In this paper I review some of the existing literature pertaining to human capital accumulation, government spending, and inequality. I outline a simple two period model of an economy in which individuals split their initial endowment between savings and investment in human capital, and consume in the second period. The government uses tax receipts to provide public education and health services which augment individuals’ human capital. I then consider the impact that the presence of credit constraints and / or heterogeneity in initial endowments of wealth might have on optimal individual and government decisions. The feasibility of other modifications and extensions to the base model is also discussed.
Introduction

Over the past few decades the central role of human capital in explanations of economic growth and development has been widely acknowledged. Theoretical contributions such as Romer (1986) and Lucas (1988) have emphasized the role played by the accumulation of human capital in the 'mechanics' of endogenously-driven growth. As education and health are key inputs into human capital, broadly defined here to mean any qualities that augment the productive possibilities of human beings, one would expect the public sector to have a potentially important role in its accumulation. This could particularly be the case in societies characterized by high levels of inequality, in which financial market imperfections or externalities may prevent optimal levels of human capital investment being reached otherwise.

In this paper I outline a simple two period model of an economy in which individuals split their initial endowment between savings (in physical capital) and investment in human capital, and consume in the second period. The government uses tax receipts to provide public education and health services which augment individuals' human capital. Specifically, a Cobb-Douglas production function is employed to model public and private investment as complementary inputs to the production of human capital. I then examine the impact that the presence of credit constraints and / or heterogeneity in initial endowments of wealth might have on optimal individual and government decisions. Simplifications of the base model are discussed together with possible modifications.

In particular, I consider the possible benefits of extending the model to incorporate overlapping generations (OLG) in a multi-period setting, and lay out a potential framework for an OLG analysis. The possibility of using this framework to simulate the evolution of growth, inequality, and human development over time in an economy with an unequal initial distribution of wealth is also flagged.

Literature review

Human capital accumulation and endogenous growth

In a seminal contribution, Lucas (1988) emphasises the interaction of physical and human capital accumulation in an endogenous growth setting. Like Uzawa (1965), and Romer (1986), he derives a model that exhibits sustained per-capita income growth from endogenous capital accumulation alone: no exogenous 'engine of growth' is required. Lucas also includes what he terms 'an external effect', $\gamma$, in his production function, so that

$$Y(t) = AK(t)^\delta [u(t)h(t)N(t)]^{1-\delta} h_a^\gamma$$

and

$$\dot{h}(t) = h(t)\delta[1 - u(t)]$$

where $u(t)$ is the amount of time each (homogenous) individual devotes to current production, $h(t)$ is his skill level, $N(t)$ is the number of workers, and $h_a$ is the average level of human capital. The stock of human capital $h(t)$ evolves such that its rate of growth depends linearly on the time an individual devotes to acquiring it. The importance of time as a
resource is common to many growth models which emphasise the accumulation of human capital. Its inclusion as a choice variable gives rise to a tradeoff faced by the representative agent between devoting more time to production (now) in order to consume more now, and investing more time in the acquisition of human capital (now) in order to consume more later.

As the above specification makes clear, there is also an externality involved in individuals’ decision to invest time in accumulating human capital. Though all benefit from the increase in output that results from an increase in \( h_a \), ‘no individual human capital accumulation decision can have an appreciable effect on it, so no one will take it into account in deciding how to allocate his time’ (Lucas 1988, p18).

What justifies the inclusion of \( h_a \) in the production function? Lucas emphasizes the importance of group interactions and the increased opportunity for exchanging ideas that obtains when a society is more educated as a whole. The notion of ‘social capital’ provides another plausible approach to \( h_a \), capturing the idea that a more educated population is more likely to understand, develop and comply with those institutions (such as the rule of law or private property rights) widely considered to be an essential part of any growth and development process. Lucas’s estimation of the elasticity of U.S. output with respect to the average level of human capital at \( \gamma = 0.4 \) reinforces the importance he attributes to \( h_a \), although it should be noted that his theoretical economy generates sustained growth regardless of whether or not this external effect is positive. That is, it is the inclusion of human capital as a productive factor rather than the external effect that leads to the possibility of constant returns in the accumulable factors (\( h \) and \( K \)) and endogenously sustainable growth.

**Government spending and human capital accumulation**

*Externalities*

Although the external effect is not necessary to obtain endogenous growth in Lucas (1988), the divergence the very fact of its existence creates between the optimal growth path and the competitive equilibrium path of the model economy suggests a role for government intervention. Zhang (1996) looks at a case where the average human capital in the economy affects the quality of education (i.e. \( h_a \) enters into the human capital accumulation function). He finds that the existence of such externalities leads individuals to underinvest in human capital and hence shows that public investments in education (such as government provided subsidies to private education) may be welfare enhancing.

*Imperfect substitutability of publicly provided services*

Barro (1990) incorporates a public sector into a model in which production exhibits decreasing returns to ‘broad capital’ (human and nonhuman capital) but constant returns to broad capital and government services taken together. His justification for including government spending as a separate argument of the production function is that private and public investments in broad capital are generally not closely substitutable, particularly in the case of public investment in nonexcludable or nonrival services. He finds that a higher tax rate reduces the after-tax return to capital, reducing the incentive to save, but that it also allows government to provide more public goods (such as infrastructure) which increase the productivity of private investment and the incentive to invest.

More recently, Blankenau and Simpson (2004) assume imperfect substitutability between private and public human capital expenditures as inputs into human capital, citing the
stylised fact that general skills acquired in primary and secondary schooling are normally financed by public investment whereas specific training is more likely to be privately financed. They specify a human capital accumulation equation of the form

$$h_{t+1} = \xi (eY_t)^\mu \theta_t^{\gamma} h_t^{1-\gamma-\mu}$$

where $\xi$ is the fraction of output devoted to public education and $\theta_t$ is the amount of private investment in human capital. They find that when taxes are nondistortionary, positive public education spending crowds out both physical capital (relative to human capital) and private human capital investment. Eventually, the negative effect on growth from crowding out offsets the positive effect of increased public education expenditures. As in Barro (1990), an optimal level of public expenditures is derived.

Glomm and Kaganovich (2003) allow for both complementarity and substitutability of public and private inputs, specifying

$$h_{t+1} = B(h_i, n_t)^\sigma (X_t + be_t)^\eta$$

where $h_i$ is the human capital of a parent, $n_t$ is the time a parent invests in educating her child, $X_t$ is the common public input received by all children, and $e_t$ is the level of private investment in human capital. Private time investment therefore complements the public input $X_t$, while the private material input substitutes for the public input. The authors cite LeGrand (1982), who argues that income related inequalities in the use of publicly provided services result in inequitable outcomes even when the public services are universally and uniformly provided, as evidence for complementarity. However, they also point to recent research by Houtenville and Conway (2001), which suggests that parents reduce their private contribution to their children’s education as the quality of public schools rises, as evidence which implies substitutability.

Inequality and human capital accumulation

Galor and Zeira (1993) explore possible linkages between income distribution and growth with a particular focus on imperfect credit markets and *indivisibilities* in the acquisition of human capital. They show that the presence of imperfect credit markets means that a more unequal distribution of wealth has an adverse impact on economic activity in the short run. Aggregate investment levels are suboptimal when poorer individuals are not able to borrow in order to obtain their preferred level of human capital. Furthermore, they show that indivisibilities in the acquisition of human capital also lead to the establishment of ‘rich’ and ‘poor’ dynasties in the long run. Initial levels of inequality can therefore have long run effects.

Tying it all together: Government spending and human capital accumulation in economies with heterogenous agents.

In what Aghion and Hewitt (1998, p.333) call ‘a pioneering contribution’ Glomm and Ravikumar (1992) outline a model which has spurred a number of advances to the literature. Referring to works such as Romer (1986) and Lucas (1988), they point out that many of the existing models at the time of writing had not adequately accounted for the large involvement of the public sector in human capital investment, despite the fact that a significant component of human capital investment is formal schooling. They also note that
the apparent preference for representative agent models meant that issues related to income distribution could not be addressed.

In an attempt to remedy these deficiencies, they present an overlapping generations model with heterogeneous agents in which human capital investment in the form of schooling drives growth. Heterogeneous individuals (who are heterogeneous in their endowments of human capital, which is linearly related to income) live for two periods and derive utility from the education which they bequeath to their children, as well as from their own consumption. Human capital is accumulated depending on the time spent accumulating it, the quality of schools (determined by expenditure), and the parents’ stock of human capital. Education is either entirely publicly provided through tax receipts or entirely privately provided. They show that public education reduces income inequality more quickly, but that higher per capita incomes are obtained through private education unless initial income inequality is sufficiently large.

Benabou (2002) focuses on the fact that redistributive policies can serve to remedy some of the problems associated with missing credit and insurance markets. He explicitly solves for the shortfall in aggregate growth achieved by a heterogeneous agent economy compared to that of a representative (homogeneous) agent economy, and shows that because decreasing returns and credit constraints imply that poorer families have a higher marginal return than wealthier ones, redistributing education resources directly or indirectly (through income taxes) can reduce this loss, although only up to a point.

Glomm and Kaganovich (2003) employ an overlapping generations economy in which human capital levels are unequally distributed amongst two period lived individuals, and human capital develops via the accumulation function described above. The government uses taxation revenue to fund public education and a PAYG social security program. They find that when public and private investments in human capital are complements, increased public education funding increases inequality. However, when public and some private inputs are substitutes, increased funding for public education reduces inequality.

The model

The following model accounts for certain characteristics specifically typical of developing economies which have rarely been analysed in combination. Firstly, government investment in social services such as health and education is included as a productive input into human capital and modeled as complementing any private investment in human capital that individuals choose to make. This is arguably a more realistic assumption for developing countries in which the quality of public services tends to be poor. Indeed, those studies (such as Houtenville and Conway 2001) which find that public and private investments in human capital tend to act more like perfect substitutes tend to be based on the experience of developed countries (such as the United States), in which the quality of public services is already high. Second, financial market imperfections in the form of credit constraints are incorporated (see cases 2 and 4). Third, as in Galor and Zeira (1993), the inherited distribution of wealth is allowed to vary away from homogeneity (see cases 3 and 4).

Consider a two period economy, in which individuals value consumption in period 2 only. In period 1 individual \( i \) receives an initial endowment \( e_{1i} \), which is subject to a flat tax \( t \). Individuals split their initial post-tax endowment into investment in human capital \( v_{1i} \) and savings \( s_{1i} \). The government uses its per capita tax revenue \( z_i = t \Sigma e_{1j}/n \) to provide public education and health services which serve to augment the human capital of each individual.
The human capital of individual $i$ is then a function of private and public investment, and develops according to

$$h^i_2 = (v^i_1)^\gamma (z^i_1)$$  \hspace{1cm} (1)$$

where $\delta, \gamma \in (0,1)$. Hence $h^i_2$ is increasing and concave in both its inputs. Furthermore, $v^i_1$ and $z^i_1$ complement each other in the production of human capital, since

$$\frac{\partial h^i_2}{\partial v^i_1 \partial z^i_1} > 0$$

Capital flows freely in and out of this economy, which is subject to the world interest rate $r$. The production technology is defined by

$$Y_2 = K^a_2 L^{1-a}_2, \text{ where } L_2 = \sum_{i=1}^{n} h^i_2$$

Since $r$ is given exogenously, $K_2$ is determined by

$$rK_2 = \alpha K^a_2 L^{-a}_2 \Rightarrow K_2 = \left(\frac{r}{\alpha L^{-a}}\right)^{1/(a-1)}$$

Now effective labour receives its marginal product, so that

$$w = (1 - \alpha)K^a_2 L^{-a}_2 = \left(\frac{1}{\alpha r}\right)^{1-\alpha}$$  \hspace{1cm} (2)$$

Hence the assumption of free capital flows means that both the wage $w = w(\alpha, r)$ and the interest rate are determined exogenously.

**CASE 1: ONE INDIVIDUAL / HOMOGENOUS INDIVIDUALS, NO CREDIT CONSTRAINTS**

The individual(s) takes $z^i_t = te^i_t$, $w$, and $r$ as given. Her problem is to choose investment in human capital $v^i_t$ and savings $s^i_t$ so as to maximise her consumption in period 2, i.e.

maximise $c^i_2 = wh^i_2 + (1 + r)s^i_1$

subject to

$(1 - t)e^i_t = v^i_t + s^i_t$

$h^i_2 = (v^i_1)^\gamma (z^i_1)$

$z^i_t = te^i_t$

Optimising individuals will equate the marginal return on investment in human capital and savings, so that
Clearly, in the absence of credit constraints, and given the complementarity between public and private investment in human capital, any government decision to raise taxes and provide more public services will increase $\frac{\partial h}{\partial v}$, leading an individual to increase her private investment in human capital until the first order condition given above is again satisfied. Note that if $s_i^*$ is negative, the individual will borrow in period 1 in order to invest $v_i^* > e_i^*$ in human capital.

An optimizing individual will therefore consume $c_2^*$ in the second period, where

$$c_2^* = w\left[ \frac{w\delta(t_e^i)^\gamma}{1 + r} \right]^{\frac{\delta}{1 - \delta}} (t_e^i)^\gamma + (1 + r) \left[ (1 - t)e_i^i - \left[ \frac{w\delta(t_e^i)^\gamma}{1 + r} \right]^{\frac{1}{1 - \delta}} \right]$$

given $t$, $w$, $r$, and $e_i^i$. The problem of a benevolent government which wishes to maximize the second period consumption of its citizens then becomes (assuming rational expectations);

choose $t$ in order to

maximise $c_2^* = w\left[ \frac{w\delta(t_e^i)^\gamma}{1 + r} \right]^{\frac{\delta}{1 - \delta}} (t_e^i)^\gamma + (1 + r) \left[ (1 - t)e_i^i - \left[ \frac{w\delta(t_e^i)^\gamma}{1 + r} \right]^{\frac{1}{1 - \delta}} \right]$

subject to $t \in (0,1)$.

Setting $\frac{\partial c_2^*}{\partial t} = 0$

yields $t^* = \frac{w^{\frac{1}{(\delta + \gamma)}} \delta^{\frac{\delta}{(\delta + \gamma)}} r^{\frac{1}{(\delta + \gamma)}}}{e_i^i (1 + r)^{\frac{1}{(\delta + \gamma)}}}$

For a maximum, we require $\frac{\partial^2 c_2^*}{\partial t^2} < 0$

and it can be shown that this condition obtains when $\delta + \gamma < 1$.

The following propositions follow:

**Proposition 1:** If $\delta + \gamma < 1$, the optimal tax rate is

$$t^* = \frac{w^{\frac{1}{(\delta + \gamma)}} \delta^{\frac{\delta}{(\delta + \gamma)}} r^{\frac{1}{(\delta + \gamma)}}}{e_i^i (1 + r)^{\frac{1}{(\delta + \gamma)}}}.$$
Proposition 2: If \( \delta + \gamma < 1 \), \( \frac{\partial t^*}{\partial \epsilon^*_1} < 0 \), \( \frac{\partial^2 t^*}{\partial \epsilon^*_1 \partial t} = -1 \)

Proposition 3: If \( \delta + \gamma > 1 \), the optimal tax rate will be at \( t = 0 \) or \( t = 1 \).

If \( \epsilon^*_1 > 0 \), \( \frac{w}{1+r} \right)^{-1}\frac{\delta}{1- \delta - (\delta + \gamma)} (1-\delta) \left( \frac{1-\delta}{1+ \delta + \gamma} \right) \), \( t^* = 1 \), otherwise \( t^* = 0 \).

[Need to check Proposition 3]
[How to interpret Proposition 2?]

CASE 2: ONE INDIVIDUAL / HOMOGENOUS INDIVIDUALS, UNABLE TO BORROW

The optimal tax problem faced by a benevolent government is more difficult under conditions in which credit is unavailable to individuals. Denote the optimal tax rate, savings rate, and investment in human capital rate in the unconstrained case by \( t^*_u \), \( s^*_u \), and \( v^*_u \) respectively. If the government chooses to set the tax rate as in the unconstrained case, \( s^*_u = (1-t^*_u)\epsilon^*_1 - v^*_u < 0 \) may obtain, which would mean the credit constrained individual would be forced to set \( v^*_u = (1-t^*_u)\epsilon^*_1 < v^*_u \). It is not at all clear that this outcome would lead to the maximisation of period 2 consumption.

A possible strategy for a government which seeks to optimize the consumption of the representative agent is the following.

Step 1: Try \( t = t^*_u \). If the credit constraint is non binding, that is if \( s^*_u = (1-t^*_u)\epsilon^*_1 - v^*_u \geq 0 \), then the results from the unconstrained case go through unhindered.

Step 2: If step 1 fails, set \( t = t^h \) such that the credit constraint becomes just binding. That is, set \( t = t^h \) so that

\[
v^*_u = v^*_u(t^h) = \left[ \frac{w\delta(t^h\epsilon^*_1)^\gamma}{1+r} \right]^{-\frac{1}{1-\delta}} = (1-t^h)\epsilon^*_1, \quad s^*_u = s^*_u(t^h) = 0
\]

This establishes a lower bound for \( t^*_c \); if step 1 fails, \( t^*_c \) is greater than or equal to \( t^h \).

Step 3: Unlike the unconstrained case, in which the cost of increases to the tax rate is the expense of interest charged on dissaving, in the constrained credit case increases to the tax rate above \( t^h \) can only be obtained at the expense of corresponding decreases to private investment in human capital. Hence the optimal tax rate is obtainable increasing \( t \) upwards from \( t^h \) until the following equality holds.
\[ w \frac{\partial h}{\partial z} \frac{\partial z}{\partial t} = -w \frac{\partial h}{\partial v} \frac{\partial v}{\partial t} \quad (7) \]

where \( v = (1-t)e_i \) and \( z = te_i \)

\[ \Rightarrow \frac{\partial h}{\partial z} = \frac{\partial h}{\partial v} \]

\[ \Rightarrow t_c^* = \frac{\gamma}{\delta + \gamma} \quad (8) \]

**Proposition 4:** When individuals are unable to borrow, the optimal tax rate \( t_c^* = t_u^* \) (given in Propositions 1 and 3) if the credit constraint is non-binding, i.e. if \( s_{1u}^* = (1-t_u^*)c_i^* - v_{1u}^* \geq 0 \), where \( s_{1u}^* \) and \( v_{1u}^* \) are defined as in (3). If the credit constraint is binding, so that \( s_{1u}^* = (1-t_u^*)c_i^* - v_{1u}^* < 0 \), then \( t_c^* = \frac{\gamma}{\delta + \gamma} \).

**CASE 3: TWO HETEROGENEOUS INDIVIDUALS, NO CREDIT CONSTRAINTS**

This model economy will be similar to that found in case 1, except it will contain two individuals, with initial endowments

\( e_i^1 = e \)

\( e_i^2 = 2 - e \)

which will give

\[ z_t = \frac{t(e_i^1 + e_i^2)}{2} = t \]

Assume utility is linear in consumption. Since the maximization of \( c_i^1 + c_i^2 \) is not dependent on \( e \), inequality will have no effect on the optimal tax rate \( t_u^* \). In the absence of credit constraints, both individuals will invest \( v_i^* = \left[ \frac{w \delta(t)^\delta}{1 + r} \right]^{\frac{1}{1-\delta}} \) in human capital, and their final period consumption will differ only by the returns they receive on their savings (or dissavings). It may be interesting to consider how inequality evolves in this economy, but this question would better be addressed in an overlapping generations framework (see below).
CASE 4: TWO HETEROGENEOUS INDIVIDUALS, NEITHER ABLE TO BORROW

This is potentially an interesting case. Since h is concave in v for any given z₁, a government seeking to maximize \( c₁ + c₂ \) will attempt to equalize the amounts of private investment in human capital as far as possible. A possible strategy for a government which seeks to maximize total second period consumption is the following:

Step 1: Set \( t = t^* \) as in case 3. If neither individual is bound by the credit constraint, then this will maximize consumption in period 2.

Step 2: If only the poorer individual is bound by the credit constraint, redistribute the savings of the richer individual to the poorer individual, until the credit constraint faced by the poorer individual becomes just binding. This progressive tax regime effectively allows the government to reverse the impact of the inequitable credit constraint and equalize private investments in human capital.

Discussion and further research

Clearly, much remains to be done (particularly with regard to case 4). Potential modifications include endogenising the interest rate (and therefore the wage) to uncover any general equilibrium effects that may be associated with government spending or credit constraints. Allowing for substitutability of public and private inputs to human capital as well as complementarity (for example, by using a similar human capital accumulation function to Glomm and Kaganovich 2003) may yield interesting results. A careful analysis of different taxation regimes on growth and distributional outcomes might also be useful.

Extending the above two period approach to incorporate overlapping generations operating in multiple periods could potentially provide much greater explanatory power. In the above analysis, the government faces what is essentially a one-shot decision, and since second period consumption is not taxed, taxation has no distortionary impact on consumption or investment. Furthermore, the two period framework limits the extent to which the effect of social externalities involved in the accumulation of human capital can be analysed.

A possible framework for an overlapping generations analysis is the following. Individuals live for two periods. In the first period, parents divide their post tax expenditure between their own consumption, investment in the human capital of their children and saving (the proceeds of which will be received by their children in the following period). The human capital of each child is a function of the average level of human capital in the parent’s generation (allowing for a possible ‘external effect’), the amount of investment their parents devote to their human capital, and government investment in public human capital augmenting services. In the second period of their lives, the children work and receive a wage on their human capital, and also receive the savings of their parents which accumulate with the interest rate. They themselves become parents and face the same decision as their own parents in period one. Each individual values his own consumption (utility) and the consumption (utility) of his child.

If the above OLG model proved tractable, the evolution of growth, inequality, and human development (for which human capital levels might be an worthwhile proxy) over time in an economy with an unequal initial distribution of wealth could be simulated. Implications for optimal government policy could then potentially be derived.
References


