AN ANALYSIS OF HUMAN CAPITAL ACCUMULATION USING THE SKILL-WEIGHTS APPROACH

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ABSTRACT
A model of human capital accumulation based upon Lazear’s (2003) skill-weights approach to firm-specific human capital accumulation is presented and analysed, with two key findings put forward. Firstly, it is found that Lazear’s approach is consistent with Becker’s idea that in cases of complete information firms will finance firm-specific human capital accumulation, and workers will finance general human capital accumulation. Secondly, it is found that there exist two symmetric equilibria for the combination of technological investments made by the firms. Which equilibrium is adopted by the firms depends on the worker’s relative efficiency at improving their own skill set. This result also shows that this model allows firms to choose the degree of firm-specificity of the skills that they employ. Lastly, there is a discussion of the potential for further study of this model.

INTRODUCTION
For the last 40 year Becker’s (1962) theoretical framework of firm-specific human capital has almost exclusively been the foundation all theoretical and empirical papers on human capital. His definitions of general and firm-specific human capital and their respective characteristics are widely accepted. One key aim of this paper is to reconcile one of Becker’s key findings from this traditional approach – that given complete information firms invest in specific human capital and workers invest in general human capital – with a new framework developed by Lazear (2003). Lazear’s approach stipulates that all skills are general skill and that firms differ in their weighting on various skills.

Another key aim of this paper is to create a model, based on Lazear’s skill-weights theory, which provides a more sophisticated and realistic picture of human capital accumulation than those based on the traditional framework. The key point of difference of this model is the inclusion of firms’ ability to make individual skills more productive through technology investments. In doing this the firms can endogenously choose how firm-specific or general their optimal set of worker skills are. This is an important development as it has the potential to give new insights into the determination of turnover rates within firms and provide previously unexplored reasons for firms to benefit or suffer from product differentiation.
A REVIEW OF FIRM-SPECIFIC HUMAN CAPITAL THEORY

Modern theory of human capital accumulation within firms is widely regarded to have began with Becker’s influential work “Investment in human capital: a theoretical analysis” (1962). Here Becker stated that human capital can be divided into two types: General human capital and firm-specific human capital. When training to increase human capital is valued equally by many firms, it is termed general human capital. Conversely, specific human capital describes situations in which human capital training has value at the firm that provides the training and nowhere else.

From these definitions it can be derived that when a worker engages only in general human capital accumulation, the worker’s post-training wage will equal the value of the worker at its current firm and other firms (as this should be the same value). Thus, the worker captures all of the return to the training and so must ultimately bear the all the costs of the general human capital accumulation.

When a worker accumulates specific human capital he does not fully capture the return to the training. This is because, by definition (of specific human capital), the worker’s current firm is not forced by outside wages offers to offer a post-training wage that reflects the increased productivity from training - for the worker does not become more productive at any other firms. However, if the firm captures all the return to the training, the worker may have an incentive to leave the firm (quit), which in turn would decrease the firm’s expected return from training. Becker (1962) finds that in this situation firms will pass on some of the gains from the productivity increase to workers through an increased in the wage in the post training period, but will recoup this expenditure by reducing the worker’s wage during the training period. This equates to the firm and the worker sharing the cost and return to firm-specific human capital accumulation.

Hashimoto (1981) developed Becker’s concepts by introducing uncertainty to the worker’s post-training productivity when looking at firm-specific human capital accumulation. Hashimoto found that when the post-training wage is decided prior to the realization of worker productivity it is efficient for the firm and the worker to share both the cost and the return to the training. This is division comes about from both the firm’s and worker’s desire to minimize separation (due to quits and dismissals).

Becker’s analysis specified that a worker’s pre- and post training wage and it level of human-capital acquisition (both general and specific) is contractible. Malcomson (1997) argues that these assumptions are far from realistic, asserting that empirical data was better captured by the assumptions that the level human capital investment is not contractible, that it is determined non-cooperatively, and that post-training wages are determined through a Nash bargaining process after productivities have been revealed.

Hart and Moore (1988) found that under the conditions laid out my Malcomson (1997) both the firms and the workers will choose to under invest in specific human capital. For
in the post-training period neither the workers nor the firm would fully capture the increase in productivity due to the training because of the nash bargaining over wages; as a result both parties have an incentive to under invest in firm specific human capital.

Lazear (2003) distinctly moved away from these traditional approaches to human capital accumulation. Lazear argues that Becker’s approach to human capital accumulation is not realistic for it is difficult to generate convincing examples of firm-specific human capital investment that could cause the large and continuing effect on earnings that are occasionally observed in the empirical data. That is, using Becker’s approach it is difficult to find convincing examples where the firm-specific component of human capital come close to the importance of the general component in the determination of a workers wage and output. However, empirical papers showed that the tenure coefficient (a reflection of firm-specific human capital accumulation) is close to the same size as the experience coefficient (a reflection of general human capital accumulation). This inconsistency prompted Lazear to develop a new approach to firm-specific human capital – the skill-weights approach.

Lazear developed a model where all human capital is “general human capital”. It is supposed that there are a variety of skills used in each job and that each of these skills is general in that it can be used in other firms are well. Furthermore, each firm is able to vary the weighting which each of these skills has in its production process. Using this approach allows for separation to result in a reduction in wages and so yield a positive coefficient on tenure in wage regressions even though each of the skills the worker has is “general”. This reduction in wage will be particularly evident when labour markets are thin and search cost for alternative employment is high, as the worker might have to settle for a firm that puts a light weight on the skills acquired at the prior firm.

Lazear presents a convincing argument for the adoption of this new approach to firm-specific human capital accumulation. The key advantages of this approach are that it provides a reasonable explanation of empirically observed high tenure coefficients, it yields all the implications for earning functions that are documented in the existing empirical human capital literature, and it may impart a more reasonable account of specific human capital.
THE MODEL

There are two identical firms (let them be called firm i and firm k). Each firm requires a combination of two skills, A and B, in their production process. Each firm employs measure 1 workers who initially has a zero quantity of skill A and skill B. The match quality of the workers with the firms (m) also effects the level of output. Each workers individual match quality with either firm is initially unknown and distributed according to an iid uniform [-1/2, 1/2]. Furthermore, all agents are risk neutral.

This model consists of two periods. In period 1 investment in technology and skill acquisition occur. In the second period wages are determined and production occurs. Without any loss of generality the discount rate each period is set to 0.

Each firm is able to invest in the improvement of two type’s technology. Investment in type A technology leads to an increase \( (z_A) \) in productivity of the use of skill A in the production process. Similarly for investment in type B technology. The combination of the firms’ technology investments effectively equates to the “weights” that each firm places on each type of skill.\(^1\) The cost of this investment in technology is \( d(z) \). Where \( d(.) \) is a convex function (sufficiently convex to ensure a unique solution), \( d'(.)>0 \) and \( d''(.)>0 \).

Each firm also makes investments in the human capital of its workers\(^2\). That is, each firm \( i (k) \) will invest a non-negative amount in training to increase the workers level of skill A and skill B by \( a_i (a_k) \) and \( b_i (b_k) \) respectively.

The training cost for each firm \( i \) is: \( C_F(a_i) + C_F(b_i) \). Let us specify \( C_F(Y) = (1/2)Y^2 \).

After observing each firm’s investment in technology and training, each firm \( i \)'s workers invest in skills A and B, increasing their skill level by \( (a_{ii}, b_{ii}) \) given that the cost of training for the worker is: \[ C_W(a_i + a_{ii}) - C_W(a_i) + C_W(b_i + b_{ii}) - C_W(b_i) \]. Let us specify \( C_W(Y) = (1/2x)Y^2 \), where \( 0<x<1 \). This cost function captures the assumption that firms are more cost effective than workers in human capital investment. As \( x \) tends towards zero, it is vastly more expensive for the worker to provide training than for the firm to. As \( x \) becomes close to 1 the difference in the cost of training to the worker and to the firm becomes small. Therefore \( x \) is a measure of the relative cost effectiveness of the firms and workers to increase the skill level of the workers.

In reality, firms can be expected to have a cost advantage in providing training to improve human capital due to economies of scale. Firms generally train multiples workers and can share the fixed cost of training over these workers. When workers bear the cost of training themselves, they are not able to reduce fixed costs in this way. Another, cost advantage for the firm can be derived from the workers work-leisure choice. Firms are able to train their workers during working hours, however if the worker

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\(^1\) Lazear (2003) took the weights of each firm to be exogenously determined.

\(^2\) Lazear (2003) implicitly assumed that firms did not invest in the human capital accumulation of its workers.
wishes to engage in training themselves they must do so outside working hours where the
opportunity cost of their time is higher.

During the training periods the firms are able to observe their own workers; consequently
at the end of this period the match quality of each worker with their current employer is
revealed and becomes common knowledge.

At this stage each firm i makes a take-it-or-leave-it wage offer to firm k (≠i)’s employees
to work in the following production period. Given this price offer, each firm i makes
take-it-or-leave-it wage offers to its own employees (these offers can differ across
workers). Each employee takes a higher wage offer. In case of tie, the worker remains
with her current employer.

The firms then engage in production. The production function is such that worker j
initially employed by firm i will have the following potential outputs.

Worker j’s output in firm i:

\[ y_{ij} = (1+z_iA_{ij})A_{ij} + (1+z_iB_{ij})B_{ij} + \alpha m_{ij}. \]

Worker j’s expected output in firm k (≠i):

\[ E[y_{kj}] = (1+z_kA_{ij})A_{ij} + (1+z_kB_{ij})B_{ij} + 0 \]

Where, \( A_{ij} = a_i + a_{ij} \),
\( B_{ij} = b_i + b_{ij} \),
\( \alpha \) is the degree to which output is affected by match quality, and
\( m_{ij} \) is worker j’s match quality with firm i.

Each firm’s post investment profits (\( \pi \)) is the summation of its output minus its wage bill
and training costs.

Each firms’ total profits (\( \Pi \)) is the firm’s post investment profits less the cost of investing
in technology.
This model can be summarized as follows:

Stage 1: Firms can invest in the “improvement” of technology. In particular, assume that each firm $i$ chooses the level of investment $(z_{iA}, z_{iB})$ where $z_{iA} \geq 0$ and $z_{iB} \geq 0$. The cost of investment for each firm $i$ is $d(z_{iA}) + d(z_{iB})$, where $d(.)$ is a convex function.

Stage 2: Firm $i$ and $k$ choose $(a_i, b_i)$ and $(a_k, b_k)$ respectively. Training cost for each firm $i$: $C_F(a_i) + C_F(b_i)$. Similarly for firm $k$.

Let us specify $C_F(Y) = (1/2)Y^2$.

Stage 3: Each worker $j$ employed by firm $i$ chooses $(a_{ij}, b_{ij})$ with the following cost: $[C_W(a_i + a_{ij}) - C_W(a_i)] + [C_W(b_i + b_{ij}) - C_W(b_i)]$.

Let us specify $C_W(Y) = (1/2)xY^2$, where $0 < x < 1$.

Stage 4: Each worker’s match quality with her current employer is realised and becomes common knowledge.

Stage 5: Each firm $i$ makes a take-it-or-leave-it wage offer to firm $k$’s employees. Given this price offer, each firm $i$ makes take-it-or-leave-it wage offers to its own employees. Each employee takes a higher wage offer. In case of tie, the worker remains with her current employer.

Stage 6: Production occurs. For worker $j$ initially employed by firm $i$. Worker $j$’s output in firm $i$ is $(1+z_{iA})A_{ij} + (1+z_{iB})B_{ij} + \alpha m_{ij}$.

Worker $j$’s expected output in firm $k$ is $(1+z_{kA})A_{ij} + (1+z_{kB})B_{ij} + 0$

Notation: $A_{ij} = a_i + a_{ij}$. $B_{ij} = b_i + b_{ij}$

ANALYSIS

Analysis of this model will be restricted to situations in which both firm choose to produce output at stage 6.

Proposition 1:

In the Stage 1 subgame when $z = 0$, $I$, $\alpha = 0$ and $(z_{iS} - z_{kS}) \geq 0 \ (S = A, B)$,

As $|z_{iS} - z_{kS}|$ increases skill $S$ will become less general. Firm $i$ will choose to invest an increasing amount in skill $S$ and the worker will choose to invest a decreasing amount in the acquisition of skill $S$.

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3 Where $I$ is a positive constant.
As $z_{iS} - z_{kS}$ approaches zero (firms have a similar weight on skill $S$ – the skill becomes more general) the firm will not invest in the workers’ skill $S$, but the worker will invest in increasing its level of skill $S$.

Thus, the both the workers’ and the firms’ investment in the workers’ skill level is consistent with Becker’s (1962) theory concerning the financing of training for general and firm-specific human capital accumulation with complete information.

**Proof:**

If firms can choose to either invest $I$ in the technologies, or not invest in the technologies (that is, $z = 0, I$), then the following combinations of firm investments in technology represents situations in which the firms have no differentiation in their production process (that is, each firm places the same weight on each skill as the other firm):

<table>
<thead>
<tr>
<th>Combination</th>
<th>Firm i $(z_{iA}, z_{iB})$</th>
<th>Firm k $(z_{kA}, z_{kB})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0,0)</td>
<td>(0,0)</td>
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<tr>
<td>2</td>
<td>(1,0)</td>
<td>(1,0)</td>
</tr>
<tr>
<td>3</td>
<td>(1,1)</td>
<td>(1,1)</td>
</tr>
</tbody>
</table>

For such combinations, where the firms invest in the technologies in an identical way, both skill A and Skill B adhere to the strict definition of general human capital. That is, the training to improve the workers’ level of either of these skills will be worth the same amount, in terms of productivity gains, at each firm.

If firm $i$ chooses $(0,0)$ and firm $k$ chooses $(0,0)$ then when firms act to maximize their post-investment profits and worker act to maximise their return to training, the following investment in skills are made:

$$a_i^* = 0, \quad a_{ii}^* = x, \quad b_i^* = 0, \quad b_{ii}^* = x$$

$$a_k^* = 0, \quad a_{kk}^* = x, \quad b_k^* = 0, \quad b_{kk}^* = x$$

If firm $i$ chooses $(1,0)$ and firm $k$ chooses $(1,0)$ then when firms act to maximize their post-investment profits and worker act to maximise their return to training, the following investment in skills are made:
If firm i chooses (I, I) and firm k chooses (I, I) then when firms act to maximize their post-investment profits and worker act to maximize their return to training, the following investment in skills are made:

\[
\begin{align*}
a_i^* &= 0 & a_{ii}^* &= x(1 + I) & b_i^* &= 0 & b_{ii}^* &= x \\
a_k^* &= 0 & a_{kk}^* &= x & b_k^* &= 0 & b_{kk}^* &= x(1 + I)
\end{align*}
\]

In each of these situations the firms invest in the technologies in an identical way, producing a situation where the two firms place the same weights on each of the skills. Hence, both skills A and B are classically general skills.

In these situations it can be found that the firms’ optimal investment in the skills \((a_i^*, b_i^*) and (a_k^*, b_k^*)\) is zero. Intuitively, this is due to the fact that under these conditions the wage bill faced by each firm will exactly equal the firm’s output. Therefore, any positive investment in workers’ skills will result in strictly negative post-investment profits.

In these identical investment situations it can be seen that when workers optimize their return to training they invest a strictly positive amount in both skills. In fact, the worker’s investment in a skill is the product of the productivity of that skill \((1 + z)\) at the firm it does not train at and the degree of efficiency of the worker bearing cost of training \((x)\).

Therefore, as the cost of training decreases for the worker (and gap between the cost faced by workers and the cost faced by firms diminishes) \(x\) approaches 1 and the worker will invest more in improving the amount of skill that they have. Furthermore, as the firm with whom the worker did not train at increases the productivity of a skill, the worker will invest more in acquiring an increasing amount of that skill as the return to that skill acquisition has increased.

These finding are consistent with Becker’s (1962) description of the financing of general human capital accumulation. To reiterate, Becker found that workers, and not the firms, will bear the cost of training to improve a worker’s level of general human capital, as it is the workers who capture the entire return to the training. Such characteristics can clearly be seen in the above cases.

If firms can choose to either invest \(I\) in the technologies, or not invest in the technologies (that is, \(z = 0, I\)), then the combination of firm i choosing (I, 0) and firm 2 choosing (0, I) would represent the investment in technology the produces the maximum amount of differentiation in the production process. Skill A is more productive at firm i than at firm
k, and skill B is more productive at firm k than firm i. Therefore, these two skills are not valued equally across the firms. In fact, the value of these skill to each firm is the most idiosyncratic possible under the restrictions.

Such a combination of technology investments equates to the skills becoming less general that in the previously explored combinations of the firms’ technology investment.

When firm i chooses \((I, 0)\) and firm k chooses \((0, I)\) and firms act to maximize their post-investment profits and worker act to maximise their return to training, the following investment in skills are made:

If \(x > \frac{1}{2} I\),

\[
\begin{align*}
a_i^* &= 0 \quad a_{ii}^* = x \quad b_i^* = 0 \quad b_{ii}^* = x(1+I) \\

a_k^* &= 0 \quad a_{kk}^* = x(1+I) \quad b_k^* = 0 \quad b_{kk}^* = x
\end{align*}
\]

If \(x \leq \frac{1}{2} I\),

\[
\begin{align*}
a_i^* &= I \quad a_{ii}^* = 0 \quad b_i^* = 0 \quad b_{ii}^* = x(1+I) \\

a_k^* &= 0 \quad a_{kk}^* = x(1+I) \quad b_k^* = I \quad b_{kk}^* = 0
\end{align*}
\]

These, optimal values indicate that if \(x\) is sufficiently large \((\frac{1}{2} I < x < 1)\) it is optimal for the firm not to invest in the skills of the worker. However, when \(\frac{1}{2} I < x < 1\), firm i and firm k’s post investment profits are strictly negative, thus in equilibrium this outcome will never occur.

When \(x\) is sufficiently small \((0 < x \leq \frac{1}{2} I)\), it is possible for both firms to experience positive profits. Further, when \(0 < x \leq \frac{1}{2} I\), firms will pay for the worker to increase the skill which they weight more heavily (than the other firm), while the worker will not invest in this skill for the worker will perceive this skill to be less general (or more specific) that the other skill in the set.

Also, workers, and not the firm, will themselves invest in the skill that their firm is less productive in. It can be seen that such a skill would be perceived as general by both the firm and the worker, for this skill is not only as productive at the other firm, but more productive.

Such findings are once again consistent with Becker (1962) theory of human capital accumulation. Becker initially found that under conditions of perfect information firms should ultimately bear the cost of firm-specific human capital accumulation. Thus, the
optimal skill investment found in this model is consistent with Becker’s theory. For in this model when the firms invest in such a way that the difference in a skill productivity between two firms is maximized the skill exhibits less of the characteristics associated with general skill and become more like a specific skill, and it is the firm and not the worker who invests in the worker’s skill accumulation.

Proposition 2:

When match quality does not affect the level of output at either firm (that is, \( \alpha = 0 \)) and firms can choose to invest either \( I \) or zero in one or both of the types of technology\(^4\) there exists a threshold valve of \( x, x' \), for which when \( x \leq x' \) the symmetric equilibrium level of investment in technology is \([ (z_{iA}, z_{iB}) = (1, 0), (z_{kA}, z_{kB}) = (0, 1) ]\)\(^5\) - the firm chose the maximum amount of differentiation in their production process, and when \( x > x' \) the symmetric equilibrium level of investment in technology is \([ (z_{iA}, z_{iB}) = (0, 0), (z_{kA}, z_{kB}) = (0, 0) ]\) – no differentiation, both firms value skills A and B in equal amounts.

This proposition says that if the firms have enough of a cost advantage (relative to the cost face by workers) in training workers, then it is optimal for the firms to differentiate their production process as much as possible. Firms will invest only in the skill that they weight most heavily (and the other firm weights the lightest), with the worker choosing to only invest in the skill that its current firm weights the least (and the other firm weights the most).

However, if the firms do not have enough of a cost advantage in training, then the firms will chose not to invest in any technology or in the workers skills, with the only investment in human capital being the training paid for by the worker themselves. For without the cost advantage in training, non-identical investments in technology will result in each firm’s wage bill exceeding their output.

Proof:

From stage 6 of the model it is known that the output of workers who trained at firm i (call them workers i) at firm i, is:

\[
y_{ii} = (1+z_{ii})A_{ii} + (1+z_{ii})B_{ii}
\]

\(^4\) Where \( I \) is a positive constant term.

\(^5\) As the firms are identical the equilibrium could also occur at \([ (z_{iA}, z_{iB}) = (0, 1), (z_{kA}, z_{kB}) = (1, 0) ]\). However, with out any loss of generality it has been assumed that firm i has an innate preference for technology A and firm k, technology B.
In stage 5 it is known that firm $k$ ($\neq i$) makes a take-it-or-leave-it offer to workers $i$, and firm $i$ makes a take-it-or-leave-it offer to its own employees. The highest wage that firm $k$ would be willing to pay workers $i$ is the value of the output of these workers at firm $k$.

Output of workers $i$ at firm $k$:

$$y_{ij} = (1+z_{kA})A_{ii} + (1+z_{kB})B_{ii}$$

Knowing that there is no further negotiation after both firms make their wage offer, firm $k$ will offer the highest wage that they are willing to pay (workers $i$ output) in order to have the best chance of securing the other firm’s workers.

As there is no renegotiation, when firm $i$ makes its take-it-or-leave-it offer it must offer a wage greater than or equal to firm $k$’s wage offer in order to guarantee the retention its workers.

Therefore, firm $i$’s wage offer for workers $i$ is:

$$w_{ii} = y_{ik} = (1+z_{kA})A_{ii} + (1+z_{kB})B_{ii} = (1+z_{kA})(a_i + a_{ii}) + (1+z_{kB})(b_i + b_{ii})$$

As firms are identical firm $k$ will also act in this way. Thus both firms will retain only the workers they had during the training period.

Workers $i$ will choose their investment in skills $A$ and $B$, $a_{ii}$ and $b_{ii}$, in order to maximize the return on this investment. That is it chooses:

$$a^*_i(a_i) = \arg\max_{a_{ii}} [w_{ii} - \text{training costs}]$$

$$a^*_i(a_i) = \arg\max_{a_{ii}} (1+z_{kA})(a_i + a_{ii}) + (1+z_{kB})(b_i + b_{ii}) - \left[\frac{1}{2\tau} (a_i + a_{ii})^2 - \frac{1}{2\tau} a_i^2 + \frac{1}{2\tau} (b_i + b_{ii})^2 - \frac{1}{2\tau} b_i^2\right]$$

The post investment profits of firm $i$ is then:

$$\pi_i = y_{ii} - w_{ii} - \text{firm i’s training costs}.$$  

$$\pi_i = [(1+z_{IA})A_{ii} + (1+z_{IB})B_{ii}] - [(1+z_{kA})A_{ii} + (1+z_{kB})B_{ii}] - \frac{1}{2} a_i^2 - \frac{1}{2} b_i^2$$

$$= (z_{IA} - z_{kA})A_{ii} + (z_{IB} - z_{kB})B_{ii} - \frac{1}{2} a_i^2 - \frac{1}{2} b_i^2$$

$$= (z_{IA} - z_{kA})(a_i + a_{ii}) + (z_{IB} - z_{kB})(b_i + b_{ii}) - \frac{1}{2} a_i^2 - \frac{1}{2} b_i^2$$
Firm i chooses its investment in skills A and B, $a_i$ and $b_i$, to maximize its post-investment profits.

$$a^*_i = \arg \max_{a_i} \left( z_{iA} - z_{kA} \right) a_i + \left( z_{iB} - z_{kB} \right) b_i - \frac{1}{2} a_i^2 - \frac{1}{2} b_i^2$$

$$b^*_i = \arg \max_{b_i} \left( z_{iA} - z_{kA} \right) a_i + \left( z_{iB} - z_{kB} \right) b_i - \frac{1}{2} a_i^2 - \frac{1}{2} b_i^2$$

Total profits including the cost associated with the investments in technology is:

$$\Pi_i = \left( z_{iA} - z_{kA} \right) A_i + \left( z_{iB} - z_{kB} \right) B_i - \frac{1}{2} a_i^2 - \frac{1}{2} b_i^2 - d(z_{iA}) - d(z_{iB})$$

When firms can choose to either invest $I$ in technologies, or not invest the following combinations of investments by the two firms is possible.

<table>
<thead>
<tr>
<th>Table 2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm k $(z_{kA}, z_{kB})$</td>
</tr>
<tr>
<td>(0,0)</td>
</tr>
<tr>
<td>Firm i $(z_{iA}, z_{iB})$</td>
</tr>
<tr>
<td>(0,0)</td>
</tr>
<tr>
<td>(1,1)</td>
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<td>(1,1)</td>
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</tbody>
</table>

As the firms are identical the symmetric equilibria of the above set of combinations can be determined from analysing the following 4 combinations:

<table>
<thead>
<tr>
<th>Table 2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm i $(z_{iA}, z_{iB})$</td>
</tr>
<tr>
<td>Combination 1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>
The post investment profits for these combinations are as follows:

If \( x > \frac{1}{2} I \),

<table>
<thead>
<tr>
<th>Combination</th>
<th>Firm i</th>
<th>Firm k</th>
<th>Firm i total profits</th>
<th>Firm k total profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>(1,0)</td>
<td>(1,0)</td>
<td>(-d(I))</td>
<td>(-d(I))</td>
</tr>
<tr>
<td>3</td>
<td>(1,0)</td>
<td>(0,1)</td>
<td>(-I^2x - d(I))</td>
<td>(-I^2x - d(I))</td>
</tr>
<tr>
<td>4</td>
<td>(1,1)</td>
<td>(1,1)</td>
<td>(-2d(I))</td>
<td>(-2d(I))</td>
</tr>
</tbody>
</table>

If \( x < \frac{1}{2} I \),

<table>
<thead>
<tr>
<th>Combination</th>
<th>Firm i</th>
<th>Firm k</th>
<th>Firm i total profits</th>
<th>Firm k total profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>(1,0)</td>
<td>(1,0)</td>
<td>(-d(I))</td>
<td>(-d(I))</td>
</tr>
<tr>
<td>3</td>
<td>(1,0)</td>
<td>(0,1)</td>
<td>(\frac{1}{2}I^2 - Ix - I^2x - d(I))</td>
<td>(\frac{1}{2}I^2 - Ix - I^2x - d(I))</td>
</tr>
<tr>
<td>4</td>
<td>(1,1)</td>
<td>(1,1)</td>
<td>(-2d(I))</td>
<td>(-2d(I))</td>
</tr>
</tbody>
</table>

When only symmetric investment combinations are considered there are two equilibria.

When \( 0 < x < \frac{I}{2(1+I)} - \frac{d(I)}{I(1+I)} \) the symmetrical equilibrium in technology investment is:

Firm i invests \((z_{iA}, z_{iB}) = (1, 0)\), and

Firm k invest \((z_{kA}, z_{kB}) = (0, 1)\) in the technologies.

When \( 1 > x \geq \frac{I}{2(1+I)} - \frac{d(I)}{I(1+I)} \) the symmetrical equilibrium in technology investment is:

Firm i invests \((z_{iA}, z_{iB}) = (0, 0)\), and

Firm k invest \((z_{kA}, z_{kB}) = (0, 0)\) in the technologies.

Assuming that when the firms are indifferent about investing or not investing in technology they will choose not invest in technology.

This result indicates that even when all the skills used in a firm is by definition a general skill. The firms can choose combinations of technology investment that effectively make the general skills firm-specific skills.
When $x < \frac{I}{2(1+I)} - \frac{d(I)}{I(1+I)}$, the workers are relatively efficient in training themselves, and as such will invest in training to increase their skill level by a strictly positive amount. Therefore, the firm can not control the total amount of training that the workers receive or their wage bill. If the firms invested idiosyncratically in the technologies then the worker will be induced to invest more heavily in the skill that their firm weights the lightest and the rival firm will weight this the heaviest (as the worker’s wage is determined by their potential output at the rival firm). As a result (if $x$ is sufficiently small) workers will invest sufficiently in the lightly weighted skill to produce a situation where a firm’s wage bill will always be larger than the level of output, and post-investment profits will be strictly negative when production occurs. To avoid such situations occurring firm much make identical investments in the technologies. When identical technology investments are made the wage bill will exactly equal the output produces for any level of total skill. Therefore, total profits (including technology costs) will be maximized when both firms do not invest in either technology.

When $x \geq \frac{I}{2(1+I)} - \frac{d(I)}{I(1+I)}$, the workers are relatively inefficient at improving their skill set through training. In fact, workers will find it optimal not to invest in the acquisition of any skills, as the cost of acquiring any positive increase in a skill will always exceed the resultant increase in their wage. As a result the worker’s firm has control over the total amount of skill their workers acquires ($A$ and $B$) and hence their wage bill. In this situation it is optimal for the firms to invest in the technologies available as idiosyncratically as possible, that is $[(z_{iA}, z_{iB}) = (1, 0), (z_{kA}, z_{kB}) = (0, 1)]$.

As $x$ is sufficiently large firm i (k) does not have to be concerned that workers will invest in skill B (A) (which would happen if $x$ were small than the threshold amount) and increase the firm’s wage bill to the point where it is not possible for the firm to produce and make strictly non-negative profits as in the previous scenario.

In this situation the firm can achieve a high output and a low wage bill by investing idiosyncratically in the technologies. For as firms can control the total skill level they can invest in skills in such a way that their workers only gain the skill which it weights the heaviest and their rivals weight the lightest – so the worker has a high potential output at their firm and a low potential output at the other firm.

This finding is significant as it brings together the traditional Becker inspired approach to human capital acquisition and Lazear’s skill-weight approach. Principally, proposition 2 demonstrates that the skill-weights approach can be adapted to create a more sophisticated view of Becker’s general and firm-specific description of human capital. The fundamental concept of the skill-weights approach is that all skills are general skills in that they can be used at other firm as well. This model demonstrates that firms are
effectively able to change these general skills into more firm-specific skills through their choice of investment in technology. For this technology investment results in firm’s having a favoured combination of the general skills. This effectively allows firms to choose whether they use general or firm-specific skills in their production process, which ultimately determines whether the workers or the firm will pay for the human capital accumulation. Such choice is not explored by in Becker’s model where the skills (general or specific) a firm used in its production process is exogenously determined.

FURTHER STUDY

Starting from the work that I have presented thus far, I plan to extent my analysis in two key areas: to extending the model to produce more realistic and interesting results and to finding compelling empirical evidence that supports the predictions made by the model. The following details more specifically the research and analysis that I plan to conduct:

- Increasing the set of the firms’ technology investment options to [No investment, Low level investment, High level investment]. This would allow for a more realistic analysis of firm’s choice of production differentiation.

- Analysis the model where match quality affects the workers’ production functions (that is, $\alpha \neq 0$). It is expected that such analysis would provide insights into the firms’ turnover rate, particularly how it may be influenced by the firms’ investments in technology.

- It can be extrapolated that when a firm becomes increasingly differentiated its production process it will also become more differentiated in its product market. It may be worth while to look at this link to see if there may be a relationship between investments in human capital, turn over rate and the reason for firms producing at socially suboptimal points of product differentiation.

- It will also be worthwhile to find both anecdotal and econometric empirical evidence to support the theoretical findings of this model.
CONCLUSION

In this paper a model based on Lazear’s skill-weights approach to human capital accumulation is presented. From this model two key results were produced.

Firstly it was found that Lazear’s skill-weights theoretical framework for addressing firm-specific capital accumulation produces results that are consistent with Becker’s idea that firms invest in specific human capital while workers invest in general human capital. This was found from analysing the stage 1 sub-game where optimal skill investment of workers and firms was determined for various degrees of differentiation in the production process resulting from different combinations of investment in technologies.

It was found that when two firms made identical investments in the technologies the firms did not invest in the workers skill level, but the worker did. When such identical investments in technology are made the skills used by the firm can be considered general in two respects: firstly they are general in that they can be used at both firms and secondly, whatever combination of the two skills the worker has it will produce the same amount of output at each firm. Therefore, Becker’s idea that worker should pay for general skill accumulation hold here.

It was also found that when the two firms invest in the technologies in such a way that the differences in their production processes are maximized, the firm will pay only to improve the skill for which it already chose to make more productive (by investing in its relevant technology). This skill, although general in that it can be used at the other firm, can be viewed as firm-specific because it is not as productive in the other firm. Thus, as the model predicts that the firm bears the cost of increasing this skill, it is consistent with Becker’s theory which claims that firms will bear the cost of firm-specific human capital accumulation. Also, as stated earlier, the worker will invest in the skill that their firm does not make more productive in the technology investment stage. This skill can be viewed as a general skill because it is more productive at the other firm. Therefore, the worker’s financing of this skill in once again consistent with Becker’s ideas.

This model allowed firms to invest in technologies to improve the productivity of the skills used in their production process. It was found that firm could effectively choose how general or firm-specific they wanted the skills they used in their production process to be. It was also found that the firms’ symmetric equilibrium in its technology investment choice depended on the efficiency with which workers can train themselves. This efficiency in worker financed training is important to the firms because it determines the degree to which the firm can control the total amount of skill the workers acquires, which in turn determines the wage bill of the firm.

There also exists great potential for further study of this model. Areas in which I plan to carry out further analysis and research include widening the firms’ choice of technology investment, incorporating match quality to study the turnover rate within the firms, the
effects that the stage 1 technology choice has on the firms’ product market, and finding empirical evidence to support the predictions of this model.
BIBLIOGRAPHY


