Access, Veto and Ownership in the Theory of the Firm

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Abstract

Ownership may not always be the best driver for investment incentives in an incomplete contract context. This paper shows that ownership has two facets (access and veto) which can be used specifically, and sometimes independently, to foster investment. Access is more efficient than ownership when assets are complements at the margin, and veto is sometimes more efficient when assets are substitutes at the margin. In particular, outside veto is more efficient than ownership because it reduces the incentive to invest on substitute assets. And joint veto is more efficient than ownership because it protects the incentives of highly productive agents while preventing them to merge the asset with substitute assets.

We discuss several implications, in particular the existence of shareholders and non-owner workers, the optimality of outside ownership, joint ownership and partnerships, hybrid governance structures, employment contracts and capital structure (debt vs equity). (*JEL* C70, D23, G30, L20)
1. Introduction

1.1. General introduction

A proper existence and distribution of property rights can eliminate the sources of high transaction costs (Alchian, 1961, Demsetz, 1967, Alchian and Demsetz, 1973). In their seminal papers, Grossman and Hart (1986, GH) and Hart and Moore (1990, H&M)\(^1\), hereafter GHM, have formalized the concept and modeled asset ownership as a substitute for contract incompleteness. In their framework *ownership* is viewed as providing ‘residual rights of control’ and more specifically the ‘ability to exclude’ others from the use of the asset (H&M). When contracts are incomplete, control over an asset gives ex-post bargaining power, hence increases ex-ante incentives to invest. Thus ownership provides incentives. But why should asset ownership be so central to the production activity? After all, most productive agents in the economy do not own assets and most asset owners (e.g. shareholders) do not produce.

Recent critiques to the GHM’s property rights theory have highlighted its limits. It does not incorporate the Berle and Means perspective of separation between ownership and control (Bolton and Sharfstein, 1998), the notion of ownership is somehow rigid and besides ‘no control’ and ‘full control’ there are other cases of intermediate control or ownership (Hart, 1995)\(^2\), ‘residual control rights’ is an ambiguous concept and may not be equivalent to ownership (Demsetz, 1998), assets are multi-attributes, each of which may have different ownership (Foss and Foss, 2001), investment incentives are not provided by ownership alone (Holmstrom and Roberts, 1998), in particular access can be an alternative (potentially complementary) way of providing incentives (Rajan and Zingales, 1998, RZ), the theory does not explain why assets are clustered in firms and why firms own assets rather than individuals (Holmstrom, 1999).

In this paper, we propose to answer these critiques and broaden the H&M framework by incorporating the view of many scholars (e.g. Schlager and Ostrom, 1992)

\(^1\) GH emphasizes the costs and benefits of ownership and control and their role in determining the size of the firm. H&M generalizes the concept to \(I\) agents and \(N\) assets and find conditions on agents and assets for an optimal integration.

\(^2\) Hart (1995) mentions a subsidiary, renting or leasing of assets, or franchising as examples. He also notes that ‘the literature has by and large not used the property rights approach to analyze intermediate forms of ownership but this is an interesting topic for future research’. This paper is an attempt in that direction.
who have defined ownership as a bundle of rights, including the right to use or access the asset (emphasized by RZ) and the right to exclude others from its use (emphasized by GHM). We take these two rights as primitives and define ownership as the right to use (we call it access) an asset and the right to exclude (we call it veto) others from using it. Integrating RZ’s intuition that access provides power, we define control over an asset as: *access, provided that no one else has veto*. In this conception (compatible with both H&M and RZ’s intuitions), access gives control and veto removes control from the other agents. We clarify here the costs and benefits of ownership and control (emphasized by GH) which determine the size of the firm. Benefits (increase of incentives) are provided by access, while costs (decrease of incentives for the other agents) are generated by veto.

In H&M assets are always *complementary* at the margin (they increase the marginal return of the agent(s) who control them together). But more assets may actually generate decreasing returns to the management or entrepreneur function (Coase, 1937). Thus in our framework, we relax the H&M assumption and allow assets to be sometimes *substitutes* at the margin.

Taking access and veto rights as primitives in the definition of ownership and control, and relaxing the assumption on asset complementarity (at the margin) gives rise to a broader framework. We find a justification for the role of shareholders (who have veto but not access) and workers/managers (who have access but not veto). Our model has implications for the boundaries of the firm and besides the owner-managed firms studied by H&M, we find optimality conditions for the existence of outside ownership (outside veto), hybrid governance structures such as franchising or licensing (multiple access on the same asset), partnerships (multiple access and joint veto on one same asset), or joint ownership (joint access and joint veto on assets).

To understand the different roles that access and veto can play here is a simple intuition. When assets are complementary at the margin and an agent gives access to her asset to a second agent, it increases incentives of the second agent (without decreasing the incentives of the first) but it also increases incentives of the other ‘external’ agents because the agent who is given access will bring the complementary asset with him in

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3 The H&M framework becomes a special case where assets are all complementary at the margin and access and veto must be allocated together.
their relationships. On the other hand, when assets are substitutes at the margin and an agent gives veto on her asset to a second agent, it increases incentives of the agent who has given veto and of all the other ‘external’ agents, because the agent who has given veto will not bring the substitute asset with her in their relationships. The separate allocation of access and veto provides a richer set of contracts and incentive mechanisms than ownership alone. We now take a simple (stylized) example to motivate our paper.

1.2. An example
In most cities, a taxi company can only operate with a license. Licenses are sold by the city which regulates the number of taxis in circulation. The owner of a license can operate a taxi company herself or lease the license to an operator who buys several cars, and leases the cars to taxi drivers. The taxi drivers typically pay a fixed fee to the taxi company and get the residual income (after paying for the petrol). So why is it that, against the prediction of the property right theory, the main agent that should be given incentives (the taxi driver) is the one that does not own anything?

A (partial) answer has already been given by RZ who have identified some conditions under which ownership may reduce incentives and shown that regulated access can be an alternative to ownership to foster incentives. But their framework is different from H&M. They assume that there is always an owner (the entrepreneur) and look at the optimal number of agents (the managers) who should have access, rather than to whether access should be given at all; their focus is the regulation rather than the allocation of access. In our view, the number of managers who should access/operate the asset is generally set by other criteria (for example only two drivers maximum can operate a taxicab) and the real question is which level of rights on the asset they should have in order to maximize their and others’ incentives. RZ consider the firm as only one asset and study the effect of complementary and substitute investments, while we view the firm as a collection of assets which can be complementary or substitutes. Unlike H&M, they consider that specific investment (i.e. specialization) may reduce the value of outside options, hence showing the deadening effect that ownership, unlike access, may have on incentives. Our model does not make any such assumptions and can cope with both H&M and RZ frameworks. While they focus on comparing ownership to access, we
analyze the notions of ownership, access and veto. Overall, RZ’s framework focuses on the optimization of internal organization rather than the external boundaries of the firm. But we join RZ in their main intuition that access may foster investment incentives.

Consider a city with three agents: Joe and Peter have a car and Lucy owns a call center. The two cabs and the call center are all complementary so that combining them always produce a higher return, *for a given level of investment*, than using them separately (i.e., two cabs combined produce a higher value since fixed costs – garage, book-keeping – can be shared, and the call center increase the value produced by a cab but has no value in itself). Joe, Peter and Lucy each are considering making a specific investment in human capital at cost 100 to each of them: they will learn the city⁴ (the streets, the shortcuts, the ‘red’ points with traffic jams,…) to increase their productivity as a taxi driver (for Joe and Peter) and as a call center manager (for Lucy). If Joe shares a taxicab with Peter (they each drive separate shifts), his additional benefit due to the investment will be 120. But if both Joe and Peter have a cab, Joe’s additional benefit will only be 80. This is because when Joe and Peter each drives a cab, they have to monitor the calls (i.e., allocate customers between them) and coordinate their routes (to cover the city efficiently). This takes time out of the productive chargeable time and imposes routes in such a way that it partially offsets the additional productivity of Joe. Two cabs decrease the marginal return on Joe’s investment (we will say that the two cabs are *substitutes at the margin*) in such a way that he will not make the investment if he owns a cab. So in this example it is better for Joe not to own a cab. So should Lucy own Joe’s cab (if Peter owns it, it will decrease his own incentive to invest)? If Lucy invests in developing her knowledge of the city, she can increase her benefit from the call center by 120. But if she also drives a cab, she can only increase her benefit by 80 (she will have to coordinate with Peter and it will partially offset her additional productivity). This is because adding a second cab decreases the marginal benefit produced with a cab and the call center (the call center is *substitute at the margin* with a cab *in presence of another cab*). Given that if Lucy owns a cab, she will drive it (a cab and the call center are complementary), she will not make her specific investment in that case. Ownership (by

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⁴ Even if this example is highly stylized, it is of some relevance as a survey conducted in March 1998 by the NSW Transport Department showed that the performance of taxi drivers in the region of Sydney lagged with respect to driver’s knowledge of locations and English proficiency.
either Joe or Lucy) is not a proper solution to induce both Joe and Lucy to invest. Rather the solution is for Lucy to have veto on Joe’s cab but not to have access to it. Outside veto by Lucy is good because it ‘ties the hands’ of Joe and protects him from merging his taxicab with another one which is substitute at the margin. But access to a complementary asset is not always good. Having access may prompt to merge two complementary assets (the call center and Peter’s cab) which may become substitutes (at the margin) in presence of a third one. In this example Lucy should have veto on Joe’s cab and Joe should have access. De-bundling ownership into access and veto gives rise to a larger set of contracts that allow maximizing the agents’ incentives. We find multiple examples in real life: shareholders have veto on the firm’s assets (it prevents managers to merge substitute assets) but do not access them, an employer has veto on the employee’s human capital (it helps the employee to specialize on her job) but the employee keeps the right to use his human capital (human capital is unalienable), a manager has veto power on the worker task,…

Our model stays close to H&M, follows the idea that ownership is a bundle of rights (Demsetz, 1967, Schlager and Ostrom, 1992) and incorporates the notion of access highlighted by RZ. The idea that a larger scope of the firm (more assets) may actually lead to decreasing returns in the management function (i.e. assets may be substitutes at the margin) comes from Coase (1937) and is explored in the work of many scholars, e.g. Williamson (1967), Rotemberg and Salomer (1994). The impact of property rights on the type of integration (inclusion, exclusion, collusion) is derived from Segal (2003). Overall, the model is highly stylized. It is restricted to a static environment and ignores the effect of repeated interactions (Halonen, 2002) and the power of relational contracts (Baker et al., 2002, Levin and Rayo, 2003). It omits the impact of financial constraints (Dewatripont and Tirole, 1994) or the potential role of authority (Aghion and Tirole, 1997). The bargaining game does not account for non linear outside options (De Meza and Lockwood, 1998) or the costs of bargaining (Milgrom and Roberts, 1990). By focusing on allocation of property rights, the framework also ignores other incentive instruments (Holmstrom and Milgrom, 1994). Our main purpose here is to propose a more elaborate view of ownership and describe its impact for the provision of ex-ante incentives. Relaxing some assumptions of the H&M framework, we are thus testing (and
we hope proving) that the model can be extended far beyond the vertical integration of physical assets to deal with more general applications.

The remainder of the paper is organized as follows. Section 2 introduces the model and the main concepts. Section 3 studies the optimal control structure and presents the main results. Section 4 discusses some applications and the consequences for the boundaries of the firm. The conclusion discusses the implications, points out some limitations and suggests directions for future research. Most of the proofs are in the appendix.

2. The model

We follow H&M (1990). We consider an economy with a set \( N = \{1, \ldots, n\} \) of risk-neutral agents and a set \( A = \{a_1, \ldots, a_k\} \) of assets. Rights over assets are allocated at date 0 and agents invest. At date 1, production takes place and the surplus is shared between the agents through an ex-post efficient bargaining process, using the Shapley value as the solution concept\(^5\). No variable, besides the allocation of rights, is contractible, but the control structure (determined by the allocation of rights) agreed upon by the agents ex ante can be enforced\(^6\). We also assume that side payments between agents are allowed, so that efficient trading at date 0 leads to a control structure that maximizes the overall surplus at date 1.

2.1. Access, Veto and Control structures

We define ownership as a bundle of two rights: the right to use or access the asset and the right to exclude or veto others from using the asset\(^7\).

\(^5\) We use the Shapley value to stick to the H&M framework, but our results would generalize to divisions of surplus other than the Shapley value and in particular to any fixed probabilistic distribution of the value among the agents.

\(^6\) In particular we rule out secret asset transfer by the agents and we assume that access at date 1 can be enforced (as in RZ).

\(^7\) This is a convenient way to summarize more complete definitions of ownership. For example, Schlager and Ostrom (1992) identify five basic property rights: access, withdrawal, management, exclusion and alienation. We combine the first three into access and the last two into veto.
We represent a *veto* structure by a mapping $\chi$ from the set of subsets of $N$ to the set of subsets of $A$, where $\chi(S)$ is the subset of assets that the coalition $S$ has veto rights on at date 1. $\chi$ satisfies the following condition:

$$\chi(S') \subseteq \chi(S) \quad \text{for any subset } S' \text{ of } S$$

The assets vetoed by a subset $S'$ of a coalition $S$ will also be vetoed by the whole coalition and we assume that the grand coalition can veto all the assets ($\chi(N) = A$).

Similarly, we represent an *access* structure by a mapping $\nu$ from the set of subsets of $N$ to the set of subsets of $A$, where $\nu(S)$ is the subset of assets that the coalition $S$ can access at date 1. $\nu$ satisfies the following condition:

$$\nu(S') \subseteq \nu(S) \quad \text{for any subset } S' \text{ of } S$$

The assets accessed by a subset $S'$ of a coalition $S$ will also be accessed by the whole coalition, and we assume that the grand coalition can access all the assets ($\nu(N) = A$).

Note that there is somehow a ‘pecking order’ between access and veto, since an agent who has veto on an asset can prevent another agent to access it without her.

The allocation of access and veto rights chosen by the agents ex-ante will characterize a *control structure* which will determine their incentives to invest. We now define the notion of control structure.

**DEFINITION.** A *control* structure is a mapping $\beta$ from the set of subsets of $N$ to the set of subsets of $A$, such that $\beta(S) = \nu(S) \setminus \chi(N \setminus S)$.

The assets *controlled* by a coalition $S$ are the assets that the coalition can access and that are not vetoed by coalitions outside of $S$. This definition combines the role of veto and

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8 This notion is similar to the notion of exclusivity provision studied by Segal and Whinston (2000). Like them, we find that allocating veto matters when the investments affect the ‘value of trade’ between the agent giving veto rights and an external party (see Proposition 3).

9 This is different from RZ who implicitly assume that $\nu(S') \supseteq \nu(S)$ for any subset $S'$ of $S$. In fact in RZ, implicitly a coalition can access an asset if *all* of its members have access ($\nu(S) = \bigcap_{i \in S} \nu(i)$) while we assume that it can access the asset if *any* of its members has access ($\nu(S) \supseteq \bigcup_{i \in S} \nu(i)$).

10 Identified by H&M.
the role of access\textsuperscript{11}. Both mechanisms are source of power and as such foster relationship-specific investment. But since we are interested in \textit{productive} investments on the assets, control without access is worthless. Hence our definition.

It is easy to see that $\beta(\emptyset) = \emptyset$ and $\beta(N) = A$ and that the assets controlled by a subset $S'$ of a coalition $S$ will also be controlled by the whole coalition:

$$\beta(S') \subseteq \beta(S) \text{ for any subset } S' \text{ of } S \quad (1.3)$$

A control structure is a veto structure when ‘control’ is fully determined by veto (i.e. $\beta_v(S) = \chi(S), \forall S \subseteq N$) and is an access structure when ‘control’ is fully determined by access (i.e. $\beta_a(S) = \nu(S), \forall S \subseteq N$).

**DEFINITION.** A control structure is an \textit{ownership} structure when ‘control’ is determined by both veto and access\textsuperscript{12} such that a coalition has access to an asset if and only if it can veto it (i.e. $\beta_o(S) = \nu(S) = \chi(S)$).

\textit{Ownership is different from veto.} In our definition, ownership is not simply the right to exclude others from the use of the asset (veto), it also requires the right to access the asset. For example, signing an exclusive contract gives veto power to the party who is given exclusivity but does not give her ownership on the other party’s asset (she does not have access to it).

In H&M the notion of ownership is defined through the concept of ‘residual control rights’. It is not very clear how this notion translates in terms of access and veto. On one hand they state that ‘the sole right possessed by the owner of an asset is his ability to exclude others from the use of that asset’ (p.1121) which seems to identify ownership with veto. But on the other hand they mention that ‘an agent’s bargaining position will depend on which assets he has access to and hence will be sensitive to the allocation of ownership’ (p.1122) and that ‘transferring ownership of an asset to party 1 increases 1’s freedom of action to use the asset as he or she sees fit’ (p.1120). Thus, despite their first assertion, it seems that their notion of ownership somehow encompasses both the right to veto and the right to use the asset. We provide here a clear definition of ownership.

\textit{Ownership is different from control.} In our conception a coalition controls a group of assets if it can access it \textit{and} no one else outside of the coalition can veto it. So ‘control’

\textsuperscript{11} Highlighted by RZ.

\textsuperscript{12} Ownership is the bundle of access and veto rights.
is different from ownership which requires both access and control\textsuperscript{13}. In their famous essay Berle and Means (1932) have highlighted that ownership is separated from control in modern corporations and control ultimately resides with managers rather than owners\textsuperscript{14}. If residual control rights are the rights to decide how the asset is to be used except the particular usages specified by contract, then we can argue that access itself may confer \textit{some} residual control rights. In particular the agent who accesses an asset may be in a position to capture some unspecified use rights over the asset. For example, there is usually no formal contract that specifies the use of a client list by a sales manager. If selling or giving away the list to a competitor is obviously not under her control, certain use rights such as which product to sell to each individual customer, or the number of lunches that the manager may take with the main customers, are likely to be unspecified and unenforceable by the ‘owner’. In these instances the manager exercises some \textit{de facto} form of residual control rights over the use of the asset\textsuperscript{15}.

In our taxi example, the government (which can determine whether the taxicab can be operated outside the contingencies outlined in the license agreement, e.g. requisition in case of war), the owner of the license plate (who may decide to renew or not the license, thereby somehow controlling the use of the car as a taxicab), the operator herself, who is the \textit{de jure} owner of the taxicab (she may decide to whom and when leasing the taxicab or how to maintain it) and the taxi driver, who leases the cab (he may decide where to drive it or how carefully to drive it) all have \textit{some} residual control rights. This simple example shows that it is difficult to assimilate ownership with residual control rights. ‘Residual control rights’ is an ambiguous concept (Demsetz, 1998). They do not always come with ownership, they can be rented (e.g. by the taxi driver) and the division of rights between the taxi driver (who has access rights) and the other parties (who have veto rights) seem more relevant to the analysis of the production activity than the division between residual and non residual rights.

\textsuperscript{13} But of course, when a coalition owns a set of assets, it controls it.

\textsuperscript{14} Hart (1995) suggests that ‘the separation of ownership and \textit{effective} control’ (we call the latter \textit{de facto} control rights) may be a better description.

\textsuperscript{15} And gets residual claim (e.g. perquisites, perks) to the asset profit stream. The difference between \textit{de jure} and \textit{de facto} residual rights of control is in the same vein as the difference between real and formal authority (Aghion and Tirole, 1997).
In H&M, control is somehow assimilated to ownership\textsuperscript{16} and the results brought by their control structure are viewed as a theory of ownership. We use here a clear distinction between control and ownership.

There is single access when a single agent can access an asset \((\nu(i) \cap \nu(j) = \emptyset)\), multiple access when several agents can access it \((\nu(i) \cap \nu(j) \neq \emptyset)\) and joint access when several agents can only access an asset jointly but not individually \((\nu(i) = \nu(j) = \emptyset, \nu(i, j) \neq \emptyset)\). Idem for veto and ownership.

Examples of multiple access are licensing or franchising. In a licensing contract an agent (the licensor) gives the right to use (access) his asset (e.g. a technology) to another agent (the licensee). The licensor keeps residual control rights on the content of the technology while the licensee gets residual control rights on the marketing. Both agents have access to the technology: they have residual control rights on different attributes of the technology (the licensor has renounced to his veto right on a part of the technology, i.e. the marketing, for the duration of the contract).

An example of multiple veto\textsuperscript{17} is a joint venture where each partner cannot control and derive revenue from the asset without the other partner (each partner can veto the other one)\textsuperscript{18}. An illustration of joint veto\textsuperscript{19} is a partnership, when each individual partner cannot veto but the group of partners can.

Finally, when a group of agents can veto an asset accessed by another group of agents it is called outside veto \((\nu(S) \cap \chi(N \setminus S) \neq \emptyset)\). An illustration of outside veto is a publicly-held company, where a group of outside agents (shareholders) can veto the agents who access the asset. Although we prefer the term outside veto\textsuperscript{20} to characterize this structure.

\textsuperscript{16} In the definition they mention that they ‘represent the ownership and control structure by a mapping…’ or in Proposition 8 state that ‘If two (or more) assets are (strictly) complementary, they should be owned or controlled together’.

\textsuperscript{17} Multiple veto can be shown to be equivalent to multiple ownership and to joint ownership.

\textsuperscript{18} H&M (1990) show that multiple veto is not efficient in their framework (Proposition HM4). This is because assets are complementary at the margin.

\textsuperscript{19} There are several specific forms of joint veto. In particular all agents who access the asset may have joint veto (unanimity) or only a group of them through a voting mechanism (majority veto) or through a stochastic system (stochastic veto). We do not study these forms specifically as the results would be similar in our framework.

\textsuperscript{20} Demsetz (1967) comments on this distinction and states that the managers are the de facto owners.
as the shareholders do not typically have access to the asset, most of the literature refers to it as outside ownership\textsuperscript{21}.

2.2. Control structure and H&M control structure

The following definition derives directly from H&M.

DEFINITION. A control structure $\beta$ is said to be an H&M control structure if $\beta(S) \cap \beta(N \setminus S) = \emptyset$ for all $S \subseteq N$

In H&M control (and ownership) is not dividable (two disjoint coalitions cannot control/own the same set of assets). This is in contradiction with the view that property rights are diverse and need not be owned by one single party (Demsetz, 1967, 1996), that assets may be viewed as a bundle of different attributes, each of which may have different ownership (Barzel, 1982) and the fact that divided property rights are frequent in the real world: copyrights, patents,, security interests, partnerships, corporations (Hansmann and Kraakman, 2002).

PROPOSITION 1. A control structure is an H&M control structure if and only if $\nu(S) \cap \nu(N \setminus S) \subseteq \chi(S) \cup \chi(N \setminus S), \forall S \subseteq N$

Proof. Take a control structure $\beta$ and a coalition $S \subseteq N$. $\beta$ is an H&M control structure if and only if $\beta(S) \cap \beta(N \setminus S) = \emptyset$ i.e. $[\nu(S) \setminus \chi(N \setminus S)] \cap [\nu(N \setminus S) \setminus \chi(S)] = \emptyset$. Thus $[\nu(S) \cap \nu(N \setminus S)] \cap [\chi(S) \cup \chi(N \setminus S)] = \emptyset$. Hence $\nu(S) \cap \nu(N \setminus S) \subseteq \chi(S) \cup \chi(N \setminus S)$

COROLLARY 1. 
(i) A control structure is not an H&M control structure if one of the following conditions holds for some $S \subseteq N$: (a) $\nu(S) \cap \nu(N \setminus S) = A$ and $\chi(S) \cup \chi(N \setminus S) \neq A$

(b) $\chi(S) \cup \chi(N \setminus S) = \emptyset$ and $\nu(S) \cap \nu(N \setminus S) \neq \emptyset$

\textsuperscript{21} May be because veto is assimilated to ownership.
(ii) An ownership structure is an H&M control structure but an H&M control structure is not always an ownership structure

An H&M control structure is a special category of control structure. H&M implicitly rule out control structures where two disjoint coalitions may access all assets if some assets are not vetoed (a) or where some agents may have (multiple) access on some assets and jointly veto them (b). Consider a simple partnership where two partners 1, 2 have joint veto on an asset $a$. Each agent can access the asset $\nu(1) = a$, $\nu(2) = a$ but no agent has veto individually ($\chi(1) = \emptyset$, $\chi(2) = \emptyset$, $\chi(1, 2) = a$). This structure is not H&M since $\nu(1) \cap \nu(2) = a$ and $\chi(1) \cup \chi(2) = \emptyset$. In H&M a partnership implies that agents cannot access the asset individually $(\nu(1) = \emptyset, \nu(2) = \emptyset, \nu(1, 2) = a)$ i.e. joint veto implies joint access. This constraint also rules out a great number of hybrid governance structures such as licensing or franchising. In a licensing structure both the licensor and the licensee have access to the technology and they jointly veto its use by other agents (the licensor vetoes the use of the technology by other agents than the licensee and the licensee vetoes the marketing of the technology by other agents in its territory).

Our notion of control differs from H&M in several ways. First we account for agents who may have veto rights but no access (e.g. shareholders)\(^{22}\). We also allow for the productive control of the same asset by two individuals. Finally, we allocate a more direct role for agents who can access an asset without owning it (e.g. employees). Overall our notion of (productive) control accounts for the fact that, in real life, managers may control the use of assets even if they do not ‘own’ them. Hence we broaden the GHM framework to incorporate the Berle and Means (1932) insight emphasizing the separation of ownership and control\(^{23}\). But we suggest an alternative, the separation between veto (by shareholders) and access (by managers): both give control.

The second part of the proposition expresses that the notion of ‘control’ in H&M is in fact different from our concept of ownership. In H&M an agent does not have to have access and veto to control a set of assets. For example, a control structure where

\(^{22}\) Shareholders are excluded in H&M (who only consider owner-managed firms) because the notion of residual control right encompasses the right to use the asset.

\(^{23}\) The necessity to incorporate both perspectives into a unified framework is highlighted and analyzed in Bolton and Scharfstein (1998).
\( \nu(S) \subset \chi(S) \) (i.e. the agents who have veto do not have to have access) is an H&M structure. So if the H&M’s notion of control structure is not equivalent to ownership, we should be careful in interpreting their result in terms of ownership\(^{24}\).

### 2.3. Optimal investment and welfare

\( \nu(S,A \mid x) \) is the value generated by a coalition \( S \subseteq N \) controlling a set of assets \( A \subseteq A \). where \( x = (x_1, \ldots, x_n) \) is the vector of investments by the agents of \( N \) (\( x_i \) is the ex-ante investment in human capital by agent \( i \)). In coalition \( S \), agent \( i \)’s marginal return on investment is given by \( \frac{\partial \nu(S,A \mid x)}{\partial x_i} = \nu'(S,A \mid x) \). The value generated will be sensitive to the relationships between the agents of the coalition and between the assets controlled by the coalition.

Given a control structure \( \beta \), an agent \( i \) will choose her level of investment \( x_i \) in order to maximize her ex-ante net benefit \( B_i(\beta \mid x) - C_i(x_i) \) where

\[
B_i(\beta \mid x) = \sum_{S \subseteq S} p(S)[\nu(S,\beta(S) \mid x) - \nu(S \setminus \{i\},\beta(S \setminus \{i\} \mid x)]
\]

is the share of value of \( i \) given by her Shapley value\(^{25}\) (\( p(S) \)

\(^{26}\) are the coefficients of the Shapley value), and \( C_i(x_i) \) is the cost of investment \( x_i \) to agent \( i \) (\( C_i \) is assumed to be a standard twice differentiable convex function). The Nash equilibrium investment \( x^e(\beta) \) is characterized by the first order conditions

\[
\frac{\partial B_i(S,\beta(S) \mid x^e)}{\partial x_i} = C_i'(x^e_i(\beta)), \text{ i.e.}
\]

\[
\sum_{S \subseteq S} p(S)\nu'(S,\beta(S) \mid x^e(\beta)) = C_i'(x^e_i(\beta)) \quad \text{for all } i
\]

(1.4)

From now on we will adopt a simplified notation by dropping the argument \( x \) whenever there is no confusion (i.e. we will write \( \nu(S,A) \) for \( \nu(S,A \mid x) \)).

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\(^{24}\) Control in H&M is also not equivalent to veto, since for example a pure access structure where \( \beta(S) = \nu(S) \) and \( \nu(S) \cap \nu(N \setminus S) = \emptyset \) (with no one having veto) is an H&M control structure.

\(^{25}\) We follow H&M (1990) in using the Shapley value as solution concept. For a non-cooperative justification of the Shapley value, see Gul (1989) and Stole and Zwiebel (1996b).

\(^{26}\) \( p(S) = \frac{(s-1)!}{n!} \) where \( s = |S| \).
We assume that $v'(S, \emptyset) \equiv v'(i, \emptyset), \forall S$ and follow H&M in assuming concavity of $v$ in $x$, superadditivity (i.e. $v(S,A) \geq v(S',A') + v(S \setminus S', A \setminus A'$), $\forall S', S, A, A' \subset A$), complementarity of investments (i.e. $(\partial / \partial x_i)v'(S,A | x) \geq 0, \forall j \neq i$), and human capital investment ($v'(S,A) = 0$ when $i \notin S$).

First-best: superadditivity implies that the maximum total value is $v(N,A | x)$ and the first-best overall surplus is achieved by maximizing $W(x) = v(N,A | x) - \sum_{i=1}^{n} C_i(x_i)$. The first best level of investment $x^*$ is given by the first order conditions:

$$v'(N,A | x^*) = C_i'(x_i^*) \text{ for all } i$$  \hspace{1cm} (1.5)

We now deviate from H&M by replacing their assumption 627 by the following.

ASSUMPTION A1. $v'(S,A) \leq v'(S',A') \leq v'(N,A), \forall i \in S, \forall S' \subset S \subset N, \forall A \subset A$

The marginal return generated by any coalition $S$ controlling a set of assets $A$ increases with the number of agents and is always (weakly) lower than the marginal return generated by the grand coalition (controlling all assets). Assumption A1 is weaker than HM6 (which in addition requires that all assets be complementary at the margin28). Nonetheless the same underinvestment results as in H&M.

**Lemma 1.** Under assumption A1, for any control structure there is always underinvestment and if every agent’s marginal return on investment increases then the equilibrium investment increases and welfare increases.

**Proof.** See Appendix

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27 $v'(S,A) \geq v'(S',A'), \forall S', \forall A' \subset A$ (hereafter HM6).

28 See section 3.2 below.
We will say that a control structure $\beta'$ is more efficient than a control structure $\beta$ if the former leads to a higher level of welfare. The rest of the paper will assume that assumption A1 holds and that there is always underinvestment\(^{29}\).

2.4. Asset complementarity and substitution at the margin

In our analysis, we will use notions of complementarity and substitution at the margin\(^{30}\), which are a convenient way of summarizing how the relations between assets affect the marginal contribution of the agents. These are of course different from the notions of ‘absolute’ complementarity or substitution of assets which describe the effect of the relations between assets on the value $v$ (in our framework assets are assumed to be complementary, since the value function is superadditive in assets). We say that two assets controlled by a coalition are complementary [substitutes] at the margin, if the marginal contribution of any agent in the coalition is higher [lower] when the coalition controls both assets than when it controls only one. Formally, $a_1,a_2$ are complementary [substitutes] at the margin when $v'(S,\{a_1,a_2\}) \geq [\leq] v'(S,\{a_1\})$ and $v'(S,\{a_1,a_2\}) \geq [\leq] v'(S,\{a_2\})$, $\forall i \in S$.

We will now study the optimal control structure and get our main results. The following section will successively cover the cases where assets are all complementary at the margin (as in H&M), where assets are all substitutes at the margin and the general case (some assets complementary, some substitutes).

3. Main results

3.1. When assets are complementary (at the margin)

ASSUMPTION A2: assets are all complementary (at the margin), i.e. $v'(S,A) \leq v'(S,A')$, $\forall i \in S$, $\forall S \subseteq N$, $\forall A \subseteq A'$

\(^{29}\) Underinvestment is not only coherent with the H&M framework. This is also a very plausible assumption in the real world, especially when focusing on human capital investment, and under-provision of incentives is a common empirical finding. In most cases, it is required to increase human capital investment through incentives and it is difficult to imagine what an overinvestment in human capital would be.

\(^{30}\) Hart (1995) defines and uses the notion of complementarity at the margin. We complement it with the concept of substitution at the margin.
H&M assume that assumptions A1 and A2 hold (A1+A2 = HM6). In this section, we will follow their framework.

In the analysis that follows we will be interested in a partial ordering of the control structures. The framework developed by Segal (2003) will be useful here. In his paper, Segal defines three types of integration contracts between agents owning resources. *Exclusion*, when an agent can exclude another agent’s resource but not use it himself (i.e. one agent gives veto power on his resource to the second agent). *Inclusion*, when an agent can use another agent’s resource but not exclude him from using it (i.e. one agent gives access on his resource to the second agent). *Collusion* when both agents merge their resources in the hands of one of them (i.e. one agent gives ownership – both veto and access – on his resource to the second agent). In this paper, we build heavily on Segal (2003). Where Segal shows that each type of integration is the best response to a particular form of relationship between the agents, we show that allocating access, veto or ownership is the best response to particular forms of asset relationships. But we consider the impact of an allocation of rights on the ex-ante incentives of all agents and thus on welfare, while Segal uses a zero-sum game where the gains of one coalition impose a negative externality on the complementary coalition (i.e. a coalition will ‘gain’ if the complementary coalition ‘looses’). His framework then applies to situations where a given coalition has ‘an institutional advantage in prior contracting’\(^{31}\) while we consider the case where agents and coalitions have equal opportunities to contract. With this focus\(^ {32}\), the results of Segal are different and sometimes opposed to ours.

Considering two control structures \(\beta\) and \(\beta'\), we will say that \(\beta\) is more *inclusive* than \(\beta'\) (or \(\beta'\) is more *exclusive* than \(\beta\)) if \(\beta'(S) \subseteq \beta(S), \forall S \subseteq N\). By extension, we will say that an allocation of rights is inclusive (resp. exclusive) if it leads to a more inclusive (resp. exclusive) control structure.

\(^{31}\) Segal gives this example: ‘if members of this coalition can meet before other players arrive at the scene’.

\(^{32}\) On the other hand Segal uses a broader solution concept (random-order value) while we stick to the Shapley value used in H&M.
LEMMA 2. Allocating access (rights) is inclusive and more inclusive than allocating ownership. Allocating veto (rights) is exclusive and more exclusive than allocating ownership.

Proof. Starting from a control structure \( \beta(S) = \nu(S) \setminus \chi(N \setminus S) \), allocating access rights to an agent \( i \) leads to a new access function \( \nu' \) such that \( \nu(S) \subseteq \nu'(S), \forall S \supseteq i \). Hence the new control structure \( \beta^a = \nu'(S) \setminus \chi(N \setminus S) \) is such that \( \beta(S) \subseteq \beta^a(S), \forall S \supseteq i \). So allocating access is inclusive. Idem for veto where \( \chi(S) \subseteq \chi'(S), \forall S \supseteq i \) so \( \chi(N \setminus S) \subseteq \chi'(N \setminus S), \forall S \not
\supseteq i \) and the new control structure \( \beta^o = \nu(S) \setminus \chi'(S) \) is such that \( \beta'(S) \subseteq \beta(S), \forall S \not
\supseteq i \). If we allocate ownership to \( i \) (i.e. veto and access on the same assets), we have a new control structure \( \beta^o = \nu'(S) \setminus \chi'(N \setminus S) \) such that:

\[
\begin{align*}
\beta^a(S) &= \nu'(S) \setminus \chi'(N \setminus S) = \beta^a(S) \quad \text{and} \quad \beta^o(S) \supseteq \nu(S) \setminus \chi'(N \setminus S) = \beta^o(S), \forall S \supseteq i \\
\beta^o(S) &\subseteq \nu'(S) \setminus \chi'(N \setminus S) = \beta^o(S) \quad \text{and} \quad \beta^o(S) = \nu(S) \setminus \chi'(N \setminus S) = \beta^o(S), \forall S \not
\supseteq i
\end{align*}
\]

Thus \( \beta^o(S) \subseteq \beta^a(S) \subseteq \beta^o(S), \forall S \).

With our definition of control, allocating more access rights can only (weakly) increase the number of assets controlled by a coalition since some coalitions get new access to some assets. Allocating more veto rights can only (weakly) decrease the number of assets controlled by a coalition, since some assets are now vetoed by additional agents. The effect of ownership is intermediary since ownership comes with both access and veto rights.

PROPOSITION 2. When assets are complementary (at the margin) (i) a more inclusive control structure is more efficient (ii) allocating access increases welfare and is more efficient than allocating ownership (iii) the optimal control structure \( \beta^* \) is the access structure where \( \beta^*(S) = \nu^*(S) = A, \chi^*(S) = \emptyset, \forall S \subseteq N \)

Proof. (i) When assets are complementary at the margin \( \nu'(S, \beta(S)) \leq \nu'(S, \beta'(S)) \) if \( \beta(S) \subseteq \beta'(S) \). Thus, by Lemma 2, \( \beta^* \) is more efficient.
(ii)-(iii) The result derives directly from Lemma 1

(iv) The most inclusive control structure will be optimal and allocating access is inclusive (and allocating veto is exclusive). So the most inclusive control structure will be obtained when \( \nu(S) = A \) and \( \gamma(S) = \emptyset, \forall S \), i.e. when \( \beta(S) = A, \forall S \).

Proposition 2 shows that the highest (second best) level of welfare is achieved with an access structure and that ownership is inefficient. In fact, given any control structure, allocating more access rights will increase welfare. It is efficient for an agent to give access on her asset to a second agent when assets are complementary because it increases the incentives of the agent who is given access and the incentives of the other agents (‘external’ to the relationship between the two agents). In the latter case the agent who is given access will bring the complementary asset in his relationship with the external agents. So open access is the best structure when assets are complementary at the margin, which seems to contradict the idea of property rights and the well known ‘tragedy of the commons’ problem.

This is because in the ‘tragedy of the commons’ agents impose negative externalities on other agents. In our framework, this would mean that agents are substitutes at the margin (when more agents access an asset, it decreases the marginal return of each agent). As we will see in the next section, substitution at the margin calls for the allocation of veto rights. But in the H&M (and our) framework, agents are complementary at the margin, and there is no need for veto.

So why is it that H&M find ownership efficient when assets are complementary? This is because the constraint on the H&M control structure (two disjoint coalitions cannot control the same asset) generates a negative externality between agents which re-establishes the need for veto rights. There is no such constraint in our model, hence no need for veto. Note that the optimal control structure is not an H&M control structure. If we restrain the scope to H&M control structures, the role of ownership as an efficient mechanism to foster incentives can be restored.

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33 \( \nu(S, A) \geq \nu(S', A), \forall i \in S, \forall S \subseteq S', \forall A \)

34 In our paper, we focus on the substitution at the margin between assets, but we might as well have studied substitution at the margin between agents. The main results of this paper would hold.

35 Since \( \nu(S) \cap \nu(N \setminus S) \not\subseteq \chi(S) \cup \chi(N \setminus S) \) (see Proposition 1).
But ownership may not be the only solution to the tragedy of the commons in presence of negative externalities. In the classical Property Rights theory\textsuperscript{36}, the solution is to internalize these externalities by allocating veto to some agents, provided that the benefits of establishing these rights are higher than their costs. These agents then get ownership (access and veto) and we move from an open access regime (or collective ownership) to a private property rights regime. But while the effect of access is always positive (it increases incentives of the agent who is given access and of the other agents), veto has two facets: a positive effect (it increases the incentives of the agents protected by their veto right\textsuperscript{37}) and a negative effect (it reduces incentives of the other agents). Rules of use or ‘governance’ among those with access to the assets may be an alternative (Smith, 2002) to the allocation of veto. Smith shows that in reality there seems to be a continuum of successful solutions between the pure allocation of veto rights and the adoption of ‘governance’ rules (the choice along this continuum depending on the relative cost structure of the different solutions). At the extreme, if we restrict $A$ to be the assets of a given firm, it may well be that the adoption of governance rules (i.e. hierarchy, authority and fiat) among the agents of the firm has a better cost structure than the allocation of veto.

In summary, we show that an access structure is optimal when assets are complementary because in our model the use of assets does not generate conflict (unlike veto). The governance structure within the firm will prevent (or limit) externalities to occur (e.g. the use of an asset by an employee will be regulated, employees will have to maintain the equipment,...) which will maintain the superiority of an access structure over an ownership structure. Note that the ‘no veto’ of the access structure is in fact equivalent to a joint veto on $A$ by $N$. In other words the agents of the firm collectively limit the amount of externalities by restricting access to the employees\textsuperscript{38}.

\textsuperscript{36} Demsetz (1967).
\textsuperscript{37} Demsetz explains that an owner ‘by virtue of his power to exclude others’ gets incentives to utilize assets more efficiently.
\textsuperscript{38} This is the difference between open access (where no one has the right to exclude others) and common property (members of a ‘clearly demarked group’ have the rights to exclude nonmembers) (Ostrom, 2000). According to Ostrom, income in a modern corporation can be seen as a common pool of resources to be shared by stockholders, management and employees, and ‘relationships within the firm are far from being ‘individual’ ownership rights’. 
For example, Christian Lacroix (a famous French fashion designer) has a personal firm whose (essentially human capital) assets are strictly complementary to the assets of the Christian Lacroix firm (hereafter CL): the latter are worthless without him. But despite the prediction of H&M’s model, Christian Lacroix does not own CL (nor even any share of the firm). CL is owned by LVMH (the worldwide leader in luxury products) together with many other complementary firms (they share common services: financial, legal, fiscal, marketing,…). Instead, his personal firm has a contract with CL, whereby he provides his services and gets access to the physical assets (with high powered incentives).

So in our framework when should an ownership structure be optimal? It can be shown that an ownership structure is (weakly) optimal when the agents are indispensable\(^4\) (the proof is straightforward and left to the reader). The intuition is that being indispensable is equivalent to having veto power (the other agents cannot work on the asset without the indispensable agent). Thus giving access to an indispensable agent is equivalent to this agent having ownership (access and veto).

So far we followed the asset complementarity assumption of H&M. We now relax it and turn to the opposite case.

3.2. When assets are substitutes (at the margin)

H&M not only assume complementarity through superadditivity (their assumption 5) but also complementarity at the margin (their assumption 6). Despite being rather strong\(^4\), the latter is at the basis of all propositions in their paper. But assets may be complementary without being complementary at the margin\(^4\). Two of Coase’s (1937) main arguments for the limitation of the size of the firm are that as firms get larger (i.e. the number of assets increase) there are ‘decreasing returns to the entrepreneur

\(^39\) Proposition 8 of H&M states that ‘if two (or more) assets are strictly complementary, they should be owned together’.

\(^40\) An agent \(i\) is indispensable to an asset \(a\) if, without agent \(i\) in a coalition, \(a\) has no effect on the marginal return for the agents of the coalition \(v'(S,A) = v'(S,A\setminus\{a\})\), for \(j \in S\), if \(i \notin S\) (H&M).

\(^41\) ‘stronger than the others’ (H&M).

\(^42\) This has been somehow pointed out by Stole and Zwiebel (1998) who mention that ‘while the characteristic function underlying many cooperative games is taken to be superadditive, this by no means implies that different parties would increase their combined Shapley value by merging’.
function’ and ‘the entrepreneur fails to place the factors of production in their best use’. In other words, adding assets reduces the marginal return on investment by the entrepreneur (what the economists call ‘diminishing returns to management’). Other scholars have also studied diminishing return to scope of monitoring i.e. ‘loss of control’ (Williamson, 1967, Calvo and Wellisz, 1978) or the fact that focusing on fewer assets brings economies of specialization and coordination (Rotemberg and Saloner, 1994). Moreover, there is multiple empirical evidence that refocusing (Berger and Ofek, 1999), spin-offs (Daley et al., 1997) and downsizing (Dial and Murphy, 1995) may increase firm performance and create value by ‘allowing managers to focus attention on the core operations they are best suited to manage’ (Daley et al., 1997). It seems to us that a model studying the boundaries of the firm should integrate this key intuition. The reasons generally invoked to justify mergers focus on pure complementarity (sharing of fixed costs, economies of scope,…) because substitution (or complementarity) at the margin is difficult to measure. But the relative un-success of mergers as outlined by the empirical literature may be partly due to this substitution effect at the margin (e.g. managers spread too thin). Adding a complementary asset to a set of complementary assets (in a superadditive game) does not ensure that the new asset is complementary at the margin with all existing assets. If it is substitute at the margin, the merger will face some trade-offs which cannot easily be resolved with the allocation of unified ownership. In that context, allocating access and veto separately may provide a solution. Anyway, the contrasting roles that substitution and complementarity may play have been highlighted in several models (Stole and Zwiebel, 1996a, RZ, Segal and Whinston, 2000, and Segal, 2003) and we feel the need to relax the assumption on complementarity at the margin. We assume now that all assets are substitutes (at the margin) before studying the more general situation where some assets are complementary and some substitutes (at the margin).

43 GH dismiss this point as ‘unconvincing’ because the owner/entrepreneur could always hire another manager. But they overlook the fact that the owner would then have to manage and monitor the additional manager…
44 Coase (1937).
45 Mailath et al. (2004) show that mergers may increase the cost of inducing managerial effort.
46 In the taxicab example we have seen that substitution at the margin has negative effects on the incentives to invest.
ASSUMPTION A3: assets are all substitutes at the margin in the absence of some agents in \( N \), i.e. \( v'(S,A) \geq v'(S,A') \), \( \forall S \subset N, \forall A(\neq \emptyset) \subseteq A', \forall i \in S \)

\[ (v'(S,\emptyset) \leq v'(S,A'), \forall S \subseteq N). \]

Assets are substitutes at the margin when an increase in the supply of assets decreases the marginal return of investment. For example, in a production function \( v(S,A) = v(x_1, \ldots, x_\nu, A) \) where \( x_i \) is the investment by \( i \in S \) and \( A = \sum a_k \), the assets are substitutes at the margin if the agents’ investments are gross \( q \)-substitutes\(^{48}\) with respect to \( A(\frac{\partial^2 v}{\partial x_i \partial A} \leq 0, \forall i \in S) \), i.e. an increase in the supply of assets decreases the marginal product of the ‘investment’ factor (the Hicks elasticity of complementarity between investments and assets is negative). A typical CES function \( v(S,A) = \left( \sum x_i^\rho + A^\rho \right)^{\rho/\rho} \) would have this property with appropriate value of the elasticity of substitution\(^{49}\).

PROPOSITION 3. When assets are substitutes at the margin (i) if \( |\beta(i)| > 1 \) for some \( i \), a more exclusive control structure increases welfare (and allocating veto is more efficient than allocating ownership) (ii) an optimal control structure \( \beta^* \) is characterized by a partition \( \left\{ N_0, N_1 \right\} \) of \( N \) such that (a) \( \beta^*(i) = \emptyset, \forall i \in N_0 \) (b) \( |\beta^*(i)| = 1, \forall i \in N_1 \)

\[ (c) \beta^*(S) = \bigcup_{i \in S} \beta^*(i) \text{ if } S \cap N_1 \neq \emptyset \]

Proof. See Appendix

When assets are substitutes at the margin, allocating veto can increase welfare and in an optimal control structure, no agent will control more than one asset. Also a coalition will only control the assets controlled by its individual members (if at least one of them controls an asset).

\(^{48}\) See Hicks (1970) or Bertoletti (2003).

\(^{49}\) I am indebted to Bertoletti for this insight.
In H&M assets are complementary at the margin and additional assets provide incentives because the outside option of the owner (i.e. the threat to withdraw her assets) increases with the number of assets that she controls. But when assets are substitutes at the margin and the agent controls more than one asset, decreasing the number of assets controlled by the agent increases her outside option. So outside veto is good because it increases the outside option of the agent (without affecting the incentives of the agent who is given veto) and it is also good for the other ‘external’ agents because the agent who has given veto will not bring her substitute asset with her in the relationship. However, when the agent controls only one asset, outside veto will actually decrease her outside option (her marginal return with one asset is always higher than with no asset). But it will also increase her marginal return in the relationship (she does not bring a substitute asset with her). So the net effect on the marginal return of the agent is unclear. Overall, in an optimal control structure no agent should control more than one asset.

The second part of the proposition tells us that, in an optimal control structure, if some agents in a coalition control one asset then the coalition will only control the assets controlled by its members (it would be inefficient for the coalition to control more assets since the assets are substitutes at the margin). On the other hand, if no agent in the coalition controls any asset, then the coalition should control no asset or one asset (controlling one asset may be more efficient in some cases).

In this economy, agents are partitioned between those who do control one asset \( N_1 \) and those who do not \( N_0 \). If \( N_0 = \emptyset \), each agent controls one asset and a coalition controls the assets controlled by its members (but several agents may control the same asset). This may only happen if the number of assets is higher than the number of agents. If \( N_1 = \emptyset \), then no agent individually controls any asset and an asset can only be controlled by a group of agents. Assuming that no asset is idle in an optimal control structure\(^{50}\), this may only happen if the number of assets is lower than the number of agents. In the general case both \( N_0 \) and \( N_1 \) will be non empty, and we will now attempt to characterize \( N_1, N_0 \).

\(-\) **Productivity and substitution**

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\(^{50}\) An idle asset would not be productive, which is suboptimal by definition.
We introduce a new statistic, the *substitution effect*, which characterizes the level of substitution between an asset \( a \) and other assets \( A \) \((A \neq \emptyset)\) and its impact on the marginal return for an agent \( i \), i.e. \( SE_i^a(S, A) = v'(S, A) - v'(S, A \cup \{a\}), i \in S \). The substitution effect characterizes the negative impact on \( i \)'s marginal return, in her relationship with the other agents of \( S \), of adding \( a \) to other assets (substitutes at the margin). It represents the managerial diminishing returns on coordination or monitoring.

The *marginal productivity* of agent \( i \) with an asset \( a \) is the difference in marginal return for \( i \) with and without the asset alone, i.e. \( P_i^a = v'(i, a) - v'(i, \emptyset) \) (by definition non negative). We will use the following definition.

**DEFINITION.** (i) An agent \( i \) is *highly productive* with an asset \( a \) if her marginal productivity with the asset alone is higher than \( n \) times the maximum substitution effect between \( a \) and any other set \( A \) of assets. That is \( P_i^a \geq nSE_i^a(S, A), \forall S, \forall A \).

Formally: \[
v'(i, a) - v'(i, \emptyset) \geq n\text{Max}_{S,A}[v'(S, A) - v'(S, A \cup \{a\})]
\]

(ii) An agent \( i \) has *low productivity* with \( a \) if, \( P_i^a \leq (1/q)nSE_i^a(S, A), \forall S, \forall A \), where \( q = [(v'(N, a) - v'(N, \emptyset))]/[v'(i, a) - v'(i, \emptyset)] \).

Formally: \[
v'(i, a) - v'(i, \emptyset) \leq (1/q)n\text{Min}_{S,A}[v'(S, A) - v'(S, A \cup \{a\})]
\]

Note that our notion of high or low productivity is taken *relative* to the substitution effect. When assets are all complementary at the margin (as in H&M), the substitution effect is non positive and the agents are all highly productive. We will now try to characterize the optimal control structure(s) in more detail.

**LEMMA 3.** *When assets are substitutes at the margin* (i) control over an asset increases incentives of a highly productive agent but decreases incentives of a low productivity agent, (ii) control by a highly productive agent is more efficient than by a low productivity agent

*Proof.* See Appendix
A direct consequence of Lemma 3 is that control over an asset should be allocated in priority to a highly productive agent (if there is such agent) and a low productivity agent should only control an asset if there is no highly productive agent with the asset who does not control any asset (otherwise it would be more efficient to give her control over the asset). In an optimal structure, there is a ‘pecking order’ in the allocation of control. As a consequence, we allow an additional assumption: an agent who does not control any asset has low productivity (with any asset). So, in which case should an agent control an asset?

This question can be answered by examining the way veto should be allocated (in the following, $S^a$ is the group of agents who access the asset $a$).

**Proposition 4.** In an optimal control structure $\beta^*$, when assets are substitutes at the margin,

(i) If an agent accessing an asset is highly productive, he should veto it ($\beta^*(i) = a$)

(ii) If a group of agents accessing an asset are individually highly productive with it, they should have joint veto on it ($\beta^*(i) = a, \beta^*(S^a) = a$)

(iii) A low productivity agent should not access an asset already accessed by an agent

(iv) If no agent individually accesses an asset, a group of low productivity agents may jointly access it. They should jointly veto it if they are collectively highly productive with it ($\beta^*(i) = \emptyset, \beta^*(S^a) = a$), otherwise there should be outside veto

(v) Unproductive agents (outside parties) may have outside veto on some assets

**Proof.** In the Appendix

**Proposition 4** justifies the existence of four types of optimal governance structures based upon the number of productive agents who access an asset and their level of productivity:

- **Owner-managed firm,** when the agent who accesses the asset is highly productive with it. The control structure is: $\beta^*(i) = v^*(i) = \chi^*(i) = a$ (single access, single veto)
- **Partnership**: several highly productive agents access an asset. $\beta^*(i) = \emptyset$, $\nu^*(i) = a_i, \chi^*(i) = \emptyset \Rightarrow \beta^*(S^a) = \nu^*(S^a) = \chi^*(S^a) = a$ (multiple access, joint veto).

- **Outside ownership**: there is no highly productive agent for the asset and a low productivity agent accesses the asset. $\beta^*(i) = \emptyset, \nu^*(i) = a_i, \chi^*(i) = \emptyset \Rightarrow \exists k$ s.t. $\chi^*(k) = a, \nu^*(k) \not\supseteq a$ (single access, outside veto).

- **Joint ownership**: no agent individually accesses an asset but a group of agents jointly does. $\beta^*(i) = \nu^*(i) = \chi^*(i) = \emptyset \Rightarrow \beta^*(S^a) = \nu^*(S^a) = \chi^*(S^a) = a$ (joint access, joint veto).

From the above we get that a low productivity agent does not control any asset ($|\beta^*(i)| = 0$) (if she accesses the asset, there is outside veto on it) and a highly productive agent controls an asset if and only if she accesses it ($|\beta^*(i)| = 1 \Leftrightarrow |\nu^*(i)| = 1$) (if she accesses it, she should also veto it). In the partition of $N$, a low productivity agent always belongs to $N_0$, while a highly productive agent may belong to $N_1$ or $N_0$.

*Proposition 4* shows the important and diverse role that veto can play in an optimal control structure when assets are substitutes at the margin. Two forms of veto play a special role: outside veto and joint veto.

Here is an intuition for outside veto. Since the value that can be generated by a coalition is superadditive in assets, there is always a desire to bring assets together. But when assets are substitute at the margin, the marginal product declines when assets are merged, so investment falls. The way to overcome this is to include other agents to give the commitment value not to merge. In this framework, this is done by giving a veto right on the asset to another agent. Allocating veto to an outside party is better than keeping ownership because it reduces the incentive to invest on substitute assets. In other words, the veto power of outside parties (e.g. shareholders) serves to prevent managers to merge substitute assets. This provides a justification for the role of outside ownership.\(^{51,52}\)

\(^{51}\) Rajan & Zingales (1998) have found a justification which is somehow related. They highlight the role of ownership by a non-investing party (i.e. a third party) in absorbing the opportunity losses from specialization that the manager would otherwise incur if she would own the asset. For them ‘those who have access, and thus the privileged right to invest, have a kind of control right which can be misused when coupled with the control rights of ownership’. This is a way of saying that access and veto may have to be disconnected. In their model outside ownership protects the incentives of the manager to specialize. In our framework, it prevents the manager to merge its asset with other substitute assets, i.e. it protects the incentives of the manager to specialize on its own asset.
But in our framework, outside veto is not always efficient (when assets are substitutes at the margin). It is efficient only when the agent has low productivity. In that case, it increases the marginal return of the agent in the relationship with the other agents who control substitute assets but it decreases her outside option with her asset. If the agent has low productivity with the asset, the overall effect is positive and her bargaining position improves. It is positive for the other agents she will bargain with since her value in the coalition increases, so the overall effect on welfare is positive. But the opposite is true for a highly productive agent.

It is worth keeping in mind that the terms highly productive and low productivity are taken relative to the substitution impact. A firm will be owned by a productive agent (owner-managed firm) if her marginal productivity with the asset alone is sufficiently high. But if her marginal productivity is lower than the substitution effect, then it will be optimal to have outside ownership\(^{53}\) (other productive agents or outside parties will have veto on the asset). A firm originally owner-managed may turn to outside ownership for two reasons: if the productivity of the owner with the asset decreases below a certain level or if the substitution effect increases above a certain threshold (e.g. if the ‘differentiation’ of the firm becomes too small). The term of outside ownership should not be viewed *stricto sensu*. In fact our model sees the emergence of third parties who get outside veto on an asset without having access on it (i.e. they do not have ownership). This is true in the case of shareholders, but a corporate vice-president making the final decision on asset allocation between two divisions would be another example\(^{54}\).

*Joint veto* plays also a special role in *Proposition 4*. The intuition here is that when highly productive agents access an asset, individual veto by one of them would reduce the incentives of the others. *A contrario*, joint veto is good because it protects productive agents who access the asset together from vetoing each other, while preventing to ‘merge’ the asset with ‘external’ substitute assets. This finds an illustration in *partnerships*. Professional partnerships are characterized by ‘individualized, autonomous day-to-day activity’ combined with ‘a system of control in which authority is

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\(^{52}\) Segal and Whinston (2000) observe that, with substitute investments, ‘it may be optimal to give ownership of the “exclusivity asset” to a noninvesting party’.

\(^{53}\) We use the term outside ownership to relate to the literature. What we really mean is outside veto.

\(^{54}\) See Bolton and Scharfstein (1998) on General Motors and Fisher Body.
shared by all members of the working group’ (Greenwood & al., 1990). The first aspect illustrates multiple access while the second represents joint veto.

Our framework also highlights that joint ownership may be optimal\(^{55}\) when assets are substitutes at the margin. We find here some commonalities with the literature on joint ownership\(^{56}\).

Together, Propositions 2 and 4 provide some indication for the boundaries of the firm and propose an answer to a major critique made by Holmstrom (1999) on the H&M model, literally that ‘the theoretical predictions concerning joint ventures, sole ownership of complementary assets and outside ownership are all quite fragile’. Overall, a firm may be defined as a collection of complementary (at the margin) assets which are substitutes (at the margin) to other firms/assets. Proposition 2 says that when assets are complementary (i.e. \textit{within} a firm), allocating access to all agents is better than allocating ownership to some. Proposition 4 says that when assets are substitutes (i.e. \textit{among} firms) outside parties may have veto power rather than ownership and productive agents should give veto on their asset to outside agents (shareholders) when their productivity is lower than the substitution impact with the other assets. Proposition 2 highlights the role of workers/managers (agents with access but no veto) when assets are complementary. Proposition 4 highlights the role of shareholders (agents with veto but no access) when assets are substitutes. Proposition 4 also characterizes the optimality of joint ownership when no agent individually accesses an asset. Both propositions show the independent role of access and veto, and that ownership may not always be the best driver for investment incentives.

So far we have focused on environments where assets are either \textit{all} complementary or \textit{all} substitutes (at the margin). But these represent rather extreme cases and, in general, the relationships between assets may be more complex.

\subsection*{3.3. General case: complementary and substitute assets}
At this stage it is important to precise the notion of complementarity and substitution between assets. When assets \(a_1\) and \(a_2\) are bilaterally complementary

\(^{55}\) In H&M, joint ownership is not optimal because assets are complementary at the margin.
\(^{56}\) Cai (2003) provides a condition for optimal joint ownership. Joint ownership is optimal when specific and general investments are substitutes, which resembles our substitutability condition on assets.
(v(S,a_1,a_2) ≥ v(S,a_1,a_2)), the presence of a third asset a_0 may in some cases reduce the complementarity between the two assets (v(S,a_0,a_1,a_2) ≤ v(S,a_0,a_1)). In other words, a_2 is complementary to a_1 in the absence of a_0 but substitute to a_1 in presence of a_0.

More generally, we will use the following definitions.

Two assets a_k,a_l are always complementary [resp. substitutes] at the margin if

v(S,A) ≤ [≥]v(S,A ∪ {a_k}), ∀ i ∈ S, ∀ A ⊆ a_k (they are independent in case of equality).

An asset a_k is complementary [resp. substitute] to an asset a_l in absence of another asset a_m if and only if ∀ i ∈ S, ∀ A ⊆ a_l, with a_m ∉ A, v(S,A ∪ {a_k}) ≥ [≤]v(S,A).

With 3 assets, a_k,a_l cannot be always complementary if a_j,a_k are always substitutes.

In our framework, when some assets are substitutes and some are complementary, we are not able to derive general conditions for an optimal structure without imposing some restrictions. So we start from a situation where some agents i,j,k,... respectively own assets a_i,a_j,a_k,... (an ownership structure) and will be interested in finding a reallocation of rights that would generate a more efficient control structure. If this new structure cannot be improved, we will have an optimal control structure. The initial set up is identical to Segal (2003) who analyzes the different types of integration between agents who originally own their own asset. But Segal studies a zero-sum game where the gains of one coalition impose a negative externality of the complementary coalition, and finds different results.

There are different ways of reallocating the rights on an asset, but they come down to four types. Agent j may give access on a_l (and renounce to her veto power, i.e. i and j have joint veto) to i (v(i) = a_i,a_j; χ(i) = a_i; ψ(j) = a_j; χ(j) = ∅; multiple access), give veto (v(i) = a_i; χ(i) = a_i,a_j; ψ(j) = a_j; χ(j) = a_j; joint ownership/multiple veto), give ownership (v(i) = a_i,a_j; χ(i) = a_i,a_j; ψ(j) = ∅; χ(j) = ∅; integration by i),
or keep separate ownership \((\nu(i) = a_i; \chi(i) = a_i; \nu(j) = a_j; \chi(j) = a_j; \) non integration\)\(^{57}\). The first allocation is inclusive, the second is exclusive and the third is collusive\(^{58}\).

**PROPOSITION 5.** When agents \(i, j, k, \ldots\) respectively own assets \(a_i, a_j, a_k, \ldots\) (i) \(j\) should give access on her asset to \(i\) if \(a_i, a_j\) are always complementary in absence of \(j\) (ii) \(j\) should give veto to \(i\) if, in absence of \(i\), \(a_j, a_k\) are substitutes for all \(a_k\) and \(j\) has low productivity with \(a_j\)

*Proof.* In the Appendix

**COROLLARY 2.** \(j\) should give ownership to \(i\) if \(a_i, a_j\) are always complementary in absence of \(j\) and if, in absence of \(i\), \(a_j, a_k\) are substitutes for all \(a_k\) and \(j\) has low productivity with \(a_j\)

Proposition 5 shows that with complementary and substitute assets, access, veto and ownership have specific roles to play to foster investment incentives. And, by Corollary 2, the optimality of integration depends not only on the complementarity between two assets (as in H&M) but also on the level of substitution between the ‘integrated’ asset and the other ‘external’ assets (more precisely whether the substitution effect is higher than the marginal productivity of the ‘integrated’ agent with his asset). These results are summarized in Table 1 below, which provides a general characterization for the optimal allocation of rights (we simplify notation by writing \(SE_j^{a_i}\) for \(\min_{S,A} SE_j^{a_i} (S, A)\))\(^{59}\).

\(^{57}\) Other allocations are possible but are redundant with those presented. In particular, \(j\) giving access on \(a_j\) to \(i\) while keeping veto and access is equivalent to non-integration and \(j\) giving access on \(a_j\) to \(i\) while keeping veto and renouncing to access is equivalent to \(j\) giving veto to \(i\) (i.e. joint ownership). This structure has been studied in detail by RZ, who analyze the effects for an entrepreneur (who does not have access, i.e. he cannot produce by himself) of giving access (while retaining veto) to different managers.

\(^{58}\) Should we say that the fourth is delusive?

\(^{59}\) Proposition 5 is compatible with Propositions 2 & 4. When all assets are complementary (top left corner) \(j\) should give access on \(a_j\) to \(i\) (Proposition 2). When all assets are substitutes (bottom), \(j\) should keep ownership if she is highly productive, but should give veto to \(i\) if she has low productivity (Proposition 4).
As we move from left to right of Table 1, the substitutability between $a_j$ and $a_k$ increases. Similarly, as we move from bottom to top, the complementarity between $a_i$ and $a_j$ increases. The allocation of rights is a function of the level of complementarity between $a_i$ and $a_j$ relative to the level of substitutability between $a_j$ and $a_k$.

Note that when $a_j,a_k$ are complementary the substitution effect is negative (by definition) so the marginal productivity of $j$ (non-negative by definition) is higher than the substitution effect. And when $a_i,a_j$ are always substitutes they are substitutes in presence of $a_k$. So Table 1 can be conveniently reduced to the following table.

In their framework, H&M focus on a special case: all assets are complementary (at the margin). Open access should be the optimal structure but it is not within the scope
of H&M structures. Nevertheless they find specific properties with some additional conditions. Because $a_i, a_j$ are always complementary, $j$ should give access to $i$. When the complementarity between $a_i$ and $a_j$ is ultimate, e.g. when $a_i, a_j$ are strictly complementary or when $i$ is indispensable to $a_j$, both the productivity of $j$ alone and the substitution effect (without $i$) are equal to zero. So $j$ may as well give veto to $i$. Overall $j$ should give ownership to $i$ and we get Propositions HM6 and HM8 of H&M.

As an illustration, consider the case of shareholders. Shareholders have a financial asset which is complementary to the physical assets of the productive agents. They should have outside veto on an asset $a_i$ if the physical asset is substitute at the margin with other assets and if the agent who accesses the asset has low productivity. But why shouldn’t the shareholders have access on the asset? The explanation is provided by Proposition 5. When their financial asset is not always complementary to the physical asset in presence of other physical asset(s), shareholders should not have access. The intuition is the following. Suppose a shareholder also has veto on a second asset $a_2$ and that the substitution between $a_i$ and $a_2$ is stronger than the complementarity between $a_i$ and the financial asset (e.g. if $a_i$ does not have a strong need for cash and is competing with $a_2$ on the same market). Then the shareholder’s incentives will be reduced in presence of $a_2$. Moreover the incentives of $a_2$’s manager will also be reduced in presence of the shareholder. So not giving access to the shareholder prevents him to ‘merge’ substitute assets ($a_i$ and $a_2$). Overall, access by managers and veto by shareholders can be viewed as joint ownership between those two parties in order to protect their respective incentives.

Our model is based on the presumption that ownership encompasses both access and veto. Another line of thought would be to consider that veto is ownership (ownership is reduced to the right to exclude others from the assets). We would then have to reinterpret our results with two main impacts. Integration would not be equivalent to ownership of one party’s asset by the other (it would require both ownership and access), and ownership would not be driven by the complementarity of assets (at the margin) but
mainly by their substitutability. In both cases the major intuitions brought by H&M would have to be re-evaluated.

4. Applications

For simplification and ease of notation we will study a three assets case\(^\text{60}\). It can easily be shown that the following results may be extended to two complementary assets substitutes to \(n-2\) assets or one asset complementary to \(n-1\) substitute assets. With three assets there are four cases: all assets are bilaterally complementary (see Proposition 2), bilaterally substitutes (see Proposition 4), two complementary assets are each substitutes to the third asset, or two substitute assets are each complementary to the third asset. We study now the two latter cases.

(a) Two complementary assets substitutes to a third asset

Applying Proposition 5 to the three agents/assets, we get the results in Table 3 below.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>(P^n_2 \leq (1/2)SE_{2}^{a_2})</th>
<th>(P^n_1 &gt; (1/2)SE_{1}^{a_1})</th>
<th>(P^n_1 \leq (1/2)SE_{1}^{a_1})</th>
<th>(P^n_2 \leq (1/2)SE_{2}^{a_2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_1, a_2) complementary and substitutes to (a_0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a_1, a_2) always complementary</td>
<td>(T1) integration of (a_2)</td>
<td>(\text{Multiple access on} a_1, a_2 \text{ by 1&amp;2 (e.g. cross licensing)})</td>
<td>(\text{Joint ownership on} a_1, a_2 \text{ by 1 &amp; 2 (e.g. merger)})</td>
<td></td>
</tr>
<tr>
<td>(a_1, a_2) substitutes in presence of (a_0)</td>
<td>(\text{Multiple veto on} a_2 \text{ by 1 &amp; 2 (e.g. JV, subsidiary)})</td>
<td>(\text{Non integration (e.g. subcontracting)})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 3

In transaction cost theory, a firm is created if the cost of transacting within the firm is lower than through the market. For H&M\(^\text{62}\), this corresponds to \(T1\) integration, and joint

\(^{60}\) As highlighted by Hicks (1970) ‘one needs at least three factors in order to exhibit the character of substitution-complementarity relationships; but three factors is enough’.

\(^{61}\) Multiple veto is equivalent to joint ownership on \(a_2\) by 1 and 2.

\(^{62}\) In H&M, \(a_1, a_2\) are always complementary, the substitution effect is non positive (since assets are complementary) and joint access is not available. The possible control structures are then: \(T1\) integration (when \(P^n_1 > 3SE_{1}^{a_1}\) and \(P^n_2 = 3SE_{2}^{a_2} = 0\)), non integration (when \(P^n_1 > 3SE_{1}^{a_1}\) and \(P^n_2 > 3SE_{2}^{a_2}\)), and joint ownership (when \(P^n_1 = 3SE_{1}^{a_1} = 0\) and \(P^n_2 = 3SE_{2}^{a_2} = 0\)).
ownership is never optimal. In our model, when two complementary assets are independently substitutes to an external asset, they should be integrated or jointly owned (i.e. constitute a firm) if they are always complementary and the productivity of one of the agents is lower than the substitution effect of her asset with the external asset. So what is a firm? H&M define a firm as a collection of (complementary) physical assets. We precise this notion. If we focus on the assets which are owned together, a firm is a collection of assets which are always complementary, for which no more than one agent does not have low productivity with her asset independently (if the agents are separately highly productive with their assets, they should own separate firms). If the group of agents has low productivity with the group of assets, there should be outside ownership. This provides an answer to the question of ‘why do firms own essentially all the nonhuman assets it uses in production’ (Holmstrom, 1999). If a firm is a collection of always complementary assets, the assets have to be bundled together (we will call $A$ the bundle of $a_1$ and $a_2$) and are not separately owned by the (low productivity) agents (otherwise the incentives of any agent would be reduced). Suppose that both 1 and 2 had low productivity with their respective asset. They jointly own $A$, i.e. none of them can individually access or veto $A$. In this case, not only are the assets clustered, but also the agents. The assets $A$ are not accessed by any individual agent but by a collectivity of agents (1 and 2, call it $f$). If $f$ is highly productive with $A$ it will own it. If $f$ has low productivity with $A$, there will be outside veto: the assets $A$ are accessed by $f$ and vetoed by both $f$ and outside parties. By induction we can extend the process to $n$ always complementary assets and $n$ agents. Suppose an asset $A = \{a_1,\ldots,a_n\}$ owned by an agent $f = \{1,\ldots,n\}$ is always complementary to an asset $a_{n+1}$ owned by an agent $f'$ the same process will take place and the two firms will merge if one of the two firms has low productivity with its asset(s). In this continuous process of mergers and acquisitions, low productivity firms are always susceptible to be acquired by another firm and look for always complementary assets in order to increase their productivity relative to the substitution effect and become highly productive. Always complementary assets are

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63 To paraphrase Coase, a firm is ‘an island of complementary assets in an ocean of substitute assets’.
64 The same analysis would apply if either one had low productivity.
clustered together and are owned by the cluster of agents who bring the assets together (plus outside parties if the owners have low productivity). If we extend the notion of the firm to the assets which are vetoed together (rather than owned together), we can remove the first part of our definition: assets do not need to be always complementary to be part of the same firm. A firm can have veto power on an asset (a JV or subsidiary) which is not always complementary (corresponding to multiple veto in Table 3 above).

- **Joint ownership**

When two complementary assets are substitutes with a third asset, joint ownership of the two complementary assets is optimal if the marginal productivity of each agent with the complementary asset is lower than the substitution effect (e.g. if the two assets are strictly complementary).

- **Vertical mergers**

When two activities (performed by separate firms) in the vertical production chain are complementary and substitutes to a third firm, from the above they should integrate (i.e. acquisition or joint ownership) if the marginal productivity of at least one of them with its asset is lower than the substitution effect. Otherwise subcontracting or joint access (e.g. cross licensing) is the best governance structure. If we interpret the substitution effect with the external firm(s) as the (negative) incentive impact of transacting through the market, we find here an illustration of the transaction cost theory. The higher the asset specificity (complementarity between the two assets), the lower the marginal productivity of the agent with its asset alone (at the extreme, when the two assets are strictly complementary, the marginal productivity of an agent with its asset alone is equal to zero). When the marginal productivity becomes lower than the incentive impact of transacting through the market (i.e. the substitution effect), integration becomes the best governance structure.

(b) **One asset complementary to two substitute assets**

Applying Proposition 5, we find the results summarized in Table 4 below.

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65 As will be seen later, the notion of assets can be extended to human capital assets and the individuals who bring their complementary human capital assets (the employees) are part of the firm $f$.

66 Williamson (1985, 1996)
TABLE 4

<table>
<thead>
<tr>
<th>$a_1, a_2$ substitutes and complementary to $a_0$</th>
<th>$P_i^a \leq (1/2)SE_i^a$</th>
<th>$P_i^a &gt; (1/2)SE_i^a$</th>
<th>$P_i^a \leq (1/2)SE_i^a$</th>
<th>$P_i^a &gt; (1/2)SE_i^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1, a_2$ always substitutes</td>
<td>Multiple veto on $a_i$ by 0 and $i$ (e.g. JV)</td>
<td>Non integration (e.g. subcontracting)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_1, a_2$ complementary in presence of $a_0$</td>
<td>$T_0$ integration of $a_i$ (e.g. acquisition)</td>
<td>Multiple access on $a_i$ by 0 and $i$ (e.g. franchising)</td>
<td>Multiple access on $a_0$ by 1&amp;2</td>
<td></td>
</tr>
</tbody>
</table>

The results in Tables 3 and 4 provide a justification and a characterization for ‘hybrid’ governance structures (Williamson, 1985, 1996). The boundaries of the firm are fuzzy. If instead of restricting the firm to the assets which are owned (i.e. vetoed and accessed) together, we extend its limits to the assets vetoed or accessed, we get a broader definition, encompassing the notion of ‘hybrid’ governance structure (one where one agent gives access or veto (but not both) on her asset to another agent). The above results provide the conditions under which hybrid structures (e.g. franchising, licensing,…) may be optimal. We analyze franchising as an example.

- Franchising
Franchising is a control structure where some agents (the franchisees) own substitute assets (their customer base), but each of these assets is complementary to the asset of the franchisor (the branded concept). Proposition 5 provides a rationale for this control structure, which is optimal when (i) the assets of the franchisees become complementary in presence of the franchisor’s asset (this is in general enforced by the exclusive territory clause in the franchising contract), (ii) the productivity of the franchisees is higher than the substitution effect (here also the exclusive territory clause may ensure that the substitution effect is kept at a level lower than the productivity of each franchisee).

- Insurance industry
Proposition 5 can also help shed some light on the organization of the insurance industry. GH comment on the positive observed correlation between the existence of ‘independent’ agents and the size of an agent’s client acquisition costs. They show that their model
predicts that an agent will own the client list when the agent’s marginal incentives are relatively important in generating contract renewal, which is in line with the observed correlation. They also comment on an alternative explanation provided by Marvel (1982): an agent will have an exclusive dealing with the company when it is more efficient for the company to advertise than for the agent. Proposition 5 helps to reconcile the two points of view. In a setting with substitute agents complementary with the insurance company, it shows that when the marginal productivity of the agent is relatively low (i.e. lower than the substitution effect) the insurance company should have some control rights on the agent’s client list. This is in line with GH’s argument. But in our framework the company should have ownership when the agent’s client list is complementary to the other agent’s list combined with the insurance company’s assets or veto rights (i.e. exclusive dealings) when it is substitute. It is plausible that an insurance company which advertises heavily will make the ‘brand’ of the contract relatively more important than the personal relationship with the agent. Thus an agent’s client list will become more substitutable to another agent’s list combined with the company’s name. In line with Marvel, it should not be surprising to find that companies with exclusive dealings contracts spend more on advertising.

(c) Other applications
The results of this paper can be used to study many other applications.
- Capital structure (financial assets)
Applied to financial assets, our framework may shed light on the capital structure puzzle. Considering that a secured debt contract gives access on the assets of the firm to the debt holder (the debt holder uses the assets of the firm to generate payment of interests and the borrower renounces to its veto power by pledging the assets) and equity gives veto power to the shareholder (voting rights), we can directly apply the results of Proposition 5:

67 In their critics of Marvel argument, GH argue that an exclusive dealing contract is ‘one method of enforcing list ownership rights’. Exclusive dealing is in fact a veto right given by the agent to the insurance company and is thus an alternative to ownership by the later.
The firm will issue debt when its productivity with the asset alone is higher than the substitution effect of the investment opportunities and its asset is always complementary to the financial asset (i.e., the firm has a high need of cash). It will issue equity when the productivity with its asset alone is lower than the substitution effect of the investment opportunities and its asset is not ‘always complementary’ to the financial asset (e.g., the firm has a low need of cash). If the firm issues debt but its marginal productivity later becomes lower than the substitution effect (e.g., as defined in the covenants), then the debt holder will get ownership on the asset.\(^{69}\)

- Employment contract (human capital assets)

Similarly, we may apply the results of this paper to human capital assets. We define human capital assets as having *inalienable access*, i.e., nobody can use agent’s \(i\)’s human capital without her consent (formally, \(\nu(i) = h_i\) and \(h_i \notin \nu(j), \forall j\)). Note that this property does not prevent agent \(i\) to give (or sell) veto power on her human capital to another agent \(j\) (e.g., the owner of a physical asset). In that case \(i\) agrees not to use her human capital without \(j\). This is our interpretation of an *employment contract*.\(^{70}\)

Also, following Williamson (1985), we will say that an employment contract (or a subcontracting contract) has *‘high power incentives’* if the employee (respectively the contractor) receives residual income. In our framework it means that the employee has

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\(^{69}\) Hansmann and Kraakman (2002) define a security interest as ‘a contingent claim on an asset that permits the holder of the interest to take physical possession of the asset and sell it to a third party upon the non-payment of the debt’ and explain that a security interest is a property right because it is enforceable against subsequent transferees of rights in the asset.

\(^{70}\) This definition is similar to Segal (2003). The GHM literature assumes that inalienability applies to the *control* (rather than access) of human capital assets. But their notion of control encompasses both veto and access.
access to the physical asset and can derive revenues from it (nobody else can veto her).

Directly applying the above results we get:

\[
P^a_1 \leq (1/2)SE^a_1 \\
P^a_1 > (1/2)SE^a_1
\]

<table>
<thead>
<tr>
<th>Human capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_0: \text{physical asset})</td>
</tr>
<tr>
<td>(h_1,h_2) always substitutes</td>
</tr>
<tr>
<td>Employment contract, Low power incentives</td>
</tr>
<tr>
<td>Subcontracting, Fixed price</td>
</tr>
<tr>
<td>(h_1,h_2) complementary in presence of (a_0)</td>
</tr>
<tr>
<td>Employment contract, High power incentives</td>
</tr>
<tr>
<td>Subcontracting, High power incentives</td>
</tr>
</tbody>
</table>

**TABLE 6**

In the debate between Coase and Alchian & Demsetz\(^{71}\) on the nature of the employment relationship, H&M propose to reconcile the two positions with the concept of ownership. The employer is the owner of the physical assets with which the employee works, unlike in the independent contractors relationship. Ownership of the physical assets confers authority to the employer and therefore the employee is more likely to do what the employer wants than in the case of independent contractors. The problem with H&M approach is that it does not explain situations where one contractor owns the physical assets with which the other contractor works (as in the case of a car manufacturer owning the dies and stamping machines with which the subcontractor will make auto bodies). In that case what is the difference with an employment relationship? **Proposition 5** provides a new perspective, closer to Coase’s argument. The employer has veto rights (i.e. control) on the employee’s human capital while this is not the case in the independent contractor relationship. In fact an employment contract corresponds to the ‘vertical integration’ of human assets by the owner of the physical assets\(^{72}\). But since, in the absence of slavery,

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\(^{71}\) Where Coase argues that the difference between an employment relationship and one between independent contractors is that the employer can tell an employee what to do, while an independent contractor must persuade the other one through prices, Alchian & Demsetz believe that both relationships are essentially of the same nature. In both cases the manager (resp. customer) can fire the employee (resp. supplier) if he is unhappy.

\(^{72}\) See also Klein (1988) for an interesting discussion on vertical integration and the difference between physical capital and human capital.
ownership of human assets is prohibited, the firm will only acquire veto power (and not access) on the human capital assets.

Therefore, when human capital assets are substitutes, an employment contract will be optimal if the physical asset is essential for the agent (or the physical asset is strictly complementary to the agent’s human capital), or the owner of the physical asset is indispensable to the agent, or the agent is unproductive with her human capital alone.

We can now precise our definition of the firm. A firm is a collection of complementary assets (physical and human) vetoed together for which no more than one agent is highly productive with one of the assets independently. In this definition employees are part of the firm, while subcontractors are not.

Our framework can be applied to a multitude of other applications. In a companion paper, we provide justification for many empirical cases (such as ownership structure of race teams in Formula 1, internal or external sales force, the structure of franchise operations, or the ownership structure of gasoline retailing).

5. Concluding remarks

Building on the Property right theory of GHM, this paper proposes a broader framework to analyze the effects of incomplete contracts. By de-bundling ownership into access and veto rights, we can study a larger class of agents (i.e. including agents who have access and no veto and agents who have veto and no access) and a larger class of contracts. Allowing for any kind of asset relationships, we can analyze a broader (possibly unlimited) class of real life situations.

Applying the framework, we find that the relationship between the marginal productivity of an agent with an asset and the substitution effect is key, and we get a set of interesting (preliminary) results: ownership is not always optimal; single ownership, joint ownership, outside ownership, and partnerships are all valid forms of optimal control structures based upon availability of highly productive agents; integration should occur only when assets are always complementary and one agent has ‘low productivity’; alternatively the allocation of access or veto is optimal depending on the productivity of the agents and the relationship between the assets. We propose a definition for the
boundaries of the firm: a firm is a collection of complementary physical and human capital assets vetoed together for which no more than one agent is highly productive with one of the assets independently. If the group of agents has low productivity with the group of assets, outside parties have veto power. In this definition employees are part of the firm, while subcontractors are not.

Of course, our framework is not immune to the usual limitations of this stream of literature. In particular, it is sensible to the solution concept and does not extend to nonlinear bargaining solutions, it focuses on the ex-ante investments effects and assumes that ex-post bargaining is efficient, and it does not consider other (important) complications such as payoffs uncertainty, risk aversion, or wealth constraints. We have limited the scope of the model by keeping the underinvestment assumption of H&M. But despite these limitations, we believe that its main merit stands out. We can now apply the power of the GHM framework to a much larger (possibly unlimited) class of situations. In the previous section, we have mentioned some of them (hybrid governance structures, human capital assets and employment contracts, financial assets and the capital structure), but we can think of many others: hierarchy (managers have veto right but not ownership on the employees), social capital, intangible assets,…

Another potential line of research would be to further extend the model and incorporate other classes of rights (e.g. alienation right, the right to sell the asset and determine who has veto rights), ultimately extending it to the five categories expressed by Schlager and Ostrom (1992). Also the results developed in that paper are linked to the notions of complementarity or substitution at the margin and to returns on investment at the margin but these are difficult to measure empirically. A promising line of research to test the pertinence of our model would be to develop empirical proxies for those concepts.

Overall, we think we have provided (partial) answers to the main critiques on the limitations of the property rights theory. Taking the ownership of rights over assets as primitives, our framework takes into account the complexity of the notions of ownership and residual control rights (Demsetz, 1998, Foss and Foss, 2001). It allows integrating the Coasian (and GHM) view of the firm, the Berle and Means perspective, and the

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73 If this seems reasonable, especially in the case of human capital investment, the possibility and impact of overinvestment would deserve to be analyzed.
Chandler’s multi-divisional corporate structure (Bolton and Scharfstein, 1998). It acknowledges that investment incentives are not provided by ownership alone (Holmstrom and Roberts, 1998). Finally it provides a more precise definition of the firm and may help to clarify why firms or shareholders ‘own’ assets rather than individuals and why the assets of the firm are clustered (Holmstrom, 1999). Nevertheless much remains to be done to fully comprehend the complex internal and external bargaining processes that determine the boundaries of the firm.
Appendix

Proof of Lemma 1.
We follow the proof of Proposition 1 in H&M. The equilibrium investment is characterized by equation (1.4), which, given assumption on human capital investment, can be rewritten as \( \nabla g(x, \beta) \big|_{x^e(\beta)} = 0 \), with \( g(x, \beta) = \sum_S p(S) v(S, \beta(S) | x) - \sum_{i=1}^N C_i(x_i) \).

(i) Equilibrium investment.
Consider two control structures \( \beta \) and \( \hat{\beta} \) so that \( v'(S, \beta(S)) \leq v'(S, \hat{\beta}(S)) \), \( \forall i, \forall S \). We have \( \nabla g(x, \beta) \leq \nabla g(x, \hat{\beta}), \forall x \). We take \( f(x, \lambda) \equiv \lambda g(x, \hat{\beta}) + (1 - \lambda) g(x, \beta) \) for \( \lambda \in [0,1] \) and define \( x(\lambda) \) to solve \( \nabla f(x, \lambda) = 0 \). By totally differentiating we get:

\[
\sum_{i=1}^N \frac{\partial \nabla f(x, \lambda)}{\partial x_i} dx_i(\lambda) + \frac{\partial \nabla f(x, \lambda)}{\partial \lambda} d\lambda = 0
\]

\[
H(x, \lambda) dx(\lambda) = -[\nabla g(x, \hat{\beta}) - \nabla g(x, \beta)] d\lambda
\]

\[
\frac{dx(\lambda)}{d\lambda} = -H^{-1}(x, \lambda) [\nabla g(x, \hat{\beta}) - \nabla g(x, \beta)]
\]

\( H(x, \lambda) \) is negative definite (by concavity of \( v \) and convexity of \( C \)) and its off-diagonal elements are nonnegative (by assumption on complementarity of investments). Thus \( H^{-1}(x, \lambda) \) is nonpositive, so \( dx(\lambda)/d\lambda \geq 0 \). Therefore \( x(0) \leq x(1) \) and \( x^e(\beta) \leq x^e(\hat{\beta}) \). So, if every agent’s marginal return on investment increases (decreases) then the equilibrium investment increases (decreases).

Setting \( \beta = \hat{\beta} \) in the above, it is easy to show that \( x^e(\beta) \leq x^e(\hat{\beta}) \) and \( x^e(\hat{\beta}) \leq x^e(\beta) \). Therefore the Nash equilibrium investments are unique.

Comparing (1.4) to (1.5), and given property (1.3), we have

\[
\sum_{S \in \mathcal{S}} p(S) v'(S, \beta(S) | x) \leq v'(N, A | x), \forall i
\]

when assumption A1 holds. Thus \( \nabla g(x, \beta) \leq \nabla W(x), \forall x \). Applying the same reasoning than in (i) and replacing \( g(x, \beta) \) by \( W(x) \), we can show that \( x^e(\beta) \leq x^* \). So when assumption A1 holds, there is always underinvestment.

Finally \( \nabla W(x^e(\beta)) \geq \nabla g(x^e(\beta)) = 0 \) and \( x^e(\beta) \leq x^e(\hat{\beta}) \) imply that \( W(x^e(\beta)) \leq W(x^e(\hat{\beta})) \) by concavity of \( W \). Thus when equilibrium investment increases welfare increases.

Proof of Proposition 3.
(i) Suppose \( \beta(i) = \{a_i, a_j\} \). Take a more exclusive control structure \( \hat{\beta} \) similar to \( \beta \) except that \( \hat{\beta}(i) = \{a_i\} \) (i.e. \( \hat{\beta}(S) \subseteq \beta(S), \forall S \)). Then \( v'(S, \hat{\beta}(S)) \geq v'(S, \beta(S)) \), \( \forall S \) and \( v'(S, \hat{\beta}(S)) \geq v'(S, \beta(S)), \forall S \), for \( j \neq i \). The marginal return of each agent will not be reduced by the change of control structure. Hence by Lemma 2, welfare will be higher under the new control structure \( \hat{\beta} \) and by Lemma 3 allocating veto increases welfare and is more efficient than allocating ownership.
(ii) From (i) we get $|\beta'(i)| \leq 1, \forall i$. By (1.3), $\beta(i) \subseteq \beta(S)$ for all $i \in S$, therefore $\beta(S) \supseteq \bigcup_{i \in S} \beta(i)$. Suppose there exists an optimal control structure $\beta^*$ such that $\beta'(S) \supseteq \bigcup_{i \in S} \beta'(i)$ for some $S$. Take $S'$ the smallest $S \subseteq N$ such that $\beta'(S') \supseteq \bigcup_{i \in S'} \beta'(i)$, with $\beta'(i) \neq \emptyset$ for some $i$ in $S'$, and consider the following control structure $\beta'$:

$$\beta'(S) = \begin{cases} 
\beta'(S) & \text{if } S \neq S' \\
\beta'(S) \setminus \{a^*\} & \text{if } S = S' 
\end{cases}$$

with $a^* \in \beta'(S') \setminus \bigcup_{i \in S'} \beta'(i)$.

When $S \subset S'$, $\beta'(S) = \beta'(S) = \bigcup_{i \in S} \beta'(i) = \bigcup_{i \in S'} \beta'(i)$ (by definition of $a^*$). Hence $\beta'(S) = \bigcup_{i \in S} \beta'(i)$ for all $S \subseteq S'$ and $\beta'(S) = \beta'(S)$ for all $S \subset S'$. Thus $\beta'$ satisfies (1.3) (since $\beta^*$ satisfies (1.3)) and $\beta'(S) \subseteq \beta'(S), \forall S \subseteq N (\beta'(S) = \beta'(S)$ for $S \neq S'$ and $\beta'(S') \subset \beta'(S'))$. $\beta'$ is more exclusive than $\beta^*$. Since assets are substitutes at the margin, $v'(S, \beta'(S)) \geq v'(S, \beta^*(S)), \forall S$ (since $|\beta'(S')| > 1$). Thus the marginal return of each agent will not be reduced by the change of control structure and by Lemma 2, welfare will be higher under the new control structure $\beta'$. So $\beta'$ is not an optimal structure which contradicts our assumption and it must be that $\beta'(S) = \bigcup_{i \in S} \beta'(i)$.

Hence, in an optimal control structure, $\beta'(S) = \bigcup_{i \in S} \beta'(i)$ if some $\beta'(i) \neq \emptyset$. If $\beta'(i) = \emptyset, \forall i \in S$, then $\beta'(S) \supseteq \bigcup_{i \in S} \beta'(i)$.

-Proof of Lemma 3.
(i) We compare a control structure $\beta$ to a control structure $\beta'$ identical to $\beta$, except that now agent $i$ controls $a$ ($a \in \beta(i)$). The impact of change for $i$ is:

$$\sum_{S' \subseteq S \atop \beta(S') = \emptyset} p(S)[v'(S, \beta(S) \cup \{a\}) - v'(S, \beta(S))] + \sum_{S' \subseteq S \atop \beta(S') \neq \emptyset} p(S)[v'(S, \beta(S) \cup \{a\}) - v'(S, \beta(S))]$$

$$= \sum_{S' \subseteq S \atop \beta(S') = \emptyset} p(S)[v'(S, a) - v'(S, \emptyset)] - \sum_{S' \subseteq S \atop \beta(S') \neq \emptyset} p(S)[v'(S, \beta(S)) - v'(S, \beta(S) \cup \{a\})]$$

which is $\geq (1/n)[v'(i, a) - v'(i, \emptyset)] - \max_{S \subseteq S' \atop \beta(S') \neq \emptyset}[v'(S, A) - v'(S, A \cup \{a\})]$, i.e. non negative if $i$ is highly productive with $a$.

This is also $\leq [v'(N, a) - v'(N, \emptyset)] - (1/n) \min_{S \subseteq S' \atop \beta(S') \neq \emptyset}[v'(S, A) - v'(S, A \cup \{a\})]$, i.e. non positive if $i$ has low productivity with $a$.

Hence control by a highly productive agent increases her incentives and control by a low productivity agent decreases her incentives.
For an agent \( j \neq i \), the impact of change is \[ \sum_{S_{ji} \in S} p(S)[v^i(S, \beta(S) \cup \{a\}) - v^i(S, \beta(S))], \]
which is equal to zero if \( j \) controls \( a \) and non negative if \( j \) does not control any asset and is highly productive with \( a \).

(ii) We suppose that two agents \( i \) and \( j \) are identical (i.e. \( v^i(S \cup \{i\}, \beta(S \cup \{i\})) = v^j(S \cup \{j\}, \beta(S \cup \{j\})) \), except that \( i \) is highly productive with an asset \( a \) while \( j \) has low productivity with \( a \). We compare a control structure \( \beta \) where \( j \) controls \( a \) to a control structure \( \beta' \) where \( i \) controls \( a \). From the above \( \beta' \) is more efficient for \( i \). The impact of change for \( j \) is:
\[ \sum_{S \in S_{ji} \in S} p(S)[v^i(S, \beta(S) \cup \{a\}) - v^i(S, \beta(S))] \]
which is non negative if \( i \) controls other asset(s) or if no other agent than \( j \) accesses \( a \).

Note that if \( j \) is indispensable, then allocating veto to her is neutral.

So, allocating veto on an asset \( a \) to an agent \( j \) who accesses it will increase welfare if no other agent accesses \( a \). A direct consequence is that if there is only one highly productive agent who accesses an asset she should also veto it.

(iii) But if other agents who have access to the asset are highly productive in her absence, the agent should not veto it (the impact of change in the above expression is non positive for \( i \)). Suppose \( S^a \) is the set of highly productive agents who can access \( a \). Then no agent in \( S^a \) should individually veto \( a \). Also any agent outside of \( S^a \) should not have veto on \( a \), since it would reduce the marginal returns of the (highly productive) agents in \( S^a \). Should all agents in \( S^a \) jointly veto \( a \)? Compare a control structure \( \beta \) where no one
can veto \( a \ (a \not\in \chi(i), \forall i) \) to a control structure \( \beta' \) where all agents in \( S^a \) have joint veto on \( a \ (\chi(j) = \emptyset, \forall j \in S^a, \chi(S^a) = a) \). For \( j \in S^a \), the impact of change is zero (no agent can veto him). For \( k \not\in S^a \), the impact is \( \sum_{\substack{i \in S^a \cap S = \emptyset \\mid \beta(S) = [a]}} p(S)[v^i(S, \beta(S) \setminus \{a\}) - v^i(S, \beta(S))] \), which is equal to zero since no agent outside of \( S^a \) has access to \( a \).

\[ \text{(iii) Should a low productivity agent access an asset already accessed by another agent?} \]

We compare a control structure \( \beta \) where an agent \( i \) and a low productivity agent \( j \) access an asset \( a \) to a control structure \( \beta' \) where only \( i \) accesses \( a \). The impact is equal to zero for \( i \) and for \( j \) is \( \sum_{\substack{i \in S^a \cap S = \emptyset \\mid \beta(S) = [a]}} p(S)[v^i(S, \beta(S) \setminus \{a\}) - v^i(S, \beta(S))] \), which is \( \geq 0 \) (\( j \) has low productivity). For \( k \not= i, j \) the impact \( \sum_{\substack{i \in S^a \cap S = \emptyset \\mid \beta(S) = [a]}} p(S)[v^k(S, \beta(S) \setminus \{a\}) - v^k(S, \beta(S))] \) is \( \geq 0 \) if \( k \) controls another asset or \( k \) does not control any asset and has low productivity with \( a \). So for any other agent \( k \not= i, j \) the impact of change is non-negative.

\[ A \text{ low productivity agent should not access an asset already accessed by another agent.} \]

Incidentally, it can also be shown that if the first agent is \emph{indispensable}, giving access on an asset to a second agent is neutral (i.e. does not increase welfare).

\[ (iv) \text{ Suppose } a \text{ is accessed by one agent } i (\nu(i) = a) \text{ who has low productivity with } a. \]

Should someone else have veto on \( a \) ? Compare a control structure \( \beta \) where \( j \not= i \) does not veto \( a \) to a control structure \( \beta' \) where she does. The impact is zero for \( j \) and for \( i \) is \( - \sum_{\substack{i \in S^a \cap S = \emptyset \\mid \beta(S) = [a]}} p(S)[v^i(S, a) - v^i(S, \emptyset)] + \sum_{\substack{i \in S^a \cap S = \emptyset \\mid \beta(S) = [a]}} p(S)[v^i(S, \beta(S) \setminus \{a\}) - v^i(S, \beta(S))] \), which is \( \geq 0 \) since \( i \) has low productivity with \( a \). For \( k \not= i, j \) the impact of change is \( \sum_{\substack{j \in S^a \cap S = \emptyset \\mid \beta(S) = [a]}} p(S)v^j(S, \beta(S) \setminus \{a\}) - v^j(S, \beta(S)) \), which is \( \geq 0 \) if \( k \) controls an asset other than \( a \) or \( k \) does not control any asset and has low productivity with \( a \). Thus outside veto on \( a \) is efficient when the agent who accesses an asset has low productivity.

Note that no condition is attached to \( j \). In particular \( j \) could be an outside party.

\[ (v) \text{ An asset not accessed by any individual agent could be accessed by a group of agents} \ (S^a) \ (\text{joint access}). \]

Comparing a control structure \( \beta \) where no agent can veto \( a \) \((a \not\in \chi(i), \forall i) \) to a control structure \( \beta' \) where all agents in \( S^a \) have joint veto on \( a \) \((\chi(j) = \emptyset, \forall j \in S^a, \chi(S^a) = a) \), the difference in marginal return is equal to zero for \( i \in S^a \), and is \( \sum_{\substack{j \in S^a \cap S = \emptyset \\mid \beta(S) = [a]}} p(S)[v^j(S, \beta(S) \setminus \{a\}) - v^j(S, \beta(S))] \) for \( j \not\in S^a \), which is also equal to zero since \( a \not\in \beta(S) \) when \( S^a \cap S = \emptyset \) (no agent outside of \( S^a \) can access \( a \)). Thus joint veto by \( S^a \) is equivalent to no veto.
What about individual veto by each of the members of \( S^a \)? Comparing a control structure \( \beta \) where the productive agents of \( S^a \) can individually veto \( a \) \((\chi(i) = a, \forall i \in S^a)\) to a control structure \( \beta' \) where the agents of \( S^a \) have joint veto on \( a \) \((\chi(S') = a)\), the impact of change for \( i \in S^a \) is \[
\sum_{S^a \neq S} p(S)[v'(S, \beta(S)) - v'(S, \beta(S))]
\]
which is equal to zero (\( S \) cannot control \( a \) in both cases since some agents of \( S^a \) are not in \( S \)). For \( j \notin S^a \) the impact is \[
\sum_{S^a \neq S} p(S)[v'(S, \beta(S)) - v'(S, \beta(S))],
\]
also equal to zero.

When agents of \( S^a \) have joint access, joint veto by \( S^a \) is equivalent to individual veto by each member of \( S^a \) and to ‘no veto’. The agents of \( S^a \) have joint ownership on the asset. In addition, if \( S^a \) has low productivity with \( a \), outside veto will increase welfare by (iv).

**Proof of Proposition 5.**

The initial control (ownership) structure where each agent owns an asset is named \( \beta_{NI} \) (no integration) and \( \beta_{NI}(i) = a_i, \beta_{NI}(j) = a_j, \beta_{NI}(k) = a_k \).

(i) When should \( j \) give access on \( a_j \) to \( i \)? We compare the marginal returns on investment between joint access (when \( j \) give access on \( a_j \) to \( i \)) and non integration. The impact is zero for \( j \). For \( i \) it is \[
\sum_{S^a \neq S} p(S)[v'(S, \beta_{NI}(S) \cup \{a_j\}) - v'(S, \beta_{NI}(S))],
\]
i.e. \( \geq 0 \) if \( a_j \) is (always) complementary to \( a_i \) in absence of \( j \). For \( k \) it is \[
\sum_{S^a \neq S} p(S)[v^k(S, \beta_{NI}(S) \cup \{a_j\}) - v^k(S, \beta_{NI}(S))]
\]
which is \( \geq 0 \) when \( a_j \) is complementary to \( a_k \) in absence if \( j \) (which is always true if \( a_i, a_j \) are always complementary).

**Hence, giving access on \( a_j \) to \( i \) will increase welfare when \( a_i, a_j \) are (always) complementary in absence of \( j \).**

(ii) When should \( j \) give veto on \( a_j \) to \( i \)? Compare the marginal returns on investment between multiple veto (when \( j \) gives veto on \( a_j \) to \( i \)) and non integration. The impact of change is equal to zero for \( i \). For \( j \) it is \[
\sum_{S^a \neq S} p(S)[v^j(S, \beta_{NI}(S) \setminus \{a_j\}) - v^j(S, \beta_{NI}(S))],
\]
i.e. \( \geq 0 \) when \( j \) has low productivity with \( a_j \) in absence of \( i \). The impact for \( k \) is \[
\sum_{S^a \neq S} p(S)[v^k(S, \beta_{NI}(S) \setminus \{a_j\}) - v^k(S, \beta_{NI}(S))]
\]
which is \( \geq 0 \) when \( a_j, a_k \) are substitutes in absence of \( i \)

**Overall, giving veto on \( a_j \) to \( i \) will increase welfare, when \( a_j, a_k \) are substitutes in absence of \( i \) and \( a_i \) and the productivity of \( j \) with \( a_j \) is lower than the substitution effect of \( a_j \) in absence of \( i \).**
References