Modelling Dynamic Conditional Correlations in Spot, Forward and Futures Returns

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Abstract:

Volatility (or risk) is a key variable in many areas of finance, and there are many applications that require an accurate estimate of volatility. One important application is in designing optimal dynamic hedging strategies. Engle (1982) proposed an autoregressive conditional heteroscedasticity (ARCH) model, which allows the conditional variance to change over time. This model has been extended over time, and has proved to be empirically successful in explaining the behaviour of stock returns, commodity prices and exchange rates. In my dissertation I will examine optimal hedging strategies for various commodities, exchange rates and stock indices. I will apply recently developed dynamic conditional correlation models as well as their static counterparts, in order to examine whether the dynamic models can be used to develop improved hedging strategies.
1. Introduction

Volatility (or risk) is a key variable in many areas of finance, and there are many applications that require an accurate estimate of volatility. For example, volatility is essential in designing optimal dynamic hedging strategies. It is also integral to the valuation of stocks, stock options and warrants. In order to obtain an accurate measure of volatility, Engle (1982) proposed the autoregressive conditional heteroscedasticity (ARCH) model. This model has been extended over time, and has proved to be empirically successful in explaining the behaviour of stock returns, commodity prices and exchange rates (see, for example, Bollerslev et al. (1992)).

In my dissertation I will examine optimal hedging strategies for various commodities, exchange rates and stock indices. Much of the previous research in this area has been undertaken at the univariate level, with particular emphasis on modelling the predictable part of the shocks to returns. Very little research has been undertaken using dynamic rather than static conditional correlations. I will examine if the recently developed dynamic conditional correlation models are superior to their static counterparts, and whether they can be used to develop improved hedging strategies.

In determining an optimal portfolio, it is important to model the conditional correlations. However, if an investor already has a portfolio, they may be more interested in modelling the conditional covariances. Therefore, the intentions of the investor may affect the choice of model for optimal decision making.

There are two main aspects of the relationship between the returns in the various markets that need to be analysed to develop an optimal portfolio, namely the riskiness associated with returns in these markets and the spillovers in these risks. For example, if the volatility in forward market returns is higher than the volatility in spot market returns, it may not be sensible to use the forward market to hedge against exposure in the spot market. In modelling dynamic risk spillovers, the CCC, VARMA-GARCH and VARMA-AGARCH models, which assume static conditional correlations in the shocks to returns, will be used. Each of these models is available in standard econometric software programs such as RATS.

Second, the conditional correlations in the returns to these markets are essential in analysing whether shocks have the same or opposite effects on the returns in different markets. For example, a strong negative correlation in the spot and forward returns would lead to a hedging strategy, whereas a strong positive correlation would lead to specialization rather than diversification of the portfolio. Modelling dynamic risk spillovers is crucial in calculating the conditional correlations, and the CCC, VARMA-GARCH and VARMA-AGARCH models will be used initially to obtain estimates of the static conditional correlations. If the conditional correlations in the shocks to these returns are time varying, the weights of the financial assets in a portfolio will also need to change over time. Consequently, the time-varying conditional correlation DCC, VCC and GARCC models will be estimated to test for the optimality of dynamic diversification versus specialization strategies.

In what follows, I will provide a brief though critical review of the development and use of the ARCH family of models with constant and dynamic conditional correlations. I will also examine several studies that analyse optimal hedge ratios. In the third section, I will discuss the data to be used in the empirical analysis, and give some concluding comments.

2. Literature Review

There are several important reasons for an accurate modelling of volatility. In finance the ability of investors to construct an optimal portfolio and to design an optimal hedging strategy are paramount, so that accurate forecasts of volatility are required. The finance industry makes wide use of the concept of Value-at-Risk (VaR), which also needs an accurate measure
of volatility in order to calculate the VaR thresholds. VaR is defined as “a quantile of the loss in portfolio value during a holding period of specified duration” (Glasserman et al., 2000), and is used to manage portfolio risk. If the threshold level is known, then it is possible to calculate the associated probability of an undesirable event occurring. Alternatively, if the probability is known, then the threshold level can be determined. Other cases where a proper model of volatility is important include investigating market efficiency, option pricing, and investigating whether volatility spillovers exist across different financial markets.

In traditional econometric models, the variance is typically assumed to be constant for a one-period forecast. In finance and monetary theory, this assumption is implausible, in practice, so that a model of time-varying variances is needed. Engle (1982) introduced a class of such models in response to this situation. The model he proposed is called an autoregressive conditional heteroscedasticity (ARCH) model, which allows the conditional variance to change over time, by proposing it as a function of past squared shocks. The ARCH model is applicable for univariate time series data. Bollerslev (1986) generalised the univariate ARCH process by modelling the conditional variance also as a function of the previous conditional variances.

Suppose the univariate conditional mean arises from the following process:

**Univariate Conditional Mean**:

\[
y_t = E(y_t | I_{t-1}) + \varepsilon_t, \quad t = 1,\ldots,n
\]

\[
\varepsilon_t = \eta_t \sqrt{h_t}, \quad \eta_t \sim iid(0,1)
\]

\[
E(\varepsilon_t^2) \leq \infty
\]

in which

\(E(y_t | I_{t-1})\) is the conditional mean of returns;

\(\varepsilon_t\) is the random shock (error) term in \(y_t\);

\(h_t\) is the conditional variance of \(\varepsilon_t\) (that is, the predictable part of \(\varepsilon_t\));

\(\eta_t\) is the unpredictable part of \(\varepsilon_t\).

The univariate GARCH(1,1) conditional volatility process is defined as follows:

**Univariate GARCH(1,1):**

\[
h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}
\]

\(\omega > 0, \alpha \geq 0, \beta \geq 0\).
for which the log-moment condition is given by:

\[ E\left( \log \left( \alpha \eta_t^2 + \beta \right) \right) < 0. \]

In the GARCH(1,1) model, \( \alpha \) represents the short-run persistence of shocks to returns, and \( \alpha + \beta \) represents the contribution of shocks to returns to long-run persistence. The GARCH(1,1) model reduces to ARCH(1) if \( \beta = 0 \).

The univariate ARCH and GARCH models have proved popular in practice, with many studies applying them to model specific economic and financial phenomena. For example, in introducing ARCH, Engle (1982) estimated the variance of quarterly UK inflation. Engle (1983) and Engle and Kraft (1983) also applied ARCH to model inflation. Engle, Lilien and Robins (1985) applied the ARCH model to estimate the conditional variance of the term structure. The univariate ARCH and GARCH models have been particularly successful in describing short-run exchange rate dynamics (see, for example, Baillie and Bollerslev (1989), Bollerslev (1987), Diebold and Nerlove (1989), Diebold and Pauly (1988), Domowitz and Hakkio (1985), Engle and Bollerslev (1986), Hsieh (1988, 1989), McCurdy and Morgan (1987, 1988), and Milhoj (1987)).

One reason for the success of these models is that they can capture the observation first documented by Mandelbrot (1963, p. 418) for asset returns that “large changes tend to be followed by large changes - of either sign - and small changes by small changes”. This phenomenon is referred to as volatility clustering.

A limitation of the ARCH and GARCH models is that they do not capture the findings of researchers, beginning with Black (1976), that “stock returns are negatively correlated with changes in returns volatility, i.e. volatility tends to rise in response to “bad news” (excess returns lower than expected) and to fall in response to ‘good news’ (excess returns higher than expected)” (Nelson, 1991, p. 349). Black (1976) refers to this phenomenon as a “leverage” effect, in which a negative shock will increase the debt-equity ratio, thereby increasing risk, whereas a positive shock will decrease the debt-equity ratio, and hence decrease risk.

Glosten, Jagannathan and Runkle (1992) developed a simple univariate GJR model that allows for such an asymmetric leverage effect.

**Univariate GJR(1,1):**

\[
\begin{align*}
    h_t &= \omega + (\alpha + \gamma I(\eta_{t-1})) \epsilon_{t-1}^2 + \beta h_{t-1} \\
    I(\eta_i) &= \begin{cases} 
        1, & \epsilon_i < 0 \\
        0, & \epsilon_i \geq 0
    \end{cases} \\
    \omega &> 0, \quad \alpha \geq 0, \quad \alpha + \gamma \geq 0, \quad \beta \geq 0
\end{align*}
\]

for which the log-moment condition is given by:

\[ E\left( \log \left( (\alpha + \gamma I(\eta_i)) \eta_i^2 + \beta \right) \right) < 0 \]
In the GJR(1,1) model, $\alpha$ captures the short-run persistence of a positive shock on the conditional variance, whereas $\alpha + \gamma$ captures the short-run persistence of a negative shock. When the conditional shocks, $\eta_t$, follow a symmetric distribution, the expected short-run persistence is $\alpha + \gamma / 2$, and the contribution of shocks to expected long-run persistence is $\alpha + \gamma / 2 + \beta$.

Nelson (1991) proposed an exponential GARCH (EGARCH) model that also allows for asymmetric leverage effects, as follows:

Univariate EGARCH(1,1):

$$\log h_t = \omega + \alpha |\eta_{t-1}| + \gamma \eta_{t-1} + \beta \log h_{t-1}, \quad |\beta| < 1$$

Nelson (1991) designed this model to counter some other limitations of the GARCH model, namely the non-negativity constraints and the belief that, with GARCH models, “it is difficult to evaluate whether shocks to variance ‘persist’ or not” (Nelson, 1991, p. 351). Many researchers have been interested in the length of persistence of the shocks to the conditional variance (see, for example, Poterba and Summers (1986), French, Schwert, and Stambaugh (1987), and Engle and Bollerslev (1986a), due to the potential impact on investment in long-lived capital goods (Nelson, 1991)). The problem in GARCH (1,1) models is that “shocks may persist in one norm and die out in another, so the conditional moments of GARCH (1,1) may explode even when the process itself is strictly stationary and ergodic” (Nelson, 1991, p. 350).

A drawback of the EGARCH model is that the presence of the absolute value of the standardized shocks, $\eta_t$, requires numerical optimization of the conditional, rather than unconditional, log-likelihood function, and that the statistical properties of the quasi-maximum likelihood function have not yet been established.

The univariate conditional volatility models presented above accommodate only the past information regarding a specific asset’s history of returns in determining the conditional variance. Consequently, these models would not be expected to be useful for analysing the volatility spillovers between stocks, markets and optimal portfolio decisions. It is possible to accommodate the lagged surprises in other assets in the portfolio by including the spillover effects across the portfolio. The efficiency in estimation and power in testing will be greater if all of the securities were to be modelled simultaneously, thereby yielding a multivariate model of the conditional covariances and correlations (Conrad et al., 1991). A multivariate approach would estimate the parameters jointly, thereby increasing efficiency.

Several multivariate GARCH models have been developed. These models can be divided into those that assume a static conditional correlation in the shocks to returns, and those where the conditional correlations in the shocks to returns are time varying. McAleer (2004) has provided a critical analysis of the typical specifications underlying the multivariate conditional mean, conditional covariance matrices and conditional correlation matrices in returns, as follows:

Multivariate conditional Mean:

$$y_t = E(y_t | F_{t-1}) + \varepsilon_t$$

$$\varepsilon_t = D_t \eta_t,$$
where \( y_t = (y_{1t}, \ldots, y_{mt})' \), \( \eta_t = (\eta_{1t}, \ldots, \eta_{mt})' \) is a sequence of independently and identically distributed (iid) random vectors, \( F_t \) is the past information available to time \( t \), \( D_t = \text{diag}(h_{1t}^{1/2}, \ldots, h_{mt}^{1/2}) \), \( m \) is the number of returns, and \( t = 1, \ldots, n \).

### Conditional Covariance:

\[
E(\varepsilon_t \varepsilon_t' | F_{t-1}) = Q_t = D_t \Gamma D_t
\]

\( \Gamma = \{ \rho_{ij} \}, \quad i, j = 1, \ldots, m. \)

in which

- \( Q_t \) is the conditional covariance matrix;
- \( \Gamma_t \) is the conditional correlation matrix.

A problem which was encountered in the early development of multivariate models was that they were computationally difficult due to the large number of parameters to be estimated. In the literature, various attempts have been made to restrict the number of parameters. For example, Diebold and Nerlove (1989) proposed a multivariate latent factor ARCH model and applied it to seven major spot rates. They found that it provided a good description of multivariate exchange rate movements. Bollerslev, Engle and Wooldridge (1988) estimated a linear diagonal GARCH model, and Engle, Ng and Rothschild (1990) proposed a factor GARCH model. Bollerslev (1990) introduced a multivariate GARCH model with constant conditional correlations (CCC) to simplify estimation and inference. He found that, compared with other multivariate models, the CCC model allowed a major reduction in the computational complexity (Bollerslev, 1990). The assumption of constant conditional correlations has been found to be empirically reasonable in a number of studies (see, for example, Cecchetti, Cumby, and Figlewski (1988), Kroner and Claessens (1991), Ng (1991), Kroner and Lastaunes (1991), Brown and Ligerde (1990), Baillie and Bollerslev (1990), and Schwert and Seguin (1990)). The CCC model assumes that the conditional variance for each return, \( h_{it}, i = 1, \ldots, m \), follows a univariate GARCH process, and can be represented as follows:

**Multivariate CCC:**

\[
h_{it} = \omega_i + \sum_{j=1}^{r} \alpha_{ij} \varepsilon_{i,t-j}^2 + \sum_{j=1}^{s} \beta_{ij} h_{i,t-j}
\]

\( \Gamma_t = \Gamma \)

where the conditional correlation matrix is defined as \( \Gamma = D_t^{-1} Q_t D_t^{-1} \).

The CCC model does not allow for spillovers between assets as the conditional variance of returns for a particular asset is a function of only the past information regarding that specific asset’s history of returns. The only multivariate analysis that occurs is in the calculation of
the conditional correlations. Moreover, the CCC model does not allow for asymmetric leverage effects.

Ling and McAleer (2003) proposed a vector autoregressive moving average (VARMA) GARCH model to allow for spillovers. This model specifies the conditional variance as follows:

**Multivariate VARMA-GARCH Model**

\[ H_t = W + \sum_{i=1}^{r} A_i \tilde{\varepsilon}_{t-i} + \sum_{j=1}^{s} B_j H_{t-j} \]

\[ \Gamma_t = \Gamma \]

where \( H_t = (h_{t1}, \ldots, h_{tm})' \), \( \tilde{\varepsilon} = (\varepsilon_{t1}^2, \ldots, \varepsilon_{tm}^2)' \), and \( W, A_i \) for \( i = 1, \ldots, r \) and \( B_j \) for \( i = 1, \ldots, s \) are \( m \times m \) matrices. This model also assumes constant conditional correlations, and does not allow negative and positive shocks to have different impacts on the conditional variance.

In order to accommodate the asymmetric impacts of positive and negative shocks, Chan et al. (2002) proposed the VARMA asymmetric GARCH (VARMA-AGARCH) specification for the conditional variance, namely:

**Multivariate VARMA-AGARCH**

\[ H_t = W + \sum_{i=1}^{r} A_i \tilde{\varepsilon}_{t-i} + \sum_{i=1}^{s} C_i I_{t-i} \tilde{\varepsilon}_{t-i} + \sum_{j=1}^{s} B_j H_{t-j} \]

\[ \Gamma_t = \Gamma \]

where \( C_i \) are \( m \times m \) matrices for \( i = 1, \ldots, r \), \( I_t = diag(I_{t1}, \ldots, I_{tm}) \), and

\[ I_{it} = \begin{cases} 0, & \varepsilon_{it} \geq 0 \\ 1, & \varepsilon_{it} < 0 \end{cases} . \]

If \( m = 1 \), the VARMA-AGARCH model collapses to the univariate GJR model.

All of the multivariate models discussed above make the simplifying assumption of constant conditional correlations. Although some empirical studies have found this to be empirically justified, other studies have found that this is not generally supported by financial data (see, for example, Tse and Tsui (2002)). Unless \( \eta_t \) is a sequence of independently and identically distributed random vectors, the assumption of constant conditional correlation will not be valid, so that it would be more likely that \( \Gamma_t = \{ \rho_{ijt} \} \) for \( i,j = 1, \ldots, m \) and \( t = 1, \ldots, n \).

In response to the need for a multivariate model that allows the conditional correlations to be time-varying, Engle and Kroner (1995) proposed the BEKK (named after Baba, Engle, Kraft, and Kroner) model, which can be represented as follows:

**Multivariate BEKK Model**
\[ Q_t = QQ' + AE_{t-1}e_{t-1}A' + BQ_{t-1}B' \]

where the second term is singular.

The dynamic conditional correlations associated with the BEKK model were not explicitly considered in Engle and Kroner (1995). However, they can be derived from \( Q_t = D_t \Gamma_t D_t \) as \( \Gamma_t = D_t^{-1} Q_t D_t^{-1} \), where \( D_t = (\text{diag}Q_t)^{1/2} \).

A major problem with the BEKK model is that, if the issue lies in designing an optimal portfolio, then the dynamic conditional correlations are of interest, not the dynamic conditional covariances, as modelled using BEKK. If the practitioner already has a portfolio, they may be interested in modelling the dynamic conditional covariances. However, the BEKK model suffers from serious computational difficulties due to the large number of parameters that need to be estimated, making it inefficient in practice. Another disadvantage of the BEKK model is that interpretation of the parameters is not straightforward, and that their net effects on the future variances and covariances are not obvious (Tse and Tsui (2002)).

Due to the dissatisfaction with BEKK, Tse and Tsui (2002) proposed a Variable Conditional Correlation (VCC) multivariate GARCH model. The VCC model uses a recursive transformation of the standardized shocks to estimate the time-varying conditional correlations, namely:

**Multivariate VCC:**

\[
\Gamma_t = (1 - \theta_1 - \theta_2) \Gamma + \theta_1 \Psi_{t-1} + \theta_2 \Gamma_{t-1}, \quad 1 - \theta_1 - \theta_2 \geq 0, \theta_1 \geq 0, \theta_2 \geq 0
\]

where the typical element in the non-singular second term, which is a lagged recursive conditional correlation matrix, is given by:

\[
\Psi_{i,j,t-1} = \sum_{t=1}^{M} \eta_{it-1} \eta_{jt-1} \left/ \left( \sum_{i=1}^{M} \eta_{it-1}^2 \sum_{j=1}^{M} \eta_{jt-1}^2 \right) \right\}^{1/2}
\]

in which \( M \geq m \). When \( \theta_1 = \theta_2 = 0 \), \( \Gamma \) is equivalent to the CCC model.

A problem with this model is that no explanation is given as to how they modified the assumption that \( \eta_t \) is a sequence of independently and identically distributed random vectors. Without explicitly relaxing this assumption, it is not possible to obtain time-varying conditional correlations.

Engle (2002) proposed the related Dynamic Conditional Correlation (DCC) multivariate GARCH model, which is given by:

**Multivariate DCC Model:**

\[
Z_t = (1 - \theta_1 - \theta_2) \overline{Z} + \theta_1 \eta_{t-1} \eta_{t-1} + \theta_2 Z_{t-1}
\]

where the second term is singular, and \( \theta_1 \) and \( \theta_2 \) are scalar parameters.
When $\theta_1 = \theta_2 = 0$, $\bar{Z}$ is equivalent to the CCC model. As $Z_t$ is conditional on the vector of standardized residuals, it would be the conditional covariance matrix. If the assumption that $\eta_t$ is a sequence of independently and identically distributed random vectors is not modified, then $Z_t$ is also the conditional correlation matrix. The problem with this model is the same as that with the VCC model, namely there is no mention of the properties of $\eta_t$ in developing the DCC model (although Engle (2002, p. 342) does state that “the errors are a Martingale difference by construction” in discussing how to estimate the model) (for further details, see McAleer (2004)).

As the DCC model does not satisfy the definition of a conditional correlation matrix, Engle (2002) calculates the appropriate dynamic conditional correlation matrix as follows:

$$\Gamma_t^* = \left\{ \left( \text{diag}Z_t \right)^{-1/2} Z_t \left( \text{diag}Z_t \right)^{-1/2} \right\}.$$ 

The standardization of $Z_t$ to obtain $\Gamma_t^*$ may also be required because the standardized residuals in the DCC model are not independently distributed.

The primary difference between DCC and VCC is that it is necessary to standardise $Z_t$ in the DCC model to obtain the dynamic conditional correlation matrix, whereas the time-varying conditional correlation matrix is calculated recursively in the VCC model.

Due to the problems associated with the DCC and VCC models, Chan et al. (2003) proposed the Generalized Autoregressive Conditional Correlation (GARCC) model. This model “motivates the dynamic structure of the conditional correlations explicitly through serial correlation in the vector of standardized shocks” (McAleer, 2004, pp. 13-14). They relax the assumption that $\eta_t$ is a sequence of independently and identically distributed random vectors, by defining $\eta_t$ to follow an autoregressive process, as follows:

**Multivariate GARCC:**

$$\eta_t = \nu_t (\eta_{t-1}, ..., \eta_{0})', \quad \nu_t \sim \text{iid}(0,1)$$

$$\eta_t = \sum_{l=1}^{L} \phi_{l}^{\dagger} \eta_{t-l} + \xi_t, \quad \xi_t \sim \text{iid}(0, \sigma^2_t), \quad i = 1, ..., m$$

This leads to a more general dynamic conditional correlation model than the DCC and VCC models, when $L \to \infty$, as follows:

$$W_t = \bar{W} + \Phi_1 \circ \eta_{t-1} \eta_{t-1} + \Phi_2 \circ W_{t-1}$$

where $\Phi_1$ and $\Phi_2$ are $m \times m$ matrices and $\circ$ is the Hadamard (or element by element) product.

Chan et al. (2003) showed that, when $\phi_0 = \phi_1 \delta_1$ and $\delta_1 \sim \text{iid}(0, \phi_2^{l-1})$, $W_t$ is the dynamic conditional correlation matrix of the standardized residuals, $\eta_t$, which are not independently distributed because of the presence of serial correlation in $\eta_t$. The GARCC conditional correlation matrix can be obtained formally through the following standardisation:
\[ \Gamma_t = \left( \text{diag}W_t \right)^{-1/2}W_t\left( \text{diag}W_t \right)^{-1/2}, \]

Chan et al. (2003) show that in practice this standardisation is not required, so that \( W_t \) is effectively the conditional correlation matrix. The standardisation for the DCC model, however, is required as \( Z_t \) does not satisfy the definition of a conditional correlation matrix of the standardized shocks.

The GARCC model does not accommodate asymmetric effects, but it can easily be modified to include such effects.

Numerous interesting practical extensions to the DCC and VCC models have been put forward in the literature (see, for example, Engle (2002), Hafner and Franses (2003), Cappiello et al. (2003), Billio et al. (2004), Billio and Caporin (2004), and Kwan et al. (2004)). An area of interest that has not yet been tackled in the literature is an examination of the asymptotic properties of the respective models (see McAleer (2004) for further details).

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The popularity of univariate and multivariate conditional (G)ARCH models is evidenced in the survey conducted by Bollerslev et al. (1992, p. 6), which found that “since the introduction of the ARCH model several hundred research papers applying this modeling strategy to financial time series data [had] already appeared”.

The research papers of particular interest to this dissertation are those which use ARCH-type models to determine an optimal hedging strategy. Hedging refers to “investing in an asset to reduce overall risk of a portfolio” (Bodie et al., 2002, p. 982). An optimal hedge ratio is the ratio “which minimizes the conditional variance of the hedged portfolio return” (Baillie and Myers, 1991, p. 117). Forward and futures markets were developed in order to allow market participants to hedge against unforeseen negative shocks. For example, a wheat farmer may wish to set the price of wheat today to rule out the possibility of a price that is too low. To do this, the farmer can enter into a forward or futures contract. A forward contract is “an agreement calling for future delivery of an asset at an agreed-upon price” (Bodie et al., 2002, p. 982), whereas a futures contract “obliges traders to purchase or sell an asset at an agreed-upon price on a specified future date” (Bodie et al., 2002, p. 982). Forward contracts are typically used for hedging by individuals who have or need the underlying asset, and are tailor-made and traded over the counter. Futures contracts are a financial asset, whereby the parties involved usually do not own or want the underlying asset, and are standardised and exchanged traded.

In order to calculate an optimal hedge ratio (OHR), many studies have assumed a constant conditional covariance matrix of cash and futures returns (see, for example, Ederington (1979), Anderson and Danthine (1981), and Hill and Schneeweis (1981)). In such situations, the OHR will also be constant over time, and is generally calculated as “the slope coefficient of a regression of the change in the logarithm of cash prices on the change in the logarithm of futures prices” (Baillie and Myers, 1991, p. 117). Several researchers have found this technique of calculating the OHR to be unsatisfactory (see, for example, Myers and Thompson (1989), Cecchetti, Cumby, Figlewski (1988), Baillie and Myers (1991), Kroner and Sultan (1993), Sephton (1993), and Manera et al. (2004)). This is to be expected, considering the number of empirical studies that have found that, for many high-frequency asset prices, the covariance matrices are conditionally heteroscedastic (see Bollerslev et al., 1992).

Such results have led to several variants of the ARCH model being used to calculate the conditional covariance between futures and spot prices to determine the optimal time-varying hedge ratio. For example, Cecchetti, Cumby and Figlewski (1988) used the univariate ARCH model, and measured the optimal hedge as the ratio of the conditional covariance to the
conditional variance of the future price. Baillie and Myers (1991) improved on this result by using a multivariate ARCH model with constant conditional correlations. Sephton (1993) also used the CCC model to calculate optimal hedge ratios for commodities on the Winnipeg Commodity Exchange. These studies have found the time-varying hedge ratios to be superior to the traditional constant hedge ratios. Manera et al. (2004) highlighted the importance of determining whether the conditional correlations are dynamic or static for hedging strategies. Using various dynamic conditional correlation GARCH models, they found that the assumption of constant conditional correlations was not supported empirically for Tapis oil spot and forward returns. Their findings suggest that “a sensible hedging strategy would consider spot and forward markets as being characterized by different degrees of substitutability” (Manera et al., 2004, p. 10). This paper extends this research by using both constant and dynamic conditional correlation models to determine the optimal hedging strategy for exchange rates, various financial and physical commodities, and some stock indices.

Although I will be concentrating on the GARCH modelling approach, it should be noted that there are other alternatives for modelling time-varying volatilities in stock returns data. One such alternative is the range of univariate and multivariate stochastic volatility models (see McAleer (2004) for a detailed discussion of these models). In my Honours dissertation, I will be concentrating on the use of GARCH models because of the heavy computational burden associated with estimating SV models, which would impose serious time constraints.

3. Data and Empirical Results

I am presently collecting the wide variety of data to be used in the dissertation.

4. Concluding Remarks

I would expect to find that the conditional correlations are not constant over time. Therefore, hedge ratios that are based on the dynamic conditional correlation models should out-perform those based on the static conditional correlation models.
References


