Abstract:
In an integrated tax system, the individual and corporate tax systems interact to remove the double taxation of dividends that otherwise occurs under a classical tax system. In this paper, we abstract from the mechanics of particular national integrated tax systems and present a simple asset pricing model for a reasonably general integrated tax framework. Specifically, we modify the standard CAPM to incorporate tax credits on risky assets and the heterogeneous valuation of these tax credits across all investors. We show that an investor’s optimal portfolio depends on the value the investor places on tax credits relative to the equilibrium value of credits.

Keywords:
CAPM; Tax; Integration; Tax Credit; Imputation; Dividend Exclusion; Utilisation Rate; Gamma.

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1. **INTRODUCTION**

Under current U.S. tax law and in contrast to many of its trading partners, corporate income distributed to shareholders in the form of dividends is generally taxed twice: first at the corporate level and then again at the shareholder level. The double taxation of dividends and the various economic distortions that can result under a classical tax system has been the subject of substantial policy debate within the U.S. and elsewhere for many years.\(^1\) The primary response of governments to date has been to switch (or at least propose a switch) to an integrated tax system. Broadly, an integrated tax system involves the integration of a nation's individual and corporate tax systems in such a way as to ensure tax is effectively imposed on distributed corporate income once and once only. There are three principal types of (distribution-related) integration systems: dividend exclusion, imputation credit and dividend deduction.\(^2\) In each case, a separate tax is levied on profits at the corporate level but part or all of this corporate level tax is then eliminated on any dividends paid out therefrom. The mechanics of elimination vary: in a dividend exclusion system, dividends are excluded from a shareholder's taxable income; in an imputation system, shareholders are allowed to claim a credit for corporate level taxes; and in a dividend deduction system, corporations are allowed to claim a tax deduction for dividends paid to shareholders.

An integrated tax system currently operates in many countries including Austria, Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, New Zealand, Portugal, Italy, Singapore, Spain, Sweden and the U.K.. A recent

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2. See U.S. Department of the Treasury (1992) for an extensive discussion of these "dividend relief" systems as well as other more comprehensive approaches to integration, which are designed to eliminate double taxation of both distributed and undistributed corporate income.
trend motivated by administrative complexities has been a move from imputation tax based systems to dividend exclusion based systems.\textsuperscript{3} In the U.S., a dividend exclusion system has also recently been proposed by President Bush.\textsuperscript{4}

Whilst the choice between the alternative integration systems ultimately depends on a number of key policy objectives including administrative simplicity and implementation flexibility, the fundamental feature common to all integrated tax systems is the provision of a valuable tax benefit to shareholders on certain dividend income. This benefit is received directly by the shareholder in both dividend exclusion and imputation systems but is received indirectly in a dividend deduction system.\textsuperscript{5} In this regard, a particularly important policy issue is whether and to what extent the benefit of integration is extended to resident tax exempt and non-resident/foreign shareholders. In this paper, we generically refer to this tax benefit, no matter the specific form of integration, as a tax credit.

A key issue for consideration, and the focus of this paper, is what is the cost of capital under an integrated tax system? Under a classical tax system, the after company before personal tax return to shareholders consists of two possible components: capital gains and dividends.

\textsuperscript{3} Commonwealth Department of the Treasury (2002) and van der Hoek (2003). For example, in 2001 Germany replaced its imputation system with a partial dividend exclusion system under which half the distributed profits are exempt from tax at the shareholder level. For further details see Strulik (2003).

\textsuperscript{4} U.S. Department of Treasury (2003). Litzenberger and Van Horne (1978) note that tax reform proposals for the elimination of the double taxation of dividends were also proposed by the Ford and Carter administrations.

\textsuperscript{5} U.S. Department of Treasury (1992 appendix C) shows that the dividend exclusion, imputation credit and dividend deduction integration systems are equivalent only if individual and corporate tax rates are equal, all shareholders are subject to tax and no corporate tax preferences exist. If any of these assumptions do not hold then the three systems produce different outcomes. For example, if corporate and shareholder tax rates differ, then a dividend exclusion system eliminates the shareholder-level tax, a dividend deduction system eliminates the corporate-level tax and an imputation system can be structured to eliminate either the shareholder-level tax or the corporate-level tax.
Comparable returns under an integrated tax system consist of three possible components: capital gains, dividends and the value of tax credits. Accordingly, an integrated tax system has the potential to introduce heterogeneity in investor returns measured on an after company before personal tax basis.

The Sharpe-Lintner CAPM was derived under an assumption of no taxes, but is conventionally applied in a classical tax system to the determination of after company before personal tax returns. In this paper, we abstract from the mechanics of any particular national integrated tax system and extend the Sharpe-Lintner model to a reasonably general integrated tax framework. We assume that investor preferences are defined over (after corporate) before personal tax returns but make no assumptions about the structure of personal taxes. In particular, we avoid complications associated with the modelling of capital gains taxes. We also allow for the heterogeneous valuation of tax credits across all investors. This is particularly important for the resulting equilibrium since it not only caters for different possible outcomes that an integrated tax system may have in relation to (resident) tax-exempt and foreign shareholders, but also for possible heterogeneity within the class of resident taxable investors. Whilst taxable residents generally extract full value of tax credits (ignoring any timing differences between receipt of the dividend and receipt of the tax credit) and non-residents generally extract no value of tax credits, this is not always the case. Since the tax credit under a dividend deduction system is received at the corporate rather than shareholder

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For example, under a dividend exclusion system, tax-exempt investors by definition receive no benefit from integration whereas, under an imputation system, the benefits of integration can readily be extended to this class of shareholder by making imputation credits refundable.

For example, the value of tax credits to resident taxable shareholders varies under an imputation system if credits are not refundable but excess credits may be offset against a shareholder's other items of taxable income. For non-resident shareholders, the value of tax credits depend on international tax considerations including domestic withholding tax liabilities, home country personal tax liabilities and if and how these two items interact through bi-lateral double tax agreements.
level, which in turn simply increases the amount of distributable profits available to shareholders, we concentrate on the cost of capital under the dividend exclusion and imputation forms of integration.

The literature on the pricing of risky assets under an integrated tax system appears in the context of a number of national imputation systems. In most cases, the analysis is based, either explicitly or implicitly, on Brennan's (1970) seminal paper on equilibrium asset pricing with heterogeneous personal taxes, in which it is assumed that investor preferences are defined over the mean and variance of after personal tax returns. Our model is closest to Wood (1997) who also assumes investor preferences are defined over the mean and variance of before personal tax returns, but he allows for only two investor classes – one which fully values tax credits and one which places no value on tax credits – and like Stulz (1981), he assumes an asymmetry in the long and short positions of investors. To the best of our knowledge, there does not appear to be any existing literature on the pricing of risky assets under a dividend exclusion system.

Other related literature on integrated tax systems includes: Stapleton and Burke (1975) and Litzenberger and Van Horne (1978) who examine dividend and capital structure policies under an integrated tax system; Fullerton, King, Shoven and Whalley (1981) who analyse the impact of alternative integration systems on the U.S. economy within a general equilibrium framework; McDonald (2001) who uses a costly arbitrage equilibrium framework to examine

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9 Such an asymmetry also introduces some indeterminacy into the resultant equilibrium asset pricing relation.
the value of German imputation tax credits across ex-dividend dates; Cannavan, Finn and
Gray (2004) who infers the value of Australian imputation credits from the relative prices of
stocks and futures and Bell and Jenkinson (2002) who examine the impact of a legislative
change in the UK which thereafter denied tax-exempt resident pension funds a full cash
refund for UK imputation tax credits. The paper is organised as follows. In section 2 we set
out our assumptions and derive a simple form of the CAPM for an integrated tax system. In
section 3, we examine the composition of investors' optimal portfolios in equilibrium and
show that a three-fund separation holds and we conclude in the final section.

2. THE MODEL

Our model is based on the standard Sharpe-Lintner CAPM framework, as described in
Brennan (1992), augmented for tax credits on risky assets available under an integrated tax
system and the heterogeneous valuation of these tax credits across all investors. We make the
following assumptions:

(i) There is a single time period. At the start of the period, each investor \( i, i = 1, \ldots, m \) is
    endowed with fractions \( e_{ij} \) of risky assets \( j, j = 1, \ldots, n \).

(ii) At the end of the period, asset \( j \) pays a dividend of \( d_j \) and a non-detachable tax credit
    of \( c_j \) with certainty. The (endogenous) initial price of asset \( j \) is \( p_{jo} \) and the
    (exogenous) uncertain ex-dividend end of period price is \( \tilde{p}_{ji} \).

(iii) Investors have homogeneous beliefs about \( \tilde{p}_{ji} = E(\tilde{p}_{ji}) \) and \( s_{jk} = \text{cov}(\tilde{p}_{ji}, \tilde{p}_{kj}) \) for all
    assets \( j, k \).
(iv) The utility of investor $i$ is defined over the mean and variance of before personal tax end of period wealth, $U(W_i, S^2_i)$ with $U_{i1} = \frac{\partial U_i}{\partial W_i} > 0$ and $U_{i2} = \frac{\partial U_i}{\partial S^2_i} < 0$.

(v) Investors may borrow or lend without restriction at the riskfree rate of $r = R - 1$.

(vi) Assets are perfectly divisible, there are no transactions costs, no restrictions on short sales and markets are competitive.

(vii) The before personal tax value of a dollar of tax credits to investor $i$ is $\gamma_i$, $0 \leq \gamma_i \leq 1$.

For simplicity, we assume $\gamma_i$ is constant and independent of portfolio choice.

Assuming a pure exchange economy, investors simultaneously and instantaneously trade at the start of the period, rebalancing their portfolios of risky assets from $e_y$ to $z_y$ with all wealth being consumed at the end of the period. The pre-trade value of investor $i$’s total endowment is then $\sum_j e_{yj} p_{j0}$ and the post trade value is $\sum_j z_{yj} p_{j0} + \sum_j (e_{yj} - z_{yj}) p_{j0}$, where the first term represents the total investment in risky assets and the second term represents the total (residual) investment in the riskless asset. The mean of investor $i$’s end of period wealth is:

\begin{equation}
\bar{W}_i = \sum_j z_{yj} (\bar{p}_{j1} + d_j + \gamma_j e_j) + \sum_j (e_{yj} - z_{yj}) p_{j0} (1 + r)
\end{equation}

and the variance is:

\begin{equation}
S^2_i = \sum_j \sum_k z_{yk} z_{jk} s_{jk}
\end{equation}
Let $Y_i$ be the $n \times l$ vector of investor $i$'s (dollar) endowment with elements $y_{ij} = e_{ij}p_{jo}$; $X_i$ be the $n \times l$ vector of investor $i$'s (dollar) demand for risky assets with $x_{ij} = z_{ij}p_{jo}$; $\mu$ be the $n \times l$ vector of expected rates of return measured on a conventional basis,\(^{10}\) with $\mu_j = (p_{ji} - p_{jo} + d_j)/p_{jo}$; $\delta$ be the $n \times l$ vector of dividend yields with $\delta_j = d_j/p_{jo}$; $\zeta$ be the $n \times l$ vector of tax credit yields with $\zeta_j = c_j/p_{jo}$; and $\Omega$ be the (symmetric, positive definite) $n \times n$ variance-covariance matrix of conventional rates of return on the risky assets with elements $\omega_{jk} = S_{jk}/p_{jo}p_{ko}$.\(^{11}\) Then, the mean of investor $i$'s end of period wealth, expressed in rate of return form, is:

\[
\bar{W}_i = \sum_j x_{ij}(\mu_j - r + \gamma_j \zeta_j) + R\sum_j y_{ij} = (\mu^T - rI^T + \gamma^T \zeta^T)X_i + R I^T Y_i
\]

and the variance is:

\[
S_i^2 = \sum_j \sum_k x_{ij}x_{ik}\omega_{jk} = X_i^T \Omega X_i
\]

where $I$ is the $n \times l$ unit vector and $^T$ denotes transpose. The decision problem faced by each investor is $\max_{X_i} U_i(\bar{W}_i, S_i^2)$. The $n$ first order conditions are $\frac{\partial U_i}{\partial X_i} = \frac{\partial U_i}{\partial \bar{W}_i} \frac{\partial \bar{W}_i}{\partial X_i} + \frac{\partial U_i}{\partial S_i^2} \frac{\partial S_i^2}{\partial X_i} = \Theta$ where $\Theta$ is the $n \times l$ zero vector and the concavity of the utility function ensures sufficiency.

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\(^{10}\) We refer to returns based on dividends and capital gains only as "conventional returns" and returns based on dividends, capital gains and the value of tax credits as "grossed-up returns".

\(^{11}\) Since tax credits are assumed certain, $\Omega$ also represents the variance-covariance matrix of grossed-up returns.
for a maximum. From (3), \( \frac{\partial W_i}{\partial X_i} = \mu - r I + \gamma \zeta \) and from (4), \( \frac{\partial S_i^2}{\partial X_i} = 2 \Omega X_i \) since \( \Omega \) is symmetric. The vector of investor \( i \)'s risky asset demands is then:

\[
X_i = \theta_i^{-1} \Omega^{-1} (\mu - r I + \gamma \zeta)
\]

where \( \theta_i^{-1} = -U_{ii}/2U_{i2} \) is a measure of \( i \)'s absolute risk tolerance. The composition of these optimal portfolios is discussed further in section 3. Market equilibrium requires all investors to choose optimal portfolios such that markets clear. Let \( M \) represent the (initial) value of the market portfolio and \( v \) be the \( n \times I \) vector of market weights with elements \( v_j \). Summing across investors and imposing the market clearing condition that the aggregate demand for each risky asset is equal to the aggregate supply, \( \sum_i X_i = Mv \) yields:

\[
\sum \theta_i^{-1} \Omega^{-1} (\mu - r I + \gamma \zeta) = Mv
\]

Let \( \theta_m = (\sum_i \theta_i^{-1})^{-1} \) be a measure of the market's absolute risk aversion and let \( \gamma = \sum_i \theta_i^{-1} \gamma_i / \sum_i \theta_i^{-1} = \theta_m \sum_i \theta_i^{-1} \gamma_i \) represent a weighted average of the value of tax credits across all investors, with individual weights based on individual levels of absolute risk tolerance. Rearranging (6) and substituting for \( \theta_m \) and \( \gamma \) gives:

\[
\mu - r I + \gamma \zeta = \theta_m \Omega \mu v
\]

Pre-multiplying (7) by \( v^T \) and rearranging allows \( \theta_m M \) to be expressed in terms of the market portfolio:
\[ \theta_m M = \left( \mu_m + \gamma \zeta_m - r \right) / \sigma_m^2 \]

where \( \mu_m = v^T \mu \) is the expected rate of return on the market portfolio, measured on a conventional basis, \( \zeta_m = v^T \zeta \) is the tax credit yield on the market portfolio and \( \sigma_m^2 \) is the variance of the market return. Further, \( \Omega v \) represents the \( n \times 1 \) vector of covariances of asset returns with the market return with \( (\Omega v)_j = \omega_{jm} \). Let \( \beta \) represent the \( n \times 1 \) vector of asset betas with respect to the market portfolio with \( \beta_j = \omega_{jm} / \sigma_m^2 \). Substituting this and (8) back into (7) gives the CAPM for an integrated tax system in relative pricing form:

\[ \mu + \gamma \zeta - rI = \beta \left( \mu_m + \gamma \zeta_m - r \right) \]

or equivalently, for all risky assets \( j; \ j = 1, \ldots, n \)

\[ \mu_j + \gamma \zeta_j = r + \beta_j \left( \mu_m + \gamma \zeta_m - r \right) \]

where \( \mu_j + \gamma \zeta_j \) is the expected grossed-up rate of return on security \( j \) and \( \mu_m + \gamma \zeta_m \) is the expected grossed-up rate of return on the market portfolio. Equation (10) corresponds to the familiar Sharpe-Lintner CAPM linear relationship between individual (excess) security returns and the (excess) return on the market portfolio adjusted for the inclusion of the value of tax credits. In particular, in equilibrium, the after corporate before personal tax expected
rate of return on any security reflects, inter alia, a (complex) weighted average of the value of tax credits across all investors in the market.\textsuperscript{12}

We have assumed a single market portfolio. In applying a CAPM to a particular country, an issue for consideration is whether the domestic capital market is segmented from or integrated with international capital markets. Our model is, in the absence of exchange rate risks, sufficiently general to cover both cases. If the domestic capital market is assumed to be segmented, then the market portfolio is a portfolio of domestic stocks and the equilibrium value of tax credits $\gamma$, is a weighted average over all investors in the domestic market, including foreign investors to the extent that they invest domestically. If the domestic capital market is assumed to be integrated, then the market portfolio is a world portfolio of stocks and $\gamma$ is a weighted average over all investors in the world market.

Finally, let $Z_i$ be the $n \times 1$ vector of investor $i$’s fractional asset demands with elements $z_{ij}$; $E_i$ be the $n \times 1$ vector of investor $i$’s endowment with elements $e_{ij}$; $P_o$ be the $n \times 1$ vector of initial asset prices with elements $p_{jo}$; $\overline{P}_i$ be the $n \times 1$ vector of expected end of period prices with elements $\overline{p}_{ji}$; $d$ be the $n \times 1$ vector of dividends with elements $d_{ij}$; $c$ be the $n \times 1$ vector of tax credits with elements $c_{ij}$; and $\Phi$ be the $n \times n$ variance-covariance matrix of end of period prices with elements $s_{jk}$. Restating (1) and (2), the mean and variance of investor $i$’s end of period wealth is $W_i = \overline{P}_i^T d^T + \gamma c^T Z_i + RP_o^T (E_i - Z_i)$ and $S_i^2 = Z_i^T \Phi Z_i$. Solving the investor decision problem and imposing the market clearing condition, $\sum_i Z_i = 1$, yields the certainty equivalent valuation equation:

\textsuperscript{12} Equation (10) is similar to the pricing relations of Officer (1994) and Lally and van Zijl (2003) posited for the Australian imputation tax system, but only the latter considers aggregation across investors.
or equivalently, for all risky assets \( j \): \( j = 1, \ldots, n \)

\[
P_{j0} = \frac{\bar{p}_j + d + \gamma c_j - \theta_m \text{cov}(\bar{p}_j, \widetilde{m})}{I + r}
\]

where \( \widetilde{m} \) is the value of the market portfolio at the end of the period.

3. PORTFOLIO COMPOSITION

In equilibrium, each investor \( i \) demands the following portfolio of risky assets:

\[
X_i = \theta_i^{-1} \Omega^{-1} \mu - r \theta_i^{-1} \Omega^{-1} l + \gamma_i \theta_i^{-1} \Omega^{-1} \zeta
\]

and invests \( (Y_i - X_i) \bar{r} l \) in the riskless asset. Equation (13) is a separation theorem whereby each investor's optimal portfolio of risky assets is spanned by three "mutual funds", \( \Omega^{-1} \mu \), \( \Omega^{-1} l \) and \( \Omega^{-1} \zeta \) with the weights based on the individual's risk aversion and tax status.

Using standard arguments, the first portfolio \( \Omega^{-1} \mu \) is the minimum variance portfolio subject to a given return, the second portfolio \( \Omega^{-1} l \) is the global minimum variance portfolio and the third portfolio \( \Omega^{-1} \zeta \) is the minimum variance portfolio subject to a given amount of tax credits. A more intuitive interpretation follows from substituting (7) into (5) and rearranging:
where $Mv$ is the market portfolio. Equation (14) suggests a clientele effect on the basis of investors' values of tax credits. Each investor holds a diversified portfolio, but one which is tilted according to the value that the investor places on a dollar of tax credits, $\gamma_i$, relative to the equilibrium value of credits, $\gamma$. In particular, investors who place a higher than average value on tax credits, typically resident taxable investors, will tend to be overweight in assets with higher tax credit yields whereas investors who place a lower than average value on tax credits, typically non-resident investors, will tend to be underweight in assets with higher tax credit yields. The tilt portfolio plays a similar role here to corresponding portfolios in Brennan (1970), Black (1974) and Elton and Gruber (1978).

4. CONCLUSION

In an integrated tax system, the individual and corporate tax systems interact to remove the double taxation of dividends that otherwise occurs under a classical tax system. In this paper, we abstract from the mechanics of any particular national integrated tax system and present a simple CAPM for a reasonably general integrated tax framework, which augments the Sharpe-Lintner model for tax credits on risky assets and the heterogeneous valuation of these tax credits across all investors. We show that each investor holds a diversified portfolio, but one which is tilted according to the value that the investor places on a dollar of tax credits relative to the equilibrium value of credits.
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