Abstract

This paper looks at how a Markov Switching model of the interest rate performs and the insights gained into the congruency of interest rate policy between Anglophile Central Banks.
Introduction

Regime switching and Markov-switching models have been studied for quite some time, and have been used in Finance since the early 1970’s. Goldfeld and Quandt (1973) were one of the first to apply this to an economic setting. In their paper, they study a situation where there are two possible regimes, described by two linear regression equations.

Neftci (1984) studied a two regime process in which one regime described rising unemployment and the other falling unemployment, with transitions between these two states described by a second-order Markov process.

Hamilton’s (1989) paper was seminal in its approach and has spawned a great deal of interest in the area. His basic approach was to use Goldfield and Quandt’s (1973) Markov switching regression to characterize changes in the parameters of an autoregressive process. In this, the unobserved state is only one of many sources of influence on the process, so that even when the economy was characterised by a growth state, the actual output may be decreasing. He used post-war U.S. real GNP data, solved the marginal likelihood function for GNP, maximised it with respect to the parameters, and then used these parameters and the data to derive statistical inference on the unobserved regimes. Hamilton found that a two-state process fitted the data well, and justified this by claiming that the two states represented a state for growth and recession.

Following on from Hamilton (1989) was a great deal of literature utilising regime switching, from modeling the Business Cycle (Durland and McCurdy (1994), Goodwin (1993), and Filardo (1994) and Layton and Smith (2000)) to exchange rates (Engle and Hamilton (1990)) and short-term interest rates.

Turner, Startz and Nelson (1989) were among the first to apply the regime-switching approach to stock market returns. They tested the hypothesis that the volatility of the market is not constant, and its variation is related to the demand for the risk premium. Turner, Startz and Nelson used the Expectation Maximisation (EM) algorithm and the monthly S&P500 composite index to show that the risk premium does indeed move in response to market volatility, which in turn implied that market returns and volatility follow different regimes.

Building on the idea of Turner, Startz and Nelson, Norden and Schaller (1993) tested whether the presumption that Markov-switching is applicable to the stock market is valid. Comparing models with regime-switching means, variances, and both means and variances, they were able to reject the null hypothesis of no switching. In a similar vein, Assoe (1998) used the idea of regime-switching to test for regimes in emerging markets as a response to various shocks created by government policy or economic reform.

The study of the volatility of asset returns by Christopeit and Cron (1997) applied a two-state Markov switching model for the volatility of asset returns. A slightly different approach was taken by Weigend and Shi (1998) who proposed that asset returns at a particular point in time follow either one of two possible Autoregressive
processes but having different means and variances. They then used a Hidden Markov Experts (HME) model to predict the full probability distribution of asset returns. Using the Expectation Maximisation (EM) method they applied the model to daily S&P500 returns, and concluded that the HME outperformed a GARCH(1,1) model in out-of-sample log-likelihood density forecasts.

**Empirical Evidence for Regime-Switching**

High persistence in the conditional variance is a common finding in two-factor models of the short rate. This means that shocks to the conditional variance do not dissipate quickly and tends to have a strong influence on the conditional variance of future horizons. Various shocks or structural changes to the economy due to oil, market crashes, changes in government economic policy and wars, leave a clear impression on the time series of interest rates, particularly those of the US. These shocks can be thought of as having distinctly a different series from when normal conditions are prevalent. These can then be described by a different function, and hence regime. When in actuality short-term rates switch between such regimes, high persistence in the volatility will result when data is averaged across the regimes, as in the case of models without Regime-Switching.

The mounting evidence that many empirical macroeconomic and financial series are plagued by parameter instability has lead to the investigation of models with time-varying parameters. This has sparked an explosion of interest in time-varying parameter models. Cai (1994), Pearson and Sun (1994) and Brenner, Harjes and Kroner (1996) find that the parameter instability of single-regime interest rate models over the 1979-82 period provides indirect evidence for regime-switching in interest rates.


Gray (1996) studies a generalized regime-switching interest rate model with state dependent mean reversion and conditional heteroscedasticity effects and find that it outperforms GARCH-type models. Naik and Lee (1997) find that the regime-switching model is better able to describe the term structure than the stochastic volatility model. The regime-switching and stochastic volatility models analysed by Naik and Lee (1997) exclude a diffusion term. Naik and Lee (1998) compare regime-switching models to stochastic volatility models and find that the former produces more reasonable term structure of volatilities, fat tails, and persistence in volatility.

Bekaert et al. (2001) find that that the term premium dynamics coupled with regime-switching effects, which proxy peso problems, lead to small sample distributions
more consistent with the data. Ang and Bekaert (2002a) find that regime-switching models replicate nonlinear patterns in the drift and volatility functions of short rates found in nonparametric approaches. Ang and Bekaert (2002b) find that regime-switching models have better out-of-sample forecasts than single-regime models. Bansal and Zhou (2002) find that a regime-switching model outperforms the Cox et al. (1985) model and a three-factor affine model where regime shifts affect the market price of risk. Smith (2002) finds evidence that either a regime switching or stochastic volatility model would adequately describe the short rate. Evans (2003) found that a three-state regime-switching model fitted UK interest rate data well. Kalimipalli and Susmel (2003) find that a regime-switching stochastic volatility process is able to capture all exogenous shocks, and fits better than a GARCH and stochastic volatility two-factor model.

Modelling of the short rate

Continuous Time Literature

Most of the literature on models of the short rate are characterised by a diffusion process. This approach has great tractability as ito calculus can be used to describe the term structure. Black-Scholes (1973) used this approach to price options, and has been found to have great use in arbitrage pricing, as well as general equilibrium approaches to pricing the term structure. Chan, Karolyi, Longstaff, and Sanders (1992) (CKLS) summarised the diffusion process used in previous literature as a general Stochastic Differential Equation:

$$dr_t = (a + br_t)dt + \sigma r_t^\gamma dB_t.$$  

where dBt is a standard Brownian motion.

Chan et al (1992) then attempted to calibrate this model to ascertain the optimal process for the short rate. Vasicek (1977) uses an arbitrage argument to derive a partial differential equation for bond prices. His derivation was sufficiently general to allow for any diffusion type of SDE for the short rate and then proceeded to derive closed form bond process for the special case of an Ornstein-Uhlenbeck process for the short rate.

It was not until CKLS that models of the short rate were calibrated, as in the past most term structure research imposed extemporaneous structures on the SDE.

The discretisation of the SDE employed by CKLS is as follows:

$$\Delta r_t = a + br_{t-1} + \sigma r_t^{\gamma} \varepsilon_t$$

where $\Delta r_t = r_t - r_{t-1}$ and $\varepsilon_t$ is a standard normal random variable.

The parameters were estimated by the Generalized Method of Moments (GMM hereafter) estimation technique developed by Hansen (1982). The key results of CKLS are that they found the short rate to be mean reverting, and the elasticity of volatility parameter to be explosive (>1).

Broze, Scaillet, and Zakoïan (1995) accounted for the discretisation bias and maximum likelihood based procedures and the indirect inference technique of
Gourieroux, Monfort, and Renault (1993), though ultimately found the bias to be fairly negligible. Aït-Sahalia (1996) estimated the implied density of discrete changes in the spot rate implied by various continuous time models, and compared these with the empirical distribution of the discrete changes in the spot rate. He finds strong non-linearity of drift, with mean reversion and higher volatility when the drift is away from the mean.

Stochastic Switching Volatility models

Lamoureux and Lastrapes (1990) show that high volatility may have been the end results of a structural change in the variance, as the resulting persistence of conditional variance represented the inability of GARCH to model shifts in the structure. Lamoureux and Lastrapes suggest that these shifts could be adequately modeled similar to Hamilton (1989). Hamilton and Susmel (1994) follow up the work of Lamoureux and Lastrapes (1990) by showing that there is a high degree of persistence when there are shocks to a GARCH model. They then suggest a first order Markov-switching ARCH process (SWARCH), which both solves the persistence problem, as well as providing a better fit than the GARCH model. Similarly, Cai (1994) considers a regime-switching ARCH model and successfully identifies regime shifts in U.S. treasury bills corresponding to the oil shock and also to U.S. Federal Reserve’s policy change.


Markov-Switching in a short-rate setting

Whereas in Regime-Switching models in general, the transitions between regimes are defined at the outset, in Markov-Switching models the regime is strictly unobservable and inference must be made as to the probability that a particular state was observed at any one time. The regression switches between regimes according to an unobserved state variable.

Duffee (1993) finds a structural break due to the Federal Reserve experiment of 1979 and postulates that the high elasticity of variance found in previous studies is a result of not accounting for this break. Cai (1994) fits a Markov-Switching ARCH model to the short rate in order to examine the issue of the persistence of volatility. He finds high volatility in the OPEC oil crisis of 1974 and the Federal Reserve Experiment of 1979.

Gray (1996) studies a generalised regime-switching model of the short rate which has both mean reversion and conditional heteroscedasticity. He finds that there is a high volatility and high interest rate regime, and a low volatility and low interest rate
regime. The three periods highlighted by the high volatility regime are the OPEC oil crises (1973-1975), the Federal Reserve experiment (1979-1983), and the immediate aftermath of the market Crash of 1987.

Smith (2000) fits a Markov-switching diffusion model with two states, attributed to the Federal Reserve experiment and the OPEC oil crisis. He finds that either a Markov-switching or a Stochastic Volatility model would adequately fit the data. An important conclusion of his paper is that interest rate volatility depends on the level of lagged interest rates. This idea will be central to the model that I select.

Most of the literature since Hamilton (1989) has assumed that regime shifts are exogenous with respect to all realisations of the regression error. This is a recent phenomena, however, as there was previous literature that dealt with endogenous switching such as Maddala and Nelson (1975).

Endogenous Switching

My paper will follow the theoretical methodology outlined by Kim, Piger, Startz (2004), in which they show that estimation methods of endogenous states, assuming they are exogenous states, produce biased parameter estimates. They then develop a method for correcting this bias, and in the process allow for an analytical characterisation of bias correction terms as well as a test for endogeneity.

Kim, Piger, Startz (2004) consider two state generating equations to link the unobserved state to the instruments. The first is an extension of the Maddala and Nelson (1975) endogenous, serially independent, switching model to the Hamilton (1989) regime switching regression and uses a probit specification. The second uses the autoregressive process described by Hamilton (1989). The bias correction procedures are found to perform well in Monte Carlo experiments.

In macroeconomic applications of Markov-Switching regressions, the estimated state variable is often correlated to the Business cycle. Indeed, it must be assumed that the shocks to the system do not come from a random outside source, but develops from a response to the system itself. In the short rate setting, it makes intuitive sense that agents react to the underlying regime, which they infer from various proxies, hence the states would be endogenous in nature. This then leads to the second aspect of my thesis, in which I study whether Reserve Banks react to the short rate in a similar way, in an attempt to maintain certain regimes across nations.

Proposed Short-Rate Model

The inspiration for the model in this paper comes from Elliott and Mamon (2002). In this they model a continuous version of the Hull-White model with a Markov chain for the mean reverting component. They argue that when interest rates are low, there is a relatively high demand for funds. This causes upward pressure on the rates, slowing down the economy and therefore reducing the demand for funds once again. Like Elliott and Mamon, I feel that the level to which the interest rates mean revert to is determined by the federal funds target set by the central bank, and varies over time.
A natural consequence of different target, or mean reverting levels, is that the level of volatility in the market will also change. Higher-mean interest rate regimes would imply higher volatility as well. As such I will model a process by which the volatility switches together with the mean reverting level.

This will be a two-state Markov Switching regression as Gray (1996), Baekaert et al (2001), Smith (2002) and numerous other studies have found this to be apt in describing the short rate. I have used the discretisation of CKLS (1992).

\[
y_t - y_{t-1} = \alpha_{S_t} + \beta_{S_t} \times y_{t-1} + \sigma_{S_t} \times \epsilon_t
\]

\[
\epsilon_t = N(0,1),
\]

\[
\alpha_{S_t} = \alpha_o (1 - S_t) + \alpha_i S_t,
\]

\[
\beta_{S_t} = \beta_o (1 - S_t) + \beta_i S_t,
\]

\[
\sigma_{S_t} = \sigma \times (1 - S_t) + \sigma \times S_t.
\]

In particular we assume that the probability that \( S_t = i \) depends on \( S_{t-1} \) and \( r_{t-1} \).

Formally:

\[
P(S_t = 1 \mid S_{t-1} = 1, ..., S_{t-j} = i, r_{t-j}) = P(S_t = 1 \mid S_{t-1} = 1, r_{t-1}) = p(r_{t-1})
\]

\[
P(S_t = 0 \mid S_{t-1} = 1, r_{t-1}) = 1 - p(r_{t-1})
\]

\[
P(S_t = 0 \mid S_{t-1} = 0, ..., S_{t-j} = i, r_{t-j}) = P(S_t = 0 \mid S_{t-1} = 0, r_{t-1}) = q(r_{t-1})
\]

\[
P(S_t = 1 \mid S_{t-1} = 0, r_{t-1}) = 1 - q(r_{t-1})
\]

The above model without any restrictions represents the time-varying transition probability Markov-switching model (TVP-MS) of Goldfeld and Quandt (1973), Diebold, Lee and Weinbach (1994) and Filardo (1994). The fixed transition probability Markov-switching model (FTP-MS) of Goldfeld and Quandt (1973) and Hamilton (1989) is described by \( p(r_{t-1}) = p \) and \( q(r_{t-1}) = q \). If \( p(r_{t-1}) = p \), \( q(r_{t-1}) = q \) and \( q=1-p \) this model then describes the fixed transition probability independent switching model (FTP-IS) of Quandt (1972). Finally, the time-varying transition probability independent switching model (TVP-IS) of Goldfeld and Quandt (1973) is characterised by \( q(r_{t-1}) = 1 - p(r_{t-1}) \).

In general I will follow the methodology of Kim, Piger, Startz (2004) in estimating the parameters. I will then test for the presence of endogeneity in the data, and the resulting bias. I will then develop an endogenous version of the model, as it is expected that there will be a significant bias in the exogenous estimates. This will be conducted using monthly US 3 month T-Bill rates from 31/01/1990 to 31/12/2003.

**Concordance**

As the central bank is believed to alter the target short rate, what is yet to be understood is whether the decision to alter the target rate is made with reference to other nations. The question arises whether central banks across nations alter their target levels, and hence set regimes, together. Using the endogenous model, I will fit
UK, Australia and Canada 3 month Treasury Bill rates, as well as the US rates. Once the inferred probabilities of being in each state has been obtained from the Markov-Switching Regression, they will then be used to determine whether these countries have short rates whose underlying regimes are the same and move together over time. Binary data can then be obtained by setting a ‘1’ if the probability of being in a particular state is above a certain probability or 0 otherwise. A concordance measure will then be used to see whether the series are moving together.

\[ C_{ij} = T^{-1} \sum_{t=1}^{T} \left( (S_{i,t} S_{j,t}) + (1 - S_{i,t})(1 - S_{j,t}) \right) \]

Where \( T \) is the sample size, \( S_{i,t} \) and \( S_{j,t} \) are the binary data from the unobserved regimes and \( C_{ij} \) measures the proportion of time the two series are in the same state. A value of around 0.7 would represent a strong co-movement of the two series.
Chapter 2 - The General Model and Estimation

In this section we present Markov-Switching model based upon the discretisation of Chan et al. (1992) of the SDE for the short rate.

This section is divided into four sections. In Section 2.1 we describe the general model presented. The algorithm used to estimate the model via maximum likelihood is developed in the Appendix. Issues relating to filtering are discussed in Section 2.2. The parameterisations needed to implement the estimation are discussed in Section 2.3.

Throughout this article, $y_t$ is the short rate, $S_t$ represents the state of the Markov Switching model and $\varphi_t$ is the filtration at time $t$.

2.1 The General Model

The general model is as follows:

\[
(1) \quad y_t - y_{t-1} = \alpha_{S_t} + \beta_{S_t} \times y_{t-1} + \sigma_{S_t} \times \varepsilon_t
\]

\[
(2) \quad \alpha_{S_t} = \alpha_0 (1 - S_t) + \alpha_1 S_t,
\]

\[
(3) \quad \beta_{S_t} = \beta_0 (1 - S_t) + \beta_1 S_t,
\]

\[
(4) \quad \sigma_{S_t} = \sigma_0 (1 - S_t) + \sigma_1 S_t.
\]

Specifically,

\[
(5) \quad S_t \in \{0,1\}
\]

where $\varepsilon_t$ is the error term at time $t$ and is a Standard Normal random variable.

Consequently, $y_t - y_{t-1}$ has a Gaussian distribution with mean $\alpha_{S_t} + \beta_{S_t} \times y_{t-1}$ and standard deviation $\sigma_{S_t}$. As such, there are two likelihoods that need to be maximised at each point in time,

\[
(6) \quad L_0 = f(y_t | S_t = 0, \varphi_{t-1}) = \frac{1}{\sigma_0 \sqrt{2\pi}} \times \exp\left(-\frac{1}{2} \left(\frac{y_t - y_{t-1} - \alpha_0 + \beta_0 \times y_{t-1}}{\sigma_0}\right)^2\right)
\]
for state 0

\[ L_i = f(y_t \mid S_t = 1, \varphi_{t-1}) = \frac{1}{\sigma_1 \sqrt{2\pi}} \times \exp\left(-\frac{1}{2} \left( \frac{y_t - y_{t-1} - \alpha_1 + \beta_1 \times y_{t-1}}{\sigma_1} \right)^2 \right) \]

for state 1

The transition probabilities for the first-order Markov Switching process are

(7) \[ Pr[S_t = 1 \mid S_{t-1} = 1] = p = \frac{\exp(p_0)}{1 + \exp(p_0)} \]

(8) \[ Pr[S_t = 0 \mid S_{t-1} = 1] = 1 - p \]

(9) \[ Pr[S_t = 0 \mid S_{t-1} = 0] = q = \frac{\exp(q_0)}{1 + \exp(q_0)} \]

(10) \[ Pr[S_t = 1 \mid S_{t-1} = 0] = 1 - q \]

This model allows the short rate to change by large amounts, reflected in large values of \( \alpha_S \) and \( \beta_S \), as well as recognising periods of high volatility found by high values of \( \sigma_S \). In essence, we expect this model to capture large changes in the short rate caused by direct intervention by central banks, as well as periods of relative stability in the short rate, absent of central bank intervention.

2.2 Filtering

We follow the procedure found in Kim and Nelson (1999) for estimating Markov Switching Models.

Step 1

To start the filter at time \( t = 1 \), we need \( Pr[S_0 = 0 \mid \varphi_0] \). The following steady-state or unconditional probabilities of \( S_t \) can be employed:

(11) \[ \Pi_0 = Pr[S_0 = 0 \mid \varphi_0] = \frac{1 - p}{2 - p - q} \]

(12) \[ \Pi_1 = Pr[S_1 = 0 \mid \varphi_0] = \frac{1 - q}{2 - p - q} \]
Now, given $\Pr[S_{t-1} = i \mid \varphi_{t-1}], i = 0, 1$, at the beginning of time $t$ or the $t$-th iteration the weighting terms $\Pr[S_t = j \mid \varphi_{t-1}], j = 0, 1$ are calculated as

\[(13) \quad \Pr[S_t = j \mid \varphi_{t-1}] = \sum \Pr[S_t = j, S_{t-1} = i \mid \varphi_{t-1}] = \sum \Pr[S_t = j \mid S_{t-1} = i] \Pr[S_{t-1} = i \mid \varphi_{t-1}]\]

where $\Pr[S_t = j \mid S_{t-1} = i], i = 0, 1, j = 0, 1$ are transition probabilities.

**Step 2**

Once $y_t$ is observed at the end of time $t$, or at the end of the $t$-th iteration, we can update the probability term in the following way:

\[(14) \quad \Pr[S_t = j \mid \varphi_t] = \Pr[S_t = j \mid \varphi_{t-1}, y_t] = \frac{f(S_t = j, y_t \mid \varphi_{t-1})}{f(y_t \mid \varphi_{t-1})} = \frac{\sum_{j=0}^1 f(y_t \mid S_t = j, \varphi_{t-1}) \Pr[S_t = j \mid \varphi_{t-1}]}{\sum_{j=0}^1 f(y_t \mid S_t = j, \varphi_{t-1}) \Pr[S_t = j \mid \varphi_{t-1}]}

where $\varphi_t = \{\varphi_{t-1}, y_t\}$
Chapter 3 - Data

Interest rate data was obtained from Datastream for Australia, Canada, UK, US. As the short rate is being modelled, the following interest rate series were obtained:

Australia: AUSTRALIA DEALER BILL 90 DAY - MIDDLE RATE
Canada: ONTARIO TREASURY BILL 3M - MIDDLE RATE
UK: UK TREASURY BILL DISCOUNT 3 MTH - MIDDLE RATE
US: US TREASURY BILL 3 MONTH - MIDDLE RATE

Monthly data is analysed in order to capture structural changes in the interest rate series. Daily data was found to be too noisy for the purpose of this study.
Chapter 4 - Empirical Results

The model fitted is estimated using maximum likelihood. The optimisation routine is performed on Matlab.

The log likelihood is calculated as a function of the parameters comprising the coefficients \((a_0, a_1, b_0, b_1, \sigma_0, \sigma_1)\) and the transition probabilities \((p, q)\). The algorithm selected was the Quasi-Newton line search.

This section will be divided as follows. Section 3.1 will present parameter estimates and their standard errors for the general model. Section 3.2 will discuss residual analysis and Section 3.3 will compare the estimated model with the actual data. Section 3.4 will present the probability series and discuss concordance between countries.

3.1 Parameter Estimates

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<th>Canada</th>
<th>UK</th>
<th>US</th>
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<td>Estimate</td>
<td>Standard Error</td>
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<td>(\sigma_0)</td>
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<td>(\sigma_1)</td>
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<td>(p)</td>
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<td>(q)</td>
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Table 3.1
Parameter Estimates and Standard errors for general model

3.2 Residual Analysis

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Table 3.2
Residual Analysis for general Model

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<th>5% Lower Bound</th>
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<td>Chi-Sq (3 df)</td>
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<td>5% value</td>
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Table 3.3
5% Cut-off values for tests
3.3 Model Fit

Graphs 3.1-3.8 (left to right)
Fitted Interest Rate and Actual Interest Rate (left column)
Fitted change in Interest Rate and Actual change in Interest Rate (right column)
3.4 Probability series and Concordance

Probability Series

Graphs 3.9-3.12 (left to right)
Probability in State 0 for Australia, Canada, UK and US (left to right)
Concordance

Concordance is a non-parametric measure of co-movement. It is measured as follows:

Let $K_{i,j}$ be a series taking on unity when the interest rate model is in state 0 and zero when it is in state 1.

The interest rate model is said to be in state 0 when the probability of being in state 0 is greater than 0.5 and in state 1 otherwise.

The Concordance statistic is calculated by

$$ C_{i,j} = T^{-1} \sum_{t=1}^{T} \left( K_{i,j} \cdot K_{i,j} \right) + \left( 1 - K_{i,j} \right) \cdot \left( 1 - K_{i,j} \right) $$

$i \neq j$

The concordance between countries in the sample is reported below. A concordance measure of 0.5 is expected between two unrelated series. A value $x$ greater than 0.5 indicates that the series’ are in the same state $x\%$ of the time and can be thought to co-vary positively. A value $x$ less than 0.5 indicates that the series’ are in different states $x\%$ of the time and can be thought to co-vary negatively.

<table>
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Table 3.4
Concordance between countries
Reference List


