Extending the Aaron Condition for Alternative Pay-As-You-Go Pension Systems
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Welfare comparisons between funded and pay-as-you-go (PAYG) or unfunded pension systems are often made using the Aaron condition (Aaron, 1966). However, the Aaron condition as usually stated is not precise enough about the exact form of the PAYG pension system. PAYG pension systems can be either of the defined benefit or defined contribution variety. They can also differ with regard to intragenerational redistribution. Here, four alternative PAYG pension systems are considered. It is shown that each system generates its own Aaron condition. In addition, the standard Aaron condition assumes that the wage rate and labour participation rate does not vary across individuals. These assumptions are also relaxed. Using US data covering the period 1933-2001, it is shown that the results of welfare comparisons are highly sensitive to different specifications of PAYG systems. (JEL H55, J13, J14)

Key Words: Aaron Condition; Pay-As-you-Go Pension; Funded Pension; Labour Participation Rate; Fertility Rate
1. Introduction

In the first decades after World War II it seemed that pay-as-you-go (PAYG) pension systems provided countries with a cheap way of supporting retirees. The generation that failed to accumulate adequate funds for retirement due to war, the great depression, and (in some countries) hyper-inflation, could receive pensions without contributing to the system. Growing populations and rapid economic growth meant that the internal rate of return of PAYG pension systems were high and that financing even the most generous unfunded system in the future seemed not that onerous. But recent declines in the fertility rates and the resulting pressures on many governments’ budgets are causing governments to reassess their pension systems.

One crucial (and much discussed) distinction between different pension systems is whether they are funded or PAYG (see e.g. Feldstein 1985, Davis 1995, Blake 2000, Bolle 2000). In funded systems each generation saves for their own retirement. Thus no inter-generational redistribution exists. In PAYG systems the current young workers finance the pension benefits of the current old generation and will in turn receive payments from the young when they themselves are old. Thus there exists an inter-generational transfer from young to old in each period.

Welfare comparisons between funded and PAYG pension systems have sometimes been made using the Aaron condition (based on work by Samuelson 1958 and Aaron, 1966), which shows how the relative performance of these systems depends on the interest rate and the growth rate of wages and fertility. However, the Aaron condition as it is usually stated assumes steady state conditions and does not take into consideration that PAYG pension systems can differ in a number of important ways.

PAYG systems (as well as funded systems) can be organized either as defined benefit (DB) or as defined contribution (DC) systems. This distinction -although not much discussed in the pension literature- is very important when discussing PAYG pension systems where contributions of one generation benefit another. In defined benefit (DB)

\[1\] For empirical examples see Davis 1995 or Blake 2000.
systems the pension benefit formula is fixed in advance but the contribution rate that is needed to finance the system will adjust depending on economic and demographic developments. In defined contribution (DC) systems it is the pension contributions that are fixed over time with the pension levels adjusting to economic and population changes.\(^2\) DB systems provide more income security for the retirees but less security about the cost of running the system (the necessary tax contributions). A DC system on the other hand provides certainty about the contribution levels (tax rates) but will provide reduced income security for retirees.

**Insert Table 1 Here**

In recent years a number of papers have considered the impact of declining fertility rates on PAYG pension systems. However, the importance of the distinction between DB and DC PAYG pension systems has been largely overlooked.

This difference between DB and DC pension systems is of particular importance when discussing the so called “pension crisis” in many OECD countries arising from declining fertility rates. While it is true that DB-PAYG systems will lead to increased public spending (deficits) as population aging occurs, this is not the case for a DC-PAYG system. In a DC-PAYG system population aging will lead to downward pressure on the level of pension benefits (thus reducing the replacement rate) rather than putting upward pressure on the budget deficit. Thus population aging in a DC-PAYG system would lead to an “old age poverty crisis” instead of a budget deficit crisis.

Another distinction is whether pension benefits are related to an individual’s prior labour market performance or not.\(^3\) In this paper, systems in which pension payments are related to prior market performance (or tax contribution) are called “earnings related” (ER) and those in which no such connection exists “flat” systems.\(^4\) Under earn-

\(^2\)The term “defined contribution pension system” is used here as in Bolle (2000) and should not be confused with the “notional defined contribution system” which is sometimes referred to as DC system in the literature.

\(^3\)Alternatively, pension payments could be linked to individual fertility levels. Kolmar (1997) shows that the traditional Aaron condition will no longer be applicable in this situation.

\(^4\)These two forms of PAYG pension systems are sometimes referred to as Bismarckian and Bev-
ings related (ER) systems individual pension benefits are linked (proportionally) to an individual’s prior labour market income - and therefore also to her prior tax payments if the pension system is funded by a payroll tax. When retiring in period $t + 1$ an old person who has worked in period $t$ receives a public pension which is proportionally linked to her tax contributions in period $t$ via the replacement rate. For funded ER systems this replacement rate is directly related to the market interest rate. In ER-PAYG systems there is generally no direct link to the market interest rate. Depending on whether the ER-PAYG system is of the defined benefit (DB) or the defined contribution (DC) variety, the replacement rate is known to the individual in advance (in the DB case) or not (in the DC case).

In all types of ER systems, high income earners receive higher pension payments than low income earners. Thus, there exists little if any intra-generational redistribution in ER-systems. In contrast, in a flat system every old person of a generation receives the same pension payments independently of her prior market performance and prior tax payments. Thus, in a flat system, there exists intra-generational redistribution. It is important to note that the difference between flat and ER systems can only be investigated within a model that allows for heterogeneous agents. Heterogeneity can arise either from differences in wage rates across individuals or due to differences in labour participation rates.

The remainder of this paper is structured as follows. First, the standard Aaron condition is derived. Then it is shown how the Aaron condition generalizes when we allow the wage rate and labour participation rate to vary across individuals. Once we allow for heterogeneity across individuals, the question then arises as to whether there is any intragenerational redistribution within the PAYG pension system. This issue is not addressed by the standard Aaron condition. We consider two varieties of DC-PAYG pension system. The first is an earnings related (ER) system with no intragenerational redistribution. The second is a flat system in which all retirees of a particular generation receive the same pension benefit irrespective of how much they have paid into the eridgean PAYG system respectively.
system. These two pension systems generate different Aaron conditions. The paper then goes on to consider DB-PAYG pensions. Again, once we allow for variations in wage and labour participation rates, a distinction can be drawn between ER-DB-PAYG and flat-DB-PAYG pension systems. These pension systems also generate their own Aaron conditions.

Finally, using US data covering the period 1933-2001, it is shown that the results of welfare comparisons are highly sensitive to the different specifications of PAYG pension systems.

2. The Standard Aaron Condition

The Aaron condition compares the rates of return of a funded and PAYG pension system.\(^5\) We consider an overlapping generations setting where each individual is assumed to live for two periods. In the first period, an individual works and in the second she is retired. Aaron (1966) assumes that the economy is in steady state (i.e. population and wages are growing at a constant rate). This assumption will be relaxed in subsequent sections of this paper. If the economy is in steady state (and there is no heterogeneity between individuals) the rate of return from DB and DC PAYG pension systems is identical and thus there will only be one type of Aaron condition.

The (gross) rate of return on a funded pension system earned by an individual is given by the gross market interest rate, \(1 + r\). The rate of return on a PAYG pension system is determined by the ratio of benefits received and contributions paid, \(q/p\), where \(q\) is the replacement rate (i.e., the fraction of net income earned when young that an individual receives upon retirement), and \(p\) is the contribution to the PAYG pension system made by a young individual. Assuming its budget is balanced with respect to the PAYG pension system, the government faces the following budget constraint in each period \(t\):

\[
w_tN_tq = (1 + \gamma)w_t(1 + n)N tp, \tag{1}
\]

\(^5\)The analysis is all done in terms of real rates of return.
where \( w_t \) and \( N_t \) denote, respectively, the wage rate and the number of young people in period \( t \). \( (1 + \gamma) \) and \( (1 + n) \) denote the growth rate of wages and population from one period to the next. The left-hand side of (1) depicts the pension liability of the government to the present old generation while the right-hand side depicts the total pension contributions of the young generation. From (1) it follows that \( q/p = (1 + \gamma)(1 + n) \). This implies that a funded pension system generates a lower rate of return than a PAYG pension system if

\[
(1 + r) < (1 + \gamma)(1 + n). 
\]  

If the inequality is reversed, then a funded pension system generates a higher rate of return than a PAYG pension system.

3. Modifying the Aaron Condition for DC-PAYG Pension Systems

(i) An Earning-Related-Defined-Contribution (ER-DC) PAYG Pension System

The Aaron condition as stated in (2) is only one of a number of Aaron conditions that could be derived for PAYG Pension systems. In steady state the ratio \( q/p \) remains constant over time, thus the rate of return on DB and DC type PAYG systems will be the same. Outside steady state however the ratio \( q/p \) will not remain constant but will reflect changes in economic and demographic developments. In DC systems \( p \) is kept fixed while \( q \) will vary over time to balance the government budget. In DB systems on the other hand \( q \) will be fixed while \( p \) will vary over time.

Aaron (1966) also assumes that the wage rate and labour participation rate is the same for all individuals of the same generation. This section considers the implications of relaxing these assumptions in DC systems. Discussion of defined benefit (DB) systems is deferred until the next section.

Once wage and labour participation rates are allowed to differ across individuals, the issue of intragenerational redistribution becomes important. Two alternative DC-PAYG pension systems are considered here. The first is an earning related (ER) system with no intragenerational redistribution.
Let \( w_i^t \) and \( z_i^t \) denote, respectively, the wage rate and labour participation rate of individual \( i \) in the young generation of period \( t \). Assuming there is no intragenerational redistribution, the government budget constraint in period \( t + 1 \) now becomes

\[
q_{t+1} \sum_{i=1}^{N_t} w_i^t z_i^t = p \sum_{j=1}^{N_{t+1}} w_j^{t+1} z_j^{t+1}.
\] (3)

As before, a young individual in period \( t \) earns a higher return on the PAYG pension system as compared to a funded system if \( q_{t+1}/p > (1 + r_{t+1}) \). In other words, the Aaron condition is

\[
(1 + r_{t+1}) < \frac{\sum_{j=1}^{N_{t+1}} w_j^{t+1} z_j^{t+1}}{\sum_{i=1}^{N_t} w_i^t z_i^t}.
\]

If we assume that the covariance between the wage rate and labour participation rate across individuals in each generation is zero\(^6\), then it follows that \( \sum_{j=1}^{N_{t+1}} w_j^{t+1} z_j^{t+1} = N_{t+1} \overline{w}_{t+1} \overline{z}_{t+1} \) and \( \sum_{i=1}^{N_t} w_i^t z_i^t = N_t \overline{w}_t \overline{z}_t \). In this case, the Aaron condition reduces to

\[
(1 + r_{t+1}) < (1 + \gamma_{t+1})(1 + n_{t+1})(1 + g_{t+1}),
\] (4)

where \( 1 + \gamma_{t+1} = \overline{w}_{t+1}/\overline{w}_t \) and \( 1 + g_{t+1} = \overline{z}_{t+1}/\overline{z}_t \).

(ii) A Flat-Defined-Contribution (Flat-DC) PAYG Pension System

Consider now a flat-DC PAYG pension system. Under this system all retirees in period \( t + 1 \) receive the same pension \( Q_{t+1} \), irrespective of how much they paid into the pension system the previous period. This means that the PAYG pension redistributes from high income individuals to low income individuals in each generation.

The government’s budget constraint in period \( t + 1 \) is now

\[
N_t Q_{t+1} = p \sum_{i=1}^{N_{t+1}} w_i^{t+1} z_i^{t+1}.
\] (5)

---

\(^6\)This assumption is less problematic for men than for women. MaCurdy, Green, and Paarsch (1990) find that both income and substitution effect of wage changes are close to zero for prime aged men. However, female labour force participation tends to be positively correlated with the wage rate (p. 1999, Ehrenberg and Smith 1996).
Individual $i$ who is young in period $t$ will prefer the PAYG system if $Q_{t+1}/(pw^t_i z^t_i) > (1 + r_{t+1})$. Using the budget constraint, this leads to the following Aaron condition:

$$(1 + r_{t+1}) < \frac{\sum_{i=1}^{N_{t+1}} w^t_{i+1} z^t_{i+1}}{N_t w^t_i z^t_i}.$$ 

This Aaron condition differs from the ones above in that now it is no longer the case that all individuals of a given generation must have the same ranking of a funded and PAYG pension. In particular, the higher individual $i$’s income (i.e., $w^t_i z^t_i$), the less likely it is that she will prefer the PAYG pension system to the funded system.

If we again assume that the covariance between the wage rate and labour participation rate across individuals in each generation is zero, then for the young individual with the average income in period $t$, the Aaron condition is the same as under an ER-DC PAYG system. i.e.,

$$(1 + r_{t+1}) < (1 + \gamma_{t+1})(1 + n_{t+1})(1 + \overline{g}_{t+1}).$$

4. Modifying the Aaron Condition for DB-PAYG Pension Systems

(i) An Earning-Related-Defined-Benefit (ER-DB) PAYG Pension System

In this section we will consider DB-PAYG pension systems. Under a DB-PAYG system, the payment received on retirement is fixed, while the tax rate on workers is adjusted to balance the government’s budget. Again a PAYG and funded pension system will be compared from the perspective of a young individual in period $t$. One important difference with DB-PAYG systems as compared with DC-PAYG systems is that, from the perspective of the young generation in period $t$, the relevant government budget constraint is the one for period $t$ not $t + 1$. This is because this is where the pension contribution rate $v_t$ is set.

If we allow wage and labour participation rates to differ across individuals, the issue of intragenerational redistribution again becomes important. We consider first an earnings related (ER) DB-PAYG system with no intragenerational redistribution. This leads to the following government budget constraint in period $t$:

$$x \sum_{i=1}^{N_{t-1}} w^t_{i-1} z^t_{i-1} = v_t \sum_{j=1}^{N_t} w^t_j z^t_j.$$ (6)
As before, a young individual in period $t$ earns a higher return on the PAYG pension system if $x/v > (1 + r_{t+1})$. In other words, the Aaron condition is

$$(1 + r_{t+1}) < \frac{\sum_{i=1}^{N_t} w_i^t z_i^t \sum_{j=1}^{N_t} w_j^{t-1} z_j^{t-1}}{\sum_{i=1}^{N_t} w_i^t z_i^t \sum_{j=1}^{N_t} w_j^{t-1} z_j^{t-1}}.$$  

If we assume that the covariance between the wage rate and labour participation rate across individuals in each generation is zero, then the Aaron condition can be rewritten as

$$(1 + r_{t+1}) < \frac{\sum_{i=1}^{N_t} w_i^t z_i^t \sum_{j=1}^{N_t} w_j^{t-1} z_j^{t-1}}{\sum_{i=1}^{N_t} w_i^t z_i^t \sum_{j=1}^{N_t} w_j^{t-1} z_j^{t-1}}.$$  

(ii) A Flat-Defined-Benefit (Flat-DB) PAYG Pension System

Consider now a flat-DB PAYG pension system. This means all retirees in period $t+1$ receive the same pension $X$, irrespective of the contribution, $v_t w_i^t z_i^t$, paid into the pension system in period $t$. The PAYG pension, therefore, redistributes from high income individuals to low income individuals in each generation. The government’s budget constraint in period $t$ is now

$$N_{t-1} X = v_t \sum_{i=1}^{N_t} w_i^t z_i^t.$$  

Individual $i$ who is young in period $t$ will prefer the PAYG system if $X/(v_t w_i^t z_i^t) > (1 + r_{t+1})$. Using the budget constraint, this leads to the following Aaron condition:

$$(1 + r_{t+1}) < \frac{\sum_{i=1}^{N_t} w_i^t z_i^t \sum_{j=1}^{N_t} w_j^{t-1} z_j^{t-1}}{\sum_{i=1}^{N_t} w_i^t z_i^t \sum_{j=1}^{N_t} w_j^{t-1} z_j^{t-1}}.$$  

Like the flat-DC-PAYG pension, not all individuals of a given generation need necessarily have the same ranking of a funded and PAYG pension. Once again, the higher the value of $w_i^t z_i^t$, the less likely it is that individual $i$ will prefer the PAYG pension system to the funded system.

If the covariance between the wage rate and labour participation rate across individuals in each generation is zero, then for the young individual with the average
income in period $t$, both the wage and labour participation rates drop out of the Aaron condition which reduces to

$$(1 + r_{t+1}) < (1 + n_t).$$

(iii) The Sustainability of DB-PAYG Pension Systems

The Aaron conditions derived above for DB-PAYG pension systems assume that the system does not collapse in period $t + 1$ due to a lack of young people. Sustainability is never an issue for DC-PAYG systems. If the fertility rate, wages or labour participation decline from one generation to the next then this simply means that retirees get a worse return from the pension system than they would have otherwise. In contrast, under a DB-PAYG system, each period the government has a fixed liability to the old generation. The main concern here is declining fertility rates in countries with generous earnings related DB-PAYG systems, such as Germany, Austria and Italy. The birth rate per woman by 2002 had fallen to 1.39 in Germany, 1.40 in Austria and 1.19 in Italy (see Central Intelligence Agency, 2002). In these countries, it is possible that to balance its budget the government may have to raise the tax rate to a level that the working population will not tolerate. Clearly, in such cases the maintenance of the pension system becomes incompatible with a balanced budget. It is important that this issue is kept in mind when interpreting Aaron conditions for DB-PAYG pension systems.

5. Implications of the Modified Aaron Conditions for the United States

It is important when comparing the welfare effects of a funded and PAYG pension system that the Aaron condition is correctly specified. This point is illustrated here using US data over the period 1933-2001 and projections to 2023. Here we assume each period in the overlapping generations model lasts 25 years. However, we will refer to cohorts rather than generations, since the gap between each cohort in our analysis is only five years. The first cohort is young in the period 1933-1958, and old in the period 1958-1983. The second cohort is young from 1938-1963, and old from 1963-1988. Later cohorts are staggered at five year intervals. The last cohort considered is young from
1998-2023 and old from 2023-2048. We show that the welfare ranking of the funded and PAYG pension system can depend on which Aaron condition is used.

Consider first the generation that is young (and working) in the period 1933-1958 and retired from 1958-1983.\textsuperscript{7} It is assumed that the midpoint of the period is representative of the period as a whole with regard to the real wage and labour participation rates. The number of young people in each cohort is proxied by the total number of births 13 years before the start of the period. In other words, the number of young workers in the 1933-1958 cohort is determined by the number of births in 1920.\textsuperscript{8} Therefore, it is assumed implicitly that there is no net immigration of young people. Immigration of old people does not matter if they are not eligible to receive the pension.

The real interest rate for each period is computed by compounding 3-month treasury bills over 25 years.\textsuperscript{9} This should be viewed as a lower bound on the rate of return on a funded pension system, and hence biases the results towards preferring the PAYG system.

We compare here the Aaron conditions derived assuming the covariance between the wage and labour participation rates is zero for individuals of the same generation. Evidence suggests that the covariance during the twentieth century was around zero for men but positive for women.\textsuperscript{10} Furthermore, it must be remembered that the population growth rate and labour participation rate are not independent of the pension system itself.\textsuperscript{11} So counterfactual comparisons of hypothetical PAYG systems with funded

\textsuperscript{7}Clearly the assumption that the period over which an individual works is of equal length to the period of retirement is unrealistic. Also, no account is being taken of increases in longevity over time. The former assumption will tend to bias the results towards preferring the funded pension system, while the latter effect will partially counteract it.

\textsuperscript{8}Data on US live births are taken from www.infoplease.com/ipa/A0005067


\textsuperscript{10}Again, see MaCurdy, Green, and Paarsch, 1990 and Ehrenberg and Smith, 1996.

\textsuperscript{11}If the rearing of children is time intensive, higher female labour participation will be associated
pension systems based on actual data are not strictly valid. However, our objective here is simply to illustrate the importance of specifying the Aaron condition correctly. The US social security system actually lies somewhere between the four PAYG pension systems described above.

The Aaron conditions for DC-PAYG and DB-PAYG pension systems are shown below in Tables 2 and 3, respectively.\(^\text{12}\) From equation (4) it follows that the (gross) rate of return on the earnings related DC-PAYG system, denoted by \((1 + M_{t+1})\), in Table 2 is equal to \((1 + \gamma_{t+1})(1 + n_{t+1})(1 + g_{t+1})\).\(^\text{13}\) As shown in section 3, this is also the rate of return for the average individual under a flat DC-PAYG system. From equation (7) it follows that the rate of return on the ER-DB-PAYG system, denoted by \((1 + M_t)\), in Table 3 is equal to \((1 + \gamma_t)(1 + n_t)(1 + g_t)\). The Aaron condition for the average individual under a flat DB-PAYG system, in contrast, simply equals \((1 + n_t)\).

It can be seen from Table 2 that for the first six cohorts (starting in 1933), the return from the DC-PAYG system (whether of the earnings related or flat variety) exceeds that from the funded system. For the cohorts that are young in 1963-1988 and 1973-1998 funded pension provision brings a higher return. In Table 3, again for the first six cohorts (starting in 1958), the return from the ER-DB-PAYG system exceeds that from the funded system. For the final three cohorts, this result is reversed. The return from the flat DB-PAYG system exceeds that from the funded flat system for only two cohorts.

Insert Table 2 Here

Insert Table 3 Here

These findings raise a number of interesting issues. First, for two of the five cohorts with lower fertility rates.

\(^{12}\)See Davis (1995), Willmore and Bertucci (1999), and Blake (2000) for alternative estimates of the traditional Aaron condition for different countries.

\(^{13}\)\((1 + \gamma_{t+1})\) denotes the growth rate of real mean wages. These data are taken from the U.S. Census Bureau, Table P-3, www.census.gov/hhes/income/histinc/p03.html. \((1 + g_{t+1})\) denotes the growth rate of the Civilian Labor Force participation rate for individuals 16 years and over. Data source: U.S. Department of Labor, Bureau of Labor Statistics, www.bls.gov/cps/cpsatabs.htm.
(1963-1988 and 1973-1998) where it is possible to compute rates of return for both ER-DC and ER-DB PAYG systems, the ranking of the PAYG and funded system depends on whether the PAYG system is of the DC or DB variety. For flat PAYG systems the result is even more dramatic. For five of the eight cohorts where it is possible to compare flat DC and DB PAYG systems, the ranking of the PAYG and funded system depends on whether the PAYG system is of the DC or DB variety. This finding illustrates how important it is to specify the Aaron condition correctly.

Second, for the five cohorts where it is possible to compare ER-PAYG systems, the return is higher on the DC PAYG system, sometimes significantly so. This difference in the outcome derives from the fact that the return on a DB-PAYG pension system for the generation that is young in period $t$ depends on changes in wages, labour participation, demographics and interest rates between periods $t-1$ and $t$, while for a DC-PAYG system the relevant comparison is between periods $t$ and $t+1$. From Tables 2 and 3 it can be seen that for the cohorts that were young in 1963-1988, 1968-1993 and 1973-1998, $(1 + n_{t+1})$ was significantly less than 1, while $(1 + n_t)$ was significantly greater than 1. In other words, the number of young people rose in the 1960s, 1970s, 1980s and early 1990s and then fell again. A DB-PAYG system pays a good return for the young generation in period $t$ when the number of young people rises in period $t$. This is because there are then a large number of workers available to pay the pensions of retirees. The number of young people in period $t+1$ does not matter to retirees in period $t+1$, as long as the system does not collapse, since their benefits are fixed. In contrast, under a DC-PAYG system, it is the contributions that are fixed. Hence the number of retirees in period $t$ is of no consequence to someone who is young in period $t$. What matters to the young generation in period $t$ is the number of young people in period $t+1$, relative to the number of retirees, since this will determine the size of the pension payment to each retiree. The same arguments explain why the flat DC and DB PAYG systems generate such different results. The results here are even more dramatic since they are not diluted by trends in wage and labour participation rates.

It is notable in Table 3 that the rate of return on the ER-DB-PAYG system is
projected to fall significantly for the last three cohorts. This is primarily due to the falling fertility rate, which eventually starts to affect retirees under an ER-DB-PAYG system as well. If the ER-DC-PAYG system was projected forward further it seems probable that its rate of return would remain below that of the ER-DB-PAYG system. It seems that the cohort that is young in the period 1988-2013 is at a threshold beyond which the rate of return on both ER-DC and ER-DB PAYG systems are likely to fall below the rate of return on a funded system. The return on flat DB-PAYG systems for the average person has always been lower than under a funded system except for the cohorts that were young in 1968-1993 and 1973-1998.

6. Conclusion

This paper has shown that comparisons of the rates of return of PAYG and funded pension systems are rather more complex than is suggested by the original Aaron condition which does not distinguish between different types of PAYG systems. The key distinctions are between DB and DC PAYG systems, and between earnings related and flat rate PAYG systems. The standard Aaron condition also fails to take account of variations in the labour participation rate both within and across generations.

To obtain actual estimates of the rates of return on PAYG systems a number of simplifying assumptions had to be made. However, two clear results emerge from the analysis. First, the result of a comparison between a PAYG and funded system can be very sensitive to how the PAYG system is specified. Second, the rate of return on a PAYG system becomes increasingly unattractive over time when the fertility rate declines, as it has in most OECD countries in the last few decades.

References


U.S. Census Bureau, Table P-3, www.census.gov/hhes/income/histinc/p03.html.


Table 1.– Different PAYG Pension Systems

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<td>ER-DB-PAYG</td>
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Table 2.– Aaron Conditions for DC-PAYG Pension Systems

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<td>1958-1983</td>
<td>1.266</td>
<td>1.306</td>
<td>1.103</td>
<td>1.822</td>
<td>1.232</td>
</tr>
<tr>
<td>1963-1988</td>
<td>1.435</td>
<td>0.866</td>
<td>1.095</td>
<td>1.360</td>
<td>1.386</td>
</tr>
<tr>
<td>1968-1993</td>
<td>1.526\textsuperscript{c}</td>
<td>0.880</td>
<td>1.044\textsuperscript{d}</td>
<td>1.402</td>
<td>1.348</td>
</tr>
<tr>
<td>1973-1998</td>
<td>1.534\textsuperscript{c}</td>
<td>0.883</td>
<td>1.028\textsuperscript{d}</td>
<td>1.392</td>
<td>1.467</td>
</tr>
<tr>
<td>1978-2003</td>
<td>1.529\textsuperscript{c}</td>
<td>1.111</td>
<td>1.002\textsuperscript{d}</td>
<td>1.702</td>
<td>1.675</td>
</tr>
</tbody>
</table>

Notes:

\(a\): The mean wage in 1945 is based on value of 1947.

\(b\): The labour participation rate of 1948 is substituted for the missing value of 1945.

\(c\): Based on extrapolation of mean wage data up to 2001.

\(d\): The labour participation rate of 2002 is substituted for the missing value of 2005.
Table 3.– Aaron Conditions for DB-PAYG Pension Systems

<table>
<thead>
<tr>
<th>Cohort</th>
<th>$1 + \tau_t$</th>
<th>$1 + n_t$</th>
<th>$1 + \bar{g}_t$</th>
<th>$1 + M_t$</th>
<th>$1 + r_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1958-1983</td>
<td>1.548</td>
<td>0.969</td>
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<td>1.541</td>
<td>1.232</td>
</tr>
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<td>1.935</td>
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<td>1968-1993</td>
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<td>1.568</td>
<td>1.076</td>
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</tr>
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<td>1978-2003</td>
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<td>2.143</td>
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<tr>
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<td>1.306</td>
<td>1.103</td>
<td>1.822</td>
<td>1.609$^p$</td>
</tr>
<tr>
<td>1988-2013</td>
<td>1.435</td>
<td>0.866</td>
<td>1.096</td>
<td>1.362</td>
<td>1.609$^p$</td>
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<tr>
<td>1993-2018</td>
<td>1.526$^a$</td>
<td>0.880</td>
<td>1.044$^b$</td>
<td>1.402</td>
<td>1.609$^c$</td>
</tr>
<tr>
<td>1998-2023</td>
<td>1.534$^a$</td>
<td>0.883</td>
<td>1.028$^b$</td>
<td>1.392</td>
<td>1.609$^c$</td>
</tr>
</tbody>
</table>

Notes:

- $^a$: Based on extrapolation of mean wage data up to 2001.
- $^b$: The labour participation rate of 2002 is substituted for the missing value of 2005.