

The Pension Protection Fund

Abstract

We develop a stylised model of the UK Pension Protection Fund (PPF), a defined benefit pension guarantee system for the UK, based on an analogy between pension liabilities and corporate debt obligations. We show that the PPF is likely to face many years of low claims interspersed irregularly with periods of very large claims. There is a significant chance that these claims will be so large that the PPF will default on its liabilities, leaving the Government with no option but to bail it out. The cause of this problem is the mismatch between pension assets (largely invested in equities in the UK) and liabilities (which are bond-like). This will cause many firms to default when their pension plans are heavily underfunded. We use our model to derive a fair premium for PPF insurance under different circumstances, to estimate the extent of cross-subsidies in the PPF between strong and weak sponsors and to show that risk rated premiums are unlikely to have a substantial effect on either the size or the lumpiness of claims. We argue that for the PPF to operate effectively, it should be introduced in tandem with strong minimum funding requirements and a lower level of benefit guarantee than at present.

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The Pension Protection Fund

The UK has recently established a Pension Protection Fund (PPF) to protect members of private sector defined benefit scheme¹ whose firms become insolvent (Department of Work and Pensions, 2004a). Many details of the fund are still to be finalised. The purpose of this paper is to identify and roughly quantify some of the main policy issues involved in the running of such a fund.

One key issue is the future solvency of the PPF, and possible claims on the public purse. The largest and best established exemplar of a Protection Fund is the Pension Benefit Guaranty Corporation (PBGC) of the United States. After a run of years of very low claims – claims over the period 1980-1999 averaged \$300m/year – the PBGC has been facing very large claims in the last three years, amounting to some \$ 9 billion in total. Its latest estimate shows a deficit of \$11.2 billion, taking account of probable claims from currently insured plans. With premium income of \$1 billion per year, and strong opposition in Congress to raising premium levels substantially, it is questionable whether the PBGC will be able to meet its obligations without Government support.

In this paper we model a generic fund to help analyze the extent to which these problems are inherent to a fund to protect defined benefit pensions.. Recognising that corporate pensions are similar to corporate debt obligations, we show that the PPF is likely to face many years of low claims interspersed irregularly with periods of very large claims when prolonged weakness in equity markets coincide with widespread corporate insolvencies. We argue that it will not be possible to build up sufficient surpluses in the PPF in the good years to pay for the bad years. It will also be difficult to raise premiums sufficiently after a run of bad years to bring the PPF back to solvency. The Government will not be able to let the PPF default, so it will be underwritten by the Government whether the guarantee is recognised formally or not.

¹ For brevity we will use the word pension to mean specifically a private sector defined benefit occupational pension.

We consider, and reject, the argument that the problem can be mitigated by levying “risk-based” premia². Relating premia to risk will have only a limited impact on moral hazard. What it will do however is ensure that the burden of making good any deficit in the PPF will fall particularly on those schemes least able to bear it, so making it more difficult to keep the PPF solvent, and increasing the likelihood of recourse to Government.

We also investigate the relation between the PPF and solvency requirements. Following the misappropriation of the assets of the pension fund of Mirror Group Newspapers, a Minimum Funding Requirement (MFR) was imposed on pension funds by the Pensions Act 1995. Following criticism of its inflexibility and its distorting effect on pension fund investment, the Government has announced it intends to withdraw the MFR. We argue that far from avoiding the need for a funding requirement, the establishment of a PPF is likely to force the reintroduction of something very similar. Once again, the analogy between pensions and corporate debt provides some insight: the MFR is similar to a covenant on secured debt.

To address these issues, we develop a simple model of a pension fund. In its basic version firm insolvency is a random (Poisson) event, with a constant hazard rate. If the firm becomes insolvent, any deficit in the pension fund is picked up by the PPF. The contribution of the firm to the pension fund follows a simple smoothing rule that ensures that any deficits and surpluses are amortised over a number of years. Fund solvency varies because of the mismatch between the assets and liabilities; the assets are partly invested in equities, while the liabilities are bond-like. The investment policy and the contribution policy are exogenous. The model shows how the premium the PPF needs to charge to remain solvent depends on key parameters such as the investment policy of the pension fund, the contribution policy, the equity risk premium and so on. The model is also used dynamically to simulate the behaviour of claims over time.

² UK legislation provides that at least half the premium should be risk-based, tied to scheme solvency, sponsor credit rating, investment policy and other factors relevant to the likelihood of a claim.

We develop a more sophisticated model in which the default rate is stochastic. Since a downturn in equity markets will not only increase pension fund deficits, but will also tend to be accompanied by an increase in insolvencies, the stochastic default model shows much greater volatility in the claims on the PPF. To model the default rate, we model the PPF as a guarantee of a corporate liability, the firm's pension promise to its employees, making use of the burgeoning literature on the valuation of default contingent securities. We use a structural model of the firm, based on Collin-Dufresne and Goldstein (2001), where the firm's assets follow a stochastic process, and the firm defaults when its leverage ratio reaches a critical level. With defaults being correlated across firms (because of the positive correlation in asset values across firms), the claims on the PPF become much more volatile. With default being correlated with deficits in pension funds (because the assets of the firm are positively correlated with the assets of the pension fund), the average rate of claims also becomes much larger.

The models we use take the firm's policy as exogenous. They do not allow us to explore the impact of the PPF on efficiency. In the final substantial section of the paper we discuss, using the model, how the existence of the PPF provides incentives which may affect behaviour – the moral hazard issue. We examine the consequences of varying premia according to the solvency of the pension fund, and the credit standing of the employer. We show that neither policy does much to mitigate the substantial wealth transfers from high credit firms to low credit firms resulting from the creation of the PPF. We conclude that to minimize the level of premia and the size of these transfers, there will need to be solvency requirements similar in form and effect to the MFR.

1. *The Nature of Pension Liabilities and Claims on the PPF*

In this section we discuss the nature of the claims on the PPF in order to explain and motivate the model we will be using. Our main concern is with the factors determining the level of the premium to be charged, and the pattern of claims over time. We model a representative firm and its pension fund. The investment policy of the fund and the contribution policy of the firm are exogenous; we consider later how they may be affected by the existence of the PPF.

Under UK law, firms offering defined benefit pensions to their employees are obliged to fund their obligations. The adequacy of the pension fund is reviewed every three years by an independent actuary who recommends to the trustees the level of future contributions required to ensure that the fund is able to meet its liabilities on a continuing basis.

The actuarial valuation is not related to solvency – ensuring that the assets of the fund exceed its liabilities – but rather to funding – setting a smooth path for contributions that will over the long term allow the fund to pay the promised pensions. In deciding whether a scheme is adequately funded, the actuary for example will make judgements about future investment returns, which are irrelevant to solvency. So a scheme that is fully funded may well be in substantial deficit³. That does not mean it will not meet its obligations, but it will need ongoing support from the employer to be sure of doing so.

If the scheme is underfunded, the actuary will recommend an increased level of contributions that will, assuming reasonable investment performance, allow the scheme to become fully funded in a number of years. The relation between the firm's financial state and its contribution policy is complex. On the one hand, a firm facing financial distress may be particularly inclined to defer contributions; on the other hand, it is precisely in these cases where a rapid return to fund solvency is of greatest importance to pensioners. Recent evidence on the relationship between pension fund solvency and the financial status of sponsoring firms is difficult to find. In the United States, Bodie *et al* (1985), found a negative relationship between the credit rating of a firm and the solvency of its pension plan in weaker firms. Orszag (2004), however, has found little evidence that weaker UK firms systematically underfund their pension plans.

Table 1 shows the median funding ratio, the total pension liability and the total unfunded pension liability for FTSE-350 companies which have defined benefit pension liabilities. It is drawn from the disclosures in their accounts of the most recent financial year, subdivided by the Standard and Poors credit rating of the

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sponsoring employer at the accounting date. Since they are computed in accordance with accounting standard FRS17, the valuation of assets and accrued liabilities approximates reasonably closely to market values at the balance sheet date.

Several patterns can be noted. The majority of pension liabilities (67%) and pension underfunding (69%) is with companies rated BBB or above, even making the conservative assumption that all non-rated companies would have credit ratings below BBB. The third column shows the median funding ratio in each rating category. There is no clear trend in funding as the credit strength of the sponsoring firm declines⁴.

Table 1 here.

Accordingly, we take the contribution policy to be independent of the firm's financial state, but to depend only on the fund's solvency level.

The potential for a large deficit when an employer becomes insolvent depends on the investment policy of the pension fund. The fund's liabilities resemble a long-dated inflation and mortality linked bond. The assets of UK pension plans are typically at least 50% invested in equities. One might expect trustees of funds that are more precarious (larger deficits, weaker employers) to be more cautious about protecting their solvency, but the evidence does not bear this out. Table 2 shows the average equity proportion of a variety of types of fund and finds no strong relationship, except that funds that are less well funded appear to invest slightly more heavily in equities. In our model, we therefore assume that the asset mix of the pension fund is independent of the firm's financial strength.

Table 2 here.

In view of the importance of equity risk, and to keep the model simple, we ignore the impact of real interest rate and mortality risk.

⁴ Financial years in the UK can end at any time between June and March. Therefore, we are examining pension liabilities at dates which may be up to 10 months apart.

2. **Modelling the Guarantee**

2.1 The Poisson Model

The PPF guarantees the pension liabilities of a set of firms. In this section, we assume that there is an infinite number of small, identical firms, and focus on one representative firm. The insolvency of a firm is modelled as a Poisson process with a hazard rate δ . δ is constant and default risk is uncorrelated across firms, so a constant fraction δdt of the firms become insolvent each period dt . The present value of the accrued liabilities of the firm's pension fund at time t , denoted by L_t , may vary over time, but it is non-stochastic. The assets of the fund have value A_t . If the firm becomes insolvent at time t , and if the liabilities of the fund exceed the assets, the PPF pays $L_t - A_t$.

In practice, pension fund liabilities are measured in several different ways. For the purpose of this model, L_t is the cost at time t of buying out the guaranteed accrued liabilities of the pension fund at that time, and A_t is the market value of the assets of the fund, after allowing for any costs of winding up. Implicitly, we are assuming that if the firm becomes insolvent, the PPF has full access to the assets of the pension fund, at least so far as they do not exceed the guaranteed liabilities, but no access to the assets of the firm itself. By topping up the pension fund's assets to equal its liabilities, the PPF can ensure that there is no further claim on the PPF from that pension fund.

The assets of the pension fund comprise a riskless bond with constant interest rate r , and an equity portfolio. The equity portfolio is just the market portfolio; the return on the market portfolio, dS/S , follows a diffusion process:

$$dS/S = (r + \alpha) dt + \sigma_m dz_m, \quad (1)$$

where z_m is a standard Brownian process, α is the market risk premium and σ_m is the volatility of the market. Under the pricing, or risk neutral, measure Q :

$$E^Q [dz_m] = -\frac{\alpha}{\sigma_m} dt. \quad (2)$$

I_t is an indicator function that takes the value 1 if the firm is still solvent at time t , and 0 otherwise. If the firm becomes insolvent, the pension fund is closed. If the firm becomes insolvent at time t (so $dI_t = -1$) and if the pension fund is in surplus at that time ($L_t \leq A_t$) then the pension fund is able to pay pensions due in full, and no liability falls on the Protection Fund⁵. If there is a deficit in the pension fund when the firm becomes insolvent, the Protection Fund takes over both the assets and the liabilities. The cost to the Fund at the time the firm becomes insolvent is thus:

$$-\int_t^\infty [L_u - A_u]^+ dI_u \text{ where } [x]^+ \equiv \text{Max}(x, 0). \quad (3)$$

Determining the Premium

The firm pays an insurance premium P_t to the Pension Protection Fund. From the Fund's perspective, insuring the firm has a present value of:

$$E^Q \left[\int_t^\infty \left(P_u I_u du + [L_u - A_u]^+ dI_u \right) e^{-r(u-t)} \right], \quad (4)$$

where the expectation is under the risk-neutral measure Q .

If the PPF is to cover the cost of claims from its premium income then, ignoring administrative costs, the present value of premium income less claims must be zero. In principle, there are many ways of levying the premium. The PBGC uses a combination of a charge per member covered and a charge proportional to the dollar size of any deficit in the scheme. In the UK, the Government has proposed taking account of other matters including the solvency of the scheme sponsor. We do not address the question of the optimum premium schedule directly. For the present, we assume that the premium is set as a constant proportion of the scheme's liabilities, and use the rate to measure the effect of changes in contribution and investment policy.

If the premium is levied at rate p :

⁵ We are implicitly assuming that the investment policy of a closed fund precludes the trustees from investing in risky assets and putting the solvency of the fund at risk.

$$P_t = pL_t. \quad (5)$$

From (4), using the Poisson default rate process and the non-stochastic nature of the liabilities, the rate p is given by:

$$p = \frac{\int_t^\infty E_t^Q \left[\delta (1 - A_u / L_u) \right]^+ L_u e^{-(r+\delta)(u-t)} du}{\int_t^\infty L_u e^{-(r+\delta)(u-t)} du}. \quad (6)$$

The premium rate is a weighted average of the expected claim rate into the future. It depends on the current solvency level of the scheme. The main focus of this paper is on the impact of different contribution schedules, investment policies and guarantee arrangements on the level of the premium. To abstract from variations caused by initial conditions, we look at processes that generate stationary distributions of insolvency rates and deficit levels, and take unconditional expectations.

With unconditional expectations, (6) simplifies to:

$$p = E^Q \left[\delta (1 - A_u / L_u) \right]^+. \quad (7)$$

The Dynamics of Scheme Solvency

The dynamics of A depend on the return on the portfolio, outflows to pensioners and inflows from contributions:

$$dA = \left[(r + x\alpha)A + (\kappa_t - \pi_t) \right] dt + x\sigma_m A dz_m, \quad (8)$$

where x is the proportion of the assets held as equity, κ is the contribution rate, and π is rate of pay out to pensioners.

We assume a constant investment policy, so x is fixed. The firm's contribution to the pension fund has two components: the first maintains the current solvency level after allowing for payments to pensioners, any change in net liabilities, and the expected return on the assets of the fund. The second component is designed to eliminate any surplus or deficit in the fund over a specified period of T years. The lower the level of T , the faster any deficit is eliminated and the lower the potential claim on the PPF. The simplest formulation that achieves this is:

$$\kappa_t = \left(\pi_t + \frac{dL_t}{dt} \frac{A_t}{L_t} - (r + x\hat{\alpha}) A_t \right) + \left(\frac{L_t - A_t}{T} \right). \quad (9)$$

$\hat{\alpha}$ is the excess return on equities assumed by the firm in setting its contribution rate; it may be identical with the true α , but is not necessarily so. Define the solvency ratio of the fund a as:

$$a_t = A_t / L_t. \quad (10)$$

Then the evolution of the solvency ratio follows the stochastic differential equation:

$$\begin{aligned} da &= dA/L - adL/L \\ &= \left[\left(r + x\alpha - \frac{dL}{Ldt} \right) a + (\kappa_t - \pi_t) / L \right] dt + x\sigma_m a dz_m \\ &= \left(\frac{1-a}{T} + x(\alpha - \hat{\alpha}) a \right) dt + x\sigma_m a dz_m. \end{aligned} \quad (11)$$

Given the investment policy and the contribution policy, the solvency ratio follows a stationary stochastic process that is independent of the behaviour of liabilities. The unconditional distribution of a at time t under the risk-neutral measure is $g^Q(a)e^{-\hat{\alpha}t}$ where g^Q satisfies the differential equation:

$$\frac{1}{2} \frac{d^2}{da^2} (x^2 \sigma_m^2 a^2 g^Q(a)) - \frac{d}{da} \left(\left(\frac{1-a}{T} - x\hat{\alpha}a \right) g^Q(a) \right) = 0. \quad (12)$$

Formula (7) then gives the fair premium rate p (expressed as a proportion of the liabilities of the pension fund) as:

$$p = \delta \int_0^1 (1-a) g^Q(a) da. \quad (13)$$

Note that the true equity risk premium, α , does not enter into the risk neutral density function g^Q or into the premium rate p . A higher equity premium raises the expected solvency level of pension schemes, but this is offset by the effect on discount rates used for valuing the PPF's liabilities which are negatively correlated with the market. However the equity premium assumed by the scheme ($\hat{\alpha}$) does enter into the

premium; the higher the assumed premium, the lower the contribution rate and the greater the expected claim on the Fund.

The premium can be compared with the unconditional objective expectation of the rate of claims as a proportion of liabilities, c , where:

$$c = \delta \int_0^1 (1-a) g^P(a) da \text{ where } g^P \text{ satisfies:} \quad (14)$$

$$\frac{1}{2} \frac{d^2}{da^2} (x^2 \sigma_m^2 a^2 g^P(a)) - \frac{d}{da} \left(\left(\frac{1-a}{T} + x(\alpha - \hat{\alpha})a \right) g^P(a) \right) = 0.$$

Extending the Model

One element of unrealism in our model is that the solvency ratio of the pension fund is not bounded above. There are limits on the degree to which the pension fund can hold assets in excess of its liabilities, imposed largely to prevent the sponsor company using the pension fund as a tax avoidance device. We can readily impose the condition in our model that a is not permitted to exceed some limit a^* . Whenever a does exceed the limit, the contribution rate is constrained to force a below the limit; this may involve negative contributions. a^* acts as a reflecting barrier.

We assume that firms are able to reclaim investment surpluses from their pension plans over the same time horizon at which deficits are amortised. In practice, firms may struggle to reclaim investment surpluses because they face pressure to improve benefits or because they do not wish to be seen ‘raiding’ the pension plan of their employees.

We have also assumed that the liabilities that are guaranteed by the PPF are identical to the liabilities used to calculate the current deficit in the fund that is amortised over the period T . In practice these two measures of liability may well differ substantially, and in either direction. Under the UK PPF there is a cap on the level of wages on which the pension is guaranteed and the PPF only guarantees 90% of deferred pensions. In addition, the definition of liabilities used by actuaries in computing funding levels generally takes account of future wage growth in computing the pension liability arriving from past service. Finally, the actuarial valuation may also

use a higher discount rate in valuing liabilities than the rate at which the liabilities can be bought out in the market⁶.

The model can readily be adapted to distinguish between the liabilities used for funding requirements and those that are guaranteed by the PPF if we assume that the ratio of guaranteed liabilities to the actuary's measure of liabilities is constant. Denote the ratio by λ . Assume also that the PPF retains a prior claim on all the assets of the fund if the firm becomes insolvent. Maintain the definition of a as the ratio of fund assets to the cost of meeting the liabilities guaranteed by the PPF. Then a mean reverts to $1/\lambda$ rather than to 1. The adjustments to the model are obvious. For example (11) becomes:

$$da = \left(\frac{1/\lambda - a}{T} + x(\alpha - \hat{\alpha})a \right) dt + x\sigma_m adz_m. \quad (15)$$

2.2 Estimating the Model

The model expresses the fair premium per dollar of guaranteed liabilities, p , as a function of seven parameters:

- $\hat{\alpha}$, the market risk premium assumed by the scheme in determining contributions
- σ_m , the volatility of the market;
- δ , the bankruptcy hazard rate of the sponsor company;
- a^* , the maximum funding ratio;
- x , the equity proportion in the fund;

⁶ According to a forthcoming report (Institute and Faculty of Actuaries Working Party, 2004), of 685 actuarial valuations surveyed in 2001 and 2002, the average valuation discount rate was approximately 140 b.p's above Government bond rates, which would likely provide the basis for any buy-out.

- T , the time over which fund deficits are amortised
- λ , the proportion of liabilities that are guaranteed.

We take $\hat{\alpha} = 6\%$, and $\sigma = 18\%$. We take the probability of the firm becoming insolvent, δ , to be 0.25%/year. This parameter is hard to estimate. Moody's provides estimates of long-term default rates by rating category. Using their global data-base for 1983-2003 (Hamilton *et al* (2004), exhibit 31) and applying it to the observed credit-rate distribution of UK pension liabilities in Table 1 suggests a 10-year cumulative default rate of 2.95%, corresponding to an annual rate of 0.30%. This may be too high as a long-term estimate since it takes as its base ratings in 2002/3 when the corporate sector was in a financially weak state. Also a firm that defaults on its debt may refinance and continue without defaulting on its pension obligations. On the other hand, by ignoring companies that have no credit rating, we are implicitly assuming that they are similar to rated companies when they are in fact likely to be substantially weaker.

. Table 3 explores the effects of varying the investment strategy (as measured by x) and the funding strategy (as measured by T) on the size of the premium. The difficulty of estimating the mean default rate means that the absolute level of premium that we obtain from our model should be treated with great caution. But since the premium is directly proportional to the default rate, the sensitivity of the premium to varying assumptions should be not be affected by the uncertainty in the default rate

Table 3 here.

The direction of the sensitivities is as one would expect; the higher the equity proportion, the larger the premium. Having a higher solvency cap does reduce the premium because the fund is allowed to build up large surpluses when the market does well. But the effect is small; raising the cap on assets from 120% to 200% of liabilities, even assuming 100% equity funding, reduces the premium by less than

2%⁷. Stricter solvency requirements, as modelled by amortising deficits over 4 rather than 10 years, has a very substantial effect, roughly halving the premium.

The assumed risk premium has a substantial impact, with a zero risk premium cutting the insurance premium by more than half in the central case. This can be interpreted in two ways. The first is that if companies, in computing their contribution rate, assume that all their assets would just earn the risk free interest rate, they would pay higher contributions for any given level of the solvency ratio, and so would on average achieve a higher solvency ratio. The burden on the Protection Fund would be lower because of the more conservative contribution policy, just as it would be with a more rapid amortization policy.

A second interpretation is to note, by comparing (12) and (14), that the premium computed using a zero risk premium is the same as the expected rate of claims (under the objective measure) when the true and assumed risk premium coincide. Taking the base case with 2/3 equity, the table shows that while the fair premium is 0.072% of liabilities each year, the (objective) expected rate of claims is less than half that level, at only 0.032% of liabilities each year. The difference between the two arises because claims on the Fund are most likely to occur when the market declines, and the cost of insuring against bad states of the world is higher than the objective probability of those states occurring.

The bottom line of Table 3 shows that restricting the guarantee to a proportion of liabilities, while retaining the PPF's senior claim on all the pension fund's assets, also reduces the premium significantly. This is not because the sum guaranteed is smaller – the premium is expressed per £1000 of guaranteed liabilities - but because the first part of any deficit in the pension fund falls fully on the beneficiaries. With two thirds equity proportion, the effect of restricting the guarantee to 90% of liabilities reduces the premium per dollar of *guaranteed* liabilities by a quarter, and so reduces the absolute level of the premium by nearly one third.

⁷ The reason that raising the cap has such a small effect is that the probability (risk adjusted) of reaching 120% solvency is rather small, so the cap does not greatly affect contribution levels.

3. A Structural Model of Default Rates

In this section we extend the model to include a stochastic default rate. There is good reason to believe that the variability of default rates is important for pension fund guarantees. The risk of default varies substantially over time and is correlated across firms. It is also negatively correlated with the equity market. This has three important implications:

- 1) a falling equity market increases both the probability of sponsor firms becoming insolvent and also the size of pension plan deficits. So stochastic default induces a positive correlation between the probability of a claim on the PPF and the size of the claim. This increases the fair premium.
- 2) the correlation between default risk and equity returns means that default risk is priced. This will further increase the difference between the (objective) expected rate of claims on the fund and the fair premium.
- 3) the correlation of default risk across firms increases the skewness of the claims process.

None of these phenomena is captured in the Poisson default model. To explore the practical significance of these issues, we need a model of default that captures correlations across firms and correlations with the equity market. We explore three possible strategies for modelling default: fitting the empirical evidence on default directly, fitting the behaviour of corporate debt spreads, and structural models of the firm. We explain why we choose to follow the structural model approach, and why we choose the structural model with mean reverting leverage of Collin-Dufresne and Goldstein (2001). We then present premium calculations and claim simulations based on this model.

3.1 Choice of Default Model

The simplest strategy for modelling default is to take historic default rates, postulate some functional form for their time series behaviour, and estimate a relationship. The problem with this is the paucity of data. Defaults are rare – fewer than 1500 defaulted issuers are included in Moody's world-wide data base between 1970 and 2003. As

shown in Figure 1, default rates are highly auto-correlated over time. This is obviously important for modelling the PPF. But basing a model purely on the limited empirical data would be hard to do with any reliability. The peaks in 1990-91 and 2000-02 would drive any analysis.

Figure 1 here.

An alternative approach is to use information from the behaviour of credit spreads. The empirical evidence does strongly support correlations in changes in credit spread across firms and strong negative correlation between credit spreads and the equity market. Pedrosa and Roll (1998) document the existence of strong common factors in credit spreads for portfolios of credits. Collin-Dufresne, Goldstein and Martin (2001) find that a 1% increase in the S&P500 index is associated with a credit spread decrease for US corporate issuers of about 1.6 basis points. Similar results are reported by Manzoni (2003) in the sterling Eurobond market who finds that a 1% increase in the FTSE 100 index is associated with a credit spread decrease of 2.1 to 3.5 basis points depending on the specification.

Building a model of default that is calibrated to bond prices is attractive because of the large amount of high quality data on the behaviour of bond yield spreads. But it faces a serious obstacle. There is mounting evidence (Elton *et al.* (2001), Huang and Huang (2003)) that credit risk accounts for only a part – according to Huang and Huang, in the case of investment grade bonds it is less than a quarter – of the yield spread. In the absence of any generally accepted explanation of why the risk-adjusted expected return on corporate bonds is higher than on default free bonds, the credibility of a model that incorporates the whole yield spread in valuing the pension fund guarantee would be in doubt.

The approach we follow is to model the default process from fundamentals, using a structural model of the firm. Merton (1974) models a risky bond as a portfolio consisting of riskless bond and a short position in a put option on the assets of the firm. This simple idea has been developed by many other authors (see Duffie and Singleton (2003) for an overview), and structural models are widely used as a basis for pricing credit sensitive instruments, though they do not appear to capture yield spreads on corporate bonds with any accuracy.

However, Huang and Huang (2003) show that structural models, when suitably calibrated, do fit the empirical data on default rather well. For our specific purpose, structural models have three other advantages: the correlation between corporate default and the behaviour of the equity market arises naturally within the model; the correlation in default rates across firms arises naturally in the model from the correlation in firms' asset values; and, unlike models based on the yield spread, the price of default risk can be computed within the model, without the need to make any assumptions about the behaviour of recovery rates.

In the previous section we had a stationary process for pension fund deficits that allowed us to compute an unconditionally fair insurance premium that is a constant proportion of the value of insured liabilities. To retain this feature, we need a structural model of default that is also stationary. The natural candidate is Collin-Dufresne and Goldstein (2001) who have a model with mean-reverting leverage ratios (hereafter 'CDG'). As in other structural models based on Merton (1974), debt is a claim on the firm's assets V . The assets follow a diffusion process with constant volatility σ_v , and the firm's leverage varies accordingly. But CDG argue that firms tend to adjust their leverage over time through their financing strategy. This causes the leverage ratio to revert to some target level.

The key variable in their model is the log leverage ratio of the firm, l . The leverage ratio is defined as the ratio of the critical asset level at which default will occur to the current asset level. CDG model the dynamics of l as a first order auto-regressive process:

$$dl = \kappa(\bar{l} - l)dt + \sigma_v dz_v. \quad (16)$$

κ determines the speed of mean reversion, and $\sigma_v dz_v$ is the innovation in the log return on the firm's assets. We assume a constant correlation between changes in firm value and changes in the assets of the pension fund, so the two stochastic process z_v and z_m have constant correlation ρ .

The log leverage ratio l is strictly negative so long as the firm is solvent; if it hits zero, the firm defaults. We have now fully specified the processes governing the claim on the PPF from an individual firm. The log leverage ratio, which determines firm

solvency, follows the stochastic process in (16) above. The pension fund solvency ratio, which determines the size of any claim that is made, is governed by the stochastic process in (11).

We need two more elements to complete the specification of the model. First, we need to specify the correlation structure of firms' asset returns. We assume a Sharpe-type "diagonal" model, where each firm's return is the market return plus a noise term that is identically and independently distributed across firms. So given two firms i and j we have:

$$dz_v^i dz_v^j = \rho^2 dt \quad \text{if } i \neq j. \quad (17)$$

We also assume that idiosyncratic risk is unpriced, so:

$$E^Q [dz_v] = -\rho \frac{\alpha}{\sigma_m} dt. \quad (18)$$

Starting with a portfolio of firms with the same leverage and the same pension funding, the pension funding level varies over time with the equity market, but remains the same across firms, while leverage ratios disperse because of firm idiosyncratic risk.

3.2 Estimating the Model

In estimating the model, we generally follow Huang and Huang (2003); their estimates are broadly consistent with CDG. Since their estimates vary slightly according to the credit rating of the bond in question, we take their estimates for an A-rated issuer (Moody's or Standard and Poor's). In particular, we take the mean reversion parameter κ to be 0.2, the asset volatility σ_v to be 24.5% and the asset risk premium 4.89%. Huang and Huang show this is consistent with an equity premium for the firm of 5.99%. Taking the equity beta to be 1, the market risk premium is also 5.99%, and the asset beta is 0.82. Using an equity market volatility σ_m of 18%, the correlation between the firm asset value and the equity market price is:

$$\rho = \beta \frac{\sigma_m}{\sigma_v} = 0.60. \quad (19)$$

Using Huang and Huang's estimate of the long term average leverage ratio of 38% gives a long run average default rate of 0.75%/year. For the reasons already discussed, this looks very high, so we have used an average leverage ratio of 31.7% which gives a long run default rate of 0.25% per year.

We compute the steady state joint density of the solvency ratio a and the leverage ratio l using a two dimensional binomial tree with births and deaths, and iterate forward in time until the default rate and rate of claims on the fund converge to their limiting values. In all the iterations we use a time step of 0.1 years.

Table 4 here.

Table 4 shows the premium and expected claims rate for a variety of parameter values. Using the same base case parameters as before (two-thirds of the pension fund invested in equity, 120% ceiling on over-funding, 10 year deficit amortization period, 100% of liabilities guaranteed) the average rate of claims is £0.93/£1000 of liabilities per year. This compares with a claims rate of £0.32/£1000 in Table 3, where the default process is Poisson. The difference – an increase of nearly 200% - is entirely attributable to the correlation between corporate defaults and the underfunding of pension schemes in the structural model.

The impact of the structural default model on the premium is still greater. With Poisson default, the fair premium in Table 3 is £0.72/£1000. With the structural default model it is more than six times as high at £4.95/£1000. The other two rows of the table show that the level of premiums, and the average rate of claim, can be reduced significantly by limiting the proportion of liabilities guaranteed (with the PPF retaining first claim on all the assets of the pension fund), and by stricter pension fund solvency requirements.

It is difficult to compare our calculated premia with actual PBGC premia, as these depend on actual pension underfunding while our calculations assume a steady state distribution of funding and firm leverage. However, the most recently available statistics (PBGC, 2002) show that in the year 2000, the PBGC collected \$807million

in exchange for insuring total liabilities of around \$1.240 trillion - a premium of \$0.65 per \$1000 of liability. Our premium is thus over 7 times greater than the PBGC premium and our expected claims nearly 50% greater. However, it would be wrong to attach too much importance to the absolute numbers. They are very sensitive to the parameters chosen, and in particular to the assumptions concerning the long run average leverage ratio. Using Huang and Huang's estimate of 38% rather than the value we have used of 31.65% would lead to fair premia that are more twice as high.

3.3 Claims distribution

The previous section established the average level of claims in the long run. The premium reflects the average long run claims experience of the Pension Protection Fund, but the variation in the claims level is also a matter of considerable concern. To investigate the variation in the claims level, we simulate the claims process, and ask: how high a claims rate can one reasonably expect over a period of say 30 years?

The simulations are carried out with the same base case as Table 4, using the structural default model, and an equity proportion of 2/3. As set out in Table 4, the fair premium is £4.95/£1000 of liabilities, while the expected level of claims is £0.93/£1000.

Table 5 shows the distribution of the thirty year worst case, using objective probabilities; it is based on one thousand simulations, with a time step of 1/10 of a year. The simulations start with the steady state distribution of firm leverage and pension fund solvency. A path for the equity market is then simulated. The liabilities of schemes grow at a constant rate that is equal to the average rate of insolvency, so ensuring that the level of insured liabilities is stationary.

Since the pension assets of all firms are perfectly correlated, and deficits are corrected by adjusting contribution policy, the initial dispersion in pension funding levels among firms quickly narrows. Firm asset value is subject to idiosyncratic risk, so while there is co-movement, there is also substantial dispersion.

In running the simulations, the first seventy years are used as a conditioning period, and the following thirty years are then used as the sample period. The conditioning period is needed to ensure that the start of the sample period is suitably randomized.

For comparison we also show comparable figures for the Poisson default case. The claims are expressed as a percentage of the average size of liabilities over the 30 year period.

Table 5 here.

The table shows how the structural model of default not only increases the magnitude of average claims, but also greatly increases their skewness. In the Poisson model, the level of claims in the worst year in 30 is less than three times the average claim level in the median case; with the structural default model, the ratio is in excess of 6. In the worst decile of thirty year periods, the contrast is even more stark. With Poisson default, the ratio is still under 4, while with structural default the ratio is over 30. The effect is strongly visible even looking at five year periods, with the worst five year period being comparable to twice the worst single year experience..

While it would be wrong to attach much precision to the numbers – we are looking at rare and extreme events – the results of the simulation do illustrate the extent to which correlated defaults across firms, and the correlation between the mean default rate and the equity market may create considerable skewness in claims experience. This has important implications for the setting of premia. If the PPF wants to build up reserves sufficient to meet claims in the worst year in 30 years with 90% probability, Table 5 suggests it would need to have reserves equal to around 25 years of average claims, or 2.5% of insured liabilities. It is hard to believe that agreement could be reached on setting the level of premiums necessary to build up such a high level of reserves.

In the absence of such reserves and of any support from Government, the PPF would need to borrow to pay claims, using its future premium income as collateral. But this alternative looks barely more palatable, since it would require premia to be raised very substantially. If for example there were claims equal to 2.5% of liabilities in one year, and they were met by borrowing that had to be repaid over 10 years, then additional premia equal to nearly three times the normal average claims level would need to be charged to repay the debt, ignoring any real interest due on the debt. This high premium would have to be charged at a time when, by assumption, the solvent firms that remain are heavily leveraged, and themselves have pension funds in substantial deficit.

If the PPF cannot weather extreme events either by way of reserves or by way of borrowing backed by increased premia, then that leaves two alternatives: default or some form of Government involvement. The PPF will have powers to reduce the amount guaranteed under extreme circumstances, but this is a route that is fraught with problems. The very name of the fund, and the fact that the Government has frequently stated that it has acted to restore confidence in the pensions promise⁸ means that it will be very difficult politically for a Government to allow the PPF to significantly reduce its commitment. It is hard to avoid the conclusion that the Government will be left as the final guarantor of defined benefit pensions.

4. *Incentives and Moral Hazard*

The model treats decisions about firm capital structure, the firm's contributions to the pension plan and the investment strategy of the pension fund as exogenous. It cannot be used to predict how far these decisions will be altered by the existence of the Pension Protection Fund, nor how they will depend on the way the premium is levied. But it can be used for making qualitative predictions.

Table 4 shows how the benefits of the PPF vary over the long term with the contribution and investment policy of schemes. The value of the PPF guarantee is greatest for those schemes where the sponsor is financially weak, the pension scheme is poorly funded, the equity exposure is high and contributions are low.

While the benefits of the PPF flow in the first place to the beneficiaries of the pension scheme rather than the firm itself, the existence of the guarantee increases the value of the pension to employees, and hence is likely to facilitate staff recruitment and retention, and to reduce the level of cash wages. If the firm wishes to maximize the value of the guarantee, it will engage in precisely the behaviour that imposes greatest costs on the PPF. This is the classical moral hazard problem of any insurance scheme.

⁸ For example "We will make sure that in future individuals in final salary schemes will never again face the injustice of saving throughout their lives only to have their hard-earned pension slashed just before they retire. The Pension protection Fund will allow individuals to save with confidence." (Andrew Smith, Secretary of State for Work and Pensions, 12 February 2004, quoted on Department of Work and Pensions website, www.dwp.gov.uk/publications/dwp/2004/pensions_bill/ppf_factsheet.pdf).

There is ample evidence from the US experience⁹ of pension fund guarantees that moral hazard is not merely a theoretical possibility, but a significant reality.

There are a number of ways to mitigate the moral hazard problem. One is to structure the insurance premium to give appropriate incentives. The challenge is to design a structure that is both credible and effective. By deferring contributions to the pension scheme, the firm is effectively borrowing from the pension plan without paying a credit spread. To discourage the firm from using the pension scheme as a cheap source of finance, the premium would need to be levied on the scheme deficit, and would need to be at a rate that is higher than alternative finance. For a firm with a credit rating just below investment grade, this would require a premium in excess of 200 basis points on the deficit.

In the short term, it is hard to see such a scheme being introduced. For a number of existing firms with pension fund deficits that are large compared with their market capitalisation, and that are financially weak, premiums at this level could well force insolvency or major restructuring. It is hard for a statutory body established to protect workers pensions to immediately put in place policies that are seen to lead directly to redundancies.

This raises the question of whether it will ever be possible to impose premium rates that are sufficiently tough to be effective. It is interesting to note that the PBGC has, since 1988, levied a premium on deficits. It is currently at the headline rate of 90 basis points. But pressure from Congress to minimize the impact on financially weak industries with badly underfunded schemes has led to a definition of deficits that is so restrictive that the effective average rate charged is no more than about 10 basis points¹⁰.

⁹ See for example Utgoff (1993) and Kandarian (2004), both former Executive Directors of the PBGC.

¹⁰ PBGC (2003) shows premium income at \$948m in 2003, and estimated end-year underfunding in insured single employer programmes of \$350 billion. Although the variable rate premium is not broken out separately in the report and accounts, it is not believed to exceed one third of the total premium income.

Suitable design of the premium structure may help to mitigate some of the moral hazard problems created by the PPF, but it is hard to believe that it will remove the moral hazard entirely. The model presented in this paper takes no account of moral hazard, and to that extent is unduly optimistic about the future solvency of the PPF.

Furthermore, a premium structure that is not just a flat rate on liabilities, but designed to bear more heavily on those schemes that pose greatest risk, will inevitably bear most heavily on those firms that are in a weak financial position. If the premium falls most heavily on the weakest firms, the PPF's ability to raise its premium income will be reduced. But as we have seen, the PPF's solvency depends critically on its ability to raise premium income after a period of heavy claims. So risk-related premiums may actually increase the likelihood of recourse to public funds.

If the threats to solvency are to be reduced, it will be necessary to consider more direct measures to contain the claims on the PPF. Effectively that means imposing strict funding requirements, possibly in conjunction with restrictions on investment mix. Requirements have one important advantage over incentives imposed through the premium: a premium is most likely to affect the behaviour of those schemes of least concern to the PPF – those with strong sponsors who can readily borrow to make good any scheme deficit – while a requirement would fall equally on all schemes.

The problem of existing underfunded schemes with weak sponsors would still need to be addressed. Sudden imposition of a funding requirement would lead to the same problems as a premium levied on deficits at a penal rate.

5. Conclusions

Our analysis does not claim to be a very accurate or even a practical method of determining a premium for the PPF. However, it does illustrate some of the problems that may be faced by the PPF in the future, and suggests ways in which the design of the PPF could be changed to accommodate these effects. Although failure of pension plans to pay people their entitlements have been unusual in the UK, it would be dangerous and wrong to conclude that failures will be rare and small in the future. The

way that pension schemes are funded, and the way that funds are invested, imply that a deep and prolonged decline in financial markets could readily lead to widespread failure. An inherent feature of the claims process of the PPF is likely to be that many years of small claims will be interspersed with rare and unpredictable periods of exceedingly large claims. These periods will coincide with periods when the stability of the whole of the financial sector is under maximum strain. We suggest that the magnitude of the claims in these unstable periods will be so large that it will not be politically feasible or economically sensible to build up reserves to meet them. When such a crisis does occur, it may well be impossible to meet claims by a steep increase in the levy on employers since they will simultaneously be facing heavy financial demands to rebuild their own depleted pension funds. It is hard to see any alternative to the Government stepping in. The Government has repeatedly made clear that it will not guarantee the PPF; in reality it will be forced to do so, and a substantial part of the cost of the scheme will actually fall to the taxpayer.

However, the major part of the cost will be borne by employers. The PPF will necessarily involve large transfers from companies that are unlikely to default to companies that may well default. These transfers are inefficient, and create opportunities for moral hazard. To minimise the cost of the insurance and to keep down the level of cross-subsidy, it will be necessary to devise incentives or rules that ensure that pension funds run by employers with weak credits keep their plans adequately funded. We argue that these rules, unless punitive, will not largely alter the current investment and funding policy of UK pension plans, and will hence not change the mismatch between assets and liabilities in UK pension plans. Premium risk-rating will therefore need to be implemented in tandem with a strong funding requirement if the cost of the PPF is not to fall to future UK taxpayers.

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Table 1: Total UK DB pension liabilities for FTSE-350 Companies

S&P Credit Rating	Number of Companies	UK Pension Liability (FRS17, £mil)	Unfunded UK Pension Liability (FRS17, £mil)	Median plan funding Ratio
AA+	2	11 816	523	0.91
AA	5	21 184	3 349	0.87
AA-	5	14 743	3 267	0.76
A+	12	32 225	5 801	0.74
A	10	21 145	4 187	0.82
A-	12	55 230	14 539	0.78
BBB+	13	13 228	3 325	0.74
BBB	14	18 977	2 427	0.81
BBB-	7	12 760	1 730	0.84
BB+	2	3 784	453	0.85
BB	3	8 711	503	0.79
Not Rated	163	63 886	14 180	0.70
Total	248	277 689	54 285	0.73

Source: Watson Wyatt Pension Risk Database. Data from published accounts for the company financial year ending between June 2002 and May 2003. Liability figures are in millions of pounds, calculated on the FRS17 basis reported in the accounts and include only UK liabilities. Figures include only those companies in the FTSE 350 which have DB plan liabilities. The credit rating is as reported by Standard and Poor's at the date of the accounts.

Table 2: Average equity proportion in pension fund asset portfolio for different pension plan types

	Below median	Above median
Pension plan assets / pension plan FRS17 liabilities	0.72	0.58
Pension plan FRS17 liabilities / Company market capitalisation	0.68	0.62
Book value of company debt / Company market capitalisation	0.66	0.63
Company market capitalisation / Book value of firm assets	0.65	0.65

Source: Authors' calculations using Watson Wyatt Pension Risk Database. Each cell shows the proportion of the plans assets invested in equities for plans below and above the median value of each plan variable. Means differ as not all data is available for every company.

Table 3: Premium with Poisson Default (£/year per £1000 of liabilities)

	Equity Proportion		
	1/3	2/3	100%
Base Case	0.418	0.722	0.956
Higher solvency cap: $a^* = 200\%$ (120%)	0.417	0.716	0.943
Stricter solvency: $T=4$ yrs (10)	0.199	0.378	0.540
No assumed risk premium: $\hat{\alpha} = 0\%$ (6%)	0.140	0.319	0.484
Partial guarantee: $\lambda = 90\%$ (100%)	0.229	0.553	0.807

Source: Authors' calculations. The base case shows the unconditional fair value premium for guaranteeing a pension fund against default when the risk of default is 0.25% per annum, equities have an expected return of 6% in excess of the risk-free rate, deficits in the fund are made up over 10 years, and the fund value is not permitted to exceed 120% of liabilities. The premium is shown as a percentage of liabilities for different investment strategies. The other lines of the table show how the cost varies as each of the input parameters is varied. Base case values are shown in parentheses.

Table 4: Premium and Average Claims with Structural Default (£/year per £1000 of liabilities)

Equity Proportion (x):	2/3		100%	
	Premium	Claim	Premium	Claim
Poisson Default	0.72	0.32	0.96	0.48
Structural Default:				
Base Case	4.95	0.93	6.28	1.25
$\lambda = 90\%$ (100%)	4.34	0.76	5.81	1.12
$T = 4$ (10)	3.38	0.67	4.54	0.94

Source: Authors' calculations. The Poisson default case is from Table 3. The Structural default model base case has the same dynamics for the solvency ratio as the Poisson model; the two also have the same expected default rate (0.245%). The first variant on the base case have only 90% of liabilities guaranteed by the PPF, and the second has an amortisation period for pension fund deficits of 4 years rather than 10. The other parameters of the models are: $a^* = 120\%$, $\sigma_m = 18\%$, $\sigma_v = 24.5\%$, $\bar{l} = -1.15$, $\kappa = 0.2$, and $\rho = 0.6$.

Table 5: Claims/£1000 in worst period in thirty years (simulation)

	Structural Default		Poisson Default	
Fair premium	4.95		0.72	
Average claim	0.93		0.32	
	1 year	5 years	1 year	5 years
Median	5.7	9.7	0.9	3.4
Top quartile	14.0	25.0	1.0	4.3
Top decile	28.9	50.7	1.1	5.8

Source: Authors' calculations. The table is based on 1000 simulations of the evolution of the distribution of firm leverage and solvency level for the population of insured firms, and shows the average and peak annual claim level over each thirty year period. The parameter values for the base case are: $a^* = 120\%$, $T = 10$, $\beta = 1$, $\sigma_m = 18\%$, $\sigma_v = 24.5\%$, $\bar{l} = -1.15$, $\kappa = 0.2$, and $\rho = 0.6$. The Poisson default case is identical except that $\rho = 0$.

Figure 1: Global issuer-weighted default rates, 1970-2003.

