Hedging Currency Risk: a Regret-Theoretic Approach

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Abstract
We present a model of optimal currency-hedging choices based on regret theory, a behavioral finance theory where investors reach optimal investment decision taking the expected pain of future regret into account. Investors are assumed to frame currency separately from asset allocation. In our model, investors exhibit narrow framing and loss aversion as in many other models; however, loss aversion does not stem from disappointment relative to some prior expectation but from regret of not having taken the best currency hedging decision. They derive utility from their global asset allocation but, in addition, they also experience regret for having chosen a currency exposure that proves, with hindsight, inappropriate. We derive closed-form optimal hedging rules for small risks and highlight the difference with the traditional expected-utility results. A significant number of institutional investors use a simple 50% hedging rule, which is often presented informally as a strategy to minimize regret. We show that this behavior is consistent with regret theory only when investors are infinitely averse to regret. In addition, we further explore normative implications of regret theory on currency hedging and explain deviations from widely accepted normative rules.

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1. Introduction

“I should have computed the historical covariance of the asset classes and drawn an efficient frontier. Instead I visualized my grief if the stock market went way up and I wasn’t in it—or if it went way down and I was completely in it. My intention was to minimize my future regret, so I split my [pension scheme] contributions 50/50 between bonds and equities.”

Harry Markowitz.¹

Regret is such a powerful negative emotion that the prospect of its future experience may lead individuals to make seemingly sub-optimal, non-rational decisions relative to the expected utility paradigm. As the opening quote suggests, the anticipation of future regret was strong enough to turn Harry Markowitz away from his very own portfolio allocation theory when faced with a financial decision on his pension plan. Regret is defined as a cognitively-mediated emotion of pain and anger when, with hindsight, we observe that we took a bad decision in the past and could have taken one with better outcome. As stated by Bell (1985), regret is a psychological reaction to making a wrong choice on the basis of actual outcomes, where a better investment decision could have been taken. Contrary to mere disappointment, which is experienced when a negative outcome happens relative to prior expectations, regret is an emotion strongly associated with a feeling of responsibility for the choice that has been made.² When experiencing regret, past actions are challenged, as their benefits are evaluated with respect to the merits of alternative foregone actions. This comparison is only made for the prevalent state of the world. When experiencing disappointment, no such comparison of past possible actions is made. Disappointment merely results from the realization of an outcome inferior to the one we had expected. Only fate or “bad luck” is held accountable for the situation, not our own past deeds. To sum it up, regret originates from comparison of outcomes across choices for an actual state of the world, while disappointment originates from the comparison of outcomes across states of the world for a given choice. Such a difference is not innocuous and, we believe, makes regret more relevant to investment decisions than disappointment.

There is an extensive literature in experimental psychology and, to a lesser extent, neurobiology that supports the assumption that regret influences decision-making under uncertainty³ beyond disappointment and traditional uncertainty measures. The psychology literature initially tried to enrich the simple economic approach by studying the experience of regret. It showed that regret was more intense when the unfavorable outcomes were the result of action rather than of inaction (Kahneman and Tversky, 1982). It subsequently moved towards the study of the anticipation of such emotions in decision-making under uncertainty, suggesting that the anticipation of such an emotion is taken into account in decision-making. The literature also pointed out that the representation, or “framing”, of outcomes (as defined by Tversky and Kahneman, 1981), played an important role in the influence of regret on decision-making under uncertainty (e.g. Harless, 1992). Regret is experienced, and taken into account in decision-making, when the outcomes of unchosen options are highly "visible", or "accessible", but less so when these outcomes are less visible. As to the neuroscience

³ For experimental psychology reviews see Gilovich and Medvec (1995), Zeelenberg et al., (2000). For neurobiological experiments see Bechara, Damasio and Damasio (2000), and Camille et al., 2004.
literature, experiments focused on the involvement of the orbitofrontal cortex in the experience of regret. Damasio (1994) showed that individuals who suffered brain lesions in this region maintained full cognitive abilities but displayed little or no emotions, among them, no feeling of regret. Camille et al. (2004) focused on the emotions of regret and disappointment among such patients and a healthy control group. The authors showed that patients did not experience regret, while they experienced disappointment with a magnitude similar to healthy individuals. In addition, these patients were shown not to anticipate the feelings of regret in selecting among lotteries, whereas healthy individuals did.

Based on this concept of regret, Loomes and Sugden (1982) and Bell (1982) derived independently an economic theory of regret. They intended to propose a theory of choices under uncertainty that explains many observed violations of the axioms used to build the traditional expected utility approach. Regret theory (RT) assumes that agents are rational but base their decisions not only on expected payoffs but also on expected regret. It predicts Allais' paradox ("common consequences effect") and many other axiom violations reported in experiments by Kahneman and Tversky (1979) and others: these are commonly referred to as the "common ratio effect", the "isolation effect", the "preference reversal effect", the "reflection effect", "simultaneous gambling and insurance". RT bears some similarities with prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) as many results of RT are consistent with the empirical observations of human behavior that constitute the building blocks of prospect theory. But prospect theory is primarily descriptive while RT is a normative theory of rational choice under uncertainty (section 2 further discusses this issue). It incorporates regret into the utility function in addition to the traditional value function of total wealth. Investors reach their investment decision by maximizing the expected value of this modified utility. So investors try to anticipate regret and take it into account in their investment decisions in a consistent manner. RT is parsimonious yet axiomatic. In contrast, utility models inspired by prospect theory typically include a disappointment term with a kink at the current investment value (zero return) where the slope of utility is higher for losses than for gains ("loss aversion"). Disappointment is easier to model as the benchmark expectation for a given investment is usually set as a fixed number (possibly the current situation), while in RT we have to wait for the realization of the "best" investment strategy in the investment decisions universe. But RT is clearly relevant to investors who compare the performance of their portfolio to foregone alternatives that they could have chosen, or to peers and benchmark portfolios whose performance could have been achieved.

With the observed evidence in favor of the influence of regret on decision-making under uncertainty as well as the axiomatic and normative appeal of RT for investment choices, it is surprising that this notion has caught so little attention in the field of finance. Indeed, it is prospect theory that has been extensively used in behavioral finance. The success of prospect theory holds as much to its descriptive power as to the ability to handpick only some features that enable to explain selected puzzles in the field. It may also be that RT is difficult to apply because of the technical difficulties associated with the optimization of an expected utility function with two attributes: value and regret. Indeed, although intuitively more appealing, applying RT to a general portfolio problem involving numerous assets seems a daunting

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5 Connolly and Zeelenberg (2002), page 212, state that "the emotion that has received the most research attention from decision theorists is regret".
technical task. This is because, regret stems from a comparison of the actual return outcome of each asset with the actual return outcome of all the other assets. In contrast, including disappointment in a utility function is much less intricate, as disappointment results from a comparison, for each asset independently, of the actual return outcome to a preset expectation return (e.g. zero, or the risk-free rate, or some other exogenous number).

In this article, we use RT to account for the observed currency risk hedging behavior of fund managers with assets invested in foreign markets. While foreign currency hedging is an important decision in its own right, it also is simple enough to allow the modeling of regret in the utility function. In addition, there is ample casual evidence that regret may indeed play a role in the observed currency risk hedging policy.

Portfolio investors have progressively accepted the argument that international diversification provides risk/return benefits. However, the currency exposure has remained an emotional decision. Attractive local-currency returns on foreign asset market can be swamped by a depreciation of the foreign currency. Conversely, the return on foreign currency can provide a major portion of the total return of international investments when the domestic currency is weak. The currency hedging decision is a simple one: what currency hedge ratio (proportion of foreign asset value hedged against currency risk) should be adopted. In other words, should international assets be fully hedged against currency risk, not hedged, or partially hedged (hedge ratio between zero and one). Traditional finance relies on expected utility maximization and uses the expected return/risk paradigm to search for such an answer. In its most simple form, if currency risk premia are nil and if asset returns are uncorrelated with currency movements; then the optimal hedge ratio that minimizes risk is 100% (see Pérol and Schulman, 1988). Of course, the optimal hedge ratio will differ from 100% if there is correlation between asset returns and currency movements, and if the currency risk premium differs from zero.

Currency hedging is a dimension where regret clearly applies. For example, an American investor who decided not to hedge currency risk would have incurred a currency loss of some 40% on its eurozone assets from late 1998 to late 2000, with a vast regret of not having hedged. Conversely a fully hedged investor would have missed the 50% appreciation of the euro from late 2001 to late 2004. Again, a vast regret of not having taken the "right" hedging decision. Regret is caused by comparing the investment outcome with the payoff that would have obtained for a different investment choice, here, a different hedging decision.

As mentioned above, regret is experienced if the outcome of unchosen options are "visible" (or "accessible"), and currency returns are highly visible and emotional. Media talk daily about the fate of the dollar. All performance reports separate the currency gains/losses on the portfolio from other sources of return. Performance relative to peers or other simple hedging strategies are important. Furthermore, everyone, even outside the sphere of finance, seems to have an opinion on the value of the dollar, especially ex-post. What might have been a reasonable hedging decision ex-ante, can be easily criticized ex-post by a board of trustees.

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6 There is extensive theoretical and empirical discussion about the existence and magnitude of currency risk premia. This is beyond the scope of this paper.

7 Global market equilibrium models have been developed by e.g. Solnik (1974), Adler and Dumas (1983) or Black (1990) to derive optimal currency hedging rules in a more general setting.

8 See also see Fisher and Statman (2004).

9 Furthermore, selling short an appreciating foreign currency leads to cash losses on the forward position that have to be covered by the sale of assets. A forced decision that is painful.
Another observation, consistent with the regret literature, is that investors "narrow frame" the currency exposure decision. As described in Kahneman and Lovallo (1993), decision makers are excessively prone to treat problems as unique; their evaluation of single risky prospects neglects the possibility of pooling risks. Rather than looking at the whole portfolio as prescribed by traditional expected utility theory, investors tend to look at individual investment decisions. They try to reach the best decision in each mental compartment. In global asset allocation, investors clearly separate the asset allocation decision and the currency risk hedging decision. Such a behavior is confirmed in a survey of Canadian pension plans. The vast majority of these plans (94%) believe that the best way to handle currency exposure is to decide first on global asset allocation and then handle the currency exposure. This confirms our assumption that the currency hedging decision is taken as a residual/ separate decision from the investment decision that creates the currency exposure. For pension funds, the currency hedging decision is sometimes delegated to a specialized currency overlay managers. In line with this behavior, we assume that investors place currency hedging in a separate mental compartment and only experience regret on their hedging decision. In our model, we will make the usual assumption that the allocation to international assets has been chosen, and that the only remaining decision is the amount of that exposure to be currency hedged.

Although, to our knowledge, this is the first attempt to apply RT to currency hedging decisions, the experience of regret in currency hedging is not news for the investment world. Several practitioners have justified a 50% naïve hedge ratio on such intuitive grounds. For example:

"The 50% hedge benchmark is gaining in popularity around the world as it offers specific benefits. It avoids the potential for large underperformance that is associated with "polar" benchmark, i.e. being fully unhedged when the Canadian dollar is strong or being fully hedged when it is weak. This minimizes the "regret" that comes with holding the wrong benchmark in the wrong conditions."


"A partial hedging policy – such as 50/50 or 70/30 – means the investor won’t ever experience the major highs of an unhedged portfolio, but won’t be subject to the lowest returns either."

"To Hedge or not to hedge", Simon Segal, SuperReview.com.au, 21 march 2003

The 50% hedge ratio is the simplest currency hedging policy that attempts to deal with regret. With unpredictable exchange rates, an unhedged or hedged policy will be wrong 50% of the time, and often by a large amount. A naïve 50% hedging policy will always be wrong and exhibit regret ex-post, but the maximum amount of regret will be cut in half. A Mellon/Russell survey conducted in 2004 shows that about one-third of the institutional


11 William M. Mercer Investment Consulting's Survey of Pension Plans On Currency Issues, September 2000 (conducted in 2000 with responses from more than 100 large funds).

12 See also Gardner and Wuilloud (1995).

13 The survey collects data from currency overlay managers and covers 563 accounts of institutional investors worldwide (304 from the U.S.A.) and 189 (111 in the U.S.A.) are found to follow the 50% policy (Harris, 2004).
investors surveyed worldwide have adopted this simple 50% hedging policy. One of our aims is to evaluate the optimal hedging policy in a regret-theoretic framework and see under what conditions one can explain deviations from the optimal 100% hedging policy. We will do so using reasonable assumptions on the experience of regret by investors and a strategy of profit maximization and regret avoidance.

We will now propose a formal analysis of the optimal currency hedging decision that incorporates regret in an expected utility optimization, and therefore deals simultaneously with traditional risk (volatility of final wealth) and regret risk. We assume that investors only experience regret on currency hedging, not on asset selection. The paper is structured as follows. In Section 2, we introduce RT and apply it to the modelling of currency hedging in Section 3. Section 4 derives closed-form hedging rules for currency risk minimization, while Section 5 derives closed-form hedging rules in the general case with expectations on currency movements and correlation between asset returns and currency movements. Section 7 concludes this paper.

2. Regret Theory

Regret theory (RT) developed by Loomes and Sugden (1982) and Bell (1982) is a theory of rational choice under uncertainty that is parsimonious yet can explain many of the observed axioms violations of traditional expected utility theory.\textsuperscript{14} Loomes and Sugden (1982) and Bell (1982) derive a modified utility function of final wealth $x$ resulting from a given investment choice, knowing that a different investment choice $y$ could have been achieved:

$$U(x, y) = v(x) + f(v(x) - v(y))$$

(1)

where $U(x, y)$ is the modified utility of achieving $x$, knowing that $y$ could have been achieved. $v(x)$ is the traditional utility function, also called value function or choiceless utility. It is the "value" or utility that an investor would derive from outcome $x$ if he experienced it without having to choose. This value function is assumed to be monotonically increasing and concave (risk aversion) as in traditional finance. The difference $v(x) - v(y)$ is the value loss/gain of having chosen $x$ rather than a foregone choice $y$. The regret function $f(.)$ is monotonically increasing and decreasingly concave\textsuperscript{15}, with $f(0) = 0$. This modified utility is defined over the ex-post (final) outcomes of investment choices; and rational investors would make choices ex-ante by maximizing the expected value of this modified utility. Loomes and Sugden (1982, 1983) and Bell (1982, 1983) conclude that this is a well-behaved parsimonious functional form that allows to take regret into account and is consistent with empirically-observed deviations from traditional expected utility theory.

This functional form has been initially derived for pair-wise choices, but it can be extended (see Quiggin, 1996) to general choice sets. Consider that an investor can select among various investments $i$, with outcome $x_i$. The modified utility of choosing investment $i$ is given by:

\textsuperscript{14} As mentioned in the introduction, Regret theory predicts Allais' paradox ("common consequences effect"), the "common ratio effect", the "isolation effect", the "preference reversal effect", the "reflection effect", and "simultaneous gambling and insurance".

\textsuperscript{15} Bell (1982, 1983) and Loomes and Sugden (1982,1983) show that several behavioral patterns which contradict traditional expected utility theory are predicted by regret theory with a function $f(.)$ that is concave for negative values of the argument and with $f''>0$, so that $f(.)$ is decreasingly concave.
\[ U(x_i) = \nu(x_i) + f(\nu(x_i) - \nu(\max \left[ x_i \right])) \]  
\[ (2) \]

where \( \max \left[ x_i \right] \) is the best ex-post outcome that can be obtained among all investments. Note that the regret term \( \nu(x_i) - \nu(\max \left[ x_i \right]) \) is always non-positive. Concavity of the regret function, \( f'' < 0 \), implies regret-risk aversion.

Rational investors choose the optimal strategy by maximizing their expected modified utility of all possible investment choices.

While RT is broadly consistent with many empirical findings of prospect theory, it cannot be as exhaustive in its ability to describe human behavior because prospect theory is a comprehensive collection of descriptive behavioral models.\(^{16}\) However, it would be unfair to judge RT on the basis of this single criterion. To its advantage, RT is built as a parsimonious and normative theory of choice under uncertainty. As such, its interest lies as much in the ability to use it in tractable and normative economic models as in its descriptive power. Numerous authors have used prospect theory in normative models of investment choices (e.g. Benartzi and Thaler, 1995; Shefrin and Statman, 2000; Barberis, Huang and Santos, 2001, Berkelaar, Kouwenberg and Post, 2004, Gomes, 2005), but the simplifying assumptions required to keep those models tractable limit their inherent descriptive power relative to their parent theory. The use of a descriptive theory in a normative model of portfolio choice often requires additional assumptions to maintain consistency. Typically only a couple of features of prospect theory can be retained (e.g. loss aversion and/or narrow framing), and often in a simplified form. In this respect, Barberis and Huang (2004) is a good “state-of-the-art” example. In a sophisticated dynamic consumption and portfolio allocation model, these authors focus on narrow framing and loss aversion by adding a piece-wise linear disappointment term for selected (narrow-framed) assets in a traditional utility function. There is a simple additive disappointment term for each of these assets. Loss aversion appears because the linear slope is higher for negative returns on these assets than for positive returns.\(^{17}\)

A regret-theoretic approach presents an alternative way of introducing emotions in investment choices. While it suffers from the criticism that it only describes one aspect of human behavior, it does so in an elegant axiomatic way. Furthermore, it goes beyond modeling disappointment, as usually done in the literature, but deals with regret which seems an important psychological trait in portfolio choices, where investors care about the outcome of their choice relative to other strategies they could have followed, passive benchmarks and peers.\(^{18}\) Besides, based on recent neurobiology experiments (Camille et al, 2004), regret appears to be a stronger emotion than disappointment, and seems more relevant to the learning process in decision making under uncertainty than disappointment.

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\(^{16}\) As Kahneman (2003, page 1456) stated in his Nobel lecture: "One novelty of prospect theory is that it was explicitly presented as a formal descriptive theory of the choices that people actually make, not as a normative model."

\(^{17}\) Actually the reference return for each asset is the risk-free rate.

\(^{18}\) Paraphrasing Braun and Muermann (2004), the ex-post assessment made by a prospect-theoretic global investor focusing on currencies would be of the sort: “I wish I had not incurred that currency loss”. In a regret theoretic framework, on the other hand, the ex-post assessment would be “I wish I had taken a better currency hedging decision.”

In accordance with our mental accounting discussion, we consider that the currency hedging decision is a residual one, once the global asset allocation has been chosen. Of their initial wealth, \( W_0 \), investors have allocated \( W_0^d \) to domestic asset and \( W_0^f \) to foreign assets:

\[
W_0 = W_0^d + W_0^f
\]

All valuations are conducted in domestic currency (e.g. the dollar for American investors).

As in all currency hedging research, we do not focus on the interaction between domestic assets and foreign currency and will make the simplifying assumption that the final value of domestic assets, \( W^d \), is non-stochastic. This is in line with the assumption that investors narrow-frame currency risk and do not pool risks.

The dollar value of foreign assets is equal to the product of the foreign-currency value of the foreign assets times the exchange rate (dollar value of foreign currency). Using log of price changes as return, the final (dollar) value of the foreign position, \( W^f \), is:

\[
W^f = W_0^f (1 + \bar{R} + \bar{s})
\]

Where \( R \) is the return of the foreign asset in its local currency and \( s \) is the percentage currency movement (e.g. changes in the dollar value of the foreign currency).

Investors decide to hedge a proportion \( h \) of the foreign assets against currency risk, by selling the foreign currency forward. Foreign assets are treated as an homogeneous asset class with a single currency. This is equivalent to saying that American investors care about an appreciation of the dollar against all currencies (a drop in the weighted average dollar value of foreign currencies). The hedge ratio can take any value between 0 (no currency hedge) and 1 (full currency hedging). As forward contracts have a zero initial value, the initial wealth is unchanged by the hedging decision. Given a hedge ratio \( h \), the final wealth value is given by:

\[
W = W_0^d + W_0^f (1 + \bar{R} + \bar{s}) - hW_0^f \bar{s}
\]

\[
W = W_0^d + W_0^f (1 + \bar{R} + \bar{s}[1 - h]) = W^f + W_0^f \bar{s}(1 - h)
\]

(3)

where \( W^f \) refers to final wealth with full hedging.

For our purpose, the global asset allocation is fixed. Hence, the value (traditional utility) of final wealth, \( V(W) \), can be written as a function of \( h \), the sole decision variable, and of the two stochastic variables \( R \) and \( s \):

\[
V(W) = v(\bar{R} + \bar{s}[1 - h])
\]

Note that derivatives satisfy the conditions \( v' = W_0^f \times V' \) and \( v'' = W_0^f \times W_0^f \times V'' \).

The modified utility can be written as:

\[
U(W) = u(h, \bar{R}, \bar{s}) = v(\bar{R} + [1 - h] \bar{s}) + k \times f(v(\bar{R} + [1 - h] \bar{s}) - v(\bar{R} + \text{max}\left([1 - h] \bar{s}\right)))
\]

(4)

Where \( v(.) \) and \( f(.) \) are monotonically increasing and concave; \( f(.) \) is decreasingly concave \( (f''<0, f'''>0) \) and \( f(0)=0 \). As mentioned above, we assume that investors only exhibit regret on the currency dimension, not on the asset allocation.

Compared to equation (2), we introduced an explicit non-negative scalar \( k \) to facilitate the analysis of the tradeoffs between value and regret. Currency hedging is easy to analyze within
a regret-theoretic approach because, with hindsight, the optimal hedging decision can only be one of two possible choices. If the foreign currency appreciated, and whatever the positive value of \( s \), the best hedging policy would have been to stay unhedged \((h=0)\). So for any positive \( s \):

\[
\max \left[ [1-h]s \right] = s
\]

If the foreign currency depreciates by any amount, the best hedging policy would have been to be fully hedged \((h=1)\). So for any negative \( s \):

\[
\max \left[ [1-h]s \right] = 0
\]

Equation (4) can be written as:

\[
\max \left[ [1-h]s \right] = \max (-h)\max (s) = \max (s)
\]

Let's focus on the impact of a currency movement \( s \). The utility \( u(.) \) is continuous and twice differentiable except in \( s=0 \). At \( s=0 \), the left-hand derivative with respect to \( s \) is equal to:

\[
\frac{\partial u}{\partial s} = (1-h)v'(\tilde{R}) + k(1-h)f'(0)v'(\tilde{R})
\]

The right-hand derivative is equal to:

\[
\frac{\partial u}{\partial s} = (1-h)v'(\tilde{R}) - kf'(0)v'(\tilde{R})
\]

At \( s=0 \), the slope on the negative side is greater than on the positive side, as the difference \( kf'(0)v'(\tilde{R}) \) is always positive. The utility function \( u(.) \) presents a kink at \( s = 0 \). Furthermore, the function \( u(.) \) is concave with respect to \( s \) (see Appendix A). The current exchange rate is a reference point and investors are more sensitive to reductions in financial wealth than to increases in financial wealth. These are common features in prospect theory. Here we have "currency loss aversion", to coin a term frequently used in behavioral finance. This results from the "narrow framing" on currency risk with regret.

In Figure 1, we illustrate the modified utility with regret aversion by assuming two simple functional forms for \( v(.) \) and \( f(.) \). Investors have a logarithmic value function and \( f(.) \) is a negative exponential in the form \( 1-e^{-\lambda} \), with \( k=1 \). For this illustrative purpose focusing on currency risk, we assume that the return on foreign assets \( R \) is non-stochastic. Without loss of generality, we take \( R=0 \). Wealth is assumed constant except for the impact of a currency movement from the current exchange rate. So the utility \( u(.) \) depicted in Figure 1 is a function of the exchange rate movements but also of the hedging decision that is taken. To contrast this modified utility with the traditional value function, we also plot the utility function without regret in dashed line on the same graph.

Let us first consider Figure 1a which assumes that no currency hedging takes place \((h=0)\). For positive values of \( s \), utility increases with \( s \) because of the increase in the value function and the absence of regret (no hedging is ex-post optimal when the foreign currency appreciates). For negative values of \( s \), utility decreases with \( -s \) because of the decrease in the value function.

\[\text{19}\] The combination of the two functions yields a very simple form for the modified utility. It is equal to a constant plus: \( \log(1+(1-h)s) - 1/\left(1+(1-h)s\right) \) for negative values of \( s \) and \( \log(1+(1-h)s) - 1/\left(1+(1-h)s\right) - s/\left(1+(1-h)s\right) \) for positive values of \( s \).
but also because investors experience regret of not having hedged. Also note that regret aversion results in higher risk aversion relative to the traditional value function, as measured by the Arrow-Pratt coefficient of absolute risk aversion. Indeed, for all negative values of s, \(-u''(s)/u'(s)\) is equal to \(1/(1+s)\) for the traditional value function and to \(\frac{1}{1+s} \times \frac{3+s}{2+s}\) in the case of the regret theoretic utility. Therefore, the regret-averse investor who does not hedge will be more risk averse in the region of losses than a traditional utility investor, whereas his risk aversion will not be changed in the region of gains.\(^{20}\)

Let us now consider the case, illustrated in Figure 1b, where the investor undertakes some currency hedging \((h=0.5)\). For positive values of s, on the one hand, the value function increases with s because of the wealth increase caused by 50% of the currency appreciation (half of it is hedged), but there is regret to have only half of the position hedged while the currency appreciates. As a result, the slope becomes smaller than for the utility function without the regret term. For negative values of s, the investor suffers a reduction in wealth (smaller than when \(h=0\)), but also a regret to have only hedged 50% of the position when it would have been optimal to have hedged fully. Overall, it is interesting to note that the Arrow-Pratt coefficient of absolute risk aversion will be lower (higher) for positive (negative) values of \(s\).\(^{21}\) A partially hedged investor displaying aversion to regret will be less risk averse in the region of gains and more risk averse in the region of losses relative to a traditional utility investor. Intuitively, regret induces the investor to move towards the ex-post optimal decision. More specifically, in the region of losses, it would have been optimal to have hedged fully (i.e. choose \(h=1\)), therefore aversion to regret makes the investor more prone to hedge (more risk averse). Conversely, in the region of gains, it would have been optimal not to have hedged at all (i.e. choose \(h=0\)). As a result, regret aversion plays in the opposite direction, making the investor less inclined to hedge (less risk averse). This is true for any partial hedge ratio \((0<h<1)\).

Figure 1c illustrates the case of full hedging. For positive values of s, the investor's wealth remains unchanged (full hedge), but he experiences regret to have missed the currency appreciation. For negative values, the value function remains unchanged (as the currency depreciation is fully hedged) and there is no regret as the optimal hedging decision was taken. This is a limit case for risk aversion. There is no currency risk in a fully hedged portfolio, hence no risk aversion. Because of the specific functional forms of \(f(.)\) and \(v(.)\), regret risk modifies traditional risk aversion in a multiplicative fashion (see footnote 20). Hence we now find a coefficient of absolute risk aversion that is equal to zero both for the traditional utility function and for our regret theory specification. Note that in all three cases there is currency loss aversion, with a kink at \(s=0\): the slope of the utility function is larger for negative values of s than for positive.

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\(^{20}\) In the general case (for any h), \(-u''(s)/u'(s)\) is equal to \((1-h)/(1+(1-h)s)\) for the traditional value function while it is equal to \(\frac{1-h}{1+(1-h)s} \times \frac{3+(1-h)s}{2+(1-h)s}\) when \(s<0\) and \(\frac{1-h}{1+(1-h)s} \times \frac{3h+(1-h)^2s}{1-2h+(1-h)^2s}\) when \(s>0\) in the case of the regret theoretic utility so that for any positive h. It is straightforward to show that we find the inequalities discussed above.

\(^{21}\) \(-u''(s)/u'(s)\) is equal to \(1/(2+s)\) for the traditional value function, while it is equal to \((1-2/s) \times (1/(2+s))\) when \(s>0\) and \((6+s)(4+s) \times (1/(2+s))\) when \(s<0\) in the case of the regret theoretic utility.
Figure 1: Utility function with regret aversion for various hedging decisions: Utility is given as a function of $s$. Dashed lines represent value function alone while full lines represent utility function with regret aversion.

**Figure 1a: $h=0$ (No hedging)**

**Figure 1b: $h=0.5$ (Partial hedging)**

**Figure 1c: $h=1$ (Full hedging)**
4. Derivations and Results: Currency risk minimization

The optimal hedge ratio is obtained by maximizing the expected modified utility with respect to $h$. It can be noted that $u(h,R,s)$ in (5) is concave with respect to $h$ (see Appendix B). To derive optimal hedging rules, we need to make specific assumptions on the functions $v(.)$ and $f(.)$ to be used as well as on the distribution of $s$. If $f(.)$ is linear, then the problem reduces to traditional expected utility maximization, as the maximization with respect to $h$ of the expected utility given in (5) reduces to the maximization of $Ev(R + [1 - h]s)$. With a linear regret function, RT always reduces to traditional expected utility theory.

In general $f(.)$ is assumed concave (regret-risk aversion). Except for very particular and unrealistic functions $v(.)$ and $f(.)$, we cannot derive explicit hedging rules and would have to resort to numerical solutions with little generality. The problem already arises in the case of maximizing expected utility traditional in portfolio theory, but there exist some interesting cases where explicit rules can be worked out. In our model, the problem is compounded by the presence of a piece-wise regret function defined over a valuation function. An ad-hoc assumption, that would make the model a bit more tractable, could be to model the regret term as piece-wise linear and defined over payoffs, not valuation of payoffs. But this simplification would not be consistent with RT and we would lose the theoretical and empirical appeal of this approach.

An interesting alternative is to study the rules for small movements in the exchange rates and asset prices. This the spirit of the approach used by Pratt (1964) to conduct his analysis of risk aversion for small risks. We can use a Taylor expansion of (5) and take its expected value, ignoring moments higher than two. We then maximize with respect to $h$ and are able to derive explicit hedging rules with interesting economic interpretation. This Arrow-Pratt approximation is valid for small risks but would not be exact for larger risks. Extending this approach to large risk requires to assume that the two-moments expansion of the modified utility provides a good approximation of the exact modified utility, which is not necessarily the case. Most of the hedging literature has been using the assumption of multivariate normal distributions for $R$ and $s$, where the first two moments of the distributions are sufficient to characterize the whole distributions. As we will compare our results to this traditional mean-variance optimization, we are quite satisfied with making the same assumption. But even, with the normality assumption, the shape of our utility function with a kink does not make the

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22 An unattractive alternative is to assume that $v$ is linear and $f$ quadratic.

23 When the utility function belongs to the HARA class and asset returns are multivariate normally distributed, there is a linear relation between optimal portfolio weights and the wealth level.

24 So $kf(R + [1 - h]s) - v(R + [1 - h]s^2)$ would be replaced by $kh$s for positive values of $s$ and $k(1 - h)s$ for negative values of $s$. This would be similar in spirit to the piecewise linear simplification in the disappointment approach of Barberis and Huang (2001, 2004). It would imply that the regret term is only concave at the kink $s=0$; investors would not exhibit regret-risk aversion elsewhere. For example a loss of $2x$ is just twice as unpleasant as a loss $x$. One can check that the risk-minimizing hedge ratio would not be affected by regret and would be similar to what obtains under traditional expected utility optimization (a hedge ratio of one).

25 A number of studies have compared the exact expected utility with its approximation obtained with a two-moments expansion under the assumption of normality; see Levy and Markowitz (1979), Kroll, Levy and Markowitz (1984) and Loistl (1976).
analysis of an exact solution tractable and amenable to economic interpretation\textsuperscript{26} unless we restrict our attention to small risks. In this case, the Arrow-Pratt approximation provides powerful results with useful economic intuition.

As mentioned before, our primary focus is on the currency-risk-minimizing behavior of investors, where investors search for the optimal hedge ratio in the absence of currency forecast and of specific assumption about the correlation between foreign asset returns and currency movements. Hence, we will now detail the derivations under the simplifying assumptions that the return on foreign assets is non-stochastic that the expected currency return is zero and that the distribution is symmetric. To simplify notations, and without loss of generality, we set $R=0$. We will provide a discussion relaxing those assumptions in the next section.

The expectation of (5) under those assumptions can be written as:

$$ Eu = Ev([1-h]\tilde{s}) + k \times E_s f(v([1-h]\tilde{s}) - v(0)) + k \times E_{\tilde{s}} f(v([1-h]\tilde{s}) - v(0)) $$

(6)

As mentioned above, our problem is well-behaved as first derivatives of $u(.)$ are well-defined and continuous, except in $s=0$, and $u(.)$ is concave in $h$ and $s$. $Eu(.)$ is concave in $h$ as shown in Appendix B.

The optimal hedge ratio satisfies the first order condition:

$$ \frac{\partial Eu}{\partial h} = 0 $$

Because $Eu(.)$ is concave in $h$, this first-order condition is necessary and sufficient for optimality. We will set the optimal hedge ratio to zero or one if it falls outside the allowed range $(0,1)$.

If $k=0$ (investor exhibits no regret), the optimization problem reduces to the traditional expected utility optimization $\text{Max}_h Ev([1-h]\tilde{s})$. Here $s$ is a pure risk (no expected return) and $1-h$ is non-negative, therefore any risk averse investor will attempt to eliminate that risk by setting $h$ equal to 1. This is the typical full-hedging risk-minimization result.

We now derive the Arrow-Pratt approximation in the presence of regret ($k>0$).

For a given hedge ratio $h$, the Taylor expansion\textsuperscript{27} of $v([1-h]\tilde{s})$ around $s=0$ is:

$$ v([1-h]\tilde{s}) = v(0) + (1-h)\tilde{s}v'(0) + \frac{1}{2}(1-h)^2\tilde{s}^2v''(0) + o(\tilde{s}^2) $$

Hence, the expected value function:

$$ Ev([1-h]\tilde{s}) \approx v(0) + (1-h)E(\tilde{s})v'(0) + \frac{1}{2}(1-h)^2E(\tilde{s}^2)v''(0) $$

(7)

The expected regret function, over $s+$:

\textsuperscript{26} The normality assumption implies that for well-behaved functions, expected utility $Eu(.)$ can be expressed as a function of the mean and variance of $s$ in (6). But we cannot infer the parameters of $Eu(.)$ from those of $u(.)$, rendering any economic interpretation impossible. However, we can do it in the case of small risks.

\textsuperscript{27} Our derivations could be made a bit more formal by taking $s=\xi s'$ and letting $\xi$ become very small, where $s'$ is a normal distribution. This is a direct application of the "compact" derivations of the approximation by Samuelson (1970).
\[ E_{ss, f}(v[1-h]\tilde{s}) - v(\tilde{s})) \approx f(0) + E_{ss}[v[1-h]\tilde{s}) - v(\tilde{s})]f'(0) + \frac{1}{2}E_{ss}[v[1-h]\tilde{s}) - v(\tilde{s})]^2 f''(0) \]

over \( s \):

\[ E_{s-} f(v[1-h]\tilde{s}) - v(0)) \approx f(0) + E_{s-}[v[1-h]\tilde{s}) - v(0)]f'(0) + \frac{1}{2}E_{s-}[v[1-h]\tilde{s}) - v(0)]^2 f''(0) \]

With:

\[ E(v[1-h]\tilde{s}) - v(\tilde{s})) \approx -hE(\tilde{s})v'(0) + \frac{1}{2}[(1-h)^2 - 1]E(\tilde{s}^2)v''(0) \]

\[ E(v[1-h]\tilde{s}) - v(0)) \approx (1-h)E(\tilde{s})v'(0) + \frac{1}{2}(1-h)^2E(\tilde{s}^2)v''(0) \]

Let's drop the argument 0 in the derivatives. Let's denote \( \overline{s} = E(\tilde{s}) = 0, \overline{s}_s = E_{ss}(\tilde{s}), \overline{s}_s = E_{s-}(\tilde{s}), \Sigma_s = E(s^2), \Sigma_{ss} = E_{ss}(\tilde{s}^2), \Sigma_{s-} = E_{s-}(\tilde{s}^2) \). Note that \( f(0)=0, \overline{s} = \overline{s}_s + \overline{s}_s = 0 \) and that \( \Sigma_s = \sigma_s^2 = \Sigma_{ss} + \Sigma_{s-} \). With symmetric (e.g. normal) distributions we have \( \Sigma_{ss} = \Sigma_{s-} = \frac{1}{2}\Sigma_s \).

The expected value function (7) becomes:

\[ Ev([1-h]\tilde{s}) \approx v(0) + \frac{1}{2}(1-h)^2\Sigma_s v'' \] (8)

Discarding moments higher than two, we get for the expected regret function over \( s^+ \):

\[ E_{ss, f}(v[1-h]\tilde{s}) - v(\tilde{s})) \approx \left[ -h\overline{s}_s v' + \frac{1}{2}[(1-h)^2 - 1]\Sigma' v'' \right] f' + \frac{1}{2}h^2\Sigma_{ss}v^2 f'' \] (9)

Similarly for \( s^- \):

\[ E_{s-} f(v[1-h]\tilde{s}) - v(0)) \approx \left[ (1-h)\overline{s}_s v' + \frac{1}{2}(1-h)^2\Sigma_{s-} v'' \right] f' + \frac{1}{2}(1-h)^2\Sigma_{s-}v^2 f'' \] (10)

The expected utility is the sum of three terms:

\[ Eu = (8) + k\times(9) + k\times(10) \] (11)

The expected utility (11) can be rewritten by grouping the terms in various powers of \( h \) as:

\[ Eu \approx v(0) + v''\Sigma_s + v'kfv'\overline{s}_s + v''kfv''\Sigma_{s-} + \frac{1}{2}v^2kf''\Sigma_{s-} - hv''\Sigma_s - hv''kfv'(\Sigma_{ss} + \Sigma_{s-}) - hv'^2kf''\Sigma_{s-} \]

\[ + \frac{1}{2}h^2v''\Sigma_s + \frac{1}{2}h^2v''kfv'\Sigma_{s-} + \frac{1}{2}h^2v'^2kf''(\Sigma_{ss} + \Sigma_{s-}) \] (12)

Where \( \Sigma_{ss} + \Sigma_{s-} = \Sigma_s \).

We now compute the first order condition for optimal hedging by setting the derivative of \( Eu(.) \) with respect to \( h \) equal to zero. This is the first order condition without constraints on \( h \).
\begin{align}
0 &= -v''\Sigma_s - v''k\Sigma_s - v''k\Sigma_{s-} + hv''\Sigma_s + hv''k\Sigma_s + hv''k\Sigma_{s-} \\
\text{Hence:} \\
\hat{h}^* &= \frac{v''(1 + k\Sigma_s + v''k\Sigma_{s-})}{v''(1 + k\Sigma_s + v''k\Sigma_{s-})} = \frac{v''(1 + k\Sigma_s') + v''k\Sigma_s}{v''(1 + k\Sigma_s') + v''k\Sigma_s} \\
\text{We can rearrange } h^*, \text{ noting that } \Sigma_{s-} = \frac{1}{2} \Sigma_s : \\
\hat{h}^* &= 1 - \frac{1}{2} \frac{v''k\Sigma_{s-}}{v''(1 + k\Sigma_s')} + 2v''k\Sigma_{s-} = 1 - \frac{1}{2} \frac{v''k\Sigma_{s-}}{v''(1 + k\Sigma_s') + v''k\Sigma_s} = 1 - \frac{1}{2} \theta \\
\text{With } \theta &= \frac{v''k\Sigma_{s-}}{v''(1 + k\Sigma_s')} + v''k\Sigma_s \\
\text{The optimal hedge ratio is equal to one, as would obtain in risk minimization without regret, minus a term linked to regret aversion. Note that } k, v' \text{ and } f' \text{ are positive (investors prefer more wealth and less regret), } v'' \text{ is negative (risk aversion) and } f'' \text{ is also negative (regret-risk aversion), so } \theta \text{ is generally positive and lesser than one. Hence, we obtain an interior solution for } h. \\
\text{As mentioned previously, when the regret function } f(.) \text{ is linear (} f''=0), \text{ we are back to traditional utility maximization and an optimal hedge ratio of 1. We introduced the scalar } k \text{ to look at the tradeoffs between value and regret. Ceteris paribus, an increase in } k \text{ reduces the optimal hedge ratio (the derivative of } h^* \text{ with respect to } k \text{ is negative). When } k \text{ is very small (value dominates regret) } h^* \text{ goes to 1 as } k \text{ goes to 0, as expected from traditional expected utility theory.} \\
\text{The case where regret totally dominates value is more complex. When } k \text{ becomes very large, } \theta \text{ tends toward:} \\
\theta_{\text{max}} &= \frac{v''f''}{v''f' + v''f''} \\
\text{This is generally smaller than 1, so that the optimal hedge ratio is generally greater than 50% even when investors care primarily about regret. The justification is that the regret function } f(.) \text{ is defined over utility losses, not over losses themselves. The value function itself is generally concave } (v''<0), \text{ so that the loss in value caused by a reduction in wealth } -\xi, \text{ is greater than the gain in value caused by a gain of the same magnitude } +\xi: v(0) - v(-\xi) > v(\xi) - v(0). \text{ As a result, investors who are averse to regret and compare their decision to the ex-post best alternative will care more about the regret in the region of losses, i.e. in case of insufficient hedging, than in the region of gains, i.e. in case of excessive hedging (opportunity loss). These investors will therefore tend to increase their optimal hedging the more concave the function } v(.) \text{ is for a given } f(.) \text{ function. If we look at the denominator of } \theta_{\text{max}}, \text{ the term } v''f' \text{ is precisely the regret-valuation of the concavity in value.} \\
\text{To account for a naïve 50% hedging policy described in the introduction, we need to resort to the following alternative assumptions. A first possibility is to define regret directly on the monetary payoffs, rather than on the utility of these payoffs. Alternatively, we may also assume that the value function is linear } (v''=0). \text{ That way, the regret valuation of the concavity in value will no longer play a role in the hedging decision, and we will find}
θ_{max} = 1$, hence $h^* = 50\%$. But these assumptions are inconsistent with regret theory or risk aversion. Finally, we may assume that the investor is infinitely averse to regret (concavity $f''$ is very large in absolute value). Under such assumptions, the investor will select the naïve 50% hedging policy. This is best illustrated by the following example. If we take the previous logarithmic value function and a negative exponential for the regret function, $f(x) = (1 - e^{-x})$, some simple calculus yields $θ_{max} = 0.5$. Hence a regret-minimizing hedge ratio of 0.75. Let’s now increase the concavity in the regret function by taking a negative exponential with a higher exponent $α : f(x) = (1 - e^{-αx})$. Then, $θ_{max}$ becomes $θ_{max} = α / (1 + α )$ which will tend to one when the concavity index goes to infinity. The higher the regret aversion, the closer we get to the naïve rule. Note that, in the case of infinite regret aversion, we need no longer assume that $k$ is large to obtain the naïve hedging policy as the optimal policy. 28 As a result, investors who care about value, but are infinitely averse to regret, will hedge only 50% of their foreign assets portfolio. With this assumption of infinite regret aversion, we believe that our regret theoretic model is similar to the “minimax regret” decision rule of Savage (1954). In this early model, Savage assumed that agents consider, for all possible decisions, the maximum regret that they may carry ex-post. Agents then select the decision that carries the smaller such “maximum regret”. Note that this decision is taken irrespective of the likelihood that such a regret may actually occur (provided that the probability is strictly positive). Intuitively, in our model with infinite regret aversion, investors care exclusively about the higher level of regret attained for any hedging decisions, whether in the region of gains or in the region of losses 29. When they hedge fully, investors anticipate that the maximum regret associated with a strong appreciation of the foreign currency, though unlikely, is so high that they reject such a hedging decision. Conversely, if they do not hedge at all, the regret associated with a strong depreciation in the foreign currency is again perceived as extremely high, even though it may be very unlikely, and is again rejected. As we assumed that the distribution of the foreign currency value is symmetric, the naïve 50% hedging policy will always be wrong and exhibit regret ex-post. However, the maximum amount of regret will be cut in half whether it is attained in the region of gains or in the region of losses.

To summarize the case of pure currency-risk minimization, a regret averse investor will always hedge less than 100%, the optimum hedging of a traditional expected-utility maximizer. However an optimal hedge ratio of 50% will only obtain for infinite regret

28 Indeed, in this case, looking at the denominator of $θ$, we observe that the first term, $v''(1 + kf'')$, is dominated by $v'^2 kf''$ when $α$ is large enough, so that $θ$ tends to 1 and $h^*$ tends towards 50%.

29 Because the quantity $\text{Exp}[-α(v([1 - h][\overline{s}]) - v(\max ([1 - h][\overline{s}])])$, where $\overline{s}$ is the realization of $\overline{s}$ that carries the higher level of regret, dominates any other quantity $\text{Exp}[-α(v([1 - h][\overline{s}]) - v(\max ([1 - h][\overline{s}])])$ for any value of $s$ different from $\overline{s}$, infinitely regret averse investors will only care about minimizing the former quantity. Indeed, for any random variable $\overline{s}$ with probability density function $g(.)$, it is straightforward to show that, for $α$ large enough, $E[f(\nu([1 - h][\overline{s}]) - v(\max ([1 - h][\overline{s}])])$ is equivalent to the term $-g(\overline{s}) \times \text{Exp}[-α(v([1 - h][\overline{s}]) - v(\max ([1 - h][\overline{s}])])$. The proof for this result is available from the authors upon request.
aversion. In general the optimal hedge ratio will be between 100% and 50%, depending on regret aversion.

5. Derivations and Results: General case

We will now consider the general case where the return on foreign assets is stochastic and where the expected currency return can be non-zero. The derivations follow the previous methodology and are given in the Appendix C. The expected value to be maximized with respect to $h$ is:

$$ Eu(h, \bar{R}, \bar{s}) = E v(\bar{R} + [1 - h] \bar{s}) + k \times E_{s} f (v(\bar{R} + [1 - h] \bar{s}) - v(\bar{R} + \bar{s})) 
+ k \times E_{\bar{s}} f (v(\bar{R} + [1 - h] \bar{s}) - v(\bar{R})) $$

(16)

where $\bar{R}$ and $\bar{s}$ are stochastic with mean $\bar{R}$ and $\bar{s}$, so that $\bar{R} = \bar{R} + \bar{r}$, where $\bar{r}$ is a random variable with zero mean, $\Sigma = E(\bar{s}^2)$ and $\Sigma_r = E(\bar{r}^2)$.

Traditional utility with no regret

To compare with the existing literature on hedging, let's first consider the special case where there is no regret ($k=0$). Then for small risks\(^{30}\), we get by developing around $\bar{R}$:

$$ Eu(h, \bar{R}, \bar{s}) \approx E(\bar{r} + [1 - h] \bar{s}) v'(\bar{R}) + \frac{1}{2} E(\bar{r} + [1 - h] \bar{s})^2 v''(\bar{R}) $$

$$ Eu(h, \bar{R}, \bar{s}) \approx [1 - h] \bar{s} v'(\bar{R}) + \frac{1}{2} \Sigma_v + 2(1 - h) \Sigma_r + (1 - h)^2 \Sigma_r v''(\bar{R}) $$

(17)

The optimal hedge ratio is obtained by setting to zero the derivative of (17) with respect to $h$. We obtain:

$$ 0 = -\bar{s} v'(\bar{R}) + \left[ -\Sigma_r - h \Sigma_s \right] v''(\bar{R}) $$

$$ h^* = 1 + \bar{s} \frac{v'(\bar{R})}{v''(\bar{R})} + \frac{\Sigma_r v(\bar{r}, \bar{s})}{\Sigma_s} = 1 - \bar{s} \frac{1}{A} + \frac{\Sigma_r v(\bar{r}, \bar{s})}{\Sigma_s} $$

(18)

Where $A$ is the Arrow-Pratt measure of local risk aversion, $-v''/v'$. This small risk approximation is a traditional result in the hedging literature. We will refer to it as the mean-variance case. It would be exact if the valuation function was quadratic or the distribution multivariate normal. Ceteris paribus, a positive expectation on the foreign currency movement reduces the optimal hedge ratio (speculative term). The lesser the risk aversion, the lower the hedge ratio (investors speculate more). Similarly, a negative covariance between foreign asset return and currency movement (the local price of the foreign asset tends to go up when the foreign currency depreciates) reduces the optimal hedge ratio (covariance term).

\(^{30}\) The derivations can be made more formal by setting $s=\xi s'$, $\tilde{R}=\xi \tilde{R}'$ and letting $\xi$ become very small, where $s'$ and $\tilde{R}'$ are multivariate normal distributions. This is the spirit of the approach of Samuelson (1970).
 Modified utility with regret

The derivations for the general case in presence of regret is given in Appendix C. The optimal hedge ratio in the general case is:

\[ h^* = 1 - \frac{\Sigma_{++}}{\Sigma_s} \times \frac{v''(1+\lambda^+ f''')}{v''(1+\lambda^+ f''') + v'^2 \lambda^+ f'''} \]

\[ + \bar{\alpha} \times \frac{v'(1+\lambda^+ f''')}{v''(1+\lambda^+ f''') + v'^2 \lambda^+ f'''} + \frac{\text{cov} (\bar{\alpha}, \bar{s})}{\Sigma_s} \times \frac{v''(1+\lambda^+ f''')}{v''(1+\lambda^+ f''') + v'^2 \lambda^+ f'''} \]

(19)

The hedge ratio is equal to one minus three terms:

a) regret term:

\[ h_{\text{regret}} = -\frac{\Sigma_{++}}{\Sigma_s} \times \frac{v'^2 \lambda^+ f'''}{v''(1+\lambda^+ f''') + v'^2 \lambda^+ f'''} \]

(20)

This is similar to the expression \(-\frac{1}{2} \theta\) of (14) in the currency risk-minimizing case, except that \(\Sigma_{++}\) will generally differ from \(\frac{1}{2} \Sigma_s\), if the expectation of \(s\) differs from zero. Hence, the previous discussion applies with one caveat. If \(\bar{s} > 0\), \(\Sigma_{++}\) will be greater than \(\Sigma\) (for a symmetric distribution) and investors will hedge less because they anticipate to experience less regret if they decide not to hedge than in the risk-minimizing case. This is because they use the current exchange rate as reference point, while they anticipate the future exchange rate to appreciate. Conversely they would hedge more if they anticipate the foreign currency to depreciate.

b) speculative term:

\[ h_{\text{speculative}} = \bar{\alpha} \times \frac{v'(1+\lambda^+ f''')}{v''(1+\lambda^+ f''') + v'^2 \lambda^+ f'''} = -\bar{\alpha} \frac{1}{A_k} \]

As in the traditional mean-variance case (18), a positive expectation on the foreign currency movement reduces the optimal hedge ratio. The lesser the risk aversion \(A_k\), the lower the hedge ratio (investors speculate more). But this is a modified risk aversion that takes regret into account. Regret-risk aversion will, overall, increase the traditional risk aversion:

\[ A_k = -\frac{v''(1+\lambda^+ f''') + v'^2 \lambda^+ f'''}{v'(1+\lambda^+ f''')} = A + \frac{-v'^2 \lambda^+ f'''}{v'(1+\lambda^+ f''')} \]

Without concavity in the regret function \(f\), the risk aversion would be similar to the traditional one. In general, regret adds to risk aversion because \(f''\) is negative. Ceteris paribus, regret-averse investors will tend to "speculate" less on their anticipations of currency movements.

c) covariance term

\[ h_{\text{cov}} = \frac{\text{cov}(\bar{\alpha}, \bar{s})}{\Sigma_s} \times \frac{v''(1+\lambda^+ f''')}{v''(1+\lambda^+ f''') + v'^2 \lambda^+ f'''} \]

Where \(\gamma\) is positive. In the trivial case of a linear regret function, \(\gamma=1\) and the covariance hedging term is identical to that in the traditional mean-variance case. In general, investors will take into account the covariance between asset return and currency movement in their
hedging decision. But with regret $\gamma$ is less than one and investors tend to deviate less from their risk-minimizing hedging policy. The intuitive explanation is straightforward. A negative correlation between foreign asset return and currency movement implies that asset returns tend to soften the impact of currency risk at the portfolio level; but regret is only measured on the currency movement itself, not on asset return. The value function in the modified utility takes into account total portfolio risk and suggests a lower hedge ratio because of the negative correlation, but this is partly dampened by regret-risk aversion on currency losses.

6. Conclusion

We present a model of optimal currency-hedging choices based on regret theory, a behavioral finance theory where investors reach optimal investment decision taking the expected pain of future regret into account. Regret averse investors derive utility from their global asset allocation (as a "traditional" utility maximizer investor) but, in addition, they also experience regret for having chosen a currency exposure that proves, with hindsight, inappropriate.

We have shown that investors who exhibit regret theoretical preferences will tend to hedge less against currency risk relative to traditional investors, when there are no speculation motives (expectations about the foreign currency movements are equal to zero) or correlation between asset returns and currency movements. This effect prevails because, in such a situation, hedging is not associated, on average, with any expected gains or losses. As a result, a traditional utility maximizer who is risk averse will eradicate the currency risk by hedging fully. In contrast, a regret averse investor will be less prone to hedge fully as, ex-post, such a decision will be associated with the regret of having hedged too much in the case of a foreign currency appreciation. Investors with extreme regret aversion will conform to the observed naïve hedging policy consisting in hedging only 50% of the foreign currency position. This hedging policy enables investors to minimize the pain associated with the “maximum regret”, irrespective of its likelihood, in the case of symmetric distributions. In general, regret averse investors will adopt a currency-risk-minimizing hedging strategy with a hedge ratio that depends on their regret aversion. The optimal hedge ratio lies between 50% (extreme regret aversion) and 100% (no regret aversion).

In the general currency hedging case, i.e. when investors have beliefs about the future currency movements and when currency risk may be correlated with the return on the foreign assets, regret will have contrasted effects on the hedging decision. The optimal hedging rule of a regret averse investor can be separated in three additive terms: a currency-risk-minimizing term, a speculative term and a covariance term. As discussed above the currency-risk minimization term will be equal to 100% (as obtains for a traditional utility maximizer) minus a regret term function or regret aversion. The speculative term reflects the fact that investors will hedge less (more) if they expect an appreciation (depreciation) of the foreign currency; but regret averse investors tend to speculate less than traditional investors. The covariance term will reduce (increase) the hedge ratio if foreign asset returns are negatively (positively) correlated with currency movements, but again this effect will be less pronounced for regret averse investor than for a traditional investor.

Risk aversion and regret aversion combine in a complex manner. This can be illustrated in the following way. For the pure currency risk-minimization motive, a regret averse investor will hedge less than a traditional investor, which could be translated as lower risk aversion. For the speculation motive, a regret averse investor will speculate less than a traditional investor, which could be translated as higher risk aversion.
We believe the regret theoretic approach applied to currency hedging decisions bears some interesting normative implications as discussed along the course of this article. More generally, we believe that regret theory may be an interesting addition to the behavioral finance theory toolbox. It can be thought of as a normative complement to prospect theory that has seen widespread use in behavioral financial modeling.
Appendix A: Concavity of $u$ with respect to $s$

Let's take derivatives with respect to $s$ of $u(h,R,s)$ in given in , where we replaced $(1-h)$ by $\gamma$:

$$u(h,\tilde{R},\tilde{s}) = v(\tilde{R} + \gamma \tilde{s}) + k \times f_+ (v(\tilde{R} + \gamma \tilde{s}) - v(\tilde{R} + \tilde{s})) + k \times f_- (v(\tilde{R} + \gamma \tilde{s}) - v(\tilde{R}))$$

The first derivative for positive values of $s$ is:

$$\frac{\partial u}{\partial s} = \gamma v'(R + \gamma s) + kf'(v(R + \gamma s) - v(R + s)) \times \left[ \gamma v'(R + \gamma s) - v'(R + s) \right]$$

The second derivative for positive values of $s$ is:

$$\frac{\partial^2 u}{\partial s^2} = \gamma^2 v''(R + \gamma s) + kf''(v(R + \gamma s) - v(R + s)) \times \left[ \gamma v'(R + \gamma s) - v'(R + s) \right]^2$$

$$+ kf'(v(R + \gamma s) - v(R + s)) \times \left[ \gamma^2 v''(R + \gamma s) - v''(R + s) \right]$$

(21)

For all values of $h$ and $R$, the second derivative $\frac{\partial^2 u}{\partial s^2}$ is negative as $v',f'>0$ and $f'',v''<0$.

The first derivative for negative values of $s$ is:

$$\frac{\partial u}{\partial s} = \gamma v'(R + \gamma s) + kf'(v(R + \gamma s) - v(R)) \times \left[ \gamma v'(R + \gamma s) \right]$$

The second derivative for negative values of $s$ is:

$$\frac{\partial^2 u}{\partial s^2} = \gamma^2 v''(R + \gamma s) + kf''(v(R + \gamma s) - v(R)) \times \left[ \gamma v'(R + \gamma s) \right]^2$$

$$+ kf'(v(R + \gamma s) - v(R)) \times \left[ \gamma^2 v''(R + \gamma s) \right]$$

(22)

For all values of $h$ and $R$, the second derivative $\frac{\partial^2 u}{\partial s^2}$ is negative.

So the function $u(.)$ is a continuous function, concave with respect to $s$ for all $s>0$ and all $s>0$. Furthermore, in $s=0$ its derivative is larger on the left-hand side than on the right-hand side. So its right-hand (or left-hand) derivative is decreasing. As indicated in Royden (1988), proposition 18, chapter 5, this is a sufficient condition for $u$ to be concave with respect to $s$.  

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Appendix B: Concavity of $Eu$ with respect to $h$

We first show that $u(.)$ is concave in $h$. Let’s take derivatives with respect to $h$ of $u(h,R,s)$ in given in (5):

$$u(h,R,s) = v(R + [1-h]s) + k \times f_{v_+} (v(R + [1-h]s) - v(R + s)) + k \times f_{v_-} (v(R + [1-h]s) - v(R))$$

The first derivative is:

$$\frac{\partial u}{\partial h} = -sv' - ksf' \times v'$$

Where the argument of $v(.)$ is $R + [1-h]s$, and the argument of $f(.)$ is $v(R + [1-h]s) - v(R + s)$ if $s$ is positive, and $v(R + [1-h]s) - v(R)$ if $s$ is negative. So $\frac{\partial u}{\partial h}$ is always positive and continuous, even in $s=0$.

The second derivative is:

$$\frac{\partial^2 u}{\partial h^2} = s^2 v'' - ks(-sf'' \times v'' - sf' v'') = s^2 (v''(1 + kf'') + kf''v'')$$

(24)

Where $v'(.)$ and $v''(.)$ are valued at $R + [1-h]s$ and $f'(.)$ and $f''(.)$ are valued at is $v(R + [1-h]s) - v(R + s)$ if $s$ is positive, and $v(R + [1-h]s) - v(R)$ if $s$ is negative.

For all values of $h$, $s$ and $R$, the second derivative $\frac{\partial^2 u}{\partial h^2}$ is negative as $v', f'>0$ and $f'', v''<0$.

Let’s now turn to $Eu$ which can be written as a function of $h$. The second derivative of $Eu$ with respect to $h$ is:

$$\frac{\partial^2 Eu}{\partial h^2} = E \frac{\partial^2 u}{\partial h^2} = Es^2 (v''(1 + kf'') + kf''v'')$$

Because $\frac{\partial^2 u}{\partial h^2}$ is negative for all values of $h$, $R$ and $s$, so is $\frac{\partial^2 Eu}{\partial h^2}$.
Appendix C: Derivation of the optimal hedge ratio

The Expected utility is given by:

\[ Eu(h, R, s) = Ev(R + [1 - h]s) + k \times E_{s'} f(v(R + [1 - h]s) - v(R + s)) + k \times E_{R'} f(v(R + [1 - h]s) - v(R)) \]

The derivations can be made more formal by setting \( s = \xi s' \), \( R = \xi R' \) and letting \( \xi \) become very small, where \( s' \) and \( R' \) are multivariate normal distributions. All covariances of \( s \) and \( R \) are of order \( \xi^2 \) and we neglect terms of order higher than 2. To simplify notations, we skip this straightforward step in the derivations presented below.

We develop the valuation function \( v(.) \) around \( \bar{R} \) and the regret function \( f(.) \) around 0. So the implicit arguments is \( \bar{R} \) for all derivatives of \( v \), and 0 for all derivatives of \( f(.) \). With the additional notations \( \Sigma_r \) and \( \text{cov}(r, s) \) for the variance of \( r \) and the covariance between \( r \) and \( s \):

\[
Ev(R + [1 - h]s) \approx v(\bar{R}) + [1 - h]\bar{v}v' + \frac{1}{2} \left[ \Sigma_v + 2(1 - h)\text{cov}(r, s) + (1 - h)^2 \Sigma_s \right] v''
\]

(25)

Over \( s+ \):

\[
E_{s'} f(v(R + [1 - h]s) - v(R + s)) \approx E_{s'} \left[ v(R + [1 - h]s) - v(R + s) \right] f'
+ \frac{1}{2} E_{s'} \left[ v(R + [1 - h]s) - v(R + s) \right]^2 f''
\]

(26)

with:

\[
E_{s'}(v(R + [1 - h]s) - v(R + s)) \approx -h\bar{v}v' - h\text{cov}_{s'}(r, s)v'' + \frac{1}{2} (h^2 - 2h)\Sigma_{s'}v''
\]

\[
E_{s'} \left[ v(R + [1 - h]s) - v(R + s) \right]^2 \approx h^2 \Sigma_{s'}v'^2
\]

Hence (26) becomes:

\[
E_{s'} f \approx \left[ -h\bar{v}v' - h\text{cov}_{s'}(r, s)v'' + \frac{1}{2} (h^2 - 2h)\Sigma_{s'}v'' \right] f' + \frac{1}{2} h^2 \Sigma_{s'}v'^2 f''
\]

(27)

Over \( s- \):

\[
E_{s'} f(v(R + [1 - h]s) - v(R)) \approx E_{s'} \left[ v(R + [1 - h]s) - v(R) \right] f'
+ \frac{1}{2} E_{s'} \left[ v(R + [1 - h]s) - v(R) \right]^2 f''
\]

(28)

with:

\[
E_{s'}(v(R + [1 - h]s) - v(R)) \approx (1 - h)\bar{v}v' + (1 - h)\text{cov}_{s'}(r, s)v'' + \frac{1}{2} (1 - h)^2 \Sigma_{s'}v''
\]

\[
E_{s'} \left[ v(R + [1 - h]s) - v(R) \right]^2 \approx (1 - h)^2 \Sigma_{s'}v'^2
\]

Hence (28) becomes:
The expected utility is the sum of three terms:

\[ Eu = (25) + k \times (27) + k \times (29) \]

Remember that \( \Sigma_s = \Sigma_s + \Sigma_{s-} \), and \( \text{cov}(r,s) = \text{cov}_{s-}(r,s) + \text{cov}_{s-}(r,s) \). Then:

\[
E_u \approx v(\bar{R}) + (1-h)v'\bar{R} + \frac{1}{2}(1-h)^2 v''\Sigma_s + \frac{1}{2} \Sigma_kv'' + (1-h)v'\text{cov}(r,s) \\
+kf'(v'\bar{R} + v''\text{cov}(r,s)) - hkf''(v'\bar{R} + v''\text{cov}(r,s)) \\
+ \frac{1}{2} h^2 v'' k^2 \Sigma_s - hv'' k^2 \Sigma_s + \frac{1}{2} v'' k^2 \Sigma_{s-} \\
+ \frac{1}{2} h^2 v'^2 k^2 \Sigma_s - hv'^2 k^2 \Sigma_s + \frac{1}{2} v'^2 k^2 \Sigma_{s-}
\]

Let's compute the optimal hedge ratio by setting to zero the derivative of \( Eu \) with respect to \( h \). This the first order condition without constraints on \( h \).

\[
0 = -\left[ v'\bar{R} + v''\Sigma_s + v''\text{cov}(r,s) + kf'(v'\bar{R} + v''\text{cov}(r,s)) + v'' k^2 \Sigma_s + v'^2 k^2 \Sigma_{s-} \right] \\
+ h \left[ v'' \Sigma_s + v'' k f' \Sigma_s + v'^2 k f' \Sigma_s \right] \\

h^* = \frac{v'\bar{R} + v''\Sigma_s + v''\text{cov}(r,s) + kf'(v'\bar{R} + v''\text{cov}(r,s)) + v'' k^2 \Sigma_s + v'^2 k^2 \Sigma_{s-}}{v'' \Sigma_s + v'' k f' \Sigma_s + v'^2 k f' \Sigma_s}
\]

\[
h^* = 1 - \frac{\Sigma_{s+}}{\Sigma_s} \times \frac{v'^2 k f''}{v''(1 + k f'') + v'^2 k f''} \\
+ \bar{R} \times \frac{v'(1 + k f'')}{v''(1 + k f'') + v'^2 k f''} \times \frac{\text{cov}(r,s)}{\Sigma_s} \times \frac{v''(1 + k f'')}{v''(1 + k f'') + v'^2 k f''}
\]

(30)
Bibliography


