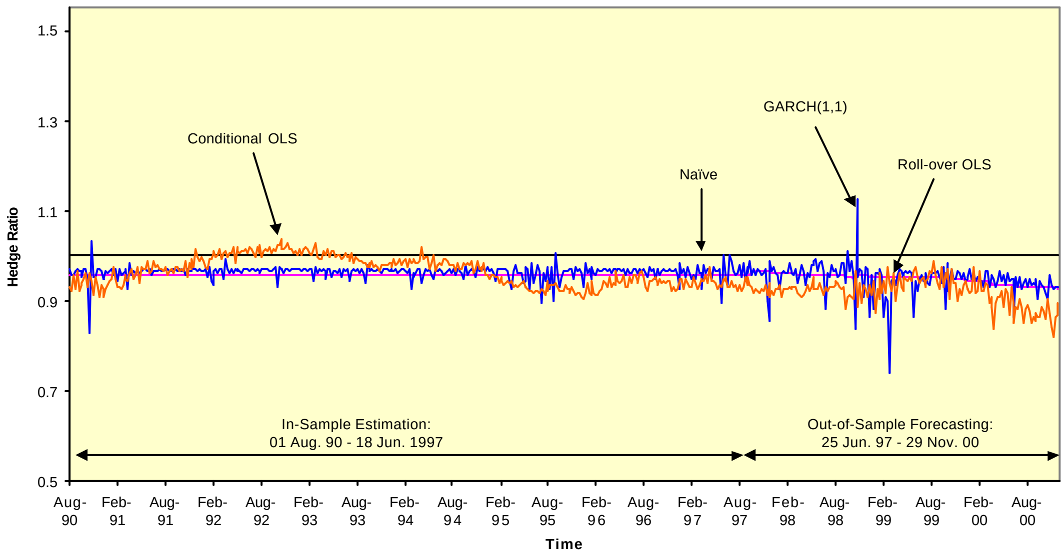

Figure 6: Swiss Franc Hedge Ratios