Exploitation versus Exploration in Market Competition

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Abstract

A general simulation model of market competition is developed to explore the effectiveness of and interactions between different types product exploration and exploitation strategies i.e. innovation, imitation and process improvement. The model, like real markets, is highly non-linear such that analytical solutions in the form of optimal marketing strategies are not possible. Hence, we use simulation experiments to examine firm survival and the effectiveness of different strategy mixes and show how these depend on the length of time it takes for each strategy to bear fruit, the speed of new product diffusion and the duration of product life cycles and the timing of new product entry. The model is implemented on the Internet and provides the basis for further experiments to examine the impact of different combinations of firm strategies on survival and performance and as a means of honing management sensitivities regarding the impact of different market response functions on the outcomes of strategy.

Introduction

"It is the unending search for differential advantage which keeps competition dynamic" (Alderson 1957 p 102). With these words Wroe Alderson, one of the pioneers of modern marketing theory begins his discussion of the essential nature of the competitive process. Firms are continually seeking to create and deliver advantage or value to customers that are better than alternatives.

They attempt to do this in many ways; in terms of market segments i.e. the customers and their particular needs and problems they target, and in terms of the means of meeting these targeted needs and problems they offer prospective customers. It is a continuing struggle because customers' needs are continually changing, in some markets faster than others, and the means of meeting customer's needs are also changing as a result of technological change and entrepreneurial action that recognizes and exploits new opportunities as they arise. As Alderson describes it, there is a continuing proliferation of opportunities as the satisfying of existing needs begets new needs and opportunities and the development of new technologies and means of satisfaction begets new problems, needs and opportunities.

This conception of competition lies at the heart of Porter's (1985) identification of three generic types of competitive strategies in terms of the twin dimensions of competitive scope and type of advantage. Competitive scope relates to the focus on a particular market segment or to a broad mass market and competitive advantage refers to the form of differential advantage offered – either low cost or some other form of differentiation matching the target markets requirements. The same concept is reflected in Kenichi Ohmae's (1982) depiction of competition in terms of a strategic triangle of three elements of a firm, customer and competitors and the way the firm seeks to offer customer advantage (or value) to target customers that is superior, in the eyes of the customer, to the customer advantage offered by competitors. The ability to offer such differential advantage in turn depends on the supplier advantage, which may be traced to various types of firm, relationship and network resources, and distinctive competencies (e.g. Dyer 1998, Hamel and Prahalad 1994, Hunt 2000, and Ritter 1999).

Our understanding of the dynamics of competitive strategy is limited because traditional analytical techniques have tended to be comparative static in nature, focusing on the way firms can compete by seeking out and occupying niches in the market place by creating and delivering a suitable market offer in terms of product/service functionalities, pricing options, promotional strategies and distribution options. Many sophisticated models of markets have been developed to guide optimal choice among the multiple dimensions of marketing strategy under various assumptions regarding the competitive situation (e.g. Carpenter et al 1988; Lilien et al 1995, Midgley et al 1997). These include various types of oligopoly models that examine the effects of different response functions among closely competing firms on firm behaviour and performance (Baye 2000).

The model presented here is different from previous models in that we focus on two fundamental types of competitive strategies: the extent to which resources are used to exploit current means of meeting customer needs and the extent to which they are used to search or explore for new way of meeting customer needs (including new needs to meet). In the next section we provide a brief overview of these strategies and their relation to marketing strategy.

Exploration versus Exploitation

This distinction between exploitation and exploration lies at the heart of competitive strategy (Nelson and Winter 1982). For example, consider Ansoff's (1965) classic competition matrix, which distinguishes between offering the same or different products to the same or different markets. Strategies focusing on the same products in the same markets are examples of exploitation strategies. All the rest involve some form of exploration, be that new means of serving existing markets, new markets for existing products or new product and market combinations.

The tradeoff between exploitation and exploration is a fundamental dimension of any strategy and is a concept that has been used to help understand the evolution of ecological systems as well as economic systems. For example, studies of social insects have revealed differences in the mix of exploration and exploitation among different types of ant colonies that have evolved in environments with more or less turbulence in the sources of food available (Bonabeau et al 1999). In stable environments with relatively fixed sources of food more resources are devoted to the exploitation of known food sources and less to exploration whereas, in more dynamic environments, ant species evolve that devote more resources to exploration. In the same way firms in a market can adopt a mix of exploration vs exploitation strategies and the optimal balance or trade-off between the two will depend on the strategies of other competitors and the nature and dynamics of the market demand. Furthermore, algorithms based on social insect foraging behavior have been used to solve complex search problems other types of approaches cannot solve (Bonabeau and Meyer, 2001)

James March (1991) summarized the key differences between exploitation and exploration strategies in the following way. “The essence of exploitation is the refinement and extension of existing competencies, technologies and paradigms. Its returns are positive, proximate and predictable. The essence of exploration is experimentation with new alternatives. Its returns are uncertain, distant and often negative”(p. 85). The strategies of exploitation versus exploration may be pursued in various ways. For example firms may choose to devote more resources to working more closely with fewer suppliers, customers and distributors i.e. exploiting existing relations or they may choose to devote more resources to finding new potentially valuable relationships.

Determining the optimal balance of exploration and exploitation strategies is impossible because of the highly non-linear systems involved and the fundamental uncertainty that arises. The benefits of each strategy revolve around the probability of success of each type of strategy and costs and sacrifices involved. This in turn depends on the nature of the market and competitive situation, including the degree of turbulence and responsiveness of the environment and the strategies adopted by competitors. There are uncertainties regarding the timing and payoffs of different strategies and there are nonlinearities resulting from the interactions among different firm strategies and between these strategies and market demand.

In order for management to deal with this kind of problem there is a need to use new ways of understanding a firm's market environment and to develop and hone managers' sensitivities regarding the effect of different market parameters on the outcomes of strategy. Traditional analytical techniques rely on mathematical methods designed to deal with linear systems in which analytical solutions are possible. But as Robert May (1976), one of the pioneers of non-linear systems analysis points out in a classic early paper published in *Nature*:

> " the mathematical intuition so developed ill equips the student to confront the bizarre behaviour exhibited by the simplest of discrete nonlinear systems … Yet such nonlinear systems are surely the rule, not the exception, outside the physical sciences" (p 467).

The same applies to management, as "students" of their own nonlinear competitive market environment. To better develop our intuition regarding the behaviour of nonlinear systems, Robert May advocated the introduction of non-linear systems into elementary mathematical education and over the last decade there has been a rapid growth and spread of general purpose simulation modelling tools such as *Starlogo*, *Stella* and *Ithink*, *SWARM* and *Repast*, that enable students, researchers and manager to build and explore models of nonlinear systems

For management, the solution is to develop simulation models of their market environment that enables them to explore the impact of different combinations of strategies and market conditions on market outcomes. In this way they can greatly enrich their intuition about such systems and be in a better position to more effectively participate in and adapt to changing market circumstances. This approach corresponds to modern developments in management theory that stress the role of participation and learning and adaptation and improvisation (Brown and Eisenhardt 1997; Chelariu, et al 2002; Moorman and Miner 1998a, 1998b; Wilkinson and Young 2002)

Hence, in line with this approach, we develop a model of competitive dynamics to examine the conditions under which different types of exploration and exploitation strategies are likely to succeed. The model allows firms to pursue different types of exploitation and exploration strategies in terms of devoting resources to improve the efficiency of supplying existing products (exploitation) as against using resources to develop new products (exploration). Two forms of exploration are considered: (a) innovation in which a firm devotes resources to the invention of new products and (b) imitation in which the firm devotes resources to copying the products offered by other firms in the market. We use simulation techniques to examine the conditions under which different mixes of exploration, including both innovation and imitation, and exploitation perform best and how this compares to the situation of a firm that devotes no resources to exploitation or exploration and simply continues to supply the same types of products in the same way.

A criticism of these types of simulation models is that they are sensitive to small changes in parameters and that "anything" can be made to happen. This sensitivity to parameter values and starting conditions is the very hallmark of the behaviour of nonlinear systems. It is from this the celebrated butterfly effect is derived, whereby the flapping of a butterflies wings in the Amazon jungle can lead to a storm system moving into New York. The problem is to investigate the pattern of behaviour that can occur under different, plausible parameter settings and to gain insight into how a system may be coaxed into exhibiting desirable behaviour characteristics. This can only be done by systematic simulation experiments because of the inherent nonlinear nature of the system and it required the advent of modern computers and software systems to permit such simulations to be run in any realistic time frame. Thus we need to abandon our prejudices borne of simulations in the past where the full repertoires of behaviour of a system could not be investigate or even contemplated. Moreover researchers in marketing are starting to make more use of this types of modelling (e.g. Goldenberg et al 2001, Hibbert and Wilkinson 1997, Midgley et al 1997, Wilkinson et al 2001).

Previous Oligopoly Models

Our model is a form of oligopoly model. Such models address the problem of interdependence among a limited number of competitors in a market and the performance of any one firm depends on what other firms are doing and how they respond to each others' actions (Baye 2000). It is beyond the scope of this paper to review this vast literature but some general observations may be made (see for example Baye 2000 for an overview). Various types of models have been proposed regarding how an individual firm should behave so as to maximize its performance under various assumptions about rivals' behaviour and responses. The models include ones based on comparative static analysis such as the Sweezey and Cournot models as well as dynamic models based on game theory.

These models focus on how firms adjust aspects of their marketing strategy for existing products such as price, quantity supplied, promotional effort etc. Depending on the response functions assumed complex dynamics can result, including chaos, with no clear winning strategies for individual competitors (Hibbert and Wilkinson 1994, Midgley et al 1997).

Another type of model is of the type pioneered by Nelson and Winter (1982) and are based on the work of Schumpeter. These are evolutionary models in which firms innovate and imitate other firms' strategies and products. These once again produce complex dynamics and admit of many possible evolutionary paths.

Models of this latter type are part of the fast growing area of complex systems analysis or complexity sciences, sometimes referred to as artificial life. They focus on dynamic systems comprising interdependent interacting elements in which order emerges in a bottom up self-organising fashion, rather than being imposed on the system by any central authority. This is exactly the form of oligopoly models. As Tefatsion (1997) characterises these types of models:

> "[T]he actions of each unit depend upon the states and actions of a limited number of other units….. The complexity of the system thus tends to arise more from the interactions among the units than from any complexity Inherent in the individual units per se" (p534)

The model developed here is designed to contribute to our understanding of the effects of innovation and imitation strategies on firm performance. We seek to examine how the outcomes from pursuing different explorations and exploitation strategies depends on how other firms in the market are competing for differential advantage. In order to do this we build a model of a market involving a limited number of competitors and use a systematic set of simulation experiments to examine how different combinations of competitors in a market affect the nature of the "optimal" competitive strategies firms should adopt. Thus we provide guidance for firms operating in real markets in terms of the appropriate competitive strategies to adopt under different competitive regimes.

Our model is different from previous models of oligopoly competition. Innovation and imitation strategies formed part of Nelson and Winter's models but they did not examine the effect on firm performance of different mixes of such strategies. Further, innovation and imitation strategies are not like the marketing strategies that form the basis of most oligopoly models as they cannot be directly observed and copied by competitors. Only the results of these strategies can be observed in the form of new products, successfully copied products and, to a lesser extent, process improvement in the form of increased productivity. Moreover, the results of such exploration strategies are delayed and uncertain, as the preceding discussion makes clear. Hence, firms have to trade off the gains from exploitation, in the form of devoting more resources to the making and selling of existing products using existing process, or to devote some resources to exploration strategies in the anticipation of future benefits. To our knowledge no existing models of market competition attempt to do this.

The simulation model developed must necessarily be a simplification of real world markets. But this is its purpose. By extracting from the real world key dynamic processes we are able to examine their role and impact in ways that are otherwise impossible. This approach to analysis is gaining increasing favour amongst scientists in many disciplines as they seek to understand the key processes underlying the development of economic, social and business systems (Anderson and Valente 2002, Casti 1997, Langton 1996, Iansiti 2002, Tesfatsion 1997, 2002) and as researchers attempt to find solutions to complex nonlinear dynamic problems. There has also been some work in marketing using such models (Goldenberger et al 2001, Wilkinson et al 2001). As Chris Langton (1996), one of the founders of the science of Artificial life observes in the context of research biology:

> "We trust implicitly that there are lawful regularities at work in the determination of this set [of realized entities], but it is unlikely that we will discover many of these regularities by restricting ourselves only to the set of biological entities that nature actually provided us with. Rather, such regularities will be found only by exploring the much larger set of possible biological entities" (p x).

The same may be said for studies of business ecosystems and markets in which we are restricted to studying the kinds of behaviour that managers happen to have manifested in the marketplace, rather than the kinds of behaviour that could exist and perhaps may be preferable.

The model in outline

The model described here is designed to enable the performance of firms to be compared when they allocate resources different mixes of exploitation and exploration strategies. The exploitation strategies are (a) producing existing products with existing technology, or (b) process improvement i.e. using resources to improve the efficiency of production for existing products. The exploration strategies are (a) innovation i.e. using resources to discover new types of products, or (b) imitation i.e. using resources to copy new products produced by competitors.

The market model is of a "closed economy"— that is, a trading environment in which a fixed total amount of resources, modeled here in terms of a firm's number of employees, is used by firms for generating product. Furthermore, a form of Say's Law applies in that supply creates demand (Sowell 1972). Demand is completely endogenously determined because the firms' employees are also the source of market demand for products. No external source of demand exists. Using the money they receive for their labour services people purchase the products offered by firms in the market. . All this takes place in successive, discrete time periods. At the beginning of each time period each firm has a budget for its labour. Each firm hires labour to the full extent of its labour budget. In the "final few moments" of each time period the following things happen:

- labour is paid by the firms in exchange for their work during that time period—at this stage labour has all the money and the firms have none;
- the firms are paid by labour in exchange for the product—all product is either sold or written off before the next time period starts—at this stage the firms have all the money and labour has none;
- the firms are now "cashed up" and they commit all of their money by hiring labour for the next time period.

If no one buys a firm's products in a period it receives no income. It will have spent all of its budget on hiring labour for that time period, will have nothing left for the next time period, and so it will go out of business. A firm's profit in a time period is the amount that it receives for selling its product at the end of that time period less the amount that it spent on hiring labour at the beginning of that time period. If a firm makes a profit during a time period then its budget is increased in the next time period and so it will hire more labour than in the previous time period. If it makes a loss then its budget is decreased and size of its labour force contracts in the next time period. The objective of each firm is to survive. The total amount of money in the economy remains constant in time and is all placed on the table at the end of each time period as described above. The size of the labour force also remains constant as does the total and per capita remuneration that labour receives. At the beginning of each time period all money is committed by firms to hiring labour.

The firms differ in the way in which they allocate resources at the beginning of each time period to the four types of strategies. The four strategies are realised by allocating labour to four job types:

- *workers* who produce product—the proportion of firm i's money spent on workers is w_i.
- *process improvers* who improve work processes by generating "process knowledge"—that is knowledge of how to produce product better—the proportion of firm i's money spent on process improvers is p_i.
- *imitators* who design processes for producing products that have been discovered by other firms—the proportion of firm i's money spent on imitators is m_i.
- *innovators* who discover new products—the proportion of firm i's money spent on innovators is n_i.

If a firm discovers a new product during a time period then, at the end of that time period, other firms may decide to attempt to copy that product.

The objective of the simulation experiments described here is to understand the effect of values for the four basic variables w_i, p_i, m_i and n_i on a firm's performance. These variables are constrained by:

$$0 \leq \{w_i, p_i, m_i, n_i\} \leq 1$$

$$w_i + p_i + m_i + n_i = 1$$

for $i = 1,..,n$ where n is the number of firms.

Structure of the model

The basic structure of the model, from the point of view of the economy, is shown in Figure 1. It owes much to [Andersen and Valente, 2002]. At the beginning of each time period a labour force of fixed size is fully employed by a number of firms at a fixed wage rate. During each time period, the total costs for each firm are the amount it spends on hiring labour. The total costs for firm i are C_i. The total costs for all firms is $\Sigma_i C_i$, and this amount of money is entirely spent on hiring labour and so this is also the amount of money that the entire labour force will spend at the end of the time period when they purchase products. In each time period firm i allocates the effort of its workers across the range of products that firm i knows how to produce. That allocation of workers will lead—as determined by each product's process knowledge—to the generation of actual product $\underline{Q}_i$ for firm i—where the underlining notation denotes a vector $\underline{Q}_i = [q_{i,1}, q_{i,2}, q_{i,3},....]$—that is, $q_{i,j}$ is amount of the j'th product that firm i produces in the time period. The total quantity of the j'th product that is available at the end of the time period is $\Sigma_i q_{i,j} = q_i$. The total output, produced by all firms, at the end of the time period is represented as the vector $\underline{\mathbf{Q}} = [q_1, q_2, q_3,...]$. The price of the various types of product is determined so that the total cost of all products is exactly the same as the amount of money that labour has to spend. That is, price is set to ensure that supply equals demand. At the end of the time period the entire labour force "goes shopping" and purchases all of the output $\underline{\mathbf{Q}}$. The $\underline{\mathbf{Q}}$ vector is unbounded in length although at any time only a finite number of entries in it will be non-zero.

Innovation takes place when one firm begins producing a product that has not been produced before. For example, if:

$$\underline{\mathbf{Q}} = [2, 3, 0, 2, \mid 0, 0, 0, 0,.....]$$

then this means that 2 units of product 1 are available, 3 units of product 2 and 2 units of product 4. New products discovered as a result of innovation are introduced to the right of the marker '|', and the marker is moved along so that it remains to the right of the most-recently-discovered product. So in the **Q** shown above product number 3 is "out of production". Suppose that, in addition to the products in **Q**, one of the firms is an innovator and that it commences production of a new product. This new product will be numbered 5. Suppose that the innovating firm produces 2 units of product 5 then the augmented product vector is:

Q = [2, 3, 0, 2, 2, | 0, 0, 0,.....]

Having reviewed the range of products that are available at the end of a time period, labour will have preferences over which particular products they desire. These preferences are expressed by labour attaching a "relative demand" measure [described below in the sub-section "Determining demand"] across the range of available products in **Q**. The *relative demand* **D** is used directly to determine the relative price per unit. For example, if the relative demand of product 1 is 0.8 and the relative demand of product 2 is 1.2 then the price per unit for product 2 will be 1.5 times the price per unit for product 1. At the end of each time period labour also places the total amount of money available, M, "on the table". M remains constant in time. The actual prices **P** are set in proportion to the relative demand so as to clear the market. So the only way in which a product will be unsold is if its relative demand, is zero. For example, suppose that the total amount of money is 100, consider the following output and relative demand vectors:

Q = [2, 3, 0, 2, 2, | 0, 0, 0,.....]

D = [30, 30, 0, 0, 25, | 0, 0, 0,.....]

This will result in product 1 being sold at 15 per unit, product 2 being sold at 15 per unit and product 5 being sold at 12.5 per unit. The 2 units of product 4 are unsold, and are written off by the firms that produced them. The total proceeds from selling at these prices is 100, which is also the total amount of money available.

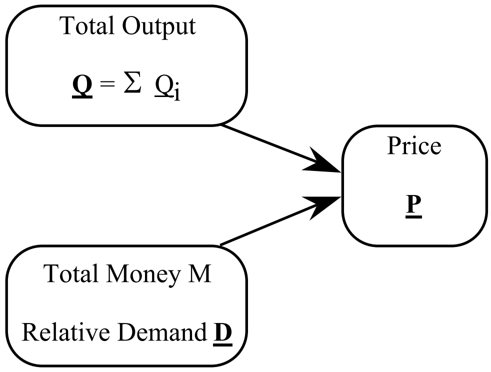


Figure 1. The model from the point of view of the economy.

Having determined the price vector $\underline{\mathbf{P}}$, the model from the point of view of firm i is shown in Figure 2. Consider the time period [t – 1, t]. At the beginning of this previous time period the firm will have carried over its revenue R_i^{t-2} derived in the previous time period and will have fully committed this revenue to hiring labour. The way in which the output vector $\underline{Q}_i^{t-1}$ and the costs C_i^{t-1} are determined for the products produced *during* the time period [t – 1, t] is described below in Figure 3. Having determined the output vector, and having calculated the price vector $\underline{P}_i^{t-1}$ so as to clear the market as described above, the revenue for firm i, which is derived at the end of the time period [t – 1, t], is:

$$R_i^{t-1} = (\, p_1 \_\, q_{i,1} \,) + (\, p_2 \_\, q_{i,2} \,) + \quad = \Sigma_j \, (\, p_j \_\, q_{i,j} \,)$$

Hence the profit for this time period, S_i^{t-1}, is determined and so is the revenue that will be carried over to the next time period. The "anti-clockwise loop" shown in Figure 2 goes "round and round" from one time period to the next.

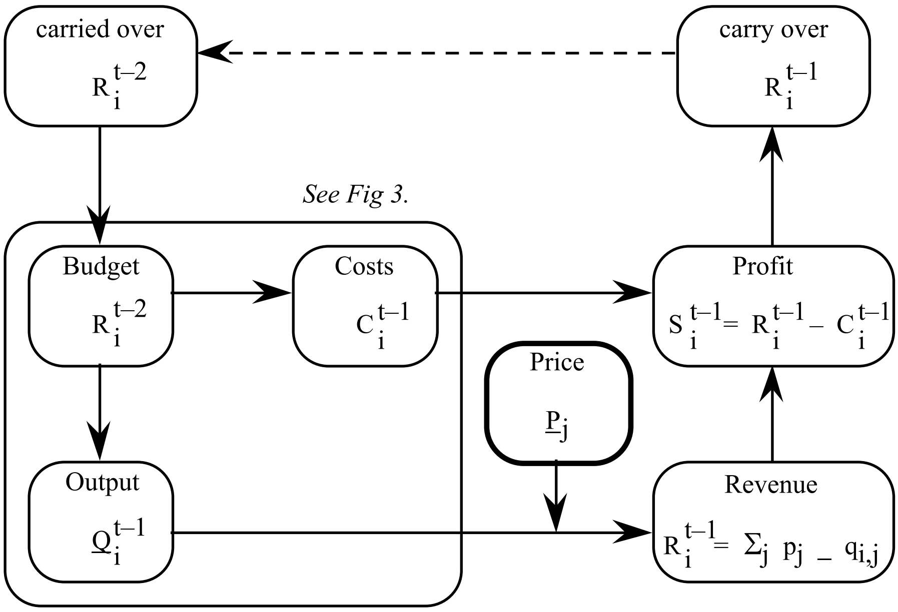


Figure 2. The model from the point of view of a particular firm i.

Figure 2 does not show how the carry over amount R_i^{t-2}, available at the start of time period [t – 1, t], generates output $\underline{Q}_i^{t-1}$ and costs C_i^{t-1} by the end of that time period. This is shown in Figure 3. The horizontal dashed line in Figure 3 divides the figure into two time periods: [t–2, t–1] in the upper part, and [t–1, t] in the lower part. First, the carry over amount R_i^{t-2} from [t–2, t–1] becomes the budget for the time period [t–1, t]. The budget R_i^{t-2} is entirely committed to hiring labour in the time period [t–1, t]. That is:

$$L_i^{t-1} = \frac{R_i^{t-2}}{c}$$

where c is the constant wage rate. For simplicity, c is set to unity. So a “unit of money” is the cost of a unit of labour for one time period. Labour is split in the proportions $w_i : p_i : m_i : n_i$ into the four categories workers, process improvers, imitators and innovators. The imitators attempt to build processes for producing products that have been discovered by other firms. If they are successful then they create a level of manufacturing expertise, or process knowledge, that is represented as a vector:

$$\underline{Imi}_i^{t-2}$$

For example:

$$\underline{Imi}_i^{t-2} = [0, 0, 1.0, 0, 0, 0, 0, 0, ...]$$

contains process knowledge with value 1.0 concerning product 3. The value 1.0 is added to the third place of the firm's process knowledge vector—this is described below. The value of a firm's process knowledge for a product will be 0.0 if the firm can not produce that product, and 1.0 if it has discovered how to produce that product by either innovation or imitation. The value of the process knowledge may then be increased to an integer value greater than 1.0 by the firms process improvers. The process improvers generate process knowledge for products that the firm already produces.

A firm's process improvers are allocated to improving the manufacturing processes for particular products. The i'th firms *process knowledge* is denoted by a vector $\underline{A}_i$. In the time period $[t-1, t]$ the process improvers may have found new process knowledge $\underline{Pro}_i^{t-1}$—as for $\underline{Imi}_i^{t-2}$ this knowledge is represented as a vector denoting the product(s) that are the subject of the generated process knowledge. Likewise the innovators $\underline{N}_i$ may discover process knowledge for new products, $\underline{Inno}_i^{t-1}$. All knowledge generated during one time period may only be used in subsequent time periods, and so each firms process knowledge available in the period $[t-1, t]$ is:

$$\underline{A}_i^{t-1} = \underline{A}_i^{t-2} + \underline{Imi}_i^{t-2} + \underline{Pro}_i^{t-2} + \underline{Inno}_i^{t-2}$$

That is, each firms process knowledge accumulates from one time to the next. It remains to describe how a firm's workers use this knowledge. Firm i's workers are distributed across the range of products that the firm can produce as represented by the vector $\underline{W}_i^{t-1}$. [The way in which this distribution is done is described below in the sub-section "Determining supply".] The quantity of output that the workers generate in the time period is:

$$\underline{Q}_i^{t-1} = \underline{A}_i^{t-1} \_ \underline{W}_i^{t-1}$$

where the _ symbol means that the vectors are multiplied together element by element.

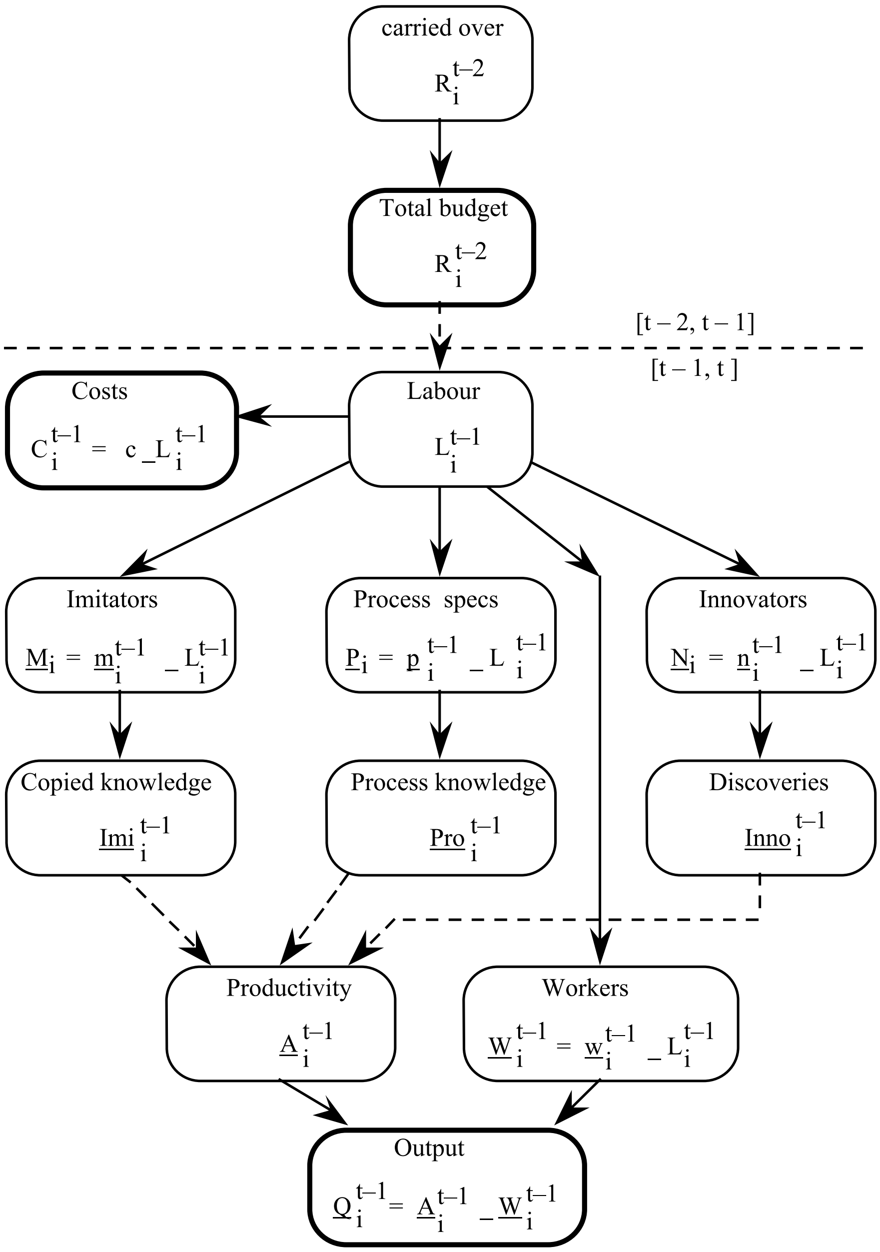


Figure 3. An allocation of resources leads to output and costs for firm i. The dashed line separate two time periods, and dashed arrows mean that the new knowledge is not available until the following time period.

The model in detail

Determining demand

The price of each type of product is determined at the end of each time period by the amount of product generated in that time period, by the total amount of money available, and by the "relative demand" for the different types of product which is determined by labour's preferences. Relative demand reflects the preferences of labour for different types of product. So a model of relative demand for each product is required to calculate unit price, as is a model of supply—ie: the product generated. Relative demand is considered now, and supply is considered in the next sub-section.

A modified Bass model or penetration model (Fourt and Woodlock 1960; Mahajan, Muller, and Bass 1990) with repeat purchase is used to model relative demand for different products in the market subject to a fixed total overall market demand. The rate of new product diffusion the rate of repurchase depends on the type of product or service, as numerous studies of new product diffusion and adoption have indicated. Thus the rate of development of the demand for automobiles is not the same as that for a new beverage because at best each member of the population will purchase one or two automobiles but may purchase a beverage repeatedly. The type of products that we have in mind in developing our model are packaged food products in a market with fixed total demand. In each time period there is a total demand for a fixed s units of product (eg: s packaged dinners). σ is called the *market size*. Given a particular product (eg; a particular packaged dinner), in a particular time period $[t-1, t]$, the *initial penetration*, P^{t-1}, is the size of the population who has purchased this product at least once either during or before this time period. In time period $[t-1, t]$, the *first-time sales*, N^{t-1}, are sales made of this product during this period to those who have not purchased this product previously. Suppose that the growth of initial penetration t is proportional, for some *penetration constant* γ, to the size of the population that has yet to purchase this product. Then initial penetration in time period [t – 1, t], P^i, satisfies:

$$P^0 = \gamma \_ \mu$$

$$P^1 - P^0 = \gamma \_ (\mu - P^0)$$

$$P^2 - P^1 = \gamma \_ (\mu - P^1)$$

Or as a continuous approximation:

$$\frac{dP}{dt} = \gamma \_ [\ \mu - P\ ]$$

Solving this differential equation gives the initial penetration:

$$P^t = \mu \_ (1 - \exp(-\gamma\, t\,)\,)$$

First-time sales is the rate of change of initial penetration. So if N^t is first time sales at time t:

$$N^t = P^t - P^{t-1}$$

and as a continuous approximation:

$$N^t = \frac{dP^t}{dt} = \mu \_ \gamma \_ \exp(-\gamma\, t\,)\,) \qquad (1)$$

First-time sales for a market of size σ = 100 and penetration constant 0.1 is shown in Figure 4.

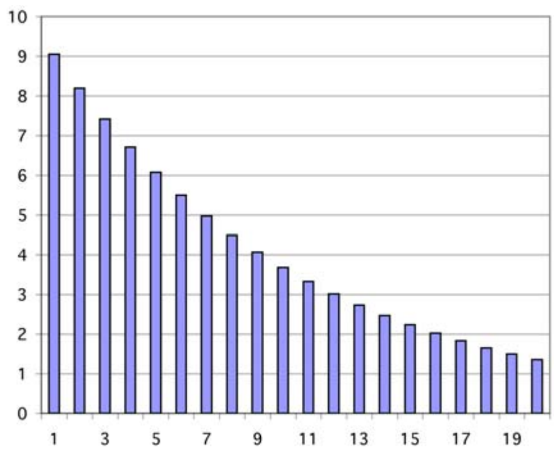


Figure 4. First-time sales, N^t, for a market of size 100 and $\gamma = 0.1$.

Now suppose that once labour has purchased a product, labour continues to purchase that product with a probability of α. That is, if T^i is total sales in time period [i – 1, i]:

$$T^{i+1} = N^{i+1} + \alpha \_ T^i$$

where N^i is first time sales in time period [i – 1, i]. Then:

$$T^0 = N^0$$

$$T^1 = \alpha \_ N^0 + N^1$$

$$T^2 = \alpha^2 \_ N^0 + \alpha \_ N^1 + N^2$$

etc

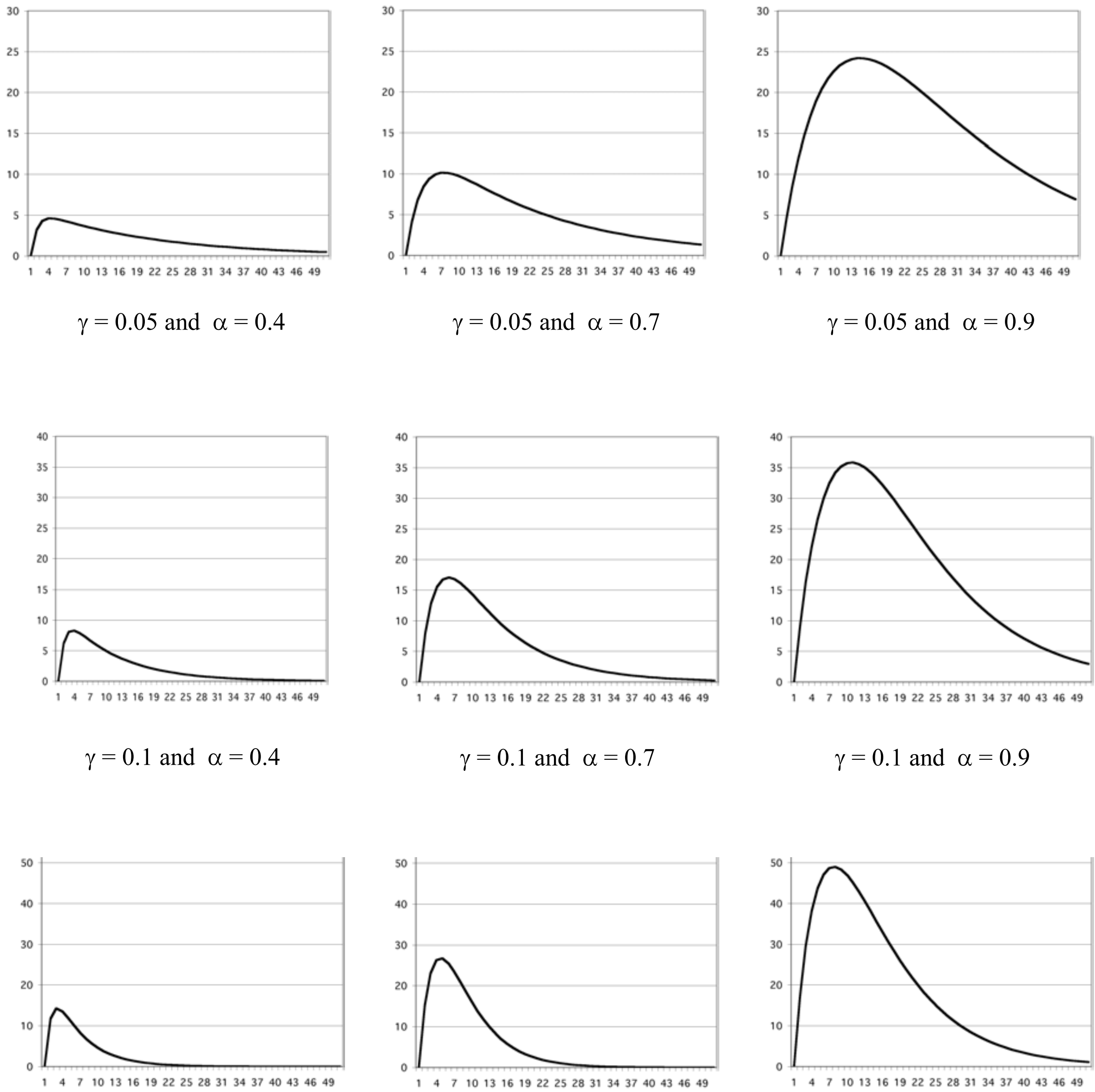
Or as a continuous approximation:

$$T^t = \int_{i=0}^{t} \alpha^{t-i} \_ N^i \_ di$$

Evaluating this using equation (1):

$$T^t = \frac{\mu \_ \gamma}{\ln(\alpha) + \gamma} \_ [\, \alpha^t - \exp(-t \_ \gamma)\,] \quad (2)$$

Which, for a market size of $\sigma = 100$ gives total sales values for each time period as shown in Figure 5 for various γ and α. The sales graphs in Figure 5 are now used to model relative demand. The discovery of a new product by an innovating firm can lead to a substantial shifts in demand for different firm's products, depending on how rapidly the new product diffuses through the market and the peak demand achieved, which depend on the values of the parameters α and γ. For example, with $\gamma = 0.2$ and $\alpha = 0.9$, there is a rapid growth in demand for a new product to nearly 50% share within 8 time periods. The choice of α and γ in the simulations described below substantially effects the speed and extent of new product diffusion and the duration of the life cycle of the product, as can be seen from the illustration in Figure 5. This affects the results of firm's using different strategies as we will show.



$\gamma = 0.05$ and $\alpha = 0.4$ | $\gamma = 0.05$ and $\alpha = 0.7$ | $\gamma = 0.05$ and $\alpha = 0.9$

$\gamma = 0.1$ and $\alpha = 0.4$ | $\gamma = 0.1$ and $\alpha = 0.7$ | $\gamma = 0.1$ and $\alpha = 0.9$

$\gamma = 0.2$ and $\alpha = 0.4$ | $\gamma = 0.2$ and $\alpha = 0.7$ | $\gamma = 0.2$ and $\alpha = 0.9$

Figure 5. Total sales for each time period for a market of size $\mu = 100$ and various γ and α.

For a given market size σ, equation (2) has two variables: α and γ. Given the values of a total sales function in the first two time periods, f^0 and f^1, it is easy to calculate α and γ:

$$\gamma = \frac{f^0}{\sigma}$$

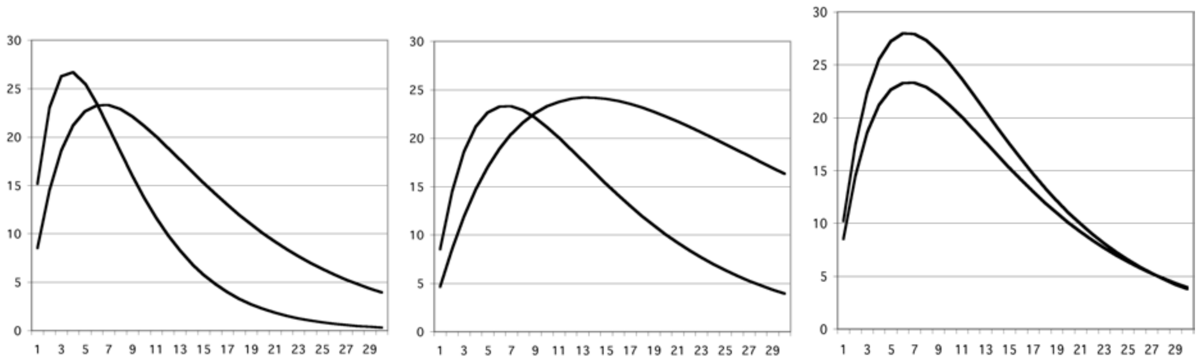
$$\alpha = \frac{f^1}{f^0} - \frac{\sigma - f^0}{\sigma}$$

and so knowing the first two values of a total sales function is to know "all there is" about it.

Returning now to the problem of modelling relative demand. The general shape of the total sales function in Figure 5 is a fair description of how interest in a new product, such as packaged foodstuffs, might be expected to develop. Equation (2), for some values of α and γ is used here to model relative demand. So each product has a *relative demand* determined by equation (2), with its own values of γ and α, for some fixed arbitrary σ, say, $\sigma = 1$. Consumers distribute their money over the different products in proportion to their relative demand **D** for each product as described above. *The prices per unit of the products is in proportion to their relative demand, and are set so as to clear the market.*

Determining supply

The only flexibility that a firm has is first to choose its parameters: w_i, p_i, m_i and n_i, and second to decide what products its workers should produce. This second question is considered now. At the beginning of each time period each firm knows all about the relative demand for the products that it is able to produce. A rational choice of which product to produce in which quantities may not be obvious. For example, consider the choices in the illustrations shown in Figure 6—all of which were produced using equation (2). In (a) and (b) the function that appears to be the most valuable turns out not to be by time 6 or 7. In (c) the function that appears to be the most valuable remains so until time 28—so choosing between those two functions is not simple as the choice will depend on whether anything "interesting" occurs between time 1 and time 28.



(a) (b) (c)

Figure 6. Choosing output on the basis of relative demand.

Suppose a firm has to choose between two products such as those whose relative demand functions are shown in any of the three pairs in Figure 6. Consider Figure 6(a) in which the graphs cross at time 6: a rational decision could be to produce the product with the higher relative demand up to time 6 and then to change to the product with higher demand after time 6. But if a firm were to behave in this way then the second product function should have been drawn starting at time 6. Some simplifying assumption is required to prevent these considerations from over-complicating this investigation. After all, the objective of this investigation is to explore changes in performance resulting from modification of the four basic parameters. *So each firm in the simulations will produce all of the products for which it possesses process knowledge, and will do so in quantities proportional to their relative demand.*

Analysis

Innovation and Trading Off Exploration and Exploitation

Suppose a number of identical firms commence production of a range of products at the same time, and each produces their air share of products —as described in the previous sub-section. As time passes the relative demand of each product will rise and then decline as determined by (2), as illustrated for a number of different values of γ and α in Figure 5. If the firms manage the production of their respective products in the same way then their market share will remain the same; customers (labour), on the other hand, will become less enthusiastic about purchasing the products but will continue to do so as there is no choice.

This situation is reminiscent of collusive oligopoly that operate through an informal "wink and a nod" or by some, possibly illegal, collusive agreement. This state of affairs will be perturbed if one of the firms discovers a new product. Initially the relative demand of this new product may rise well above that of the established products whose product life cycles may now be well into their decline stages. On introduction of this new product the innovating firm will benefit by being a monopolist and gain from the demand for this new product, but monopolistc pricing strategies are ruled out by our market clearing price system.

But, innovation comes at a price. In order to innovate, the i'th firm must allocate some of its labour to innovation by setting $n_i > 0$. Suppose that two firms commence production of the same, single product at the same time. Suppose that the first firm allocates all of its labour as workers, $w_1 = 1$, and that the second firm divides its labour equally between workers and innovators, $w_2 = n_2 = 0.5$. The role of the innovators in the second firm is to discover a new type of product, and, until they do, they contribute nothing. If the innovators fail to discover a new type of product then the size of the second firm will decline. Suppose that both firms initially have 100 units of labour that are paid one unit of money each per time period, and suppose that each firm's process knowledge is 1.0 for each product. At the end of the first time period the first firm will produce 100 units of product and the second firm 50. So two thirds of the revenue of 200 will go to the first firm and one third to the second. In the second time period the first firm will employ 133 units of labour and the second firm 67. The continuing development of their respective sizes is shown in Figure 7. In nine time periods the second firm is less that one per cent of its original size. However, if the second firm had set $w_2 = 0.9$ and $n_2 = 0.1$ then it would have taken 52 time periods for the size of the firm to become less than one per cent of its original size. In general if L_2^{t-1} is the size of the second firm at the end of time period [t – 1, t] then:

$$L_2^t = \frac{TR \_ (1 - n_2) \_ L_2^{t-1}}{TR - (L_2^{t-1} \_ n_2)}$$

where TR is the total revenue, ie: 200 in the example above. One advantage of defining relative demand using equation (2) is that a firm can only go out of business "analytically" by choosing to have no workers; ie: by setting $w_i = 0$. The size of the firm illustrated in Figure 7 becomes smaller and smaller without actually reaching zero.

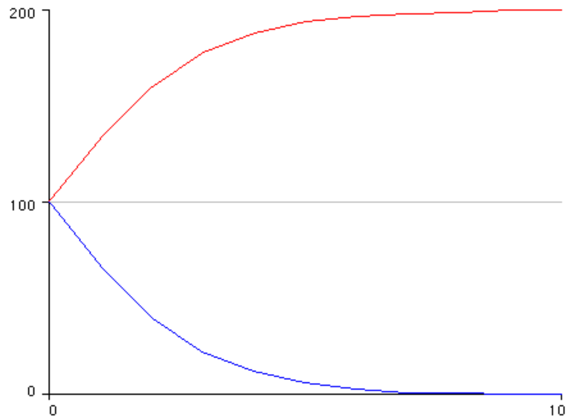


Figure 7. The development of the sizes of a non-innovator firm and an innovator firm each of which has an initial size of $\nu = 100$. The innovator firm allocates 50% of its labour to innovation, fails to discover a new product and dies.

Suppose that the second, innovating firm discovers a new product in time period 10. This may or may not be a good thing in the medium term as Figure 8 illustrates. Even if a new product is valuable, an innovating firm may find that it is so depleted that it is unable to produce sufficient quantities to benefit from this new product in the medium term. The problem here is classic tradeoff between exploration and exploration. . It is a problem of trading off the future benefits to exploration, in this case innovation, against the more immediate benefits of greater exploitation of the demand for existing products, i.e. devoting more resources to producing current products. The tradeoff in turn depends on the timing of new product discoveries and the speed and size of the market that will result. For example, Figure 7 shows that setting n_i = 50% results in a firm having no resources left to exploit the results of its innovation.

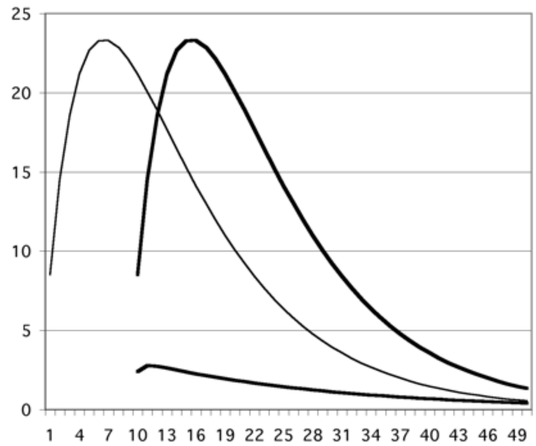


Figure 8. Two new products introduced at time 10—the figure shows the relative demand of the three products.

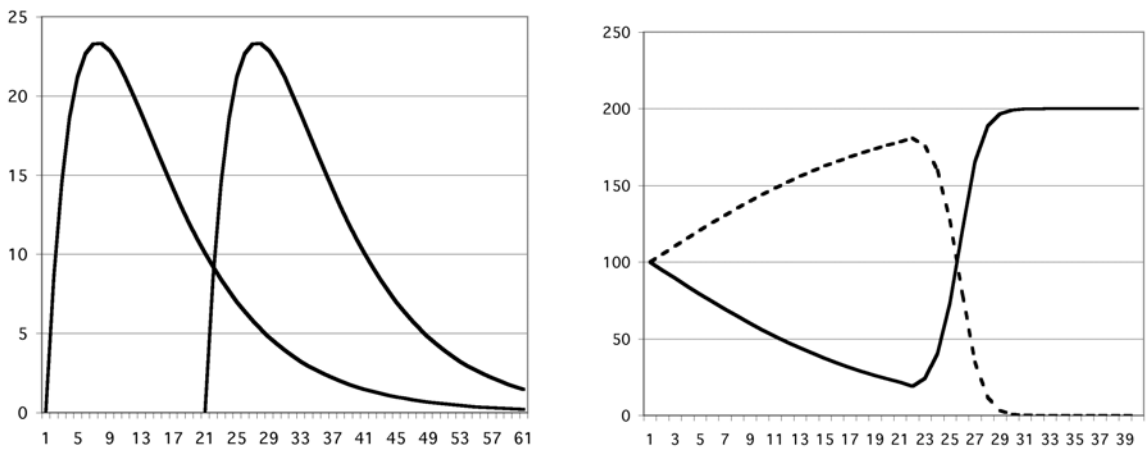
In rough terms, we think of a time period as being one week. The rate at which the i'th firm can expect to discover a new product will depend on the amount of person-hours spent on innovating, and this will depend on both its total labour force and on the proportion of its labour force allocated to innovation, n_i. For example, suppose that at the beginning there are two firms that have the same size and that the first firm sets $w_1 = 1$, and that the second sets $w_2 = 0.9$ and $n_2 = 0.1$. If the mean of the random discovery process is twenty weeks then by time 20 the second firm will be 23.8% of its original size—by time 30 it would have shrunk to 9% if its original size.

The trade-off between innovation and exploiting the demand for existing products depends on the speed and amount of market penetration for new products, as well as the time between new product discoveries noted already. This can lead to first mover advantages and disadvantages, as is illustrated in the following example.

First Mover Effects

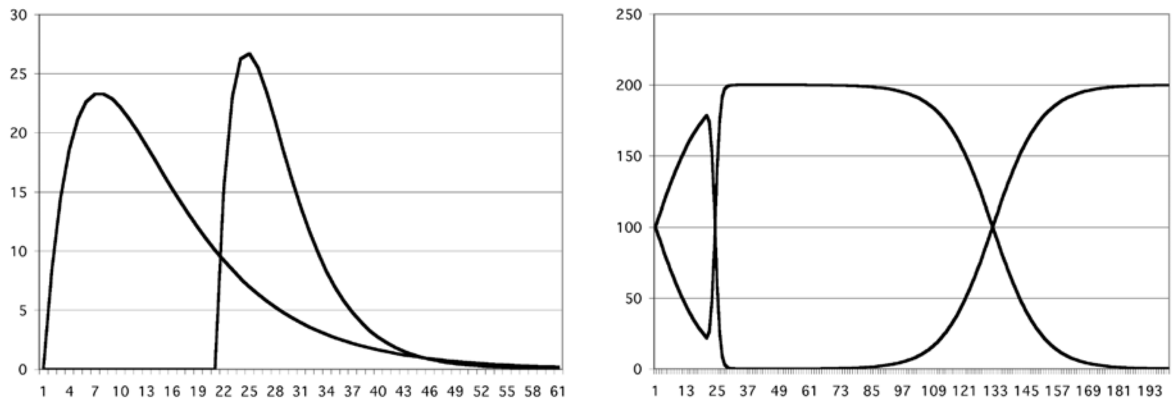
Suppose two firms start at time zero each producing a product with $\gamma = 0.1$ and $\alpha = 0.8$, and that in time period [20, 21] the second firm discovers a new product with the same γ and α. The relative demand curves are shown in Figure 9(a). From time 21 onwards the second firm will produce the original product and its new product in quantities that are proportional to their relative demand—as described above. How will the second firm fare? Not badly it would seem, as is shown in Figure 9(b). The second, innovating firm's size reaches a low of 19 in time period [21, 22] and then turns up quite sharply reaching 99.6% of the labour force by time period [29, 30]. The first, non-innovating firm has virtually been obliterated by time 40. The problem for the first firm is that the second firm has a new product whose relative demand remains above that of the only product that the first firm can produce. Thus innovation is rewarded.

However, both firms may survive depending on the pattern and size of diffusion of the new product. Figure 10 shows the results for a market in which the parameter settings for the original product's demand function are $\gamma = 0.1$ and $\alpha = 0.8$ and for the new product for the second firm they are $\gamma = 0.2$ and $\alpha = 0.7$. Under these conditions, by time 200 the beneficial effect of the second, innovating firm's new product has passed and it is obliterated. The reason for this being that the graphs in Figure 10(a) cross at time 45, after which the original product has a higher relative demand, but the second firm continues optimistically to make a mix of both products. What happens within 200 time periods is shown in Figure 10(b).



(a) (b)

Figure 9. For two firms a new product with $\gamma = 0.1$ and $\alpha = 0.8$ is introduced at time 20. (a) shows the resulting relative demand of the two products and (b) shows the resulting sizes of the two firms.



(a) (b)

Figure 10. For two firms a new product with $\gamma = 0.2$ and $\alpha = 0.7$ is introduced at time 20. (a) shows the resulting relative demand of the two products and (b) shows the resulting sizes of the two firms.

Clearly, the benefits of a new product must be great enough to allow the innovating firm to recover its investment. The insight from the simulation is how this depends on the timing of the new product introduction and from the pattern of growth of demand over time of the new product *relative to existing products*. In Figure 10, if the new product entered later it would sustain its superior demand over the existing product for longer but this would be at the cost of increased investment in innovation. In a later section we examine in more detail the return on investment from different types of strategies and how this is depicted in the simulation results.

In the simulations described here, *a firm discovers an innovation when the total amount of its labour periods allocated to innovation reaches an innovation target. The innovation target may be a fixed amount or may be randomly determined in some interval.* Once a firm has discovered an innovation, its "innovation counter" is decreased by the amount of the innovation target for the next discovery.

If two innovating firms are competing then what is the optimal value of n_i? This will depend on the values of γ, α and the innovation threshold. These proved difficult to calculate. Perhaps this is due to computational round-off errors, or perhaps there are only "approximate" optima.

These matters are not resolved here and await further research investigation. The optimal percentage to allocate to innovation, for various innovation thresholds and different values of the two demand parameters, are shown in Table 1. The table shows us that the higher the innovation threshold the greater the percentage of resources that should be devoted to innovation. It also shows that the more rapid and greater is product diffusion [see Fig 5] the lower the optimal value for the innovation allocation n_i. The latter result occurs because there is more to be gained (and hence lost) by focusing resources on exploiting the demand for existing products or any new product developed than there is to devoting resources for developing new products.

	α = 0.4			α = 0.7			α = 0.9		
	γ = 0.05	γ = 0.1	γ = 0.2	γ = 0.05	γ = 0.1	γ = 0.2	γ = 0.05	γ = 0.1	γ = 0.2
20.0	4.0	5.0	6.4	2.5	2.8	3.9	1.3	1.8	2.5
25.0	4.1	4.9	6.0	2.7	3.5	5.0	1.5	1.9	2.7
30.0	4.2	5.7	7.0	3.3	4.2	4.9	1.7	2.5	3.3
35.0	4.8	7.0	8.5	3.4	4.3	5.7	1.8	2.5	3.4
40.0	4.9	6.5	9.5	3.5	4.9	6.5	2.0	3.0	4.0
45.0	5.5	7.0	9.0	4.0	5.5	7.0	2.5	3.5	4.4
50.0	6.0	8.0	10.0	4.4	5.5	8.0	2.6	3.5	4.9
55.0	6.0	8.5	10.5	4.4	6.0	8.0	2.5	3.8	4.9
60.0	6.5	8.0	11.5	5.5	6.5	9.0	3.0	4.0	5.0
65.0	6.5	8.0	11.0	5.0	6.5	8.0	3.0	4.0	5.5
70.0	6.5	9.0	12.0	5.5	7.0	9.0	3.5	4.5	6.0
75.0	7.0	9.0	13.0	5.5	7.0	10.0	3.5	4.5	6.0
80.0	7.0	10.0		6.0	7.5	10.0	3.6	5.0	6.5
85.0	7.0	11.0		6.0	8.0	11.0	4.0	5.0	7.0
90.0	7.0	10.0		6.0	8.5	12.0	4.0	5.0	7.0
95.0	7.5	11.0		5.5	9.0	11.0	4.0	5.5	7.5
100.0	8.0			7.0	9.0		4.0	5.5	7.0

Table 1. Optimal values of n_i for values of innovation threshold between 20 and 100 where $\gamma = 0.1$ and $\alpha = 0.7$.

Imitation and Trading Off Exploitation and Exploration

The kind of imitation described here is that of using labour to copy new products introduced by other firms, rather than, say, purchasing technology from other firms. So *if a firm allocates resources to imitation then they can only be employed as imitators if there is a product in production that their firm can not produce.* If a firm allocates resources to imitation then those resources are re-applied to the worker category until a new product is discovered when the imitation allocation is fully applied to imitation until the required knowledge is found. On observing a new product discovered by another firm the imitators work until they can produce that product, at which time it becomes an output of their firm with a process knowledge value of 1.0. As for innovation, *imitations are discovered when the total amount of its labour periods allocated to imitation reaches an imitation target. The imitation target may be a fixed amount or may be randomly determined in some interval.* Once a firm has discovered an imitation, its "imitation counter" is reduced by the imitation target value for the next discovery. There is little to be gained by investing in imitating a product if that product's relative demand is low. Figure 6 shows that selecting the "best" relative demand function is not simple. For want of a better criterion, *an imitating firm will choose the most recently discovered product to attempt to imitate.*

If two imitating firms are competing then what is the optimal value of m_i? For this to make sense at least one firm needs to be innovating, as otherwise there will be nothing to imitate. If neither firm is innovating then all of the products in the system are those that were there at the beginning. So the reward for a firm that invests in imitation will be products that are as old as the products that it already has. Unless the relative demand functions for the imitated products are significantly different from those of its existing products such an investment will not be worth while. If the relative demand functions of all products are the same then imitation is certainly not worth while. So the question of an optimal value of m_i only really makes sense if both of the firms are innovating. Consider two firms, each of which allocates an optimal 7% to innovation with an innovation threshold set at 70.0—as indicated in Table 1, $\gamma = 0.1$ and $\alpha = 0.7$. The firms now allocate differing proportions of staff to imitation for different values of the imitation threshold.

The result is zero even for small values of the imitation threshold. It appears that if a firm allocates the optimal proportion of its resources to innovation, n_i, then imitation is not worth doing. Therfore we consider what happens if two firms have an identical, sub-optimal allocation to innovation and different allocations to imitation. Suppose that both firms allocate 4% to innovation [threshold 70] and different amounts to imitation [threshold 20]. A local optimal imitation allocation is around 8%—which does better than 7% and 9%. But 0% does better than the 8%! It seems that if two firms are both innovating to the same extent then imitation is not worth while—it is better to allocate resources to the generation of new products. But this does not mean that imitation is not worth while generally. Below we show that, under certain circumstances, an imitating firm can live off an innovating firm.

A related question is the level of imitation that leads to the slowest decline. If two firms allocate 6% to innovation [threshold 70], with he first allocating 0% to imitation [threshold 5], then 22% allocated to imitation for the second firm leads to the slowest decline with the firm's size at 0.2% of its original size at time 100. This is slower even than allocating a very small amount to imitation, say 0.1%. The reason for this is not clear to us at present and serves as a demonstration of the sometimes non-intuitive outcomes of nonlinear systems behaviour that Robert May (1976) describes.

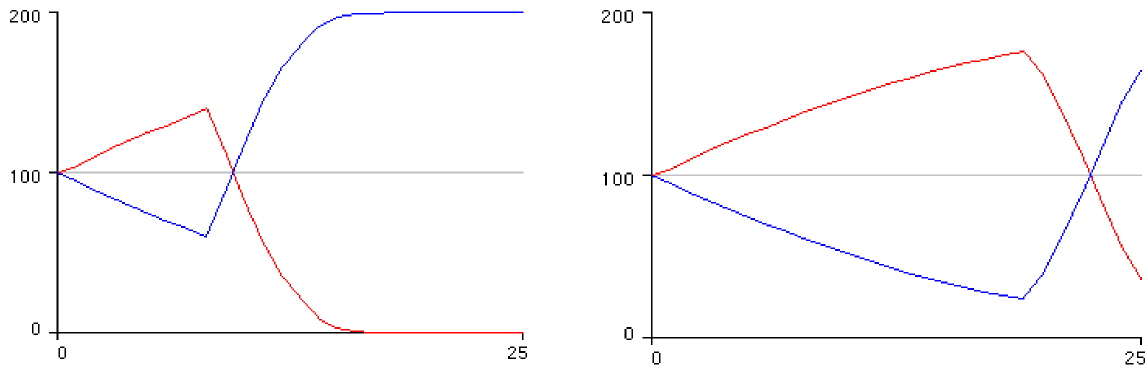
Process improvement: Trading off further exploitation

Investment in process improvement enables a firm to produce output at lower cost. Each firm has a level of process knowledge for each product. Firm i's workers are distributed across the range of products that the firm can produce as represented by the vector $\underline{W}_i^{t-1}$. The quantity of output $\underline{Q}_i^{t-1}$ that the workers generate in the time period is:

$$\underline{Q}_i^{t-1} = \underline{A}_i^{t-1} \_ \underline{W}_i^{t-1}$$

where $\underline{A}_i^{t-1}$ is the firms process knowledge, and _ means that the vectors are multiplied together element by element. Each firm's process knowledge for a given product either remains constant or increases from one time to the next. Initially the process knowledge is set to 1.0 in which case "one worker will produce one unit of product in one time period". If a firm allocates labour to process improvement then it is reasonable to allocate those process improvers to (one of) the most recent product(s) that the firm has learned to produce. In this way an improving firm will derive returns from such investments for products whose relative demand is likely to be large. In any case, this makes sense as the more recent the product the greater the likelihood of deriving benefit from investing in process improvement for that product.

The model chosen for process improvement is similar to that chosen for both innovation and imitation. That is, an investment of a certain proportion of labour on the improvement of a particular product until the labour/time exceeds a set improvement threshold will cause the process knowledge for that product to increase by one. The reason for this choice is to ensure a uniform basis for innovation, imitation and improvement. For example, if two firms are both producing their own single product with the second firm allocating 10% of its labour to process improvement, then this leads to an initial decrease in the size of the second firm. Figure 11(a) shows what happens with a process improvement threshold of 50—the two graphs cross in time period [33, 34]. Figure 11(b) shows a threshold of 100. These calculations are invariant to the values of alpha and gamma.



(a) (b)

Figure 11. Two firms with an initial size ν = 100, the second firm allocates 10% of its labour to process improvement. In (a) with a process improvement threshold of 50, and in (b) with a process improvement factor of threshold of 100.

If two improving firms are competing then what is the optimal value of p_i? Suppose that two firms allocate 5% to innovation [threshold 70]. Then the more allocated to improvement [threshold of 20] the better, up to 30% when a difference of 5% is not sufficient to dominate within 160 time units although the equilibrium reached is alarmingly unstable. See Figure 12.

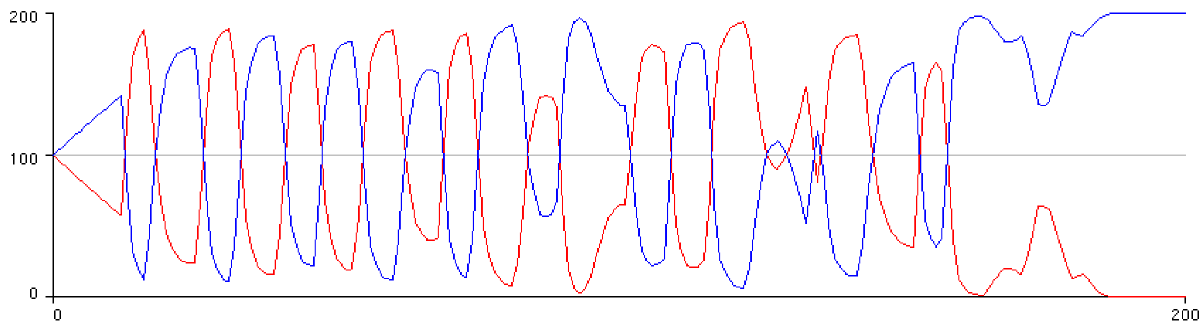


Figure 12. Two firms with an initial size ν = 100, both firms allocate 5% to innovation [threshold 70], the first firm allocates 30% to improvement [threshold of 20] and the second firm allocates 25%.

The Return on Investment of Different Strategies

The return on investment (ROI) of different strategies may be indicated in terms of the area beneath the relative demand curve for a product. For example an innovator invests for a period of time and discovers a new product. That directly benefits the innovator until an imitator learns to imitate that product, at which time the innovator will share the benefit with the imitator. The innovators relative benefit is shown as the hashed area in Figure 12(a). Likewise the imitators relative benefit is shown in Figure 12(b).

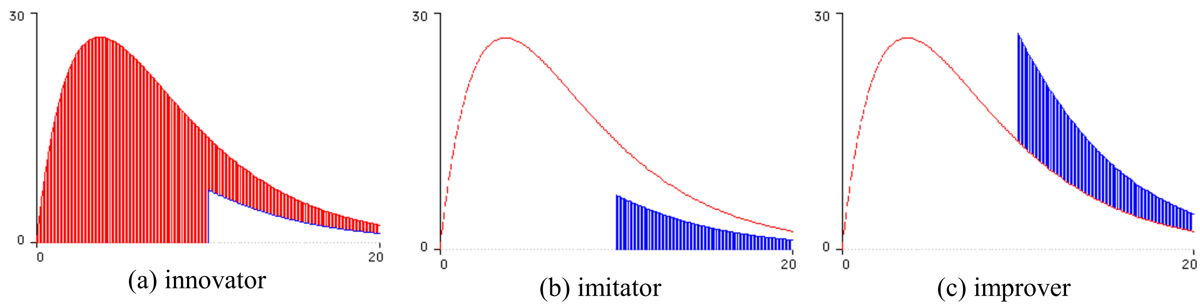


Figure 12. Return on investment for an innovator, an imitator and an improver.

If an innovating firm is competing with an imitating firm then, when the innovating firm discovers a new product, it benefits entirely from the revenue derived from selling that product until the imitating firm learns how to produce that product and thereafter the two firms share the revenue from that product. So the difference between the sales volumes derived by the innovating firm and the imitating firm is related to the area beneath the being-imitated product's relative demand curve from its beginning to the time at which the imitating firm learns how to imitate that product. The area beneath a relative demand curve (2) from the beginning to time t is:

$$\int_{x=0}^{t} \frac{\mu \_ \gamma}{\ln(\alpha) + \gamma} \_ \left[\alpha^{x} - \exp(-x \_ \gamma) \right] \_ dx$$

$$= \frac{\gamma \_ \mu}{\ln(\alpha) + \gamma} \_ \left[\frac{\alpha^{t}}{\ln(\alpha)} + \frac{\exp(-t \_ \gamma)}{\gamma} - \frac{1}{\ln(\alpha)} - \frac{1}{\gamma} \right] \qquad (4)$$

which gives an area under the entire curve of:

$$\frac{\gamma \_ \mu}{\ln(\alpha) + \gamma} \_ \left[- \frac{1}{\ln(\alpha)} - \frac{1}{\gamma} \right]$$

For example if $\gamma = 0.1$ and $\alpha = 0.7$ then the area under the entire curve is 448.142, and the expression (4) tends asymptotically to this value.

The process improver will invest in improving production for a recently discovered, or copied, product. When the investment in process improvement exceeds the improvement threshold the process knowledge for that product will increase by 1.0. This is illustrated in Figure 12(c).

What Figure 12 shows is that if an innovating firm, an imitating firm and an improving firm are coexisting in a moderately stable way then we expect the innovation threshold to be greater than the process improvement threshold, which in turn will be greater than the imitation threshold. This turns out to be the case.

Model Constraints

Before continuiing with the analysis some necessary constraints on model parameters needs to be explained. .The first constraint on the parameters follows from the use of discrete time in the simulations. The constraint is that each firm can have at most one of each of innovation, imitation and improvement in any time interval. To ensure that this constraint is satisfied it is sufficient to ensure that the product of the total labour and the percentage allocated to each of endeavour is less than the threshold for that endeavour. For example, if the total labour is 200, and a firm allocates 10% to imitation then the imitation threshold must be greater than 20.

The model economy described here is fundamentally unstable due to the shape of the relative demand curves which when an innovation, imitation or improvement is found may substantially increase the fortunes of the firm involved. This suits our purpose which is to investigate the, consequently fairly rare, regions of stability.

The next constraint is to ensure that the alpha and gamma parameters are chosen so that these regions of stability are not too small. This can be achieved by ensuring that when an innovation is made by a firm the relative demand curve of the most recent existing product for that firm does not declined to a level that threatens the survival of the firm. Otherwise Innovators would go out of business before any innovation ever occur. For example, suppose that the steady state size of firms is 100, and that the innovation threshold is 100 then a firm that allocates 10% of its labour to innovation can be expected to discover an innovation approximately every ten time units. If $\gamma = 0.1$ and $\alpha = 0.7$ then the relative demand curve has the shape shown in Figure 5. Those diagrams show time extending from zero to 50. So a new innovation made at ten to fifteen time units would occur when the previous innovation's curve is well past its peak but still sufficient to permit the innovator to survive.

We also attempt to ensure that the alpha and gamma parameters are chosen so that the regions of stable coexistence of different types of strategies are not too large. This can be achieved by ensuring that when an innovation is made by a firm the relative demand curve of the most recent existing product for that firm "has decayed a fair amount". By "a fair amount" we mean that an innovating firm will not discover a new product before the relative demand curve of its "previous" new product reaches its maximum value. Setting the derivative with respect to t of equation (2) to zero:

$$t_{max} = \frac{1}{\ln(\alpha)+\gamma} - \ln\left(\frac{-\gamma}{\ln(\alpha)}\right) \tag{3}$$

The values of t_{max} for the nine relative demand curves illustrated in Figure 5 are given in Table 2. In so far as the maximum value of the relative demand curve is useful for determining "interesting" values for innovation effort, the values in Table 2 can be used as a guide. For example, suppose that a firm has a product whose relative demand curve has $\gamma = 0.1$ and $\alpha = 0.7$, suppose that this firm has initial size $\mu = 100$ and 10% of its labour allocated to innovation. If the innovation effort is set at 50, or there abouts, and if the imitating firm's size remains at its initial value, then the imitating firm will discover how to make the product when the relative demand curve of its old product is at its peak.

		α		
	t_{max}	0.4	0.7	0.9
γ	0.05	3.357	6.407	13.464
	0.1	2.714	4.954	9.741
	0.2	2.125	3.692	6.772

Table 2. Values of t_{max} for various values of γ and α.

What is a "good" value for the innovation target when, say, $\gamma = 0.1$ and $\alpha = 0.7$? This function is illustrated in Figure 5. If the innovation target is high then by the time a new product is discovered the relative demand of the "old" products will be very low and so the impact of the new product will be large and destabilising. These experiments address destabilising effects, but if they are too great then the location of the stability points is very sensitive. The relative demand curve for $\gamma = 0.1$ and $\alpha = 0.7$ reaches a maximum value of 17 at about time 5. By time 13 it has dropped back to around 10, and by time 35 it has dropped back to around 1.18. So if a firm were to discover a new product at time 35 then the new product would be competing with an old product whose relative demand was around unity and falling—highly unstable behaviour would result. As a general rule if the time to discover a new product is greater than "lower teens" then highly unstable performance should be expected.

Interactions Between Innovation, Imitation and Process Improvement Strategies

The interplay between imitation, innovation and improvement is quite complex. For example consider three firms—the first allocates 10% of its labour to innovation, the second 15% to imitation, and the third 5% to each of imitation and improvement. The innovation effort threshold is set at 80, the imitation threshold at 20, and the improvement threshold at 30. All products have $\gamma = 0.1$ and $\alpha = 0.7$. Then:

- Firm 1 has an initial product number 0
- Firm 2 has an initial product number 1
- Firm 3 has an initial product number 2
- Firm 3 starts improving product number 2 in time interval [0,1] process knowledge = 1.0
- Firm 2 starts imitating product number 2 in time interval [0,1]
- Firm 3 starts imitating product number 1 in time interval [1,2]
- Firm 2 discovers how to imitate product number 2 in time interval [2,3]
- Firm 2 starts imitating product number 0 in time interval [3,4]
- Firm 2 discovers how to imitate product number 0 in time interval [5,6]
- Firm 3 discovers how to imitate product number 1 in time interval [8,9]

- Firm 3 discovers how to improve product number 2 in time interval [9,10] process knowledge = 2.0
- Firm 3 starts imitating product number 0 in time interval [9,10]
- Firm 3 starts improving product number 1 in time interval [10,11] process knowledge = 1.0
- Firm 1 discovers a new product number 3 in time interval [13,14]
- Firm 2 starts imitating product number 3 in time interval [14,15]
- Firm 3 discovers how to imitate product number 0 in time interval [15,16]
- Firm 3 discovers how to improve product number 1 in time interval [16,17] process knowledge = 2.0
- Firm 3 starts imitating product number 3 in time interval [16,17]
- Firm 3 starts improving product number 0 in time interval [17,18] process knowledge = 1.0
- Firm 2 discovers how to imitate product number 3 in time interval [17,18]
- Firm 3 discovers how to imitate product number 3 in time interval [19,20]
- Firm 3 discovers how to improve product number 0 in time interval [24,25] process knowledge = 2.0

and the resulting firm sizes are shown in Figure 13.

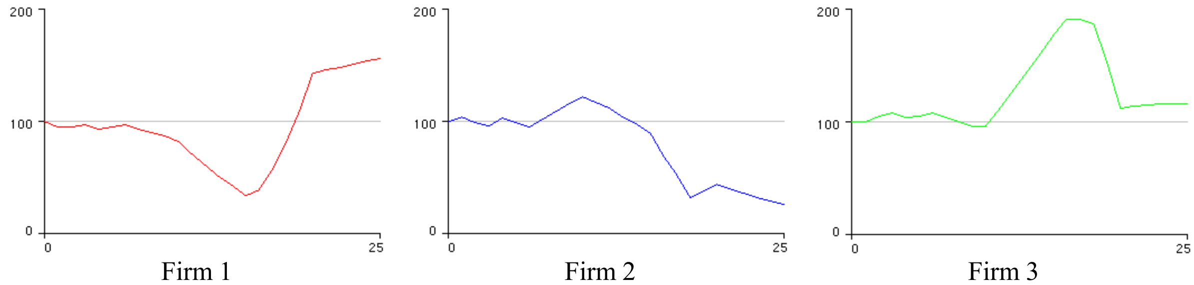


Figure 13. Three firms with an initial size of 100, the first firm allocates 10% of its labour to innovation, the second allocates 15% to imitation, and the third allocates %5 each to imitation and improvement with , $\gamma = 0.1$ and $\alpha = 0.7$ for all products. The innovation effort threshold is set at 80, the imitation threshold at 20, and the improvement threshold at 30.

The model allows us to explore the conditions under which firms in a market adopting different exploitation and exploration strategies can survive and when they do not. When different firms can coexist for long periods of time we can define the market as stable. For the purposes of analysis we will define a market as stable if all firms are still in business at time 250. This is a very pragmatic definition but, given the tendency of the system to destabilise, it is quite reasonable. For example, consider two firms. The first firm allocates 10% of its labour to innovation, and the second 5%. Suppose the innovation threshold = 40, $\gamma = 0.1$ and $\alpha = 0.7$. How much should the second firm allocate to imitation if it is to compete with the first? The answer is none. Why? Table 1 shows that the optimal value for n_i with these parameters is 4.9. So the second firm will dominate the first without any investment in imitation. By time 45 the size of the first firm is practically zero.

Now consider two firms where the first allocates 5% to innovation and the second 5% to imitation. Given a value of the innovation threshold, what values for the imitation threshold lead to joint survival? That is, under what circumstances can an imitating firm "live off" an innovating firm? The result may at first appear counter-intuitive in that as the innovation threshold *increases* the stable imitation threshold *decreases*. The reason for this is that in a stable configuration the imitation threshold should be at a level so that the imitating firm discovers how to imitate in good time but not too soon. If it discovers an imitation early in the innovation cycle then it will have nothing else to imitate and so will allocate all of its labour to workers and so may kill the firm from which it derives its inspiration. Further, the greater the innovation threshold the longer the time between innovation discoveries, the greater the relative demand of the discovered product, and the sooner the imitator imitator must learn to imitate the product. Table 3 shows sample values and Figure 15 shows the respective sizes of the firms—the imitating firm dominates.

Innovation threshold	Imitation threshold
10.0	9.6762085
15.0	10.123603
20.0	9.366529
25.0	8.693429
30.0	7.92837
35.0	7.215324
40.0	6.624544
45.0	6.3362412
50.0	5.8482556
55.0	4.824529
60.0	4.6543684
65.0	4.5592747
70.0	4.454127
75.0	4.164027

Table 3. Optimal values of the imitation threshold for values of the innovation threshold between 10 and 75 where two firms invest 5% in innovation and imitation respectively, and $\gamma = 0.1$ and $\alpha = 0.7$.

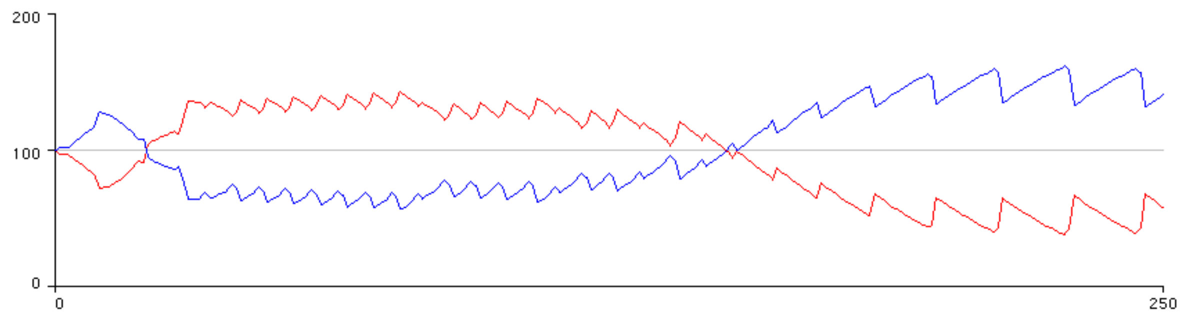


Figure 15. Two firms compete. An innovating firm allocates 5% to innovation with a threshold 40, an imitating firm allocates 5% to imitation with a threshold at 6.624544. There is no randomisation of the parameters: $\gamma = 0.1$, $\alpha = 0.7$ and $\nu = 100$.

Next consider two firms where the first allocates 5% to innovation and the second 5% to improvement. Given a value of the innovation threshold, what values for the improvement threshold lead to joint survival? That is, under what circumstances can an improving firm survive against an innovating firm? The improving firm will be making further and further improvements to its production of the one product that it knows how to produce. It is no surprise that the stable values are very hard to find—a very small change alters the outcome completely. Table 4 shows sample values and Figure 16 shows the respective sizes of the firms—the innovating firm dominates. The graph in Figure 16 shows sizes up to time 50 only and is not stable—locating the stable solution is very time consuming and probably beyond double precision arithmetic.

Innovation threshold	Improvement threshold
10.0	50.827827
15.0	48.68846
20.0	47.29402
25.0	48.145622
30.0	51.364643
35.0	50.897617
40.0	50.4034
45.0	50.0
50.0	55.0
55.0	55.0
60.0	55.0
65.0	55.0
70.0	55.35309
75.0	55.904945
80.0	60.0

Table 4. Optimal values of the improvement threshold for values of the innovation threshold between 10 and 80 where two firms invest 5% in innovation and improvement respectively, and $\gamma = 0.1$ and $\alpha = 0.7$.

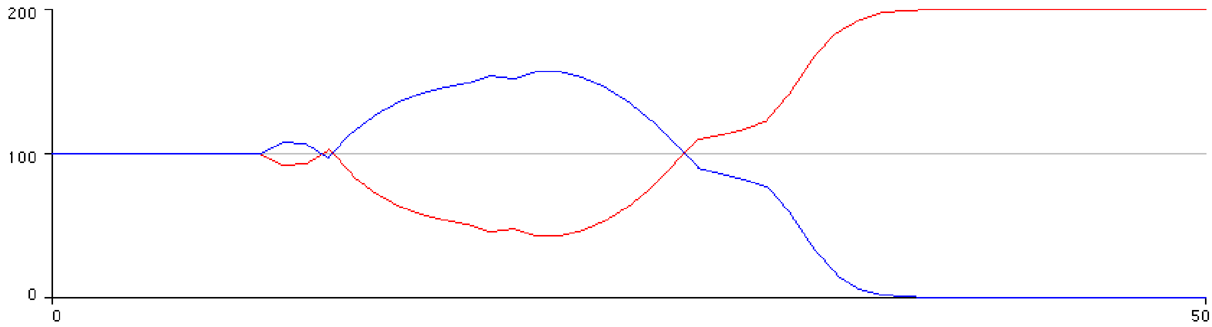
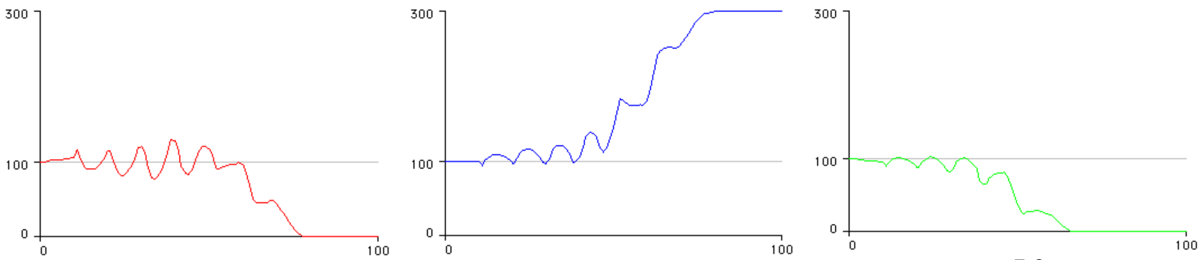


Figure 16. Two firms compete. An innovating firm allocates 5% to innovation with a threshold 40, an improving firm allocates 5% to improvement with a threshold at 50.40341. There is no randomisation of the parameters: $\gamma = 0.1$, $\alpha = 0.7$ and $\nu = 100$.

Greater than Two Competitors

So far we have focused attention on two firm markets. Some additional simulation results involving more than two competitors will serve to indicate further the complexity of the market system. If there are two firms with an innovation threshold at 70 then the optimum innovation allocation is around 7%. What happens if there are more than two firms? Constraining the allocations to be 0.5% apart, to prevent roundoff errors from confusing the picture, if there are three firms the maximum is around 7.3%. The graphs for three firms will innovation allocations of 6.8%, 7.3% and 7.8% are shown in Figure 17.



$n_1 = 6.8$ $n_2 = 7.3$ $n_3 = 7.8$

Figure 17. Three innovating firms compete. The innovating firms have innovation at 6.8%, 7.3% and 7.8%. There is no randomisation of the parameters: innovation threshold = 70, $\gamma = 0.1$, $\alpha = 0.7$ and $\nu = 100$.

Suppose that there is one innovating firm that allocates 7% to innovation with an innovation threshold of 70. Suppose that three imitating firms compete with this one innovating firm with an imitation threshold of 5. Then if the imitation allocations are constrained to being integers 1% apart the lowest imitation allocation dominates for values in the range [6,7,8] to [17,18,19].

Outside that range the single innovator dominates. So it appears that for imitators the winning strategy is to allocate less than competing imitators to imitation. The graphs for the three imitating firms with an allocation of 11%, 12% and 13% are shown in Figure 18.

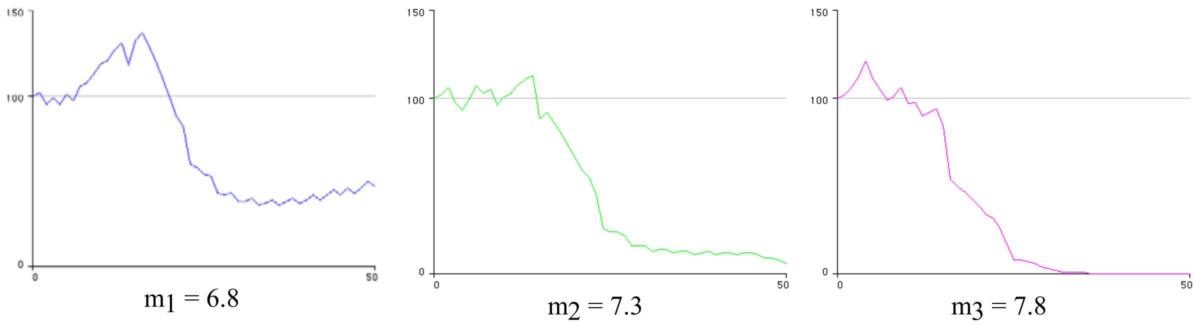


Figure 18. Three imitating firms compete with one innovating firm. The innovating firm allocate 7% to innovation with a threshold of 70.. The imitating firms allocate 11%, 12% and 13% with an imitation threshold of 5, $\gamma = 0.1$, $\alpha = 0.7$ and $\nu = 100$.

Suppose that there are four firms who are prepared to allocate 12% to other than workers and that:

- the first allocates 6% to imitation and 6% to process improvement
- the second allocates 6% to innovation and 6% to process improvement
- the third allocates 6% to innovation and 6% to imitation
- the fourth allocates 4% to each of innovation, imitation and process improvement

There is no randomisation of the parameters: innovation threshold = 80, imitation threshold = 12, improvement threshold = 85, $\gamma = 0.1$, $\alpha = 0.7$ and $\nu = 100$. The sizes of the four firms are shown in Figure 19. That Figure also shows what happens if the innovation threshold = 84, imitation threshold = 13, improvement threshold = 90. In both examples the fourth firm with the mixed strategy across innovation, imitation and improvement performs the worst. Once again the detailed explanation and understanding of this result is not clear.

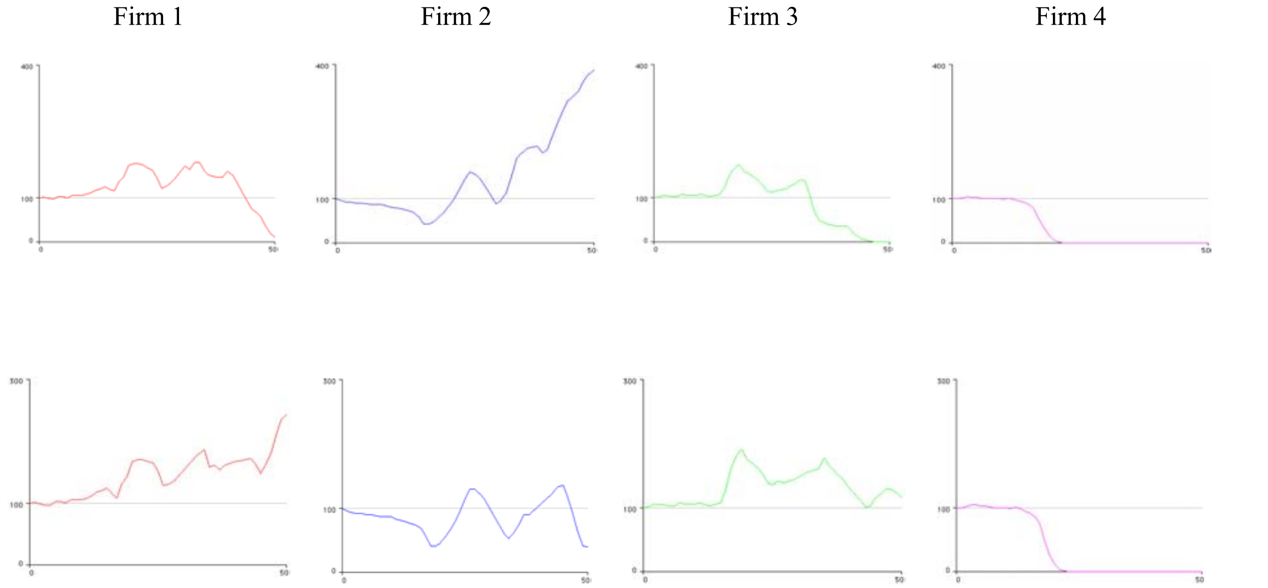


Figure 23. Size of four firms when: the first allocates 6% to imitation and 6% to process improvement, the second allocates 6% to innovation and 6% to process improvement, the third allocates 6% to innovation and 6% to imitation, and the fourth allocates 4% to each of innovation, imitation and process improvement. There is no randomisation of the parameters: innovation threshold = 80, imitation threshold = 12, improvement threshold = 85, $\gamma = 0.1$, $\alpha = 0.7$ and $\nu = 100$. The second row shows what happens if the innovation threshold = 84, imitation threshold = 13, improvement threshold = 90.

Conclusion

Our paper shows how the choice of exploration versus exploitation strategies is a complex problem without any analytical solutions in the form of optimal strategies for individual firms. This arises in part because innovation, imitation and process improvement are not deterministic processes and because of interactions among the strategies of different firms. This makes the whole market system a highly non-linear one. As a result we have to utilise simulation models to examine the conditions under which different strategies are successful or not in terms of firm and competitor survival and ROI.

We have show how survival depends on the timing of innovation, imitation and process improvement and the speed of diffusion and level of penetration of products. These factors affect the trade-off between the more shorter term gains from exploiting existing or recently developed new products against the more distant gains from developing new products. Optimal rates of allocating resources have been detected under different threshold and market demand conditions in markets with two firms competing and these appear to dominate any imitation strategies. We have also found conditions under which different strategies can co-exist, such as when an imitator can live off an innovator firm and when a process improver can live with an innovator. Finally we have begun to examine results for competition among more than two firms.

Much remains to be done in utilizing and extending the model and we have made it available on the Internet for other researchers to use, as detailed in the appendix. Further simulation experiments are required to further develop our understanding of the dynamics of competition and the trade-off between exploration and exploitation strategies under different conditions. So far our model has fixed strategies for each firm for the duration of the simulation. A next step would be to build in response functions whereby firms modify the amount of resources they devote to exploration and exploitation strategies. The challenge is also to undertake studies of the evolution of existing markets to see the extent to which the model can replicate know patterns of development and to allow the model to be calibrated against real market parameter values in particular industries.

Appendix: Model Implementation on the Internet

The "economy" described above has been implemented as a Java applet and is available on the World Wide Web at:

`http://www-staff.it.uts.edu.au/~debenham/research/evolution1/`

The use of Microsoft Internet Explorer with Java enabled is recommended.

The applet window is in three parts—see Figure 14:

- a "blue boxes" in the top section in which the basic parameters are set.
- a "pink boxes" in the middle in which each firm's labour is assigned.
- a "white boxes" at the bottom that controls the graphical presentation.

The general idea is that the blue boxes in the top half of the applet should be set before the system is run and may not be changed without initialising the system—ie: by setting time back to zero. On the other hand the values in the pink boxes may be changed during a run—this enables a firm's labour deployment strategy to be modified as things progress. When a run commences, all of the firms in the simulation produce the same number of products that are unique to each firm. This initial number of products is set in the top row. The specification of γ, α, innovation threshold, imitation threshold and improvement threshold are given as ranges. If, for example, the range for γ is set to [0.1, 0.1] then $\gamma = 0.1$. If the range is set to [0.1, 0.2] then γ will be set to a random number in this range. The random distribution used is a truncated normal distribution—that is, quantities more than 2 standard deviations from the mean are discarded. The left-hand button "Reset" sets the values in the pink boxes to zero. The "Time = 0" button should be used to initialise the system before a run. The four buttons "Step 1",.., "Step 100" move time forward by the designated amount and the resulting sizes of the firms should appear on the lower right-side of the applet window. Once the program has been run, the "Draw" button should open another window with a graph of the resulting firm sizes. The three "white" text boxes at the bottom of the applet window may be used to control the size and proportions of the graphical output.

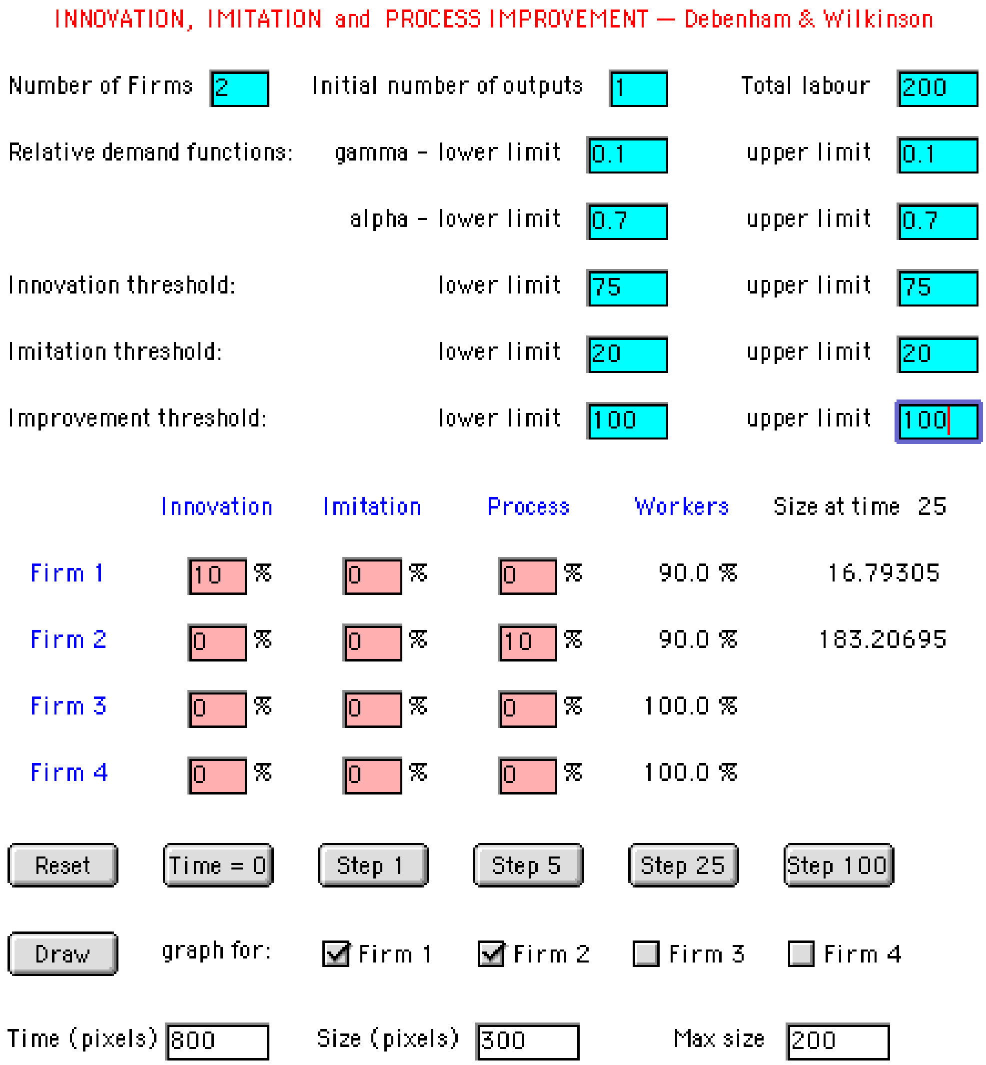


Figure 14. The Java applet.

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