Co-Signed Loans versus Joint Liability Lending in an Adverse Selection Model

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The first version of the paper was prepared when the first author was visiting the University of Groningen. The SOM Research School and N.W. O. provided funding for this work. The authors bear complete responsibility for any errors.
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Abstract
This paper develops an asymmetric information model that provides an economic rational for co-signing. It is shown that banks can solve adverse selection problems by offering a co-signing contract that induces a risky and a safe firm to group together. The equilibrium co-signing debt contract strictly Pareto dominates an equilibrium without a co-signer if the latter entails rationing. The debt contract is such that a safe firm will apply for a joint loan, which will be co-signed by a risky firm.

JEL classification: O1, O16, I3. Keywords: Co-signing, Group Lending, Joint Liability, Asymmetry of Information, Credit Rationing.
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I. INTRODUCTION

Formal credit markets in underdeveloped areas, where borrowers do not have enough (acceptable) collateral, often do not function well. The reason is that asymmetric information between financiers and those who need financing may lead to adverse selection and moral hazard problems (e.g. Stiglitz and Weiss, 1981). The lack of access to formal credit implies a loss in national output, because productive opportunities are not being utilized by a properly functioning credit market. Therefore, it is highly important to better understand how lending schemes can be developed that may come around the negative consequences of asymmetric information.

The literature considers different mechanisms by which adverse selection and moral hazard problems can be solved. Some authors point to the need of government intervention by means of subsidies or taxes (see e.g. Gale, 1990). The problem of this solution is that government interventions are not able, in general, to circumvent the information problems more than the private sector can. Hence the disastrous targeted credit programs of the 60s and 70s in many developing countries. Other authors show that introducing collateral as an additional instrument for banks can solve the adverse selection problem. A basic reference is Bester (1985) who shows that a separating equilibrium with no credit rationing will result if banks compete by choosing collateral requirements and the lending rate simultaneously. However, this solution assumes that
collateral is available, which is often not the case, particularly not for the poor in many developing countries.

Stimulated by the widespread adoption of group lending schemes, such as the Grameen Bank (see e.g. Morduch, 1999 and Ghatak and Guinnane, 1999), and the success of those programs, several theoretical papers show that joint liability loans may have an advantage over individual loans. These institutions offer group loans where borrowers are jointly liable for all loans taken up by group members. Most of these papers focus either on the role of joint liability in alleviating ex-ante moral hazard problems (e.g. Chowdury, 2005; Conning, 1999 and 2005; Laffont and Rey, 2003, and Stiglitz, 1990) or the role of joint liability in ameliorating ex-post moral hazard problems (e.g. Besley and Coate, 1995). Basically, these papers argue that if a bank writes a contract in which borrowers are made jointly liable for the repayments of the loans, each borrower has an incentive to monitor her peer. This may provide incentives to the group members to invest in safe projects, and thus may reduce problems of moral hazard. This community-based monitoring may be a cost efficient system of monitoring since people living close to each other have better information about borrowers than banks (Varian, 1989).

There are also some theoretical papers that have explored the key mechanisms that give joint liability loans an advantage over individual loans in solving adverse selection problems (e.g. Armendáriz de Aghion and Gollier, 2000; Gangopadhyay, Ghatak and Lensink, 2005; Ghatak, 2000, and Laffont and N’Guessan, 2000). These papers show that joint liability lending may solve adverse selection problems and
improve efficiency. The key to this result is that debt contracts containing a joint-liability component provide incentives for assortative matching (Becker, 1993) implying that similar types of firms group together (Ghatak, 2000). Consequently, the assortative matching property enables banks to price discriminate between (pairs, or sets of) borrowers since the group of risky firms is less willing to accept an increase in the joint-liability component than the group of safe firms.

This paper adds to the literature on group loans in the context of adverse selection problems. The main novelty is that we explore whether adverse selection problems can be overcome when borrowers are offered co-signed loans. Co-signed loan arrangements are ubiquitous, both in developed and developing countries. For instance, in Vietnam about 40% of the number of formal credit debt contracts is backed by a guarantor (calculated from the Vietnam Living Standard Survey in 1998; see Tra and Lensink, 2005). In addition many micro finance institutions in developing countries provide individual loans that are guaranteed by a co-signer (Ledgerwood, 1999). In developed countries also, co-signing is an important phenomenon. In the United States, so-called Morris Plan banks required a loan applicant to find co-signers (Phillips and Mushinski, undated). Morris Plan banks are profit-making industrial banks that supply consumer credit to low and middle income individuals. These institutions were especially important in the beginning of the twentieth century, but there are still some Morris Plan chartered banks in the US. In addition, many credit cooperatives and credit unions, both in the developed and the developing world impose co-signing requirements. This was e.g., the case in nineteenth-century German cooperatives (Banerjee, Besley and Guinnane, 1994).
The literature distinguishes between co-signed loans and joint liability lending. However, this distinction is not theorized, which is unfortunate given the importance of co-signed arrangements in practice. Often it is even assumed that co-signed loans and joint liability lending work in exactly the same way. However, there is an important difference between joint liability lending and co-signed loans. In contrast to the standard joint liability debt contract, which assumes that community (or group) members are jointly liable for all loans, the co-signing principle implies that the co-signer is liable for the loan applicant, but the loan applicant is not liable for the co-signer. In fact, a co-signed loan is an extreme form of joint liability lending. We will also examine whether co-signed loans or joint liability loans are to be preferred if a finance organization (say a microcredit organisation) is faced with adverse selection problems.

There are only a few theoretical papers that have analyzed co-signed loans. Banerjee, Besley, and Guinnane (1994) show that co-signed loans may reduce ex-ante moral hazard problems. In addition, Bond and Rai (2004) compare joint liability lending and co-signed loans in their ability to ameliorate enforcement problems and thus ex-post moral hazard. The model that comes closest to our model is Besanko and Thakor (1987). The main aim of their paper, in line with Bester (1985) is to analyze whether and how collateral can be used to come around the adverse selection problem. They show that in the case where the borrower’s wealth imposes a binding constraint on collateral, an equilibrium involving a co-signer strictly Pareto dominates an equilibrium not involving a co-signer. A major difference between their model and ours is that we focus on co-signed loans in the context of group lending. In our model, the co-signed loan contract specifies
that two individuals simultaneously apply for a loan, while only one of the borrowers is liable for the loan of the other. In other words, the primary applicant and the co-signer invest and thus apply for a bank loan that is sufficient for the financing need of both the primary applicant and the co-signer. In Besanko and Thakor (1987), the co-signer does not invest and therefore does not borrow. Our model enables one to investigate endogenous peer selection if a co-signed “group” loan is offered, whereas in Besanko and Thakor (1987) the risk profile of the co-signer does not play a role. Moreover, our model explains why a borrower of a certain risk type is willing to become a co-signer, whereas in all other existing co-signing models this is not spelled out.

We will show that a particular form of a co-signing debt contract results in an outcome that is equal to the optimal outcome with full information. This debt contract with a co-signer is such that the risky firm becomes liable for the failure of the safe firm, i.e. the risky firm becomes the co-signer. This implies that, in contrast to the standard literature on joint liability debt contracts, the proposed co-signing debt contract leads to non-assortative matching. Our analysis also suggests that a debt contract with a co-signer is, from a social point of view, more efficient in solving adverse selection problems than a debt contract with joint liability. Most importantly, this paper provides an economic rationale for co-signing, and proposes an alternative loan arrangement to joint liability lending in the context of group loans.

The paper is organized as follows. Sections II and III deal with stand-alone firms in a model with full information and asymmetric information, respectively. We will compare the social surplus for both cases. Section IV derives a co-signing debt contract
that induces safe firms to group together with risky firms. We will show that such a debt contract solves the adverse selection problem and leads to a social surplus that is equal to the social surplus in the case of perfect information. Section V analyzes the consequences of a full (two-sided) joint liability contract in the context of our model. Section VI discusses the specific conditions in our model and argues that they are no more restrictive than those already in the literature. Section VII concludes.

II. THE FULL INFORMATION MODEL

There is a continuum of risk neutral entrepreneurs in the interval [0,1]. There is one risky project available with each entrepreneur, so that entrepreneurs and projects are interchangeable. There are two types of projects, and hence, entrepreneurs. The project types are distinguished by their probability of success $p$. Let $p_t$ be the probability of success of project type $t$, $t = r, s$, and $R_t$ be the output when the project succeeds. Both types of projects yield a zero return if unsuccessful.

A.1: $0 < p_r < p_s < 1$ and $p_s R_s = p_r R_r = \mu^2$.

Using the second order stochastic dominance as a definition of risk (Rothschild and Stiglitz, 1970), we term the $s$ entrepreneurs as the safe borrowers and the $r$ entrepreneurs as the risky borrowers. Also observe that, A.1 implies that $R_r > R_s$. Each project requires a unit of investment. Entrepreneurs do not have initial
wealth, so they cannot self-finance their projects. We assume that the costs of issuing equity and bonds are prohibitively high, which implies that the entrepreneur needs bank loans.

Let $\rho$ be the opportunity cost of the bank's resources.

A.2: $\mu > \rho \geq 1$

A.3: *The bank is risk neutral and makes zero profits.*

A.2 says that all projects that are financed at the opportunity costs of bank funds are viable. Entrepreneurs need to decide on undertaking the project and borrowing bank funds or, not undertaking the project. If it does not undertake the project, profits are zero.

A.4: *There is an unlimited supply of bank funds.*

A.4 implies that the supply of loans is perfectly elastic at a cost $\rho$. If the firm undertakes the project type $t$, $t = r, s$ and borrows from the bank, profit equals $\mu - p_t d_t$, where $d_t$ is the (limited liability) debt claim of the bank from firm type $t$, $t = s, r$. Under full information, and zero (expected) profit for banks, $p_t d_t = \rho$.

Since under full information the costs of bank funds ($\rho$) are lower than $\mu$ the firm will undertake the project. The entrepreneur’s profit is $\mu - \rho$. The total surplus, $S$, generated by these projects will be the sum of profits by the safe and risky entrepreneurs,
since banks exactly cover costs. Let the proportion of safe and risky projects be the same.\textsuperscript{4} We then have

\[ S = \frac{1}{2}(\mu - \rho) + \frac{1}{2}(\mu - \rho) = \mu - \rho \]

This is the optimal outcome from a social point of view if the price mechanism works perfectly.

III. THE MODEL WITHASYMMETRIC INFORMATION

So far we have been assuming that the project types are common knowledge. In this section, we assume that this is private knowledge to the entrepreneur. It is in the interest of risky firms to pose as safe firms, as the (zero bank profit) debt claim from a safe firm is less than that from a risky firm. One way of preventing firms from misrepresenting themselves is to give debt contracts that are contingent on the revenues realized. Recalling that \( R_r > R_s \), it is possible to think of a debt contract where the debt claim is higher for a successful risky project compared to a successful safe project. However, this means that even when the borrower pays up, the bank has to verify the revenue earned by the borrower. Alternatively, if the debt repayments are different depending on whether the outcome is \( R_r \) or \( R_s \), it is in the interest of the entrepreneur to report that return which is associated with the lower repayment amount. This entails an additional monitoring cost to the bank. In keeping with most models of this genre, we assume that it is prohibitively costly for the bank to verify returns in all states.\textsuperscript{5} This assumption rules
out contracts contingent on project returns. We, however, assume that the bank is able to verify whether a project has yielded positive or zero return, so that there is no strategic default.

Given our assumptions on equal proportions of safe and risky projects, if all projects are undertaken and take bank finance, the bank has to assume that the probability of getting repaid is the average probability of success, \( \bar{p} = (p_s + p_r) / 2 \). Since the bank cannot distinguish between good and bad projects, it offers the same contract to everybody. If \( d \) is the bank claim, and the bank loan equals 1, the bank will put \( \bar{p}d = \rho \). The return to an entrepreneur of type \( t \), \( t = s, r \), is,

\[
\mu - p_t \frac{\rho}{\bar{p}}
\]

This expression allows us to focus on the major problem we want to address. If projects are undertaken, the expected profit to the entrepreneur is \( \mu - \frac{p_t}{\bar{p}} \rho \). Alternatively, if firms do not invest, profits are zero. We make the following assumption:

\[\text{A.5: } \mu < \frac{p_s}{\bar{p}} \rho\]

Observe that, given A.2 and A.5, \( [\mu - \frac{p_s}{\bar{p}} \rho] < 0 < [\mu - \frac{p_r}{\bar{p}} \rho] \). Thus, while safe firms will not invest and borrow from the bank, the risky firms will. But the bank should know
this and, hence, cannot assume that the probability of success on a project to which it has lent is \( \bar{p} \). The bank is better off putting a debt claim that satisfies \( p_r d = \rho \), instead. If all risky projects take all their money from the bank, the total demand for bank loans will be \( (1/2) \) since the measure of risky firms is \( (1/2) \). We state the following result.

**Proposition 1:** Suppose the bank cannot identify the project type, which is privately known to entrepreneurs. Under A.1-A.5, the bank gives a loan equal to \( 1 \) for a debt claim of \( (\rho / p_r) \). All risky firms borrow from the bank and the safe firms do not. The total surplus with the entrepreneurs is less than that in the full information case.

**Proof:** We only need to show that the total surplus is less than the full information case. In this scenario, where safe projects are not undertaken and not funded and the risky projects are undertaken and funded through a debt contract the surplus with the entrepreneurs is:

\[
S = \frac{1}{2} [\mu - \rho] < \mu - \rho
\]

The last term on the right of (2) is the same as the last term in equation (1) \( \blacksquare \)

Note that this result differs from the Stiglitz-Weiss (1981) result.\(^6\) In our model there is no excess demand for credit since the risky get exactly the amount of credit they demand, and the safe do not demand any credit at the price of \( \rho / p_r \). In contrast, the model of Stiglitz and Weiss implies that some people demanding credit at the current price are under-served.
IV. DEBT CONTRACTS WITH A CO-SIGNER

In the previous section we showed under what conditions the return to entrepreneurs is affected when project types are private information to the project owners. In this section we analyze the consequences of debt contracts with a co-signer (CC). We present the analysis in two steps. First, assuming that firms have no other choice than to accept a debt contract with a co-signer and that the cosigner has enough funds (if her project is successful) to pay her own debt obligations and that of her partner, aggregate payoffs will be maximized if a risky firm becomes the co-signer in a group consisting of a risky and a safe firm. Next, we analyze for which parameter values there exists a CC that will lead to higher payoffs for safe borrowers.

*Endogenous group formation for a given debt contract with a co-signer*

We use a CC of the following form. If a potential borrower, together with a co-signer, goes to a bank, the primary applicant will apply for a loan that is sufficient for the financing need of herself and the co-signer. The bank is willing to offer this loan if the primary applicant and the co-signer promise to pay $d_{CC}$ each. This promise has to be kept both in case they are successful and in case the co-signer is successful and the primary applicant fails. In the latter case, the co-signer pays the obligation of her failing partner (equal to $d_{CC}$), along with her own obligation (also equal to $d_{CC}$). If the co-signer fails and the primary applicant succeeds, then only the primary applicant meets the debt obligation (equal to $d_{CC}$). We make the following assumptions:
A.6: Borrowers know each other’s types.

A.7: Side transfers are allowed.

A.6 is a common assumption in the literature on peer monitoring (see Ghatak, 2000). Community members are assumed to have better, and more, information about each other than the bank has. Our model intends to deal with borrowing communities in less developed countries, where community members know each other well, and try to form a group, or try to find a co-signer, within the same community. Moreover, we would like to compare our debt contract with a co-signer with a standard joint liability debt contract, if used in a similar situation. Therefore, in line with the joint liability literature we assume that borrowers have much more information about each other than the bank has. This assumption is also made in Besanko and Thakor (1987). A.6 is an extreme representation of this actual situation. We abstract from problems that may arise if potential group members, or primary applicants and potential co-signers do not know each other. See e.g. Armendáriz de Aghion and Gollier (2000) and Laffont and N’Guessan (2000) for group lending models dealing with adverse selection problems if borrowers do not know each other.

We consider the two entrepreneurs together, and denote the payoffs they jointly make to the bank as $D(m,n)$. Define the ordered pair $(m,n)$ to mean that $m$ is the co-signer. Since we are considering a pair of entrepreneurs, we have four possible states ---
both successful, only one successful (and there are two such states) and, neither
successful. Then the CC contract gives the following payoff to the bank,8

\[
D(m,n) = \begin{cases} 
2d_{cc} & \text{with probability } p_m p_n \\
2d_{cc} & \text{with probability } p_m (1 - p_n) \\
d_{cc} & \text{with probability } (1 - p_m) p_n \\
0 & \text{with probability } (1 - p_m)(1 - p_n)
\end{cases}
\]

If \( ED(m,n) \) denotes the expected (combined) payoff to the bank, then

\[
ED(m,n) = [p_m(2 - p_n) + p_n]d_{cc}
\]

The joint profit of the pair \((m,n)\) is given by

\[
\Pi(m,n) = p_mp_n(R_m + R_s - 2d_{cc}) + p_m(1 - p_n)(R_m - 2d_{cc}) \\
+ (1 - p_m)p_n(R_n - d_{cc}) \\
= 2\mu - d_{cc}[p_m(2 - p_n) + p_n] \\
= 2\mu - ED(m,n)
\]

**Lemma 1:** Let \( d_{cc} \) be given. Then \( \Pi(r,r) > \Pi(r,s) > \Pi(s,r) > \Pi(s,s) \).

**Proof:** From (5),

\( \Pi(r,r) > \Pi(r,s) \iff p_r(2 - p_r) + p_r < p_r(2 - p_s) + p_s \iff p_r < 1 \), which is true.

\( \Pi(r,s) > \Pi(s,r) \iff p_r(2 - p_s) + p_s < p_s(2 - p_r) + p_r \iff p_r < p_s \), which is true. And, finally,

\( \Pi(s,r) > \Pi(s,s) \iff p_s(2 - p_r) + p_r < p_s(2 - p_s) + p_s \iff p_s < 1 \), which is true.
**Lemma 2:** Let $d_{cc}$ be given. Then $E\Pi(r,s) > \frac{1}{2} E\Pi(r,r) + \frac{1}{2} E\Pi(s,s)$.

**Proof:** Using (5), we know that, to prove the hypothesis, we need

$$p_r(2 - p_r) + p_s < \frac{1}{2} [p_r(2 - p_r) + p_r] + \frac{1}{2} [p_s(2 - p_s) + p_s]$$

$$\Leftrightarrow (p_s - p_r)^2 < (p_s - p_r) \Leftrightarrow (p_s - p_r) < 1$$

which is true.

**Proposition 2:** Under A1-A7 and assuming that borrower payoffs are sufficient to pay their own borrowing cost as well as the borrowing costs of their partner, that borrowers have no other choice than to accept a given debt contract with a co-signer, the following holds: a safe firm makes a profit if she induces a risky firm to co-sign her debt. This combination maximizes aggregate payoffs for all possible matches.

**Proof:** Any firm has two types of options, forming a pair with another firm of its own type, or with the other type. Pairing can be of two forms --- cosigning the partner’s project or having the partner cosign one’s own project. From Lemma 1, we know that the total profit of pair-type $(r,s)$ dominates the pair $(s,r)$. Therefore, whatever $(s,r)$ can achieve, $(r,s)$ can do better for each firm since side payments are allowed. So, if risky and safe get together, the combination will be given by $(r,s)$ and not $(s,r)$. 

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Suppose the pair \((r, r)\) gets formed and let the individual profit of a partner be \(\alpha\) and of the other \(\Pi(r, r) - \alpha\). If \(\alpha\) is less than \(\frac{1}{2} \Pi(r, r)\), then two such firms in different pairs can break away, form a new pair and pay each of themselves an amount \(\frac{1}{2} \Pi(r, r)\). Now suppose, there is only one pair where \(\alpha < \frac{1}{2} \Pi(r, r)\), and all other pairs give each member exactly \(\frac{1}{2} \Pi(r, r)\). The partner who gets \(\alpha\), can offer a return

\[
\frac{1}{2} \Pi(r, r) + \frac{1}{2} \left[ \Pi(r, r) - \alpha \right]
\]

to any one member from the other pairs and break that pair. The \(\alpha\) partner can now get \(\alpha + \frac{1}{2} \left[ \Pi(r, r) - \alpha \right] > \alpha\). A similar argument immediately implies that a safe firm is guaranteed a return equal to \(\frac{1}{2} \Pi(s, s)\), by forming a pair with another safe firm.

Now if a safe firm has to induce a risky firm to cosign its loan, it has to ensure that the risky partner gets at least \(\frac{1}{2} \Pi(r, r)\). Also, what is left over for the safe firm itself, is at least as much as \(\frac{1}{2} \Pi(s, s)\). Putting the two together, we have,

\[
\Pi(r, s) - \frac{1}{2} \Pi(r, r) \geq \frac{1}{2} \Pi(s, s).
\]

Observe that, if the risky firm wants to induce the safe firm to allow it to cosign the loan we get the condition
\[ E\Pi(r, s) - \frac{1}{2} E\Pi(s, s) \geq \frac{1}{2} E\Pi(r, r) , \] which is the same as the one above. From Lemma 2, we know that this condition always holds in the CC. ■

Proposition 2 argues that if only cosigned loans (as defined in equation (3)) are being offered, and firms must get together in pairs to obtain the loan, then risky and safe firms will go together, so that non-assortative matching will result. It also implies that non-assortative matching for groups in which the risky borrower is the co-signer maximizes aggregate expected outcome of all borrowers over all possible matches.

The intuition for our outcome can be explained further by considering the direct payoffs (before side transfers) of the different types of firms, if they are a co-signer or the primary applicant. The direct payoffs for a primary applicant are independent of the composition of the group. For firm \( i \) \((i = r, s)\) the direct payoffs of being the primary applicant equal \( \mu - p_id_{cc} \). On the other hand, the direct payoffs for the co-signer depend on the group composition. For co-signer \( i \) \((i = r, s)\) the direct payoffs in a group with firm \( j \) \((j = r, s)\) equal \( \mu - p_id_{cc}(2 - p_j) \). Thus, without side payments, nobody prefers to become the cosigner. In addition, it is also clear that a co-signer prefers to have a safe partner. More importantly, a co-signer only has to pay for the partner if the co-signer herself has succeeded. Since the probability of success for a safe borrower is higher than for a risky borrower, the safe borrower benefits more from not being a co-signer. So, both types of borrowers gain from not being a co-signer, but this gain is higher for the safe firms. This implies that safe borrowers can make a profit by grouping with a
risky firm who promises to co-sign her debt. The risky firm will not be hurt since she will be kept on her participation constraint determined by the profits she would get if grouped with another risky firm.

**Parameter values for which the CC solves the adverse selection problem**

So far, we have not allowed a firm to remain unpaired. Recall that, the safe firm has the possibility not to invest and get a profit of zero. Similarly, the bank could always give a loan contract \( d = \frac{\rho}{\rho} \) that allowed the risky firm to get \( \mu - \rho \). Let \( E\pi_j(r,s) \), \( j = r, s \) denote the amount received by firm \( j \), when the pair \((r,s)\) is formed. If \( E\pi_r(r,s) = \alpha \), then \( E\pi_r(r,s) = E\Pi(r,s) - \alpha = \beta \). Individual rationality, or participation by the firm in the paired, or co-signed loan market demands that \( \alpha \geq 0 \) and \( \beta \geq \mu - \rho \). To complete our analysis we have to show that the participation constraints can indeed be satisfied for both types of firms.

We begin by describing the contracts the bank gives out. The bank offers two contracts --- a standalone contract and a CC (co-signed contract). The standalone contract is available to a single (unpaired) firm while the CC is available to a pair. Remember the pair is defined only when one (and only one) of them has agreed to co-sign the loan of the other. The firm taking the standalone contract is offered a loan of one unit against a repayment amount \( d \). Each of the paired firms in a CC contract is also given a loan of one unit (total of 2 units of investment to the pair) against a promised payment of \( d_{cc} \).
per unit of loan. The conditions of payment on the CC are given by (3). The standalone is a standard debt contract (see Section II).

**C1:** *The standalone contract and the CC satisfy*

(a) \[ d = \frac{\rho}{\rho} \]

(b) \[ d_{cc} = \frac{2\rho}{p_r(2 - p_s) + p_s} \]

**The participation constraints**

We need to check whether there exist parameter values for which the payoff of the CC contract for each borrower exceeds the payoff of the standalone contract and the payoff of not investing. In Section II we have shown that a safe firm will never take the standalone contract. It can do better by not investing, where it gets a return of zero. Thus, if a safe firm wants to invest and to borrow, it must be paired with some other firm. It has also been shown that a risky firm can take the standalone contract.

We now show that, indeed, \( \alpha \geq 0 \). First recall that the risky firm will be willing to cosign a safe firm if it gets \( \frac{1}{2} E\Pi(r, r) \). Given C1 and (5),

\[ \frac{1}{2} E\Pi(r, r) = (\mu - \rho) \frac{p_r(2 - p_r) + p_s}{p_r(2 - p_s) + p_s} > (\mu - \rho) \]

The strict inequality follows because
\[
\frac{p_r(2-p_r)+p_r}{p_r(2-p_s)+p_s} < 1
\]
\[
\Leftrightarrow p_r(2-p_r)+p_r < p_r(2-p_s)+p_s \Leftrightarrow p_r(p_s-p_r) < p_s-p_r \Leftrightarrow p_r < 1
\]

So, if the risky firm is given an amount equal to \(\frac{1}{2} \Pi(r, r)\) its participation constraint will be satisfied. The amount left over for the safe firm is then

\[
\Pi(r, s) - \frac{1}{2} \Pi(r, r) \geq 0
\]
\[
\Leftrightarrow \mu + d_{cc}\left[\frac{1}{2}\{p_r(2-p_r)+p_r\} - \{p_r(2-p_s)+p_s\}\right] \geq 0
\]
\[
\Leftrightarrow \mu \geq \frac{2\rho}{p_r(2-p_s)+p_s}\left[-\frac{1}{2}\{p_r(2-p_r)+p_r\} + \{p_r(2-p_s)+p_s\}\right]
\]
\[
\Leftrightarrow \mu \geq \frac{\rho}{p_r(2-p_s)+p_s}\left[p_r(1+p_r)+2p_s(1-p_r)\right] = \rho\left[1 + \frac{(p_s-p_r)(1-p_r)}{p_r(2-p_s)+p_s}\right]
\]
We introduce this as an assumption:

A.8: \(\mu \geq \rho \left[1 + \frac{(p_s-p_r)(1-p_r)}{p_r(2-p_s)+p_s}\right] > \rho\)

**The limited liability constraint**

For a CC to be meaningful, the firm that becomes the co-signer must have a high enough return, when successful, to pay not only its own debt obligation, but that of its partner also. For our purposes we must ensure that \(R_r \geq 2d_{cc}\). This will always be the case if

A.9: \(\frac{\mu}{\rho} \geq 4 \frac{2+p_s}{p_r(1-p_r)}\)
Observe that the inequality is more likely to be satisfied if $\mu$ is sufficiently high compared to $\rho$.

**Proposition 3:** Let $A.1$-$A.9$ hold and the bank gives contracts specified by $C.1$. Recall that there are equal numbers of safe and risky firms. Then, a risky firm takes on the liability of the safe firm. The total surplus with the entrepreneurs is the same as in the situation where the bank knows the project types.

**Proof:** From Proposition 2 and $A.9$, we already know that a safe firm can induce a risky firm to take on its liability. The fact that the total surplus with the entrepreneurs is the same as in the situation where the bank knows the project types follows directly from comparing total group profits with (2). Thus, if a risky firm takes over the liability of a safe firm, the surplus with the entrepreneurs is the sum of the profits made by each pair $(r,s)$. The profit of each pair is

$$E\Pi(r,s) = 2\mu - d_{cc}[p_r(2 - p_s) + p_s] = 2\mu - 2\rho$$

Given that the measure of such pairs is half, it follows that,

$$S = \mu - \rho$$

which equals (2).

Finally, we need to prove that the range on $\mu$ is feasible. There are three assumptions on the relationship between $\mu$ and $\rho$, $A.5$, $A.8$ and $A.9$. The participation constraint ($A.8$) and the limited liability constraint ($A.9$) are likely to be satisfied if $\mu$ is
sufficiently high compared to $\rho$. However, A.5 requires that, given $p_s$ and $p_r$, $\mu$ and $\rho$ are close. For A.5 and A.8 to hold simultaneously, we need

$$p_s \frac{\rho}{\rho} > q \geq \rho \left[ 1 + \frac{(p_s - p_r)(1 - p_r)}{p_r(2 - p_s) + p_s} \right].$$

Since $p = \frac{1}{2}(p_s + p_r)$, this holds if

$$\frac{2p_s - (p_s + p_r)}{p_s + p_r} > \frac{(p_s - p_r)(1 - p_r)}{p_s + p_r(2 - p_s)}.$$

which is always true. For A.5 and A.9 to hold simultaneously, we need

$$p_s > \frac{2p_r^2}{(1 - p_r)}.$$ So, $p_s$ should be sufficiently high as compared to $p_r$.  

V. A COMPARISON WITH A JOINT LIABILITY DEBT CONTRACT

In a full (two-sided) joint liability contract (JC), both firms are liable to pay the other’s debt obligation if its project succeeds. Thus, if $d_{JC}$ denotes the claim in a JC contract, then the payoff to the bank is given by

$$D(m, n) = \begin{cases} 
2d_{JC} & \text{with probability } p_m p_n \\
2d_{JC} & \text{with probability } p_m (1 - p_n) \\
2d_{JC} & \text{with probability } (1 - p_m) p_n \\
0 & \text{with probability } (1 - p_m)(1 - p_n)
\end{cases}$$

Unlike in (3) above, here the pair must pay the bank the obligations of both partners if any one is successful. If $ED(m, n)$ denotes the expected (combined) payoff to the bank, then
The joint profit of the pair \((m,n)\) is given by

\[ E\Pi(m,n) = p_m p_n (R_m + R_n - 2d_{JC}) + p_m (1 - p_n) (R_m - 2d_{JC}) + (1 - p_m) p_n (R_n - 2d_{JC}) = 2\mu - 2d_{JC} [p_m + p_n - p_m p_n] = 2\mu - ED(m,n) \]

Suppose the total surplus generated is \(\mu - \rho\). For this to happen with zero profit for the bank, a pair must together make a profit equal to \(2(\mu - \rho)\).

**Case 1:** Suppose \(d_{JC}\) is such that risky firms take the standalone contract and safe firms form pairs with the same type. Then, for banks to make zero profits, it must be that

\[ d = \frac{\rho}{p_r} \quad \text{and} \quad d_{JC} = \frac{2\rho}{2p_s - p_r^2}. \]

But then,

\[ E\Pi(r,r) = 2\mu - d_{JC} [2p_r - p_r^2] > 2(\mu - \rho), \]

the inequality following from the fact that \(2p_r - p_r^2\) is less than \(2p_s - p_r^2\). Hence, the risky firms will get together and form pairs. The bank will then receive an expected return on the loan that is less than \(2\rho\) from risky pairs and exactly \(2\rho\) from safe pairs while advancing a loan of 2 units to each pair. The bank will make a negative profit, which contradicts the zero profit condition of banks.
Case 2: Suppose the risky firms form pairs and the safe firms take the standalone contract. Then, for a (firm) surplus of $\mu - \rho$, it must be that

$$d = \frac{\rho}{p_s} \quad \text{and} \quad d_{JC} = \frac{2\rho}{2p_r - p_r^2}.$$  

It is immediately clear that if a risky pair broke up and each took the standalone contract, they will (together) make more than $2(\mu - \rho)$ while as a pair they make exactly $2(\mu - \rho)$. Once again, the bank will make negative profits.

Case 3: The final case to consider is one where a risky and a safe firm get together under a joint liability contract. Then,

$$d = \frac{\rho}{p_r} \quad \text{and} \quad d_{JC} = \frac{2\rho}{p_r + p_s - p_s p_s}.$$  

As in the case of co-signing contracts, we need to show that for this case to be valid, it must be that

$$E\Pi(r, s) - \frac{1}{2} E\Pi(r, r) \geq \frac{1}{2} E\Pi(s, s) \iff 0 \geq (p_s - p_r)^2,$$

which is not possible.

Putting Cases 1, 2 and 3 together, a joint liability contract such that the bank makes zero profit and the total surplus with the firms is $\mu - \rho$, is impossible.

There are two possibilities to solve this problem. The first one is to differentiate between the amount a firm has to pay when both are successful, and the amount a firm has to pay if one succeeds and the other fails. This is assumed in Ghatak (2000). In
particular, Ghatak assumes that firms pay $r$ for their own loan and pay $c$ in case the other fails. However, the solution requires that $c > r$ (Gangopadhyay, Ghatak and Lensink, 2005). So the pair pays more to the bank when one of them fails and the other succeeds, $c + r$, compared to the case when both succeed, $r + r$. This makes it rational for the succeeding firm to pass on an amount $r$ to the failing firm, who then pays to the creditor its committed payment. Thus, while the creditor expected to get $c + r$ with some probability, this argument suggests that the creditor may never get this payment. Another possibility is explored in Gangopadhyay, Ghatak and Lensink (2005). They show that the separating contracts can be made incentive compatible even if $c = r$. However, this requires that banks make positive profits out of loans to safe firms. The joint liability contract then still leads to higher welfare than individual liability contracts, but the full information outcome for safe borrowers cannot be achieved. In contrast, as we have shown above, aggregate welfare if borrowers take on our co-signing contracts will be equal to the aggregate welfare obtained under perfect information.

VI. ARE THE CONDITIONS TOO RESTRICTIVE?

An important parameter restriction in our model results from the limited liability constraint ($R_s \geq 2d_{cc}$). This constraint implies that, although borrowers do not need to have collateral, it is required that borrowers have enough income flow to cover the lending costs. This may in practice impose a constraint on the loan size since a borrower may have increasing difficulty fulfilling (all) her liabilities when loan sizes become larger. However, note that the same constraint appears in a joint liability contract. It is
even so that the limited liability constraint in a joint liability contract is probably more restricting since in that case the constraint needs to hold for all borrowers. In a co-signing contract, the constraint only holds for a co-signer, who happens to be the more risky firm. In the joint liability contract, both the risky and the safe firm have to be able to pay the other’s obligation. Given \( R_r > R_s \) (follows from A.1), a lower bound on \( R_r \) is weaker than that on \( R_s \).

The positive result for our co-signing contract also depends on the assumption that borrowers have perfect information about each other. This creates the possibility that co-signing is more efficient than a standard individual debt contract since the co-signer and the primary loan applicant take over the monitoring role of the bank via peer monitoring. This implies that our model might work better in socially cohesive communities. In other words, there need to be close ties between the primary applicant and the co-signer. Note that, the advantages of joint liability systems also stem from the assumption that borrowers know each other’s types and that they can observe each other without costs. Besanko and Thakor (1987), the only other paper available in which co-signing in the context of an adverse selection problem is analyzed, also assume that the co-signer knows the borrower better than the bank.

Our results are also based on the assumption that the proportions of risky and safe borrowers are the same. In the context of our analysis this implies that we abstract from matching frictions that may cause situations where borrowers cannot match with their preferred partners. For a theoretical and empirical analysis of the consequences of matching frictions, see Sadoulet and Carpenter (2001). However, given our standalone
and co-signed contracts, C1, our results hold as long as the proportion of risky projects is at least as great as that of safe projects. All safe firms are then paired with risky projects, and the rest of the risky projects take the standalone contracts.

VII. CONCLUDING REMARKS

This paper develops an asymmetric information model to analyze the relevance of a debt contract with a co-signer. It shows that banks can solve adverse selection problems caused by asymmetric information by offering a co-signing contract that induces a risky and a safe firm to group together. The debt contract is such that a safe firm will apply for a joint loan, which will be co-signed by a risky firm. In line with Besanko and Thakor (1987) our model provides an economic rationale why low risk borrowers use co-signers. However, in contrast to Besanko and Thakor (1987) our model also shows that it only pays for the risky borrowers to become co-signers. The co-signing debt contract will increase aggregate welfare as compared to individual lending. The co-signing contract even leads to an increase in aggregate welfare that is equal to the outcome in case of perfect information.

Although a co-signing contract has some similarities with a full two-sided joint liability contract, there are some important differences. One of the main suggestions emerging from our analysis is that co-signing debt contracts induce non-assortative matching. By contrast, the standard joint liability literature assumes assortative matching in group formation. We argue that a co-signing contract may be a good alternative for joint liability contracts. Both types of contracts may solve (part of) the problems
associated with asymmetric information between borrowers and financial institutions. However, the co-signing contracts we propose will lead to the full information outcome, whereas in the context of a similar model environment joint liability contracts will not. This is important information for micro-finance organizations if they need to make a decision regarding the use of co-signed or joint liability loans.

The results of our paper are based on a simple static asymmetric information model. For our purpose of making a first attempt to analyze co-signing debt contracts, a static model suffices. However, for further research it is important to make the model dynamic.
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1 Note that Armendáriz de Aghion and Gollier, 2000, in a model where borrowers in a group are uninformed about each others’ types, show that non assortative matching may result and that even then joint liability lending may improve efficiency.

2 The assumption of equal expected returns for all projects is in line with Stiglitz and Weiss (1981), but in contrast to De Meza and Webb (1987). De Meza and Webb (1987) assume that payoffs in states are all the same for all projects, while the expected returns differ.

3 We assume that a bank (debt market) exists, and abstract from the possibility that endogenous intermediary coalitions emerge which evaluate projects ex ante in such a way that firm incentives are affected, leading to a Pareto-optimal allocation. For this type of models, see e.g. Boyd and Prescott (1986) and Williamson (1988). More in general, our model does not consider possible monitoring activities of banks which may improve the efficiency of the decentralized outcomes with asymmetric information (see e.g. Diamond, 1984 and Ramakrishnan and Thakor, 1984).

4 This is a simplifying assumption. In general, one can have $\eta$ to be the number of safe projects and $(1-\eta)$ to be the number of risky projects.

5 For costly state verification models see Townsend (1979) and Gangopadhyay and Mukhopadhyay (2002).

6 The Stiglitz-Weiss (1981) outcome leads to underinvestment from a social point of view. Note that some authors, e.g. De Meza and Webb (1987), show that asymmetric information may also lead to overinvestment. In the latter paper there is no credit rationing. This is basically a consequence of their assumption that payoffs in different states are the same for all projects. In a recent paper, De Meza and Webb (2000) show that excessive lending from a social point of view may also occur in combination with a credit-rationing equilibrium.

7 The formal definition is given below.

8 Recall that we are assuming that the successful state of the co-signer gives her enough revenue to repay both claims.

9 Note that in an earlier version of the paper we allowed for an alternative, more expensive source of funds. In contrast to this version of the paper the firms did not choose between investing and not investing, but between borrowing bank loans or taking funds from the more expensive source. The results of that version of the paper were similar to the results of this version of the paper. The main difference being that the viable set of parameters was then determined by the profits in case of borrowing from banks as compared to the profits in case of borrowing from the alternative source. Now the viable set of parameters is determined by a comparison of the profits in case of investing and profits in case of not investing, which comes down to a relationship between the opportunity costs of bank funds and expected returns. In the former case the condition with respect to the needed gap between the probability of success for safe and risky firms is not relevant.

10 As a footnote, one can point out that here, the standalone contract is not necessary but is stated for completeness because it guarantees that risky firms can always make $\mu - \rho$ on their own.