Modelling of utility dynamics based on attribute-importance learning, or preference structure formation, has been recently conceptualised under the goal-contingency framework. The purpose of the current paper is to derive and test a quantitative model of preference structure formation founded on the goal-contingency framework and the associative learning theory from psychology. The model is calibrated in two experiments based on the E-T 3000 project. Both experiments test the model’s predictions in the context of repeated consumer interaction with initially unfamiliar product features of an Internet-based decision-support software agent. Experiment one presents subjects with a simplified learning task and measures utility and preference estimates over time for a wide range of contingency relations between the subjects' goal and the agent's alternative attributes. Experiment two replicates the design of experiment one, but adds complexity to the learning task by introducing random noise into the diagnosticity of attributes in the stimulus. Outcomes of the two experiments verify the utility dynamics and the effect sizes predicted by the model. This lends support to the model. Implications for model development and further applications are provided.
**Introduction**

Preference formation is an important area of research in marketing. Preference formation has implications for new product development (Oren and Rothkopf, 1984), market entry (Carpenter and Nakamoto, 1989), product diffusion (Bergmann and Valimaki, 1997) and market structure over time (Bockenholt and Dillon, 1997). Several articles in marketing have modelled dynamics of consumer preferences. For example Roberts and Urban (1988) assumed that consumers who face attribute level ambiguity, but are able to acquire information about the product through experience or word of mouth, alter their initial estimates of the mean and variance of product attribute levels. This in turn results in altered choice probabilities. Chatterjee and Eliashberg (1990) in addition considered consumer heterogeneity, the degree of risk aversion and consumer responsiveness to successive information as influences on utility and choice dynamics. Heilman, Bowman, and Wright (2000) modelled changes in intrinsic product utility as a result of the product category experience and the influence of the marketing mix variables. These studies indicate the general approach taken to quantitative modelling of consumer utility dynamics in the marketing literature.

Such quantitative models derive utility dynamics based on the assumption of a consumer’s learning about product attribute levels and stable preference structures. This is consistent with traditional multiattribute utility theory (Lancaster, 1966), where utility is expressed as a combination of attribute importance weights (preference structures) and individual attribute levels (see equation (2)). Therefore the models retain the traditional formulation (McFadden, 2000) by assuming stable and innate attribute importance weights (Roberts and Urban, 1988; Chatterjee and Eliashberg, 1990). Instead, dynamics are introduced into utility evaluations by modelling changes in the consumer’s perception of product attribute levels.

On the other hand descriptive models of preference construction (Bettman, Luce, Pyne, 1998) suggest that consumers may have fuzzy or non-existent preference structures when they encounter new product attributes. Such models argue that consumers construct their preference structures at the time of choice and according to various decision rules and heuristics given the information available in the market. The limitation of the descriptive approach is that it does not calibrate the model parameters to provide quantifiable predictions about consumer behaviour.
In contrast the quantitative model we propose in this study suggests that starting from a situation of fuzzy or non-existent preference structures consumers can learn about attribute importance through repeated interaction with product attributes in the market. Hence, the utility dynamics in our model result from changes in a consumer’s actual preference structures, rather than changes in the perception of the attribute levels. The contribution of our approach is the algebraic model of preference structures formation that can predict the equilibrium utility levels and the dynamics towards that equilibrium based on the consumer’s repeated interaction with product attributes, even when perception of the attribute levels remains constant.
Conceptual Background and Theory

The quantitative model which we develop in this paper is based on the goal-contingency framework (Chylinski et al., 2004). The framework relates preference structure formation to learning of the contingency relations between a consumer’s goal and product attributes in the market.

The notion of contingency is founded on probabilistic causation theories (Ells, 1991). Probabilistic causation theories suggest the conditions under which the effect a product attribute has on achievement of the consumer’s goal can be determined based on the observed co-occurrence of the goal and the product attributes in the market. The central idea is that causes change the probability of their effects, all else being equal (Skyrms, 1980). Thus, a consumer can determine if an attribute alters the chances of achieving his/her goal by calculating the conditional probabilities of goal achievement given the occurrence and non-occurrence of the attribute in the market. Following Allan (1980) we represent the contingency between a goal, \( Y \), and the attribute, \( X_k \), while keeping all else constant, as:

\[
\Gamma(Y | X_k) = P(Y | X_k) - P(Y | \overline{X_k})
\]

(1)

Where: \( \Gamma(Y | X_k) \) is the contingency relation between \( X_k \) and \( Y \) the two separate binary events corresponding to a product attribute and the consumer’s goal respectively. \( P(Y | X_k) \) is the conditional probability of the goal \( Y \) given attribute \( X_k \).

Consumers’ estimates of the contingency in equation (1) (which we represent as \( \Gamma^*(Y | X_k) \)) have been shown in the psychology literature to be influenced by associative learning; that is classical and operant conditioning\(^1\) (Shanks, 1985, 1987, 1991, 1995; Shanks and Dickson, 1987; Lopez et al., 1998). These findings in psychology provide the foundation for the process of preference structure learning which we adopt in our model. This process is illustrated in figure 1.

\(^1\) For a review of classical and operant conditioning see (Wasserman and Miller, 1997)
Figure 1.

The goal-contingency framework of preference structure formation.

The classical conditioning applies in our model, because a consumer’s repeated interaction with product attribute information creates conditions for co-occurrence (i.e.: pairing in the classical conditioning sense) of information about the product attribute and achievement of the consumer’s goal. Perception of such co-occurrences (and non co-occurrences) by the consumer leads to formation of relative frequency estimates of conditional probabilities required to compute equation (1) (Hinton and Sejnowski, 1986 Shanks 1985, Wasserman and Berglan, 1998).

The operant conditioning process works in the model through the feedback effect generated by the decision outcomes. The effect an attribute has on achievement of the consumer’s goal results in affective responses of satisfaction or dissatisfaction following each decision outcome (McQuitty, Finn, and Wiley, 2000; Churchill and Surprenant, 1982; Oliver and Dessarbo, 1988). In the behavioural sense, these responses act to reinforce or punish the preference structures that led to the choice of the particular attribute. When the same attribute is encountered again, the reinforced preference structures exert stronger, and the punished preference structures weaker, influence on subsequent decisions. In this way preference structures are shaped by the attribute outcomes.

Thus, the conceptual model in figure 1 provides a framework through which to understand preference structure formation, and a learning process to apply within that framework.

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2 Or information about those outcomes.
3 For more detailed discussion of the psychological antecedents of the goal-contingency model see Chylinski et al. (2004).
Model

The following quantitative model is an algebraic representation of the goal-contingency framework in figure 1. We begin by stating the relation between the goal-contingency and the more traditional multi-attribute utility theory. Based on this relation we state the model under conditions of equilibrium, assuming perfect information and no limits to individual information processing. Having established the model under equilibrium we progressively relax the assumptions to propose a dynamic form of the model based on a consumer’s imperfect learning in repeated exposures to information about goal achievement and product attributes in the market.

Relation of Multi-attribute utility to goal-contingency

The concept of multi-attribute utility (MAUT) provides the starting point of our model. We define the utility \( U(Z) \) of a product \( Z \) as:

\[
U(Z) = \sum_{k=1}^{K} w_k(x_k) x_k
\]

where: \( U(Z) \) is the utility of product \( Z \) (such that \( Z = (x_1, x_2, ..., x_K) \)); \( x_k \) is the level of attribute \( k \); \( w_k(x_k) \) is a function that assigns a consumer’s unique importance weight to each level of attribute \( k \).

In this paper we will look at the most common case of equation (2) where \( w_k(x_k) \) is invariant with changes in \( x_k \). Hence, in the future we can denote \( w_k(x_k) \) by \( w_k \).

According to the above formulation the importance weights play a central role in terms of determining attribute utility. They indicate the consumer’s judgment of the importance of attribute \( X_k \) at a level \( x_k \), and influence the overall shape of the utility function. However, determinants of attribute importance are not precisely defined under MAUT. MAUT assumes that consumers can and do make attribute importance judgments, but the psychological antecedents behind those judgments are not specifically modelled.
The notion of multiattribute utility can be related to the goal-contingency framework in figure 1. The goal-contingency framework views utility of any attribute \( k \) as a function of the consumer’s goal and the contingency association of that goal with the attribute. That is:

\[
U(X_k = x_k) = f_i(\Gamma^e(Y = y | X_k = x_k), U(Y = y))
\]

where: \( U(X_k = x_k) \) is the utility of attribute \( X_k = x_k \), \( \Gamma^e(Y = y | X_k = x_k) \) is the consumer’s estimate of the underlying contingency relation between goal \( Y = y \) and attribute \( X_k = x_k \) defined in equation (1); \( U(Y = y) \) expresses the value of the consumer’s goal.

Equations (2) and (3) can be related to represent the MAUT attribute importance in terms of goal-contingency. That is, we suggest that attribute importance can be considered a function of the estimated contingency and the value of the consumer’s goal:

\[
w_k(x_k) = f_i(\Gamma^e(Y = y | X_k = x_k), U(Y = y))
\]

The core conjecture of our model, which we examine in the following sections, is that the relationship in equation (4) is subject to change over time due to a consumer’s learning of the contingency between the product attribute and the achievement of the goal. That is, we argue that:

\[
w_k(x_k, t) = f_i(\Gamma^e(Y = y | X_k = x_k), U(Y = y))
\]

where: \( w_k(x_k, t) \) is the consumer’s judgment of attribute importance at time \( t \), \( \Gamma^e(Y = y | X_k = x_k) \) represents a consumer’s estimate of contingency at time \( t \).

However, before we analyze the dynamics of preference structure formation we first define our model under the equilibrium conditions, where there is no change over time and consumers have perfect information. This scenario represents the model of rational utility maximising.
Equilibrium Analysis: Case of a Single Binary Attribute and a Single Binary Goal

Consider the simple case of a single binary consumer goal $Y = (1,0)$ and a single binary product attribute $X_k = (1,0)$. In this case, following from equation (1), the contingency can be represented as:

$$\Gamma(Y = 1 | X_k = 1) = P(Y = 1 | X_k = 1) - P(Y = 1 | X_k = 0) \quad (6)$$

where: $-1 \leq \Gamma(Y = 1 | X_k = 1) \leq 1$ is the contingency between achievement of the goal ($Y = 1$) and the attribute $k$ at level 1 ($X_k = 1$); $P(Y = 1 | X_k = 1)$ represents the conditional probability of $Y = 1$ given $X_k = 1$; and $P(Y = 1 | X_k = 0)$ represents the conditional probability of $Y = 1$ given $X_k = 0$.

If the value to a consumer of achieving his/her goal can be expressed by $U(Y = 1)$, and the value of not achieving the goal can be expressed by $U(Y = 0)$, then the expected utility of the consumer observing $X_k = 1$, $E(U|X_k = 1)$ relative to not observing it, that is $X_k = 0$, may be written in terms of the expected utility when $X_k = 1$ occurs, $E(U|X_k = 1)$, and the expected utility when $X_k = 0$ occurs, $E(U|X_k = 0)$:

$$E(U | X_k = 1) = P(Y = 1 | X_k = 1)U(Y = 1) + P(Y = 0 | X_k = 1)U(Y = 0)$$
$$= P(Y = 1 | X_k = 1)U(Y = 1) + \{1 - P(Y = 1 | X_k = 1)\}.U(Y = 0) \quad (7)$$

And:

$$E(U | X_k = 0) = P(Y = 1 | X_k = 0)U(Y = 1) + \{1 - P(Y = 1 | X_k = 0)\}.U(Y = 0) \quad (8)$$

Hence:

$$U(X_k = 1) = E(U | X_k = 1) - E(U | X_k = 0)$$
$$= \{P(Y = 1 | X_k = 1) - P(Y = 1 | X_k = 0)\}.U(Y = 1) - \{P(Y = 1 | X_k = 1) - P(Y = 1 | X_k = 0)\}.U(Y = 0) \quad (9)$$

Substituting (6) equation (9) simplifies to:

$$U(X_k = 1) = \Gamma(Y = 1 | X_k = 1).\{U(Y = 1) - U(Y = 0)\} \quad (10)$$
Equation (10) provides the algebraic interpretation of the goal-contingency framework for the case of a single binary goal and a single binary product attribute under conditions of perfect information and no change over time. It states that utility of observing $X_1=1$ is the product of the actual contingency relation between the attribute at level 1 and achievement of the consumer’s goal, and the value of the goal itself. We refer to the goal-contingency framework under the above conditions as the equilibrium interpretation of the model.

**Equilibrium Analysis: Case of Multiple Binary Attributes**

In most market environments consumers are likely to face multiple potential predictor attributes. In the case of more than one attribute, the role of consumers’ preference structures is to form comparative utility judgments that take into account the presence of other product attributes.

We can capture the effect of alternative predictor attributes by analysing a two attribute situation, where $X_1=(1,0)$ and $X_2=(1,0)$ are the two attributes. In this case the contingency between $Y=1$ and $X_1=1$ while in the presence of $X_2=x_2$ can be described by:

\[ \Gamma(Y=1 \mid X_1=1 \text{ at } X_2 = 1) = P(Y=1 \mid X_1=1, X_2 = 1) - P(Y=1 \mid X_1=0, X_2 = 1) \]  
\[ \Gamma(Y=1 \mid X_1=1 \text{ at } X_2 = 0) = P(Y=1 \mid X_1=1, X_2 = 0) - P(Y=1 \mid X_1=0, X_2 = 0) \]

That is, in this model we have removed the assumption of independence between the specific predictor attributes implied by equation (2). The effect of attribute one in achieving the consumer goal may depend on the level of attribute two if the left hand sides in equations (11) and (12) are not equal (see Cheng and Holyoak, 1995).

Following the logic in equation (10) and using the results in equations (11) and (12) we can represent the utility of attribute one, while in the presence of an alternative predictor attribute, as:

\[ U(X_1=1 \mid X_2 = 1) = \Gamma(Y=1 \mid X_1=1 \text{ at } X_2 = 1).\{U(Y=1) - U(Y = 0)\} \]  
\[ U(X_1=1 \mid X_2 = 0) = \Gamma(Y=1 \mid X_1=1 \text{ at } X_2 = 0).\{U(Y=1) - U(Y = 0)\} \]

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4 Ceteris Paribus.
which collapses to equation ((10)) if attribute two does not moderate the effect of attribute one on achievement of the consumer goal.

Equations (13) and (14) provide the algebraic representation of the goal-contingency framework for two specific binary product attributes and a single binary consumer goal. By replacing $X_2$ by a vector of alternative attributes the above formulas extend to the multi-attribute case. However, for simplicity of exposition in the following sections we continue with the two-attribute analysis.

Relation of Goal-Contingency to MAUT

In order to specify the relation of the goal-contingency model to the more traditional multi-attribute utility theory, we note that equation (2) does not allow for interaction between attributes, so we use the simpler form of the contingency equation (10).

Assuming attribute independence the MAUT model suggests that:

\[ E(U \mid X_k = 1) = w(X_k = 1). (X_k \mid X_k = 1) \]

\[ E(U \mid X_k = 0) = w(X_k = 0). (X_k \mid X_k = 0) \]

Therefore, substituting (15) and (16) into (9) we obtain the MAUT representation of attribute utility:

\[ U(X_k = 1) = w(X_k = 1). (X_k \mid X_k = 1) = w(X_k = 1) \]

Equating (17) to equation (10) we can see that:

\[ w(X_k = 1) = \Gamma(Y = 1 \mid X_k = 1) [U(Y = 1) - U(Y = 0)] \]

This means that given our assumptions, the evaluative weight on $X_k=1$ is represented by the product of its contingency in achieving the consumer’s goal and the value of the goal itself. The notable result in (18) is that given utility of the goal is constant, the evaluative weight of an attribute will vary only when the contingency in the environment varies.
Dynamic Analysis

Having represented the goal-contingency model at equilibrium, we now relax the assumption of perfect information and analyse how consumer learning can affect the predictions of the model over time.

Perfect Learning Representation of the Goal-Contingency Model

Consider a situation where a consumer knows the value of achieving his/her goal, but does not know the contingency between the product attributes and the goal. Also assume that the consumer successively encounters information (in $t$ number of exposures) about the product attributes, $X_1=(1, 0)$ and $X_2=(1, 0)$, and the goal $Y=(1, 0)$.

Introduction of successively occurring information allows interpretation of the variables $Y$, $X_1$, and $X_2$ in terms of associative learning theory. In the classical conditioning sense a consumer’s goal ($Y$) can be interpreted as the unconditioned stimulus ($US$), which represents the value of reinforcement to the consumer. Attributes ($X_1$ and $X_2$) can be interpreted as the conditioning stimuli ($CS$). Thus, a sample of $t$ exposures in which the information about product attributes and the consumer’s goal occurs in temporal and spatial proximity (that is, is paired together), will lead to conditioning of the association $P(US|CS)$ (Hinton and Sejnowski, 1986). In terms of the ‘goal|attribute’ notation, this association is equivalent to learning $P(Y=y|X_1=x_1, X_2=x_2)$, which provides the consumer with an indication of the underlying conditional probability of his/her goal given alternative product attributes in the market. Assuming no limitations to information evaluation, the consumer’s estimate of the conditional probability based on the observed pattern of events in the environment can be expressed via the relative frequency approximation of $P(Y=y|X_1=x_1, X_2=x_2)$ at any exposure $t$.

That is:

$$P^*(Y = 1 | X_1 = x_1, X_2 = x_2) = \frac{\sum_{n=1}^{t} I(Y = 1, X_1 = x_1, X_2 = x_2)}{\sum_{n=1}^{t} I(Y = 1, X_1 = x_1, X_2 = x_2) + \sum_{n=1}^{t} I(Y = 0, X_1 = x_1, X_2 = x_2)}$$

(19)

where $P^*(Y = 1 | X_1 = x_1, X_2 = x_2)$ is the relative frequency interpretation of the conditional probability at time $t$; $I$ is an indicator function that expresses the instance of pairing of events.
in the environment, such that: \( I(Y = y, X_1 = x_1, X_2 = x_2)_n = 1 \) if \( Y = y, X_1 = x_1, X_2 = x_2 \) at exposure \( n \), 0 otherwise.

Equation (19) represents an accurate, or unbiased, approximation of the conditional probability relation between a consumer’s goal and the product attributes in the market based on the observed events over time.

Note: \( \sum_{n=1}^{t} I(Y = 1, X_1 = x_1, X_2 = x_2)_n + \sum_{n=1}^{t} I(Y = 0, X_1 = x_1, X_2 = x_2)_n = t \), since every time \( X_1 = x_1 \) and \( X_2 = x_2 \) occurs, which is \( t \) times, \( Y \) must be either 0 or 1. Therefore, equation (19) may be expressed as:

\[
P^*(Y = 1 | X_1 = x_1, X_2 = x_2)_t = \frac{\sum_{n=1}^{t} I(Y = 1, X_1 = x_1, X_2 = x_2)_n}{t}
\]

Since, in the following equations the argument \( Y=1|X_1=x_1,X_2=x_2 \) is repeated without change for simplicity of notation we denote it \( y(.) \), as such we can represent

\[
P^*(Y = 1 | X_1 = x_1, X_2 = x_2)_t \quad \text{as} \quad P^*(.)_t.
\]

Given the simplified notation, equation (19) can be expressed as a weighted average of the consumer’s previous approximation of the conditional probability and the event \( I_t \) (i.e.: the consumer’s estimate of probability based on the last observation of paired events). That is:

\[
P^*(.)_t = \beta_t I_t + (1 - \beta_t) P^*(.)_{t-1}
\]

where \( \beta_t = 1/t \).

From equation (21) we can deduce that:

\[
\Delta P^*(.)_t = \beta_t [I_t - P^*(.)_{t-1}]
\]

where \( \Delta P^*(.)_t \) is the updating of the consumer’s relative frequency approximation of conditional probability.

Equation (22) implies that the consumer will move his/her estimate in the direction of the difference between the most recent observation and his/her previous estimate by an amount
inversely proportional to the number of pieces of information he/she already has. That means as \( t \to \infty \), \( \Delta P^*(\cdot) \to 0 \). Therefore, as \( t \to \infty \), \( P^*(\cdot) \) converges to an equilibrium level. We assume that equilibrium occurs in the limit when \( P^*(\cdot) = P^*(\cdot - 1) = P(\cdot) \); such that \( P(\cdot) \) is the actual conditional probability in the environment. This implies that as \( t \to \infty \), \( P^*(\cdot) \to P(\cdot) \).

Substituting (21) into (11) and (12) the individual consumer’s perfect estimate of the contingency at time \( t \) can be represented as:

\[
\Gamma^*(Y = 1 | X_1 = 1 \text{ and } X_2 = 1, X_1 = 1, X_2 = 1, X_1 = 0, X_2 = 1) = \Gamma^*(Y = 1 | X_1 = 1, X_2 = 0) = \Gamma^*(Y = 1 | X_1 = 1, X_2 = 0) - \Gamma^*(Y = 1 | X_1 = 0, X_2 = 0)
\]  
(23)

\[
\Gamma^*(Y = 1 | X_1 = 1 \text{ and } X_2 = 0, X_1 = 1, X_2 = 0, X_1 = 0, X_2 = 0)
\]  
(24)

Therefore, equations (23) and (24) introduce dynamics into consumer preference structures by incorporating learning from experience. Based on the properties of (21), equations (23) and (24) can be interpreted as the unbiased process of preference structure formation.

Further, substituting the equations (23) and (24) in (13) and (14) respectively we obtain the unbiased estimate of utility at time \( t \):

\[
U(X_1 = 1 | X_2 = 1) = \Gamma^*(Y = 1 | X_1 = 1, X_2 = 1) \cdot \{U(Y = 1) - U(Y = 0)\}
\]  
(25)

\[
U(X_1 = 1 | X_2 = 0) = \Gamma^*(Y = 1 | X_1 = 1, X_2 = 0) \cdot \{U(Y = 1) - U(Y = 0)\}
\]  
(26)

This means that given the consumer knows the reinforcement value of his/her goal, the dynamics of utility are driven by the unbiased learning of the contingency between product attributes and the consumer’s goal.

**Relating Dynamics of Goal-Contingency to MAUT**

Referring back to equation (18), the above analysis also implies that under the assumptions of traditional multi-attribute utility theory (specifically the independence of product attributes) the estimate of the evaluative weight on attribute \( X_i = 1 \) at time \( t \) will be influenced by the consumer’s unbiased preference structure at time \( t \), such that:

\[
w(X_i = 1) = \Gamma^*(Y = 1 | X_i = 1) \cdot \{U(Y = 1) - U(Y = 0)\}
\]  
(27)

---

5 In the current analysis we assume \( P(\cdot) \) remains constant over time.
Imperfect Learning Representation of the Goal-Contingency Model

The above model assumes unbiased, and accurate updating of contingency judgments by a consumer. However, there is evidence in the psychology literature (e.g.: Shanks and Dickson, 1987), which suggests that the learning process may deviate from the predictions of the unbiased model due to consumers’ limited ability to perceive, integrate and process the observed information. This creates a situation where only part of the observed information is applied to update the consumer’s judgment at any exposure. In this section we analyse a situation where there is only partial learning over time. As such, we relax the assumption of unlimited consumer capacity to process information.

In keeping with the associative learning literature (Kamin, 1969) we assume that learning occurs when there is a discrepancy between the consumer’s initial estimate and the observed contingency in the environment. That is, the consumer will learn if his/her prior information about similar categories, attributes or other related features does not exactly reflect what the consumer observes. As such, if at any point in time the consumer’s prior estimate differs from what the consumer observes we would expect the consumer to change his/her estimate. Any change in the estimate that reduces the difference between the estimate and the observations can be called learning. Hence, as the consumer learns the magnitude of the difference between his/her estimate and the observation decreases. We can represent this type of associative learning by:

$$
\Delta P^e(t) = \lambda [P^*(t) - P^e(t-1)]
$$

(28)

where $P^*(t)$ is the unbiased approximation of the conditional probability, defined as in equation (21), that represents the events experienced by the consumer up to time $t$. $P^e(t)$ is the consumer’s estimate of conditional probability at time $t$. This estimate can differ from $P^*(t)$ because of the consumer’s potentially inaccurate initial estimate. $\Delta P^e(t)$ is the updating of a consumer’s estimate at exposure $t$; and $\lambda$ is the rate of learning parameter that moderates the extent of updating of the consumer’s estimate at each exposure $t$.

In keeping with Lopez et al. (1998) we assume that $\lambda$ is constant over time. This allows a parsimonious formulation of the model. The implication of this assumption is that the change
in the perceived relation between $Y$, $X_1$, and $X_2$ is a constant proportion of the discrepancy between the consumer’s imperfect estimate and the unbiased approximation.

Rearranging equation (28) we obtain the consumer’s estimate of achieving his/her goal ($Y=1$), given $X_1=x_1$ and $X_2=x_2$ at any point in time:

$$P^e(.)_t = \lambda P^e(.)_t + (1 - \lambda)P^e(.)_{t-1}$$  \hspace{1cm} (29)

The solution to equation (29) can be represented as:

$$P^e(.)_t = P^e(.)_0(1 - \lambda)^t + \lambda \sum_{n=1}^{t} P^e(.)_n (1 - \lambda)^{t-n}$$  \hspace{1cm} (30)

Analysing some of the key properties of equation (30) we can show\(^{6}\) that $\Delta S_t = S_t - S_{t-1}$ converges to zero (where $S_t = \lambda \sum_{n=1}^{t} P^e(.)_n (1 - \lambda)^{t-n}$) when $|1 - \lambda| < 1$. When the change in $S_t$ is zero (i.e.: $\Delta S_t = 0$) equilibrium is achieved, however this only occurs in the limit when $S_{t-1} = S_t = P^e(.)_t = P(.)_t$. This implies that as $t \to \infty$, $P^e(.)_t \to P(.)_t$.

The result in equation (30) also suggests that if $\lambda = 1$ (i.e.: learning is perfect), then $P^e(.)_t = P^e(.)_t$. This means that equation (21) is a special case of equation (30) with perfect learning.

Now, based on the results in equations (11) and (12), the consumer’s estimate of contingency at the time $t$ can be expressed as:

$$\Gamma^e(Y=1 | X_1 = 1 \text{ at } X_2 = 1)_t = P^e(Y=1 | X_1 = 1, X_2 = 1)_t - P^e(Y=1 | X_1 = 0, X_2 = 1)_t$$  \hspace{1cm} (31)

$$\Gamma^e(Y=1 | X_1 = 1 \text{ at } X_2 = 0)_t = P^e(Y=1 | X_1 = 1, X_2 = 0)_t - P^e(Y=1 | X_1 = 0, X_2 = 0)_t$$  \hspace{1cm} (32)

Substituting the result in equation (29) into (31) and (32) the equation reduces to:

$$\Gamma^e(.)_t = \lambda \Gamma^e(.)_t + (1 - \lambda)\Gamma^e(.)_{t-1}$$  \hspace{1cm} (33)

\(^{6}\)Based on the ratio test: $$\rho = \lim_{t \to \infty} \frac{S_{t+1}}{S_t}$$
And its solution can be expressed as

\[ \Gamma^*(.)_i = \Gamma^c(.)_0(1 - \lambda)^i + \lambda \sum_{n=1}^{t} \Gamma^c(.)_n(1 - \lambda)^{i-n} \]  

(34)

Note that equation (33) implies that the rate of learning, \( \lambda \), between \( P^c(Y=1|X_1=1,X_2=x_2)_t \) and \( P^c(Y=1|X_1=0,X_2=x_2)_t \) is the same. This assumption is plausible in situations where the stimuli for \( P^c(Y=1|X_1=1,X_2=x_2)_t \) and \( P^c(Y=1|X_1=0,X_2=x_2)_t \), as well as the learning context within which they occur are similar. However, it is easily relaxed, though at some cost to the parsimony of the model.

Relating (34) to (25) and (26), we can represent the goal-contingency model under the assumptions of imperfect information, limited consumer information processing capacity, and learning over time by:

\[ U(X_1 = x_1 | X_2 = x_2)_t = \Gamma^*(.)_t \{U(Y = 1) - U(Y = 0)\} \]  

(35)

Relation of the Imperfect Learning Goal-Contingency Model to MAUT
The model in equation (35) implies that the utility of any attribute \( k \), will change over time as a consumer’s preference structure changes due to learning over time. Under the multi-attribute representation of utility this change is captured by the change over time in the attribute’s evaluative weight. That is:

\[ w_k(x_k), x_k = \Gamma^*(Y = 1 | X_k = x_k)_t \{U(Y = 1) - U(Y = 0)\} \]  

(36)

Equation (36) suggests that utility dynamics can occur even when attribute levels are known, or alternatively when attribute level ambiguity is not a factor. This occurs because contingency estimates, \( \Gamma^*(Y = 1 | X_{jk} = x_{jk})_t \), are updated over time as the consumer encounters information about clearly specified attribute levels.
Empirical Testing

To test our model we used two experiments. The objective of experiment 1 was to test subjects’ sensitivity to different levels of contingency strength between the predictor attribute $X_1$ and goal $Y$, while in the presence of alternative predictor attribute $X_2$ not related to the achievement of the goal. The aim was to determine whether this sensitivity was consistent with the predictions of the model and to investigate how it altered as the subject gained more information about the relation over time.

The objective of experiment 2 was to replicate the design of experiment 1 with the added complication of random noise in the conditioning task. Hence, whereas in experiment 1 subjects saw blocks of product attributes and outcomes in which the presence or absence of an attribute matched exactly with the tested contingency in terms of the outcome of goal-achievement, in experiment 2 the tested contingency provided the mean association and stimuli were generated by sampling from a distribution with this mean. Experiment 2 increased the difficulty of the learning task and provided a test of the robustness of the model.

Stimuli:
Figures 2, 3, and 4 present screen shots of the main stimuli used in each experiment. Figure 2, screen 1, represents pairing in the classical conditioning sense, of attributes $X_1$ and $X_2$ with subjects’ goal $Y$. Subjects were asked to quickly make buy decisions using an automated trading website called Equity-Trade.com. In each decision they purchased a single equity stock out of the 4 short-listed on the screen (see screen 1, figure 2). Subjects’ goal ($Y$) was in line with the stock purchase recommendations provided by an automated agent on the Equity-Trade.com website. The recommendations could either be correct or incorrect. The correctness/incorrectness of the recommendations was paired (see screen 2, figures 3, and 4) with the presence or absence of each of the agent’s attributes$^7$.

---

$^7$ The case of the agent providing recommendations with $X_1=0$ and $X_2=0$ was explained by reference to the effect of the underlying base program of the E-T agent 3000.
Figure 2.

*Decision task (Screen 1): Stimulus pairing product attributes with subject’s goal.*

![Diagram showing a decision task screen with product attributes and a statement of goal.]

- Internet browser interface
- Statement of goal (Y)
- Short-list of equity stocks
- E-T agent 3000 interface

**Attribute 1 \( (X_1=1) \)**

**Attribute 2 \( (X_2=1) \)**

**Recommend**

---

Figure 3.

*Feedback (Screen 2): Feedback on subject’s performance and performance of the agent.*

*Stimulus representing pairing of product attributes with reinforcement.*

![Diagram showing feedback on subject's performance and agent's performance.]

**Simulation results summary**

**Feedback on agent's correct performance**
Figure 4.

*Feedback (Screen 2): Stimulus demonstrating product attributes paired with punishment.*

![Feedback on agent's incorrect performance](image)

**Design of experiment 1.**

The within subjects design of experiment 1 is summarized in tables 1. The design was based on 12 replications of treatment each followed by an observation over the course of the experiment. Table 1 shows how each single treatment was constructed.

**Table 1.**

*Experiment 1, within subjects design: composition of treatment.*

<table>
<thead>
<tr>
<th>Composition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item ($I$)</td>
<td>Screen 1 followed by Screen 2. This represents pairing of attribute(s) with the goal, followed by pairing of reinforcement or punishment with the attribute(s). Each item corresponds to the indicator function $I_i$ in equation (19).</td>
</tr>
<tr>
<td>Batch ($B$)</td>
<td>4 items related to the same attribute(s). The order of item presentation within a batch was randomised.</td>
</tr>
<tr>
<td>Treatment ($T$)</td>
<td>4 batches containing all combinations of attribute levels. The order of batches within a treatment was randomised.</td>
</tr>
<tr>
<td>Observation ($O$)</td>
<td>Observation of subjects’ estimates of conditional probability and utility after each treatment. The order of questions within an observation was randomised.</td>
</tr>
<tr>
<td>Replication ($R$)</td>
<td>Treatment followed by observation.</td>
</tr>
<tr>
<td>Experiment ($E$)</td>
<td>Twelve successive replications.</td>
</tr>
</tbody>
</table>
Each subject was shown an item (corresponding to a service description as in figure 2, and an outcome related to his/her goal such as in figure 3). This was presented three more times with the service description staying the same, but the outcome varying. These four items formed a batch. In a single treatment subjects saw four batches corresponding to presence and absence of both attributes in a random order. After each treatment an observation was taken, where subjects estimated the utility and the probability of the relation between each attribute and achievement of the subject’s goal. Each set of treatment and observation was replicated twelve times to enable the dynamics of utility to be studied.

In addition at the start of the experiment initial stimulus was presented in order to control subjects’ prior estimates of conditional probability and the contingency of each attribute at the common level of zero (see Appendix). At the end of the experiment measures of subject’s involvement, accuracy, and perceived difficulty of the experiment were also taken along with standard demographic information (see Appendix). This resulted in the complete within subjects design.

The between subjects design of experiment 1 is summarized in table 3.

Table 3

<table>
<thead>
<tr>
<th>Condition</th>
<th>Programmed conditional probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma$</td>
<td>$P(Y=1</td>
</tr>
<tr>
<td>1</td>
<td>+1.00</td>
</tr>
<tr>
<td>2</td>
<td>+0.75</td>
</tr>
<tr>
<td>3</td>
<td>+0.50</td>
</tr>
<tr>
<td>4</td>
<td>+0.25</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>-0.25</td>
</tr>
<tr>
<td>7</td>
<td>-0.50</td>
</tr>
<tr>
<td>8</td>
<td>-0.75</td>
</tr>
<tr>
<td>9</td>
<td>-1.00</td>
</tr>
<tr>
<td>10</td>
<td>+0.50</td>
</tr>
<tr>
<td>11</td>
<td>+0.50</td>
</tr>
</tbody>
</table>

The number of replications was set at 12 based on results of model simulations and extensive experimental pretests, both of which indicated that 12 replications would provide sufficient information for subjects to compute relative frequency estimates of conditional probabilities. On the other hand sufficient observations were also necessary in order to make the experiment sensitive to any learning curve effects and to allow estimation of the model.
In order to investigate the effect of the extended range of contingencies on consumer’s preference structure formation eleven different treatment conditions were included in experiment 1. The first nine conditions were defined by different levels of contingency (Γ). Conditions 10 and 11 on the other hand corresponded to the same contingency level as condition 3 (i.e.: Γ = +0.5), but were defined through different combination of the underlying conditional probabilities. Conditions 3, 10 and 11 represented a manipulation check on the effect of the arrangement of conditional probabilities on subjects’ estimates of the contingency.

**Measures:**

Instructions (reproduced in appendix A), presented each time before the measures were taken, instructed subjects to assume (in contrast to their task during treatment) that Equity-Trade.com was now set to auto-execute its recommendations such that it automatically bought for each subject the equity stocks it recommended. This controlled for the effect of individual subject’s performance during treatment as well as any other external information effects. In particular it made it credible to setup scenarios with negative (less than random) performance of the Equity-Trade.com recommendation agent.

The question asked to elicit subjects' estimates of conditional probabilities was: "Estimate the long-term performance of the agent shown on the right [picture of the Equity-Trade.com agent]. On average how well would the agent perform with these characteristics over the next 100 trades?"

Subjects made their responses to the conditional probability questions by entering a number between 0 and 100 in the fields provided as shown in figure 5.
The questions asked to elicit subject’s estimates of attribute utility were based on the comparison of two alternative configurations of the software agent (figure 6). These questions followed a set of instructions that explained the pricing policy at the Equity-Trade.com site. In essence the pricing policy stated that if the Equity-Trade.com agent made a correct purchase decision on behalf of the subject (i.e.: purchased the best equity stock), then the subject would be paid $100 by the Equity-Trade.com site. On the other hand if the Equity-Trade.com agent made an incorrect purchase decision on behalf of the subject (i.e.: purchased an equity stock other that the best stock), then the subject would receive $0, but would not lose any money. That is the Equity-Trade.com site absorbed any gains or losses from other than optimal purchase decisions by its recommendation agent. This setup controlled the value of goal achievement at $100. The question, which was asked to elicit subjects’ attribute utility evaluations, was:

"Over the next 100 trades what amount per trade would you pay (or have to be paid) to make the alternative agent [picture of the agent] the same value as the baseline agent [picture of the agent]?".

Estimates of the conditional probability were coded as $P(X_1X_2)$, where $X_1 = (1,0)$, and $X_2 = (1,0)$ depending on the presence or absence of each attribute in the pictured software agent. For example, the question screen above was coded as variable P11.
Subjects made their responses to the attribute utility questions by entering a number (implicitly limited by the Equity-Trade.com pricing policy) between -100 and 100 in the fields provided as shown in figure 6.

**Design of experiment 2.**

Experiment 2 was designed to replicate the within subjects design of experiment 1 with the added complication of random noise in the stimulus. Therefore, with the exception of the different pattern of presentation of items (I) see table 1, (specifically the random outcomes with respect to achievement of goals) the setting, stimuli, task, measures, apparatus and procedure in experiment 2 were exactly the same as in experiment 1.

The aim was to determine to what extent random noise influenced associative learning and what effect it had on formation of preference structures over time.

Table 4 summarises the programmed contingencies and their respective conditional probabilities in each condition of experiment 2.

---

10 The attribute utility responses were coded as $U^bX_1^aX_2^bX_3^a$, where $b = \text{baseline, } a = \text{alternative}; \ 1 = (1,0), \ 0 = (0,0), \ X_1 = (1,0), \ X_2 = (1,0)$. Therefore, the above comparison was coded as variable $U0111$ representing comparison of the baseline agent with attributes $X_1=0, X_2=1$ with an alternative agent with attributes $X_1=1, X_2=1$. 
Table 4.

*Experiment 2. Experimental conditions and their programmed contingencies.*

<table>
<thead>
<tr>
<th>Group</th>
<th>Effective Γ</th>
<th>Effective Γ</th>
<th>Effective Γ</th>
<th>Effective Γ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P(Y=1</td>
<td>X₁=1, X₂=1)</td>
<td>P(Y=1</td>
<td>X₁=0, X₂=1)</td>
</tr>
<tr>
<td>1</td>
<td>-0.75</td>
<td>0.23</td>
<td>1.00</td>
<td>-0.73</td>
</tr>
<tr>
<td>2</td>
<td>-0.50</td>
<td>0.27</td>
<td>0.81</td>
<td>-0.52</td>
</tr>
<tr>
<td>3</td>
<td>-0.25</td>
<td>0.27</td>
<td>0.52</td>
<td>-0.27</td>
</tr>
<tr>
<td>4</td>
<td>0.00</td>
<td>0.27</td>
<td>0.27</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>0.27</td>
<td>0.21</td>
<td>0.31</td>
</tr>
<tr>
<td>6</td>
<td>0.50</td>
<td>0.56</td>
<td>0.83</td>
<td>0.48</td>
</tr>
<tr>
<td>7</td>
<td>0.63</td>
<td>0.68</td>
<td>0.83</td>
<td>0.14</td>
</tr>
<tr>
<td>8</td>
<td>0.75</td>
<td>0.73</td>
<td>1.00</td>
<td>0.27</td>
</tr>
<tr>
<td>9</td>
<td>0.88</td>
<td>0.85</td>
<td>1.00</td>
<td>0.14</td>
</tr>
</tbody>
</table>

The contingencies (Γ) in table 4 represent the ‘true’ mean of random numbers from the binomial distribution. However, the actual contingencies achieved at the end of replication twelve, based on the relative frequency of items in experiment two, are shown under the heading “effective Γ”.

**Subjects:**

A sample size of 300 subjects (15 per condition in each experiment) was selected from the population of undergraduate and MBA students from a major Asia Pacific university. The selection procedure involved posting flyers around the university campus and advertising the study in lectures of various faculties at the university. A sure gain of $20, plus a chance of winning a major prize of $500 were offered to subjects as compensation for participating in the experiment. Once the sample was obtained subjects were then randomly assigned to experimental conditions and were tested individually at the time of their convenience within the set period allocated for the experiment.
Estimation

The goal-contingency model makes two key predictions about attribute utility. Firstly, the model predicts that at equilibrium attribute utility can be represented by the product of the contingency between the attribute and achievement of the consumer’s goal, and the value of the goal itself (equation (13) and (14)). Secondly, the model predicts that given constant goal value the trajectory towards equilibrium can be expressed according to a smooth contingency learning curve (equation (34)). We test the two predictions separately.

Testing the Equilibrium Model Predictions

The goal-contingency model makes a specific prediction about the size of the utility effect that we would expect to see if consumers behaved in a way that maximises achievement of their goal. That is, based on equations (13) and (14) the model predicts that:

\[
U(X_k = x_k \mid X_h = x_h) = \Gamma_k(\cdot)\{U(Y = 1) - U(Y = 0)\}, \ k \neq h
\]  

(37)

This means that as contingency between the attribute \( k \) and goal fulfillment \( U(Y=y) \) changes in the environment, so does the consumer’s estimate of the attribute utility. In order to test whether the prediction in equation (37) holds across the contingency spectrum, we fit the regression in equation (38) to the data at each experimental replication, and tested the hypothesis \( H_1 \) that \( \alpha = 0 \), while \( \beta = 1 \).

The regression equation which we used to test \( H_1 \) can be represented as follows:

\[
(U(X_1 = 1 \mid X_2 = x_2) \mid \Gamma_{1,j}) = \alpha + \beta \{\Gamma_{1,j}U(Y = y)\} + \varepsilon
\]

(38)

Where: \( (U(X_1 = 1 \mid X_2 = x_2) \mid \Gamma_{1,j}) \) is the utility of the predictor attribute \( X_1 \), while in the presence of the alternative attribute \( X_2 \), given the underlying contingency on attribute 1 in the experimental condition \( j \), i.e.: \( \Gamma_{1,j} \); \( \alpha \) is the intercept term; \( \beta \) is the trend coefficient; and \( \varepsilon \) is the error term.

In this test we used a comparative statics approach to determine if the hypothesis \( H_1 \) held at different points in time though out each experiment.
The results of the regression analysis for each replication of experiment 1 and 2 are presented in Table 5.

Table 5
Linear regression results of U0010 and U0111 on the predicted equilibrium effect in each replication of experiments 1 and 2.

<table>
<thead>
<tr>
<th>Experiment 1</th>
<th>95% Confidence</th>
<th>95% Confidence</th>
<th>Experiment 2</th>
<th>95% Confidence</th>
<th>95% Confidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replication</td>
<td>Low(α)</td>
<td>High(α)</td>
<td>Sig.</td>
<td>Low(β)</td>
<td>High(β)</td>
</tr>
<tr>
<td>1</td>
<td>2.85</td>
<td>-3.07</td>
<td>0.76</td>
<td>0.44</td>
<td>0.35</td>
</tr>
<tr>
<td>2</td>
<td>3.92</td>
<td>-2.85</td>
<td>10.69</td>
<td>0.25</td>
<td>0.48</td>
</tr>
<tr>
<td>3</td>
<td>4.32</td>
<td>-1.71</td>
<td>10.36</td>
<td>0.73</td>
<td>0.63</td>
</tr>
<tr>
<td>4</td>
<td>0.14</td>
<td>-5.89</td>
<td>6.17</td>
<td>0.96</td>
<td>0.75</td>
</tr>
<tr>
<td>5</td>
<td>3.92</td>
<td>-1.98</td>
<td>9.81</td>
<td>0.20</td>
<td>0.70</td>
</tr>
<tr>
<td>6</td>
<td>1.34</td>
<td>-4.39</td>
<td>7.08</td>
<td>0.68</td>
<td>0.79</td>
</tr>
<tr>
<td>7</td>
<td>4.08</td>
<td>-1.13</td>
<td>9.30</td>
<td>0.87</td>
<td>0.79</td>
</tr>
<tr>
<td>8</td>
<td>5.33</td>
<td>0.22</td>
<td>10.44</td>
<td>0.88</td>
<td>0.80</td>
</tr>
<tr>
<td>9</td>
<td>5.42</td>
<td>0.16</td>
<td>10.69</td>
<td>0.88</td>
<td>0.80</td>
</tr>
<tr>
<td>10</td>
<td>5.80</td>
<td>0.16</td>
<td>11.04</td>
<td>0.85</td>
<td>0.77</td>
</tr>
<tr>
<td>11</td>
<td>4.81</td>
<td>-0.02</td>
<td>9.35</td>
<td>0.91</td>
<td>0.83</td>
</tr>
<tr>
<td>12</td>
<td>6.04</td>
<td>0.13</td>
<td>11.96</td>
<td>0.85</td>
<td>0.75</td>
</tr>
</tbody>
</table>

The results in Table 5 show positive and statistically significant (at statistical significance of .05) response coefficients ($\beta$), as well as small and to a large extent statistically non-significant intercept terms ($\alpha$) at each replication of the two experiments.

Figure 7 summarises the 95% confidence intervals for the regression parameters. Notably, the response coefficients ($\beta$) tended to be higher in the later (7-12) as compared to the earlier (1-6) replications, and approached (but did not reach) the expected value of 1. Therefore, statistically the specific expectation of $\beta = 1$ was not supported. However, $\beta > 0$ was statistically significant. This means the effect was consistent with its expected direction. Furthermore, the general pattern of results suggests smaller deviations from the exact model prediction in the later replications of each experiment.
The regression results in table 5 indicate presence of the statistically significant positive linear trend in utility estimates across the contingency spectrum (i.e.: $\beta > 0$). This means that positive contingencies resulted in positive utility evaluations and negative contingencies in negative utility evaluations. Furthermore, stronger contingencies (such that: $|\Gamma_1 = \phi | > |\Gamma_1 = \phi |$) resulted in greater absolute utility estimates.

Therefore, even though the strict predictions about the exact parameter estimates were not evidenced in each experiment, there still is enough evidence to suggest sensitivity to goal-contingency in subjects’ evaluations of utility. This sensitivity evidences a consistent and diminishing (with each replication) tendency for underestimation of utility in the positive, and overestimation of utility in the negative contingency conditions. This pattern is well illustrated in figure 8.
Figure 8.

*Plot of the predicted values from regression of U0010 and U0111 relative to the equilibrium model predictions at selected replications 1, 2, 3, 5, 7 and 12 for experiments 1 and 2.*

The plots in figure 8 suggest a pattern of convergence of utility estimates, across the contingency spectrum, towards the predicted equilibrium effects in the later experimental replications. The general underestimation of utility in the positive, and overestimation of utility in the negative, contingency conditions to some extent can be explained by the influence of the few outlier cases (conditions –100 and –50 in experiment 1, and conditions 50, 75 in experiment 2). However, the remaining deviation from the predicted trend is interesting and deserves further investigation in following studies. This deviation appears to indicate existence of a diminishing, but a consistent conservative bias in the utility estimates compared with the utility maximizing predictions of the rational model of consumer behaviour. The results also hint at the presence of utility dynamics across the contingency spectrum.
Testing the Dynamic Model Predictions

The interpretation of contingency learning in equation (33) suggests that dynamics of utility in our model (equation (35)) are primarily based on the partial adjustment pattern of preference structure formation. This means that the model predicts a smooth trajectory of utility estimates towards the equilibrium over time.

The adjustment pattern of preference structure formation in equation (33) also suggests that preference structure is updated at each exposure to information about an attribute and goal-achievement. However, the design of experiments 1 and 2 did not capture subjects’ estimates of utility at each exposure to the stimulus; rather observations were taken only after every 16 exposures. In order to account for the expected learning between the experimental observations, in estimating the model we use the solution to the continuous analog of equation (33) written as (39):

\[ \Gamma^*(t) = r\Gamma^*(t) + c \exp(-\lambda t) + e_t \]  

(39)

where: \( r \), \( c \) and \( \lambda \) are model parameters, while \( e_t \) is an error term.

The equation (39) is analogous in derivation and its properties to equation (33). Substituting (39) into (35) gives us the continuous analog of the goal-contingency model.

To test the dynamics of utility, we note that the value of consumer goal in both experiments was fixed at $100 (i.e.: { (1) (0)} = \{U(Y = 1) - U(Y = 0)\} = $100). Therefore, we analysed equation (39) as the only driver of the dynamics in the data.

In order to determine if the specific interpretation of the contingency learning in equation (39) provides a good representation of the dynamic patterns in the data, we first compare equation (39) to other plausible statistical models of contingency learning in a nested models comparison. Then, we generalise equation (39) under the mixed effects format and investigate its parameters relative to our prior expectations based on the conceptual framework.
Nested Models Comparison.  

In order to test the appropriateness of equation (39) as a model of preference structure dynamics we examine its fit relative to other more complex and simpler statistical models (see table 5).

Table 5.  
Summary of the models included in the nested comparisons.

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturated</td>
<td>$\Gamma^c(t) = \sum_{t=1}^{T} \mu_t J_t + e_t$, $\mu_t = \text{the mean contingency estimate at time } t$, $J_t = \text{the indicator variable such that } J_t = 1 \text{ if } t = n, 0.$</td>
<td>Model interpolating the exact pattern of contingency learning observed in the data.</td>
</tr>
<tr>
<td>Monotonic 1</td>
<td>$\Gamma^c(t) = \sum_{t=1}^{T} \mu_t J_t + e_t$, subject to $\delta_i &lt; 0$ or $\delta_i &gt; 0$ depending on the direction of convergence, where $\delta_i = \mu_{t+1} - \mu_t$</td>
<td>Model assuming monotonic learning.</td>
</tr>
<tr>
<td>Monotonic 2</td>
<td>$\Gamma^c(t) = \sum_{t=1}^{T} \mu_t J_t + e_t$, subject to $\delta_{t+1} - \delta_t &lt; 0$, $\delta_i &gt; 0$; and $\delta_{t+1} - \delta_t &gt; 0$, $\delta_i &lt; 0$.</td>
<td>Model assuming monotonic learning in the first and negative second derivative.</td>
</tr>
<tr>
<td>Exponential</td>
<td>$\Gamma^c(t) = r \Gamma^c(t) + c \exp(-\lambda t) + e_t$; equation (37)</td>
<td>Exponential model based on the assumption of surprise-based learning.</td>
</tr>
<tr>
<td>Null</td>
<td>$\Gamma^c(t) = r + e_t$</td>
<td>Model assuming no learning ($\lambda = 0$).</td>
</tr>
</tbody>
</table>

Results of the model comparisons for experiment 1 are presented in the table 6, and for experiment 2 in table 7.
Table 6

Experiment 1. Comparison of the five nested models according to the total and the average AIC and BIC results.

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>BIC</th>
<th>AIC</th>
<th>BIC</th>
<th>AIC</th>
<th>BIC</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturated</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1766.92</td>
<td>2188.40</td>
<td>1743.68</td>
<td>2165.16</td>
<td>587.78</td>
<td>1009.25</td>
<td>617.16</td>
<td>1038.63</td>
</tr>
<tr>
<td>Average</td>
<td>160.63</td>
<td>198.95</td>
<td>158.52</td>
<td>196.83</td>
<td>53.43</td>
<td>91.75</td>
<td>56.11</td>
<td>94.42</td>
</tr>
<tr>
<td>Monotonic 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1653.17</td>
<td>1863.91</td>
<td>1620.79</td>
<td>1812.37</td>
<td>440.42</td>
<td>555.37</td>
<td>499.27</td>
<td>564.34</td>
</tr>
<tr>
<td>Average</td>
<td>150.29</td>
<td>169.45</td>
<td>147.34</td>
<td>164.76</td>
<td>40.04</td>
<td>50.49</td>
<td>45.39</td>
<td>59.03</td>
</tr>
<tr>
<td>Monotonic 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1642.08</td>
<td>1820.89</td>
<td>1607.16</td>
<td>1763.61</td>
<td>454.41</td>
<td>576.94</td>
<td>486.27</td>
<td>567.91</td>
</tr>
<tr>
<td>Average</td>
<td>149.28</td>
<td>165.54</td>
<td>146.11</td>
<td>160.33</td>
<td>41.31</td>
<td>52.63</td>
<td>44.21</td>
<td>55.24</td>
</tr>
<tr>
<td>Exponential</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1611.13</td>
<td>1710.50</td>
<td>1584.95</td>
<td>1690.32</td>
<td>446.14</td>
<td>551.51</td>
<td>462.47</td>
<td>567.84</td>
</tr>
<tr>
<td>Average</td>
<td>146.47</td>
<td>156.05</td>
<td>144.09</td>
<td>153.67</td>
<td>40.56</td>
<td>50.14</td>
<td>42.04</td>
<td>51.62</td>
</tr>
<tr>
<td>Null</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1724.15</td>
<td>1759.27</td>
<td>1671.95</td>
<td>1707.08</td>
<td>818.23</td>
<td>1163.07</td>
<td>1127.66</td>
<td>1472.50</td>
</tr>
<tr>
<td>Average</td>
<td>156.74</td>
<td>159.93</td>
<td>152.00</td>
<td>155.19</td>
<td>39.33</td>
<td>42.52</td>
<td>43.17</td>
<td>46.36</td>
</tr>
</tbody>
</table>

* Total over eleven conditions; average per condition.

The results in table 6 show that the exponential model (equation (39)) performed better than the other models for the predictor attribute $X_1$ (variables $\Gamma_{0010}, \Gamma_{0111}$), while the null model tended to provide a better description of the patterns in the data on the alternative attribute $X_2$ (variables $\Gamma_{0001}, \Gamma_{1011}$)\(^{11}\) for which no learning was expected\(^{12}\). Table 7 shows the relative rankings of the five nested models. The AIC rankings especially were consistent with the progressively greater level of restriction on the learning pattern imposed by the top three models. The BIC rankings on the other hand, which entailed a greater penalty for the number of parameters, showed that only the exponential model outperformed the null model. Table 7 presents the results of the model comparisons in experiment 2.

Table 7

Experiment 2. Comparison of the five nested models according to the total and the average AIC and BIC condition results.

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>BIC</th>
<th>AIC</th>
<th>BIC</th>
<th>AIC</th>
<th>BIC</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturated</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1724.39</td>
<td>2069.23</td>
<td>1532.99</td>
<td>1877.83</td>
<td>818.22</td>
<td>1163.07</td>
<td>1127.66</td>
<td>1472.50</td>
</tr>
<tr>
<td>Average</td>
<td>191.59</td>
<td>229.91</td>
<td>170.33</td>
<td>208.64</td>
<td>90.91</td>
<td>129.23</td>
<td>125.29</td>
<td>163.61</td>
</tr>
<tr>
<td>Monotonic 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1637.42</td>
<td>1781.10</td>
<td>1419.57</td>
<td>1531.32</td>
<td>724.02</td>
<td>842.15</td>
<td>1020.63</td>
<td>1129.19</td>
</tr>
<tr>
<td>Average</td>
<td>181.93</td>
<td>197.90</td>
<td>157.73</td>
<td>170.47</td>
<td>80.44</td>
<td>93.57</td>
<td>113.40</td>
<td>125.46</td>
</tr>
<tr>
<td>Monotonic 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1629.21</td>
<td>1747.35</td>
<td>1432.02</td>
<td>1550.16</td>
<td>728.53</td>
<td>828.71</td>
<td>1027.05</td>
<td>1122.84</td>
</tr>
<tr>
<td>Average</td>
<td>181.02</td>
<td>194.15</td>
<td>159.14</td>
<td>172.24</td>
<td>80.72</td>
<td>92.07</td>
<td>114.11</td>
<td>124.76</td>
</tr>
<tr>
<td>Exponential</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1615.48</td>
<td>1701.69</td>
<td>1404.61</td>
<td>1490.82</td>
<td>723.05</td>
<td>809.26</td>
<td>1053.39</td>
<td>1139.66</td>
</tr>
<tr>
<td>Average</td>
<td>179.49</td>
<td>188.07</td>
<td>155.68</td>
<td>165.04</td>
<td>80.33</td>
<td>89.91</td>
<td>117.04</td>
<td>126.23</td>
</tr>
<tr>
<td>Null</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1785.33</td>
<td>1791.76</td>
<td>1444.35</td>
<td>1473.26</td>
<td>731.54</td>
<td>760.38</td>
<td>1032.61</td>
<td>1061.34</td>
</tr>
<tr>
<td>Average</td>
<td>196.13</td>
<td>199.33</td>
<td>160.50</td>
<td>163.69</td>
<td>81.29</td>
<td>84.48</td>
<td>114.73</td>
<td>117.93</td>
</tr>
</tbody>
</table>

* Total over nine conditions; average per condition.

\(^{11}\) With the exception of the AIC result on variable ($\Gamma_{1011}$).

\(^{12}\) The exception for attribute $X_1$ was the control condition (zero contingency) where, again, we would expect no learning.
The results of the model comparisons in experiment 2 (table 7) were not as consistent as those in experiment 1. In relation to the predictor attribute $X_1$ (variables: $\Gamma^{0010}$, $\Gamma^{0111}$) the exponential model performed better than the other models on all variables, except for the BIC scores on variable $\Gamma^{0111}$, which marginally favoured the null model. In relation to the alternative attribute $X_2$ (variables $\Gamma^{0001}$, $\Gamma^{1011}$) the null model was consistently dominant on the BIC criteria. The AIC scores, however, tended to favour either the exponential model (variable $\Gamma^{0001}$) or the monotonic 1 model (variable $\Gamma^{1011}$). The main difference between experiments 1 and 2 was introduction of noise in the pattern of contingency. We suspect that noise in the stimulus made learning necessary in some conditions before judgments of attribute irrelevance could reliably be made by subjects. This may have influenced the pattern of results on attribute $X_2$.

In comparison to experiment 1, the average AIC and the BIC results in experiment 2 were higher suggesting that introduction of noise in the stimulus resulted in a less consistent learning pattern and a worse performance of the exponential model.

**Mixed-effects Representation of the Goal-Contingency Model**

Results of nested model comparisons show that the exponential model in equation (39) on average explained the dynamic patterns in the data better than the other models in the comparison, when the attribute contained information about goal achievement that was unexpected by subjects. We now build on this result and investigate the effect of different levels of contingency on the rate and accuracy of contingency learning. In order to allow generalizability of results we formulate the model in equation (39) under the mixed effects framework. That is, we examine heterogeneity of preference structure learning at the individual and at the condition (contingency) levels. In full notation the mixed effects model can be expressed as:

$$\Gamma^e(i, j, t) = r_{i,j} \Gamma^* (i, j, t) + c_{i,j} \exp(-\lambda_{i,j} t) + e_{i,j,t}$$

where: $\Gamma^e(i, j, t)$ denotes the estimate of contingency on the given attribute at time $t$ for individual $i$ in condition $j$; $r_{i,j} \Gamma^* (i, j, t)$ is the asymptotic level of the estimated contingency for individual $i$ in condition $j$; $\Gamma^* (i, j, t)$ is the relative frequency level of contingency observed by
individual \( i \) at time \( t \) in condition \( j \); \( r_{i,j} \) is the accuracy of learning parameter, such that \( r_{i,j} - 1 \) can be interpreted as an individual learning bias. Term \( c_{i,j} \) influences individual’s prior expectation of the contingency, such that the overall prior expectation is given by \( r_{i,j} \Gamma^*(\cdot)_{i,j,0} + c_{i,j} \). Term \( \lambda_{i,j} \) denotes the rate of contingency learning by individual \( i \) in condition \( j \). \(-\lambda_{i,j} c_{i,j}\) is the marginal effect of \( \Gamma^*(\cdot)_{i,j,0} \); and \( e_{i,j,t} \) is the error term.

Expectation of parameters in the equation (40):

1. \( \Gamma_{i,j} = 1 \). The size of the parameter \( r_{i,j} \) is expected to equal 1 based on the normative model prediction that individual preference structures will exactly reflect the underlying contingency relation between the consumer’s goal and the given attribute\(^{13}\). Any deviation from 1 would represent the extent of the learning bias (i.e.: \( r_{i,j} - 1 \)) relative to the predictions of the rational economic model\(^{14}\).

2. \( c_{i,j} = -\Gamma^*(\cdot)_{i,j,0} \). We expect \( c_{i,j} \) to equal \( -\Gamma^*(\cdot)_{i,j,0} \) based on the assumption that an individual’s prior expectation of contingency at time \( t = 0 \) for any unfamiliar attribute is zero; that is neither favourable nor unfavourable. This expectation is also reinforced by the initial manipulation introduced in the experimental design.

3. \( (\lambda_{i,j} \mid \Gamma = \phi) > (\lambda_{i,j} \mid \Gamma = \varphi) \) for \( |\phi| > |\varphi| \). No specific prediction about the level of the parameter \( \lambda_{i,j} \) is explicitly suggested by the model. However, given the possibility of easier discrimination of an attribute’s performance in stronger contingency conditions (i.e.: conditions further away from zero) the speed of learning \( (\lambda_{i,j}) \) might be expected to be greater in the stronger contingency conditions.

\(^{13}\) Note that our previous analysis based on the comparative statics has already suggested a possible bias in the estimates of utility. However, our interest is to analyse the extent of this bias relative to the theoretical predictions of the model.

\(^{14}\) The model in chapter 3, equation (3.3.16), implicitly assumes that preference structures exactly reflect the contingency between attribute and goal achievement.
Analysis of the Model Parameters

In order to test the above parameter expectations we fit the model in equation (40) to the data for attribute $X_1$ (variables $\Gamma_{0010}$ and $\Gamma_{0111}$) using the nonlinear mixed effects (nlme) module in S-plus (Insightful Corp, 2002). The results at the experiment level of analysis are presented in table 8 for experiment 1 and in table 9 for experiments 2.

Table 8.

Experiment 1; No Noise. NLME results for equation (40) over all conditions, and separately over the negative only and the positive conditions.

<table>
<thead>
<tr>
<th>Contingency conditions</th>
<th>Effect: Fixed</th>
<th>Random</th>
<th>Fixed</th>
<th>Random</th>
<th>Mixed Model (41)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1 \leq \Gamma \leq 1$</td>
<td>0.776 &lt;.001</td>
<td>0.037</td>
<td>0.015</td>
<td>0.559</td>
<td>-0.137 0.997 31.617</td>
</tr>
<tr>
<td>$\Gamma &lt; 0$</td>
<td>0.789 &lt;.001</td>
<td>0.059</td>
<td>0.500</td>
<td>0.031</td>
<td>0.182 0.522 0.059 0.197</td>
</tr>
<tr>
<td>$\Gamma \geq 0$</td>
<td>0.920 &lt;.001</td>
<td>0.022</td>
<td>-0.423</td>
<td>0.001</td>
<td>0.085 0.532 0.003 0.095</td>
</tr>
</tbody>
</table>

Table 11

Experiment 2; Noise. NLME results for equation (40) over all conditions, and separately over the negative only and the positive conditions.

<table>
<thead>
<tr>
<th>Contingency conditions</th>
<th>Effect: Fixed</th>
<th>Random</th>
<th>Fixed</th>
<th>Random</th>
<th>Mixed Model (41)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1 \leq \Gamma \leq 1$</td>
<td>0.621 &lt;.001</td>
<td>n/a</td>
<td>-0.001</td>
<td>0.985</td>
<td>[0.089] [0.993] n/a 4019.976</td>
</tr>
<tr>
<td>$\Gamma &lt; 0$</td>
<td>1.261 &lt;.001</td>
<td>0.079</td>
<td>0.474</td>
<td>0.026</td>
<td>0.203 0.060 0.786 0.219</td>
</tr>
<tr>
<td>$\Gamma \geq 0$</td>
<td>0.593 &lt;.001</td>
<td>0.059</td>
<td>-0.515</td>
<td>0.704</td>
<td>0.162 1.659 0.512 0.175</td>
</tr>
</tbody>
</table>

The first point to note about the results in tables 8 and 9 is that in comparison to experiment 2 the model (40) tended to perform better in experiment 1 in terms of convergence and the
parameter estimates. The second point of note is that due to different directions of learning between the positive and the negative contingency conditions, the model estimates across all conditions did not provide meaningful results.

However, statistically significant estimates were achieved in experiment 1 when we split the analysis along the positive and the negative contingencies. The fixed estimates of parameter $c$ showed the expected differences in sign, while the rates of learning ($\lambda$) appeared similar between the positive and negative contingencies. Interestingly, the fixed estimates of parameter $r$ suggested that subjects in the negative contingency conditions exhibited a negative bias (significant at .05 level) in their estimates of contingency. That is, they consistently overestimated the contingency in the negative part of the contingency spectrum. There was also an indication of a statistically significant negative bias in the positive contingency spectrum based on variable $I_{0111}$. However, the bias was smaller than in the negative part of the contingency spectrum and was not replicated on variable $I_{0010}$. In experiment 2 the results tended to remain largely statistically not significant even when split along the positive and negative contingency.

Analysis of equation (40) by individual conditions showed statistically non-significant estimates of both parameters $c_j$ and $\lambda_j$ in the two experiments. The results also indicated co-linearity between $c_j$ and $\lambda_j$. However, despite the lack of statistical significance, estimates of parameter $c_j$ were consistently close to those expected based on the assumption of zero prior contingency estimate on an unfamiliar attribute; i.e.: $-\Gamma^*(.)_{j,0}$ (see figure 7).
Figure 7.
Plots of the estimated values of parameter $c_j$ across conditions for experiments 1 and 2 respectively.

**Experimental Model Parameter $c$ Fixed Effect**

**Experiment 1.** Variable: $\Gamma_{0010}, \Gamma_{0111}$

Exponential Model Parameter $c$ Fixed Effect

\[ y = -0.0104x + 0.0779 \]

$R^2 = 0.926$

**Experiment 2.** Variable: $\Gamma_{0010}, \Gamma_{0111}$

Exponential Model Parameter $c$ Fixed Effect

\[ y = -0.0084x + 0.1461 \]

$R^2 = 0.3853$

Given these results on parameter $c_j$, we reduced the number of parameters in the model by substituting $-\Gamma^e(\cdot)_{i,j,\theta}$ for the parameter $c_{i,j}$ such that:

\[
\Gamma^e(\cdot)_{i,j,t} = r_{i,j} \Gamma^e(\cdot)_{i,j,t} - \Gamma^e(\cdot)_{i,j} \exp(-\hat{\lambda}_{i,j,t}) + e_{i,j,t} \tag{41}
\]
We then fit the model in equation (41) to the data (variables \( \Gamma_{0010} \) and \( \Gamma_{0111} \)) in experiments 1 and 2. The results are presented in table 10 for experiment 1 and table 11 for experiment 2.

Table 10

Experiments 1; No noise. NLME results for equation (41) over all conditions, and separately over the negative only and the positive conditions.

<table>
<thead>
<tr>
<th>Contingency conditions</th>
<th>Parameter: ( \Gamma_{0010} )</th>
<th>Parameter: ( \Gamma_{0111} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1 \leq \Gamma \leq 1)</td>
<td>(r) Fixed: 0.882, p-value &lt;.0001, StdDev 0.036 | (\lambda) Fixed: 0.623, p-value &lt;.0001, StdDev 0.089</td>
<td>(r) Fixed: 0.855, p-value &lt;.0001, StdDev 0.036 | (\lambda) Fixed: 0.707, p-value &lt;.0001, StdDev 0.088</td>
</tr>
<tr>
<td>(\Gamma &lt; 0)</td>
<td>0.786, p-value &lt;.0001, StdDev 0.059 | 0.726, p-value 0.0002, StdDev 0.144</td>
<td>0.781, p-value &lt;.0001, StdDev 0.158 | 0.708, p-value 0.009, StdDev 0.242</td>
</tr>
<tr>
<td>(\Gamma \geq 0)</td>
<td>0.962, p-value &lt;.0001, StdDev 0.054 | 0.522, p-value 0.001, StdDev 0.235</td>
<td>0.913, p-value &lt;.0001, StdDev 0.211 | 0.706, p-value 0.016, StdDev 0.276</td>
</tr>
</tbody>
</table>

Table 11

Experiments 2; Noise. NLME results for equation (5.3.3) over all conditions, and separately over the negative only and the positive conditions.

<table>
<thead>
<tr>
<th>Contingency conditions</th>
<th>Parameter: ( \Gamma_{0010} )</th>
<th>Parameter: ( \Gamma_{0111} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1 \leq \Gamma \leq 1)</td>
<td>(r) Fixed: 0.731, p-value 0.0047, StdDev 0.257 | (\lambda) Fixed: 0.861, p-value 0.016, StdDev 0.334</td>
<td>(r) Fixed: 0.855, p-value &lt;.0001, StdDev 0.036 | (\lambda) Fixed: 0.706, p-value 0.016, StdDev 0.276</td>
</tr>
<tr>
<td>(\Gamma &lt; 0)</td>
<td>0.868, p-value &lt;.0001, StdDev 0.087</td>
<td>(r) Fixed: 0.706, p-value 0.016, StdDev 0.276 | (\lambda) Fixed: 0.706, p-value 0.016, StdDev 0.276</td>
</tr>
<tr>
<td>(\Gamma \geq 0)</td>
<td>0.670, p-value 0.0152, StdDev 0.274</td>
<td>0.913, p-value &lt;.0001, StdDev 0.211 | 0.706, p-value 0.016, StdDev 0.276</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Contingency conditions</th>
<th>Parameter: ( \Gamma_{0111} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1 \leq \Gamma \leq 1)</td>
<td>(r) Fixed: 0.855, p-value &lt;.0001, StdDev 0.036 | (\lambda) Fixed: 0.706822, p-value &lt;.0001, StdDev 0.088</td>
</tr>
<tr>
<td>(\Gamma &lt; 0)</td>
<td>0.788, p-value &lt;.0001, StdDev 0.082</td>
</tr>
<tr>
<td>(\Gamma \geq 0)</td>
<td>0.609, p-value &lt;.0001, StdDev 0.059</td>
</tr>
</tbody>
</table>
The results based on equation (41) show an overall improvement in the AIC and the BIC values, as well as statistically significant parameter estimates in both experiments. This implies that equation (41) is a better model. It also implies that our expectation of $c_{i,j} = -\Gamma^*(\gamma_{i,j,0})$ is plausible.

In relation to our expectation of the parameter $r$, the results from equation (41) show a negative bias ($r - 1$) in across the contingency spectrum in both experiments. In experiment 1 the bias was strongest in the negative part of the contingency spectrum. In experiment 2 the reverse result was apparent. This was influenced by conditions 50 and 75 (see table 12), which to a certain degree were affected by few known outlier cases. Furthermore, based on the analysis of the 95% confidence intervals on the estimates of the parameter $r$ in experiment 2, the stronger negative bias in the negative part of the contingency spectrum could not be ruled out.

In order to test our expectation of the parameter $\lambda$ we fit the models in equations (40) and (41) by contingency condition. Tables 14 and 15 present the results for experiments 1 and 2 respectively.

Table 14

*Experiment 1; No noise. Contingency condition results of the nlme analysis, equation (41)*

<table>
<thead>
<tr>
<th>Condition</th>
<th>$r$: Fixed</th>
<th>Random</th>
<th>$\lambda$: Fixed</th>
<th>Random</th>
<th>AIC</th>
<th>BIC</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>-100</td>
<td>0.776 &lt;.0001</td>
<td>0.089</td>
<td>0.938</td>
<td>0.007</td>
<td>0.219</td>
<td>238.3197</td>
<td>257.4104</td>
<td>-6.974</td>
</tr>
<tr>
<td>-75</td>
<td>0.986 &lt;.0001</td>
<td>0.076</td>
<td>0.451</td>
<td>0.038</td>
<td>0.188</td>
<td>81.14269</td>
<td>100.2334</td>
<td>-9.292</td>
</tr>
<tr>
<td>-50</td>
<td>0.452 0.030</td>
<td>0.184</td>
<td>0.532</td>
<td>0.254</td>
<td>0.391</td>
<td>195.5869</td>
<td>214.6776</td>
<td>-7.584</td>
</tr>
<tr>
<td>-25</td>
<td>0.381 0.019</td>
<td>0.321</td>
<td>1.096</td>
<td>0.435</td>
<td>0.788</td>
<td>194.6407</td>
<td>213.7314</td>
<td>-8.733</td>
</tr>
<tr>
<td>-10</td>
<td>0.500 [&lt;.0001] n/a [2.757] [0.013] n/a</td>
<td>[47.300] [56.879] 0.012</td>
<td>0.012</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>1.155 0.055</td>
<td>0.371</td>
<td>0.113</td>
<td>0.849</td>
<td>0.568</td>
<td>207.141</td>
<td>226.232</td>
<td>6.733</td>
</tr>
<tr>
<td>50/50</td>
<td>0.859 &lt;.0001</td>
<td>0.120</td>
<td>0.786</td>
<td>0.055</td>
<td>0.294</td>
<td>94.84178</td>
<td>113.9325</td>
<td>-8.883</td>
</tr>
<tr>
<td>50/75</td>
<td>1.266 0.000</td>
<td>0.246</td>
<td>0.239</td>
<td>0.573</td>
<td>0.377</td>
<td>179.7404</td>
<td>198.8311</td>
<td>-10.276</td>
</tr>
<tr>
<td>50/100</td>
<td>1.032 &lt;.0001</td>
<td>0.123</td>
<td>0.271</td>
<td>0.184</td>
<td>0.406</td>
<td>209.7264</td>
<td>228.8171</td>
<td>-8.145</td>
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<tr>
<td>75</td>
<td>1.006 &lt;.0001</td>
<td>0.095</td>
<td>0.506</td>
<td>0.067</td>
<td>0.233</td>
<td>207.141</td>
<td>226.232</td>
<td>6.733</td>
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<td>100</td>
<td>0.889 0.000</td>
<td>0.239</td>
<td>0.634</td>
<td>0.044</td>
<td>0.285</td>
<td>120.9042</td>
<td>139.9949</td>
<td>-8.666</td>
</tr>
</tbody>
</table>

* S-Plus nlme did not converge. Fixed effects based on SPSS non-linear analysis.

The results were derived using the REML algorithm in S-plus. The starting values for the reported results were $r = 0.5, \lambda = 0.5, c = -0.5$ in positive conditions, and $c = 0.5$ in negative conditions. Conditions highlighted by "*" indicate situations in which REML did not converge. In those situations fixed effects only, based on the SPSS nonlinear Levenberg-Marquardt algorithm, are reported in square brackets.
Experiment 1. Mixed model results, Γ0111 (equation 42)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Parameter</th>
<th>Effect: Fixed</th>
<th>λ:</th>
<th>Value</th>
<th>p-value</th>
<th>StdDev</th>
<th>Mixed Model (42)</th>
<th>Change (42) - (41)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-100</td>
<td>r:</td>
<td>0.770 &lt;0.001</td>
<td>0.889</td>
<td>0.066</td>
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<td>-75</td>
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<td>0.966 &lt;0.001</td>
<td>0.842</td>
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<td>203.338</td>
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<td></td>
<td>0.436 0.021</td>
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<td>0.401</td>
<td>110.224</td>
<td>[119.803]</td>
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<td>*-25</td>
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<td>[0.881] &lt;0.001</td>
<td>n/a</td>
<td>[0.345]</td>
<td>n/a</td>
<td>[188.364]</td>
<td>[197.942]</td>
<td>84.282</td>
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<tr>
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<td>1.350 0.0061</td>
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<td>50.05</td>
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<td>0.858 &lt;0.001</td>
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<td>50.075</td>
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<td>1.038 &lt;0.001</td>
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<td>0.328</td>
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<td>-19.093</td>
</tr>
<tr>
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<tr>
<td>75</td>
<td></td>
<td>0.967 &lt;0.001</td>
<td>0.860</td>
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<td>0.212</td>
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<td>-19.093</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>0.846 &lt;0.001</td>
<td>0.821</td>
<td>0.001</td>
<td>0.175</td>
<td>157.900</td>
<td>176.991</td>
<td>-19.093</td>
</tr>
</tbody>
</table>

* S-Plus nlme did not converge. Fixed effects based on SPSS non-linear analysis.

The results of the model estimates by contingency condition show that the change in the AIC and BIC scores between the two models based on equations (40) and (41) favoured equation (41) across both experiments, while statistically significant estimates of λ tended to occur only in the very strong contingency conditions (|Γ| ≥ 0.75). Interestingly this result was also
accompanied by a consistent reduction in the variability of the $\lambda_i$ random effects as contingency strength increased (see figure 8).

Figure 8

*Plots of the random effects for parameter $\lambda_i$ across conditions for experiments 1 and 2 respectively.*

The results summarised in figure 8 suggest that even though there appeared to be no consistent effect on the mean rates of learning as contingency strength increased, which is contrary to our expectation, there was some indication of the greater consistency of the learning patterns at the extreme ends of the contingency spectrum.
Plots of the Predicted Values over Time.

The ability of the models in equations (40) and (41) to track the dynamic patterns in the data can be illustrated by plotting the predicted values from each model in relation to the mean contingency responses observed in each contingency condition (see figures 9 and 10).

In figures 9 and 10 the predicted values for equations (40) and (41) are coded as eq. (40) and eq. (41) respectively. Variables $\Gamma_{0010}$ and $\Gamma_{0111}$ represent the mean of subject’s contingency estimates on the respective variable in each condition.

Figure 9

*Experiment 1. Condition level plots of the predicted values from equations (40) and (41) relative to the mean estimates of contingency in each experimental condition.*
The plots in figure 9 show two apparent features of the models (40) and (41) in experiment 1. Firstly, both models tend to reflect the average patterns of contingency estimates quite well. Secondly, the actual learning patterns across conditions varied such that in the weak contingency conditions ($|\Gamma| \leq .25$) the learning curves were flat indicating little or no learning, whereas in the strong contingency conditions ($|\Gamma| \geq .50$) distinct learning curves were evident.
Figure 10 shows the condition level plots for experiment 2. The use of the letter ‘S’ in front of a variable name (e.g.: $S\Gamma_{0010}$) indicates the mean contingency level programmed on the variable in each condition of experiment 2. This corresponds to $\Gamma^*(.)_{j,i}$ in equations (40) and (41).

Figure 10

*Experiment 2. Condition level plots of the predicted values from equations (40) and (41) relative to the mean estimates of contingency in each experimental condition.*

Experiment 2 Noise.
The plots in figure 10 show that both models (40) and (41) did reflect the actual patterns of contingency learning in experiment 2. This is particularly important, because the variation in the stimulus due to noise in experiment 2 had a significant effect on subjects’ estimates of contingency over time.

Therefore, based on the plots in experiments 1 and 2 (figures 9 and 10) we conclude that on the whole both exponential models provided a good approximation to the average learning patterns observed in the sample, while the simpler model (equation (41)) had the advantage of fewer parameters.

Discussion

In this study we proposed a quantitative representation of the goal-contingency framework and tested the model’s predictions across a wide range of contingencies in two related experiments. The results suggest that the conceptualisation of utility at equilibrium, as the product of the contingency between an attribute and a consumer’s goal, and the value of the goal itself, is a reasonable predictor of subjects’ utility evaluations. This relation was most reliable in the later experimental replications when subjects were highly familiar with the product attribute. Under such conditions the goal-contingency model allows objective prediction of the size of the potential utility evaluations across the entire contingency spectrum. This means that utility can be determined apriori based on the interaction of the consumer goal with the product attributes in the market. Therefore, to create value for consumers, managers should first understand consumers’ goals, and then design contingency
relations between their product attributes and achievement of those goals. Positioning of products thus implies positioning of attributes relative to consumer goals.

The second main result of the study is that the predicted equilibrium levels of utility are not achieved instantly. They require time to develop as consumers learn about the contingency between product attributes and their goals. The results of the dynamic model analysis suggest that the trajectory towards equilibrium can be approximated by the representation of the goal-contingency model in equation (39). In the current sample equation (39) on average provided the best estimate of the learning patterns in the data when the attribute predicted goal achievement. When the attribute was not related to goal achievement, the null model (i.e.: a model that assumes no learning; \( \lambda = 0 \)) tended to provide a better fit. The implication of this result is that the specific formulation of utility dynamics based on preference structure formation due to associative learning of contingency provides a reasonable model of utility dynamics. It also suggests that when consumers’ goal(s) and the product attributes are well defined and understood by consumers, the dynamics of utility are primarily driven by the process of contingency learning; that is preference structure formation.

The analysis of the parameters of the dynamic model indicated that our initial expectation of consumers’ estimate of zero contingency for new product attributes was plausible. Therefore, in our study when consumers encountered a new product attribute they generally had no have positive or negative preferences towards it. This was the cease even though the attribute was objectively positively or negatively related to achievement of the consumer goal. We can interpret this as the lack of innate preference structures for new product attributes. Hence, the standard economic assumption (see McFadden, 2000) of pre-existing preference structures may be an oversimplification of actual consumer behaviour.

Our second expectation, based on the predictions of our utility maximizing model, that subjects can learn the exact underlying contingencies regardless of the contingency strength, was not verified. The results indicated a change in the consumers’ contingency estimates over time in the direction of our prediction. However, at the end of each experiment a consistent negative bias in the estimates of contingency remained.
This means that consumers’ judgments of contingency were more conservative than what would be expected under utility maximizing behaviour. The implication of this result is that the average long term value of a product attribute may tend to be underestimated by about 10% when the attribute has a positive contingency relation to achievement of a consumer’s goal, and overestimated by up to 20% when the attribute has a negative contingency relation.

Analysis of the average speeds of learning showed no consistent trend relative to increased contingency strength, which was contrary to our expectation. What we found however was greater consistency of individual rates of learning in the stronger contingency conditions. More consistent learning is indicative of a greater percentage of consumers who reliably form preference structures for initially new product attributes. This result may have implications for market entry and subsequent product diffusion. The speed at which consumers attain the potential attribute utility, that is crystallize their preferences, may depend on factors other than contingency. The possible factors (to be investigated in following studies) could include the exposure frequency and conditioning schedules. On the other hand the consistency strength appears to affect the likelihood that consumers will reliably form preferences for a given product attribute. Thus, our results would imply that product diffusion might be aided by strong contingency, not in terms of faster preference structure formation, but in terms of the percentage of customers who are likely to form preference structures for the new product attributes.

Taken together the results presented in this study suggest that starting from a position of indifference, over time consumers consistently learn to approximate the value of new product attributes by estimating the contingency between an attribute and achievement of their goal. That is, they form their preference structures over time. This result was replicated under a wide range of contingencies and under conditions of random noise in the stimulus. Therefore, the result is robust. Furthermore, our results also indicate that the evolution of preference structure formation can be modeled based on equations (40) and (41).
Limitations and Extensions

The main limitation on the conclusions of the current study is the simple nature of the learning scenario investigated in experiments 1 and 2. Both experiments relied on a single, binary, stable, and well-defined consumer goal, and used one binary predictor and one binary alternative attribute. This simple scenario was useful for demonstrating the main principles of the goal-contingency framework, however generalisation of the results to considerably more complex market situations, which would likely include multiple and multi-level predictor attributes, as well as competing and/or fuzzy consumer goals, could not be determined based on the results of the current study alone.

Another significant limitation of the study was the restricted sample size of 15 subjects per each experimental condition. Given the level of variation between subjects’ responses, these restricted sample sizes likely influenced some of the test results in the study especially in relation to statistical significance of hypotheses tests and the estimation of the model parameters. The overall pattern of results was not critically affected; but individual tests could have had more statistical power given larger sample sizes.

A natural extension of the study would be to broaden the scope of the goal-contingency model to include multi-level, and/or continuous, multiple predictor attributes, as well as multiple and multi-level consumer goals. This might lead to a richer understanding of preference formation. Such extensions would likely lead to investigation of inference formation and learning of complete utility function under the goal-contingency framework, which might also require investigation of attribute interaction and cue competition effects, as well as competition and interaction among consumer goals.
The following study has been designed to test the effect of a stock market decision support software program called “E-T agent 3000”.

The study has two components:

1. **The task**, where you choose equity stocks with the help of the decision support program (E-T agent 3000).

2. **Questions**, where you need to indicate how much you value the decision support program (E-T agent 3000).

   • In order to respond to the questions (component 2) you will need to pay attention at the task stage (component 1).
   • The study is about one hour long, hence it will test your ability to stay focused, therefore we ask you to be conscientious and concentrate throughout the duration of the whole study.

• The E-T agent 3000 can operate in two ways. It analyses a stock market each trading period and either:

   1. **Generates a recommendation only** for you to buy one stock in that period.

   OR

   2. **Automatically buys** one stock based on its own recommendation (ie.: it auto-executes its recommendations).

   • In the Task component (1) the E-T agent 3000 will provide recommendations only, and you will choose which stocks you want to buy yourself.
   • In the Questions component (2) you will be asked to evaluate the E-T agent 3000 assuming that you allowed it to automatically buy stocks for you (ie: to auto-execute its recommendations).
• **Therefore to reiterate:**

1. In the Task component you choose the stocks and the agent provides recommendations only

2. In the Questions component you evaluate the agent assuming you let it automatically execute its recommendations and buy stocks for you.

The following slides explain the task component of this study.

---

Page 4

• **Imagine** you are a private equity share trader buying and selling shares for short-term gain via the Internet service called Equity-Trade.com. You will be trading in a simulated market that resembles a stable major real-life stock market.

• **Your goal** as the private share trader is to pick one stock out of four displayed on your screen that will make the most capital gain* in a single trade.

*Capital gain is the increase in value of a stock between the time it is bought and the time it is sold.

To determine the value of a decision in any single trade assume that:

• **If you pick correctly you get $100** for that trade.
• **If you pick incorrectly you get $0** for that trade.
To help you achieve your goal of picking the correct stock you can use recommendations generated by the Internet based program called E-T agent 3000.

In the task stage the program will provide you with a recommendation only to buy one stock out of the four displayed on your screen.

E-T agent 3000 makes its recommendations based on:

1. **Neural Net** processing; and/or

2. **Temporally Continuous** stock monitoring.

These are referred to as the agent’s characteristics.

It is not necessary to understand what these are exactly, but to learn more click “?” button

---

It is not necessary that you understand what these characteristics are exactly.

*Neural Net* refers to a control mechanism that makes use of a large number of interconnected processing elements and allows reasoning with incomplete information.

*Temporally Continuous* refers to a process that continuously monitors stock price movements and continuously adjusts algorithm based estimates towards an optimum solution.
At each trade the agent’s characteristics will be presented to you either switched on (enabled) or switched off (disabled).

Thus at any trade you may be presented with an agent that has only Neural Net as its characteristic, or only Temporally Continuous, or both together, or neither characteristic.

Knowing which characteristic is switched on (enabled) at any trade may be useful for you in determining whether to accept the agent’s recommendation or to go with your own guess during the task.

By selectively choosing between recommendations and your own guesses you should aim to maximise achievement of your goal.

The following slides explain the E-T agent’s interface.
• Given that short term share trading is not much different from gambling, the only information you will be given about the stocks is what will be displayed in a table similar to the one shown in the bottom right hand corner of the screen.

• **To choose an equity stock** you simply click on the button next to the stock symbol displayed in a table

• **Results.** After you made your selection the computer will run the market simulation and calculate the stock with the greatest capital gain for that trade. It will then display the result.

---

**Example of the Table:**

<table>
<thead>
<tr>
<th>Participating Stocks</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock Symbol</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>DDF</td>
</tr>
<tr>
<td>2</td>
<td>HGGH</td>
</tr>
<tr>
<td>3</td>
<td>NNF</td>
</tr>
<tr>
<td>4</td>
<td>SWQ</td>
</tr>
</tbody>
</table>

The button. Clicking this button will buy stock SWQ

---

**Page 10**

• Shown below is a screen that will be displayed at the end of each simulation. The screen below indicates that the agent made the correct recommendation.
Your ability to stay focused on a single task will be tested by this study. Please be conscientious and pay attention to the task and the questions throughout the duration of the whole study.

If you have understood all the previous instruction, then press the continue button to begin the study, otherwise go back and read the instructions again. Once you click continue you will not be able to see these instructions again.
Initial screen \( (S^{\text{Initial}}) \)

### Summary of agent's past performance

#### Agent Characteristics

**Recommendation** | **Result**
---|---
**BUY**  |  |  
ADS | SDR | ✓
CCR | NTI | X
MDQ | SSE | ✓
RTF | ASY | ✓

### Conditional probability observation screen \( P(Y=1|X_1=1,X_1=0) \)

**Estimate the LONG-TERM performance of the agent shown on the right.**

On average how well would the agent perform with these characteristics over the next 100 trades?

Enter a number between 0 and 100 to indicate your estimate of the agent's performance:

Scale:

- 0 = Always wrong
- 100 = Always correct

Use zero (number = 0) to indicate completely flawed performance (ie. the agent is always wrong)

Use one hundred (number = 100) to indicate flawless performance (ie. the agent is always correct)

Use numbers greater than zero but less than one hundred (0 < number < 100) to indicate performance that is neither completely flawed nor absolutely flawless, but rather is somewhere in-between the two extremes.
Attribute utility question instructions

Page 1

In the next questions assume that:

1. The agent automatically executes its recommendations (ie.: purchases equity stocks on your behalf, using its auto-execute function).

In calculating your responses assume that in each period:

2. If the agent makes a correct buy decision you get $100
3. If the agent makes an incorrect buy decision you get $0, but you do not lose anything either.

That is when the agent buys a stock other than the stock with the greatest capital gain Equity-Trade.com absorbs any loses/gains. But when the agent picks the stock with the greatest capital gain Equity-Trade.com pays you $100.

Page 2

The questions will ask you to:

• Compare the value of a Baseline configuration of the agent to an Alternative configuration of the agent, and to

• Calculate an amount which if you paid [or received] would make the value of the Alternative configuration (+/- the amount) equal to the value of the Baseline configuration.

The next screen shows and explains an example of the actual question that you will see.
Example of a question screen:

**1. Compare Alternative to Baseline**

**Baseline** (TC alone)

This agent makes buy decisions

<table>
<thead>
<tr>
<th>Audit Characteristics</th>
<th>_</th>
<th>_</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC</td>
<td>_</td>
<td>_</td>
</tr>
</tbody>
</table>

Example: Temporarily Continuous TC

**Alternative** (NN & TC)

This agent makes buy decisions

<table>
<thead>
<tr>
<th>Audit Characteristics</th>
<th>_</th>
<th>_</th>
</tr>
</thead>
<tbody>
<tr>
<td>NN</td>
<td>_</td>
<td>_</td>
</tr>
<tr>
<td>TC</td>
<td>_</td>
<td>_</td>
</tr>
</tbody>
</table>

Example: Temporarily Continuous TC

What **amount per trade** would you pay [or have to be paid] to make Alternative **the same value** as Baseline?

Enter negative number when you pay
Enter positive number when you get paid

---

To calculate the amount consider:

1. **How valuable is the Baseline** to you on average per period.
   (Remember: if the Alternative agent [right] is correct you get $100, incorrect get $0)

2. **How valuable is the Alternative** to you on average per period.
   (Remember: if the Baseline agent [left] is correct you get $100, incorrect get $0)

3. From 1 and 2 **work out the amount** to pay (-) or receive (+) that will make the value of the Alternative equal to the value of Baseline.

**That is:**

(1.) Baseline $ value = (2.) Alternative $ value +/- (3.) the amount

**Example:**
Say you value Baseline at $10 p.p., but you value Alternative at $15 p.p., then to make these equal you would pay extra $5 p.p to make these equal, ie.: Baseline $10 = Alternative $15 - Amount $5
Attribute utility question screen $U(X_1|X_2)$.

2. Compare Alternative to Baseline

**Baseline** (NN alone)
This agent makes buy decisions for you.

Think of all the previous rounds, and think how much you value the Baseline agent.

**Alternative** (NN & TC)
This agent makes buy decisions for you.

Think of all the previous rounds, and think how much you value the Alternative agent.

Over the next 100 trades, what amount per trade would you pay [or have to be paid] to make Alternative agent the same value as Baseline agent? Enter minus (-) when you pay Enter number when you get paid
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