Reverse Mortgages as Retirement Financing Instrument: An Option for "Asset-rich and Cash-poor" Singaporeans

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April 2004

ABSTRACT

The unique way of financing housing through the mandatory savings system in Singapore has created a class of "asset-rich and cash-poor" Singaporeans. This paper provides a framework to assess the viability of a reverse mortgage (RM) market so that such instruments may be harnessed as a source of financing retirement income for home owners. Based on different cost of capital, we estimate the probability of loss for both the private supplier and public provider of RMs. The probability of loss is computed by three major components: choice of replacement ratio and property growth rate; forecast of cohort survival probability by joint-life; and generation of yield curves to discount the future cash flows. The stochastic forecast of survival probability is estimated using the Lee-Carter demographic model based on the abridged life tables. The discount factor for future cash flows are generated from stochastic interest rates. Our simulation results indicate that based on the benchmark scenario, RM instruments by private providers are likely to achieve about 50% replacement ratio for the 4-room public housing owners. However, the market may be missing if a replacement ratio of 70% is required.

Keywords: reverse mortgage market, replacement ratio, probability of loss, risk free interest, breakeven annuity

1. Introduction

Housing equity forms a large fraction of the non-pension wealth for elderly households in many developed countries.[1] If there is a mechanism to unlock housing equities, it will help to alleviate poverty among elderly homeowners and to finance retirement expenditures. Essentially a reverse mortgage (RM) is designed to allow property owners to obtain loans to purchase retirement annuities, using their residential assets as collateral; and repaying the loans by selling the house upon their death. It thus affords an alternative means for elderly homeowners to borrow against the financial equity embodied in their homes, while sparing them from the emotional disruption of moving out of or selling their abodes.

There are many forms of RM, but the basic idea is that the property will be reverted to the RM supplier at the end of a period or upon the death of the reverse mortgagor. In conventional mortgages, the loan quantum is dependent on the borrower's ability to pay. However for RM, the loan quantum depends not only on the age and sex of the homeowners, but also on the appraised value of the property, the projected rate of house price appreciation and the levels of interest rates.[2] There are also costs involved in taking out these RM loans. RMs differ in terms of the types of loan advance and the time frame. The loan advance can be taken as a lump sum or as a regular income stream. The terms of the RM can either be fixed-term or tenure. In a fixed-term RM, the period of loan advance is usually fixed at either 10, 15 or 20 years.

[1] For instance, in the US in 2000, about 33% of the total financial asset is in housing assets (Poterba, 2001). It is even higher for Japan at 63.9% (Noguchi, 1997).

[2] For example, in US the minimum qualifying age for RM is set at 62 years, and the average age is around 72 years. In Singapore, the average qualifying age is 62. Gender is important since male and female have different life expectancy.

Empirical work by Kutty (1998) indicates that the use of home equity conversion mortgage products could possibly raise about 29% of the poor elderly homeowners in the US above the poverty line. In addition, the equity released could potentially help to finance long term care among the elderly, where relatively large sums of money are required (Gibbs, 1992).[3] The usefulness of such a scheme is conceivably greater in countries where land prices and values of residential assets are extremely high (for example, Japan) or where there is a skewed investment portfolio towards home ownership (for example, Australia) or where there is a deliberate public policy towards home ownership (for example, Singapore).[4]

While some empirical studies support the potential effect of RM, others do not. For example, Hancock (1998) examines the impact of equity release scheme on the net income of older homeowners in Britain and finds that the increased income is not significant for some of the oldest homeowners. Although theoretically, there is potential in unlocking housing wealth to help alleviate poverty and to meet health care needs of the elderly, in reality, RM markets have remained weak. Mayer and Simmons (1994) and Caplin (2002) attribute this to the substantial loads in the RM market because of moral hazard and adverse selection problems. Other major barriers include product designs, availability of information, bequest motives and the desire to keep house equity as precautionary savings. These authors conclude that to unleash the potential of RM instrument, there is a need for policy makers to provide institutional and legal support for

[3] Sheiner and Weil (1993) find that besides shocks to family status, health shocks also contribute to the decline in home equity at older ages.

[4] The share of land assets in real assets is 83.4% in Japan and 36% in US. (Noguchi, 1997). Australian home ownership is in excess of 70% (Beal, 2001), whereas home ownership in Singapore is in excess of 90%.

the RM market. For RM to operate effectively in Japan, Mitchell and Piggott (2003) highlight the need to facilitate information on housing values and transactions and credit worthiness of borrowers.

Simulations by McCarthy et al. (2002) indicate that a typical Singapore worker would have around 75% of his retirement wealth in housing asset. Such a concentration surpasses that of an American elderly household who would have only 20% of their retirement wealth in housing asset. In fact, some attempts have been made in Singapore to unleash housing assets as alternatives to finance retirement needs. In January 1997, the NTUC Income, a local insurance firm, launches the first RM scheme. But the market has remained thin, with only 180 customer base. The average monthly draw down is $1800; and the average property value is $1.6 million.[5] RM remains unpopular as it is only available to private property owners and not to the public housing owners. In addition, the high profit margin set by the private provider reduces the monthly annuity payouts to RM buyers. In this paper, we shall explore whether RM is a viable option for "asset-rich and cash-poor" elderly Singaporeans who are owners of public housing.

Most RM studies have focused on the demand side of the market and examine the effect of equity release on net incomes. For examples, Merrill et al. (1994), Hancock (1998), Venti and Wise (1991, 2001) and Mitchell and Piggott (2003) calculate the tenure or life RM that would provide the homeowners with monthly payments over the borrower's remaining life after retirement. In these studies, the maximum amount of RM

[5] See Inter-Ministerial Committee on Ageing Population, 1999, p. 77.

loan is calculated and then the lump-sum loan is converted to lifetime annuities with monthly payments. The analyses then focus mainly on the demand for RM to augment the incomes of the elderly homeowners. Our approach is different because we consider both the demand and supply sides of the RM market. They include the replacement ratio, the initial appraised value of the property, the growth rate of house price appreciation, the survival probability of homeowners, various costs of capital and the probability of loss.

The organisation of the rest of the paper is as follows. In Section 2, we provide an overview of the housing market in Singapore, focusing on the unique way of financing housing through the compulsory savings mechanism. Section 3 describes the methodology and calibration procedure used in the Monte Carlo experiments. The calibration of the retirement annuity consists of three major components. The first part assesses the adequacy of the monthly annuity payments to finance retirement at the first breakeven month for different initial property values and appreciation rates. Second, the Lee-Carter (1992) model is adopted to forecast cohort survival probability at each post-retirement age for the household using the abridged life tables for Singapore. Third, the discount/accumulation factor for future cash flows are generated from three interest rate models, including two deterministic and one stochastic interest rate environments. Section 4 presents simulation results and analysis on the first breakeven month and the probability of loss for both the private supplier and the public provider of RM. The final section concludes the study and draws some policy implications on the role of a public supplier.

2. Public Housing in Singapore

After independence from the British in 1959, the new Singapore government was set to solve the housing shortage which saw many living in slums. The Housing Development Board (HDB) was set up in 1960 to build "emergency" public housing on state-owned land. These 1-room to 3-room apartments were leased to citizens at standardised and affordable rates, which averaged $20 and $30 per room in suburban and urban zones respectively.[6] However in February 1964, in line with the strategy of making private home ownership an investment in the stake of the country, HDB introduced a scheme to encourage existing tenants to own their flats. Under the Home Ownership Scheme for the People, HDB offered subsidized mortgage loans with an attractive repayment scheme. The loan quantum was set at 80% of the price of a new flat with repayment periods of either 5, 10 or 15 years. However, the home-ownership rate remained low at only 5% by December 1965. This was due to the low purchasing power of households at that time.

To further ease financing difficulty on the demand side, the government introduced a unique system that is closely integrated to the compulsory savings scheme or the Central Provident Fund (CPF) system. The CPF, which was instituted in 1955, was originally a retirement savings scheme. It is a fully funded and a defined contribution system. Under the CPF, every employee and employer is required to contribute a proportion of the wage to the CPF which is credited directly into the

[6] All dollars used in this paper refers to Singapore dollars, US$1 ≅ S$1.70.

employees' personal accounts.[7] In September 1968, the CPF introduced the Approved Housing scheme which allowed HDB purchasers to withdraw their savings with CPF to finance the purchase of public housing. Funds can be withdrawn for down-payment, stamp duties, mortgage payments and interest incurred for the purchase. The CPF Approved Housing Scheme marked the beginning of a series of schemes in which mandatory savings were used in relation to housing finance.[8] It also set off a gradual liberalisation of CPF from merely a retirement vehicle to instruments to help finance merit goods consumption such as education and health care. [9]

In order to achieve a nation of homeowners, HDB also implemented the supply-side regulations and subsidies. First, the option to rent was made unattractive or effectively unavailable for the majority due to strict eligibility criteria. Second, public housing was priced affordably through government supply and price discounts, enabling buyers to purchase HDB flats at below market prices. However, unlike most merit good programs, these subsidies are not financed primarily from taxes or other government revenues, but rather from land rents that are captured through state ownership and acquisition.[10] Indeed, nearly 80% of the land in Singapore is owned by the state. Under

[7] In 1955, the rate of contributions was only 10%. Since 1968, the rate has increased, rising to a peak of 50% in 1984. The rate is currently graduated according to age, with an average rate of 36%. More details on the CPF and the CPF-HDB link are discussed in the next section. Also, see Chia and Tsui (2003a) for the institutional details regarding the CPF.

[8] In 1981, the use of CPF savings was extended to the purchase of private residential private property. See Phang and Tan (1991) for a chronological account of the liberalization of the use of retirement savings in the CPF for housing finance in Singapore.

[9] See Chia and Tsui (2003b) for the link between compulsory savings and the financing of health care in Singapore using the medical savings accounts.

[10] With this arrangement, the supply-side price discount has little impact on government expenditure and is not a significant expenditure item. According to Asher and Phang (1997), receipts from land rent have enabled the government to keep expenditure on housing at no more than 2% of total government expenditure in any fiscal year.

the Land Acquisition Act, the government is empowered to acquire land at its discretion from private land owners and at prices below market prices.[11] Compared to private sector developers who have to purchase or cost land at market rates, producer costs for public sector housing is thus lower and HDB is able to sell its flats at below market prices.

The success and sustainability of HDB to build public housing and pricing flats at affordable prices is due to the strong institutional support from the government. HDB's annual deficit is fully covered by a government grant. The cumulative government grant received since the establishment of HDB amounted to $10,533 million. In the fiscal year 2000/2001, HDB received $920 million to cover the deficit. (HDB, Annual Report 2001). HDB also receives two main government loans to finance its operations. First, the Housing Development Loans is used to finance the development programmes and operations. Interest rate is pegged at two percentage points above the floating CPF interest rate and with a repayment term of 20 years. The second is the Mortgage Financing Loans which in turn finance the mortgage loans granted to the purchasers of HDB flats. The ability of HDB to obtain loans from the government at below market rates enables HDB to offer subsidised mortgage financing rate for its buyers. The mortgage financing rate is pegged at 0.1% above the interest rates paid by the CPF for the compulsory savings and is about 2% below the housing mortgage interest rates of commercial banks. [12]

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[11] For example, between 1973 and 1987, the government acquired land under the Land Acquisition Act at l973 rates rather than at market rates of compensation. For details, see Phang (1996).

[12] The CPF interest rate on the ordinary account is pegged to the average of 12-month fixed deposit and month-end savings rates of the local banks rate.

The HDB Home Ownership Scheme and its link to the CPF housing financing scheme, together with many supply-side instruments, have skewed the housing tenure choice towards owner occupation, particularly towards the owner-occupied public housing. This is evident from the fall of HDB rental occupancy of 100% in the early 1960s to 76% at the end of 1970 and to 38% in 1981 and finally to just 7% in 2002. After almost four decades since the inception of Home Ownership Scheme in 1964, 85% of Singapore's population now resides in public housing; with 93% of the public housing residents owning the units they occupied. (See Figure 1). Furthermore, HDB flats constitute 80% of the residential housing stock in Singapore. [13]

== Insert Figure 1 ==

One consequence of the owner-occupied housing policy is that housing becomes the most important non-financial assets for Singaporeans. This can be gleaned from Table 1. Compared to France (47%), Japan (40%), US (28%) and UK (34%), Singapore has the highest ratio of household residential property assets relative to total assets (at 51%). This is also true for housing assets relative to personal disposable income and GDP. In the National Survey of Senior Citizens (1995), 63.1% of elderly aged 60 and above reported housing among their assets and 48.4% cited their own house as their most important asset. While the provision of early withdrawal from CPF savings has helped in housing finance, it has diluted its original intent as a retirement savings scheme, thereby reducing the accumulated amount available for retirement needs. Indeed, the CPF-HDB

[13] See Singapore, Department of Statistics (2003), *Statistical Highlights*.

link has created a class of "asset-rich and cash-poor" households, whose savings are "plastered on the wall". The issue is how to unlock the housing equity and whether RM is a viable option to finance post-retirement needs for these "asset-rich and cash-poor" elderly.

== Insert Table 1 ==

3. Modelling Reverse Mortgages

Our model consists of three main parts, namely the treatment of the monthly annuities in terms of replacement ratios, the use of joint life survival probability and various interest rate models. It builds on the approach by Tse (1995b) but we make four extensions. First, we provide an economic interpretation of the pre-supposed level of life annuity which is set according to different target replacement ratios to ensure adequate post-retirement living. Second, like Tse (1995b), Lachance and Mitchell (2002) and Chia and Tsui (2003a), the Cox-Ingersoll-Ross (CIR) model is used to generate yield curves for discounting and accumulating cash flows. But instead of using the 6-month rates, 3-monthly interest rates are generated. This facilitates the computation of the monthly accumulated loan value and expected profit. Our approach also allows death to happen monthly instead of in the middle of the year as assumed by Tse. Another distinctive difference in our approach is that, unlike Tse's analysis which is based on a single period life table, we forecast mortality rates using the Lee-Carter (1992) stochastic demographic model and further construct the cohort mortality rates using the approach by Bourbeau et al. (1997). More discussion is given in Section 3.3. Finally, Tse's assessment of the RM market is mainly based on some assigned profit margin issued by private RM suppliers. However we also assess the viability of RMs based on risk-free interest rates, thereby allowing some non-profit organisations who have access to low-cost capital to launch the RM instrument in the case of an incomplete market. As such, our findings bear policy implications for government intervention in the reverse mortgage market.

The nature of the RM market entails lenders granting various guarantees to borrowers. First, RM instrument has a residency guarantee in which the mortgagors are allowed to remain in their property until death, regardless of the loan amount. Second, under income guarantee, the monthly annuity payment will continue as long as the homeowner lives in the home. Third, under repayment guarantee, repayment will only occur after the demise of the last couple, thereupon the property is sold. Fourth, being a "non-recourse" loan, the accumulated loan value cannot exceed the accumulated property value and the mortgagor's other assets cannot be used to repay the loan.

All these guarantees inevitably spell different risks for the lender, which are reflected by three different interest rates. The risks for the lender include the longevity of the borrowers. The longer the life expectancy of the lender, the higher is the probability of loss for the lender as the accumulated loan value may exceed the accumulated property value. [14] As repayment is over a longer time frame, there are risks associated with volatilities of interest rates and property prices.

[14] See Phillips and Gwin (1993) and Mitchell and Piggot (2003) for an excellent exposition of the risks facing the RM suppliers. Besides longevity risk, interest rate risk and property appreciation risk, they also discuss specific house appreciation risk and expense risk.

It is crucial to distinguish three types of interest rates used in our model.[15] First, the cost of capital, denoted by r, is a risk-free rate of interest, which represents the opportunity cost of using funds. Second, an interest rate i is used to discount the future value of loan and repayment cash flows. In the context of reverse mortgages, this interest rate can be interpreted as the lending rate which includes risk premium to reflect the uncertainty to the lender in the event that at the time of repayment, the accumulated loan value exceeds the accumulated property value. This is partly due to the uncertainty on property appreciation rates and uncertainty on mortality which affects the length of residence. We assume that $i = r + 0.02$. Third, an interest rate y is used to discount the loan balance that incorporates the necessary profit margin. It can be regarded as the cost of borrowing for the lender to finance the RM loan. We assume that $y = r + 0.01$ for the private supplier. The spread between y and i also reflects the intermediary role of the lender who has access to lower cost fund at y but charges the borrower i for the use of fund[16].

We assume that the elderly do not have any outstanding mortgage and that a tenure joint life RM is taken up by a married elderly, both of the same age. The eligible couple will then receive a fixed monthly annuity at the beginning of each month till the end of life. Furthermore, we assume that the couple will not move out of their home. Although death is a random process, for convenience we assume that death occurs at the end of the month. The monthly payout will continue upon the death of one spouse. Only

[15] For example, Boehm and Ehrhardt (1994) show that compared to other types of interest-bearing assets, interest rate changes are riskier for RMs. Both Tse (1995b) and Mitchell and Piggot (2003) also incorporate the different risks when modelling interest rates.
[16] It is not necessary to fix the spread at 1%. It can vary with the level of interest rate. The spread assumed here is meant for illustration and may not reflect the actual risk in the market.

upon the death of the last survivor would the loan be repaid through the sale of the property. However, the property sale will initiate one month after death and that the sale will be completed after three months.

3.1 Present value of estimated profit

The private supplier provides a RM loan to the homeowner in the form of monthly life annuity payout. The monthly payout is accumulated with interest until the repayment period. Let *A* denotes the fixed monthly payout generated from the RM. For exposition purpose, we assume that the elderly receive the first monthly annuity payout at age 62 and continue to receive the payout at the beginning of each month till death of the last survivor. We assume a maximum life span of 105 years, so that *t* can take any value from 1 to 528 months. Upon the death of the last surviving spouse in month *t,* the total accumulated loan balance (L_t) is:

$$L_t = \left(\sum_{j=1}^{t} AU_j \right) B_t \qquad t = 1, \ldots, 528 \qquad (1)$$

where U_j is the accumulation factor used to sum up the level monthly annuity *A* from age 62 to the time of death at time *t*. Denoting i_j as the nominal interest rate which reflects the supplier's cost of capital at month *j*, U_j can be expressed as follows:

$$U_j = \prod_{n=j}^{t} \left(1 + \frac{i_n}{12} \right) \qquad (2)$$

As it takes four months to complete the sale, B_t in equation (1) is the additional accumulation factor given by:

$$B_t = \prod_{n=1}^{4} \left(1 + \frac{i_{n+t}}{12} \right) \qquad (3)$$

In the simulations, deterministic and stochastic interest rates are generated to accumulate and discount the future cash flows.

Besides interest rate charges, the supplier also levies other administrative costs, including an origination fee for initiating the RM instrument. We assume that the origination fees, denoted by λ, are fully borne by the RM buyer and are borrowed from the supplier who incorporates the amount into the loan. As in Tse (1995b), we also set the origination fee at 1% of the appraised value of the property. Besides the origination fee, closing costs are incurred at the time of sale of the property. Such costs cover fees for title search and title insurance, legal and appraisal services, surveys and inspections, mortgage taxes, credit checks and other related transaction costs.[17] The closing cost, denoted by τ, is set at 3.5% of the initial appraised value of the property, which is the usual rate charged in Singapore.

The accumulated value of the property net of all transaction costs is given by P_t as follows:

$$P_t = P_0\left((1-\lambda)(1+\alpha)^{(t+1)/12} \quad - \quad \tau U_t^*\right) \tag{4}$$

$$U_t^* = \prod_{j=1}^{t+4}\left(1+i_j/12\right) \tag{5}$$

where P_0 is the initial appraised property value; α represents the rate of property appreciation or the annual growth rate of the appraised value of the property. U_t^* is the accumulation factor to compute the cash flow of the property, including the four-month

[17] In the United States, the Home Equity Conversion Mortgage (HCEM) imposes 2% (of home value) for origination and closing fee and 2% insurance premium. Besides these costs, other loading factors include insurance cost at 0.5 % over and above the interest rate charges.

lags to complete the sale of the house. i_j is the interest rate charged by the supplier at month j.

In what follows we consider the profit maximizing behaviour of the RM suppliers under uncertainty. There are two major sources of uncertainties. Besides the stochastic interest rate movements, the other source comes from the time of death of the last survivor which in turn determines the time of repayment. The supplier would then compare the accumulated net value of the property (P_t) with the value of the accumulated loan (L_t) to assess the profitability of supplying the RM instrument. During the initial months of the launched RM, the accumulated loan is definitely smaller than the accumulated value of property. However, as the RM progresses and as the time of residence lengthens, the accumulated loan becomes bigger and subsequently it may exceed the accumulated value of the house. As such, it is necessary to compute the first breakeven month $m*$ such that at $t = m*$, the accumulated loan (L_{m*}) is greater than the accumulated net value of the property (P_{m*}). Hence, for all $t > m*$, we have $L_t > P_t$. In other words, if the borrower survives $m*$ months or longer, the accumulated net property value will fall short of the accumulated annuity payouts, thereby the supplier incurs a loss. As the probability of loss is dependent on the mortality of the RM holder, we define the probability of loss to the supplier (γ) as:

$$\gamma = \sum_{t=m^*}^{528} {}_{t-1|1}q_{\overline{62}} \qquad (6)$$

where ${}_{t-1|1}q_{\overline{62}}$ denotes the probability that death of the last surviving spouse occurs within month t conditional on having survived (t-1) months from age 62.

We next examine the flow of funds for the supplier in terms of receipts and costs. As a "non-recourse" loan, the total value of the loan cannot exceed the sale value of the property. Thus at any repayment month t, the receipt (Q_t) or the maximum claim amount by the supplier is the minimum of the accumulated loan L_t and the accumulated appraised net value of the property P_t , that is,

$$Q_t = \min \{L_t , P_t \} \tag{7}$$

The present value of the cost to the RM supplier for providing the monthly level payout A up to the period t is C_t, such that:

$$C_t = \sum_{j=1}^{t} A W_j \tag{8}$$

where $y_0 = 0$ and

$$W_j = \prod_{n=1}^{j} \left(1 + \frac{y_{n-1}}{12}\right)^{-1} \tag{9}$$

We assume that the private supplier has access to a lower cost of capital. The spread between y_t and i_t then represents the intermediary role of the RM supplier who charges the borrower at a higher rate for the use of fund, taking into account the risk premium and profit margin.

Hence, the present value of profit (π_t) generated at month t which is discounted to age 62 is given by:

$$\pi_t = Q_t / V_t - C_t \tag{10}$$

where V_t is the appropriate discount factor for Q_t given by:

$$V_t = \prod_{n=1}^{t+4} \left(1 + \frac{y_n}{12}\right) \tag{11}$$

The mean present value of profit (*MPVP*) is obtained by weighing π_t in equation (10) by the mortality of the last survivor to obtain:

$$MPVP = \sum_{t=1}^{528} \pi_t \; {}_{t-1|1}q_{\overline{62}} \tag{12}$$

In what follows we highlight the major procedures required to calibrate the discount factors and the mortality rates of the last survivor respectively.

3.2 Interest rates

The calibrated cash flows of monthly retirement expenses for the elderly couple are to be discounted by the appropriate yield curve. As there are no consensus for modeling interest rates[18], we follow Tse (1995a), Lachance and Mitchell (2002), and Chia and Tsui (2003a) to generate stochastic short-term interest rates using the discretized version of the Cox, Ingersoll and Ross (1985) short-term model: [19]

$$r_{t+1} = r_t + \theta\,(r_a - r_t) + \beta r_t^{1/2}\,\varepsilon_{t+1} \tag{13}$$

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[18] See "Term-Structure Models" in Campbell, Lo and MacKinlay (1997, Chapter 11).

[19] Tse (1995a) is the only available empirical study on the stochastic behaviours of 3-month Treasury-bill rates in Singapore and finds that the CIR model provides reasonable replicates of the yield curves. We have also tried out other stochastic models and found that the CIR model is more adequate.

where r_t is the short-rate at time t. The second term on the right-hand side of (3) captures the deterministic trend which consists of the long-term average interest rate, r_a , and the speed of mean-reversion, θ, respectively. For $\theta > 0$, r_t is expected to decrease and revert to r_a if the current rate is above the long-run mean, and vice versa. The third term captures the stochastic part, consisting of independently and identically distributed standard normal random variable ε_{t+1}; and β is the volatility parameter.

For comparison, we also employ two deterministic interest rate models to discount the future cash flows: the constant yield curve (CYC) model and the fixed yield curve (FYC) model. The CYC has a flat annual rate for all durations. Choices include 2%, 3% and 4%, respectively. The FYC is based on the average of the available historical rates for the government bonds since 1988. They comprise 2.3% for 3-month bills, 2.5% for 1-year bills, 3.0% for 2-year bonds, 3.9% for 5-year bonds, 3.8% for 7-year bonds, 4.3% for 10-year bonds and 3.9% for the 15-year bonds, respectively. We use the 15-year rate as proxy for spot rates with longer durations. Spot rates for other durations below 15 years are obtained by the method of interpolation.

3.3 Mortality of the last survivor

Appropriate mortality rates of the last survivor are required to compute the present value of profit for the RM suppliers as given in equations (10) to (12). The Lee-Carter (1992) demographic model and the Bourbeau and Legare approach are adopted to forecast cohort mortality rates at each post-retirement age for the household by sex using the abridged life tables for Singapore. We follow the approach by Chia and Tsui (2003a) to construct the required probabilities by age and by sex starting from age 62, up to and including 105.

Basically the construction takes three main steps. First, we predict the future mortality rates of the elderly based on the abridged life tables using the following Lee-Carter model (1992, 2000) for the male and female elderly of age 60 to 85 and above, such that,

$$ln\ m_{x\,t} = a_x + b_x\, k_t + \varepsilon_{xt} \tag{14}$$

$$k_t = \mu + \varphi\, k_{t-1} + \eta_t \tag{15}$$

where $m_{x\,t}$ is the central death rate in age class x in year t; a_x is the additive age-specific constant, reflecting the general shape of the age schedule; b_x is the responsiveness of mortality at age class x to variations in the general level; k_t is a time-specific index of the general level of mortality; μ and φ are parameters; ε_{xt} is the error to the actual age schedule, assuming to follow a normal distribution with zero mean and a constant variance; and η_t is the white noise. The Lee-Carter model has been successfully applied to the G7 countries to forecast life expectancy at birth. See Tuljapurkar et al. (2000). For those elderly aged above 85, the interpolation method proposed by Wilmoth (1995) is used.

Second, we calibrate the abridged cohort life tables based on the predicted mortality rates obtained from the previous step using techniques developed by Bourbeau et al. (1997). Third, we convert the calibrated abridged cohort mortality rates into annual mortality rates using Pollard (1989)'s methodology, and then to monthly mortality rates.

The probability that the last survivor dies within the t^{th} month given survival up to the (t-1)[th] month from age 62 can be computed as follows:

$$ {}_{t-1|1}q_{\overline{62}} = {}_{t-1}p_{\overline{62}} - {}_{t}p_{\overline{62}} \tag{16} $$

where ${}_{t}p_{\overline{62}}$ denotes the probability of survival of the last spouse up to t months from age 62. For computational simplicity, we assume that the married elderly couples are of the same age and the events of deaths are independent in probability. And ${}_{t-1|1}p_{\overline{62}}$ can be further related to the mortality rates and survival probabilities of the female and male as below[20]:

$$ {}_{t}p_{\overline{62}} = 1 - {}_{t}q_{62}^{F}\ {}_{t}q_{62}^{M} = 1 - (1 - {}_{t}p_{62}^{F})\,(1 - {}_{t}p_{62}^{M}) = {}_{t}p_{62}^{F} + {}_{t}p_{62}^{M} - {}_{t}p_{62}^{F}\,{}_{t}p_{62}^{M} \tag{17} $$

where ${}_{t}q_{62}^{F}$ and ${}_{t}q_{62}^{M}$ denote the mortality rates of the female and male elderly who dies within t months starting from age 62; while ${}_{t}p_{62}^{F}$ and ${}_{t}p_{62}^{M}$ denote the corresponding survival probabilities respectively

4. Calibration and Simulation Results

Monte Carlo experiments are conducted to explore the feasibility of launching RM instruments for the public housing owners in Singapore. We obtain the simulated

[20] See Chapter 10 of Jordan (1975) for details.

values of L_t and P_t using equations (1) and (4) by alternative model parameterisations, including different initial home values (P_0), rates of property appreciation (α), interest rate paths to reflect different risk premium, and joint cohort survival probabilities. All computations and estimations are coded in Gauss. It is possible to generate infinitely many interest-rate paths by the CIR model as described in equation (13), but we have confined our simulation to 5,000 runs.

We choose the 4-room public housing owners as the benchmark household. This is supported by the empirical evidence that the largest proportion of HDB owners (about 39%) in year 2002 are in 4-room flats. In fact, 68% of the residents are living in HDB 4-room or larger flats or private housing, up from 52% in 1990. While a decade ago, the greatest proportions of the HDB owners (about 35%) were in 3-room flats. Because of greater affluence, 42% of these 3-room HDB owners have moved to 4-room or larger flats or private properties in 2000.[21] Table 2 shows the price range of the new 4-room flats offered by HDB for different residential zone and town in 2000. In pricing new flats for sale, HDB takes into consideration the affordability factor. The prices of new 4-room flat are pegged to the average household income levels to ensure that at least 70% of all household can afford to purchase a new 4-room flat. It also shows the average valuation of resale flat at market prices, which varies according to location.[22]

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[21] See Singapore, Department of Statistics (2001), Table 10.

[22] Since March 1971, a resale market in HDB flats emerged when owners of HDB flats were allowed to sell their flats at market prices. The government intervenes in the HDB resale market by setting the minimum occupancy period requirement before resale is possible. In 1971, the period was set at three years and was extended to five years in 1973. It has remained so until 1979 when it was relaxed to two and half year. The active resale HDB market has led to the sentiment that public housing has become "a cash cow for the milking of housing subsidies". (*The Straits Times*, April 19, 1997). As of January 2003, the minimum occupancy period was further reduced to one year.

== Insert Table 2 ===

Table 3 describes the average floor area of the different public housing types and the profile of the households in terms of the average annual household income for different public housing types. The demand for RM is assessed by comparing monthly annuity payments to the replacement incomes which are proportions of pre-retirement household incomes. We set the benchmark mean household income at $3719 and the median household income at $3000.[23]

== Insert Table 3 ===

Table 4 summarises the value of parameters used in the benchmark scenario. We first estimate the accumulated loan amount (L_t) in equation (1) by setting various monthly annuity payouts, starting from $900 and increasing it in steps of $100. For each level of payout, we compute the associated net accumulated property value P_t which is given in equation (4). As long as P_t exceeds L_t , the supplier will make a profit. However at a first breakeven month $m*$, such that $t > m*$, L_t will exceed P_t. As can be observed from Table 5, our simulation results are consistent with the intuition that the first breakeven month for the supplier is sooner for the higher than the lower annuity payouts. Table 5 also tabulates the probability of loss for different levels of monthly annuity payouts, using equation (6).

[23] The Census of Population 2000 indicates that for the general population, the average household income in 2000 is $4,943 and the median household income is $3,607. But for HDB dwellers, the average household income is lower at $3,719 and the median household income is $3000. (HDB, 2000).

== Insert Table 4 ===

The present value of profit (*PVP*) in equation (10) depends on the stochastic interest rate fluctuations. Using the CIR model as described in Section 3.3, we generate a yield curve to obtain the corresponding *PVP,* and the whole process is run 5000 times to obtain the mean value of *PVP* (*MPVP*). Table 5 also tabulates the values of *MPVP,* the standard deviation of *PVP* and its values at the 5^{th} and 95^{th} percentiles as well as the probability of loss and the first breakeven months at various monthly level annuities.

== Insert Table 5 ==

Table 5 shows that for an initial property value of $240,000 and a monthly annuity level of $1500, the probability of loss for the supplier is only 0.0367, with a later first breakeven month, occurring at m^* = 478. However, when the monthly annuity payout is increased to $1600, the probability of loss will be much higher at 0.5374, with the first breakeven month occurring sooner at m^* = 350. If the private RM supplier is risk averse and prefers a smaller probability of loss, then he will set a lower level of monthly annuity, say at $1000. But in this instance, there may be no demand for the RM as the replacement ratio will be much lower than the expected 70%.[24] Hence, the completeness of the RM market depends on setting an annuity payout that is adequate to finance retirement expenditure, while containing the probability of loss to the supplier.

[24] There is no single acceptable replacement ratio. McGill et al., (1996) recommend using a replacement rate of 73%. In Canada, financial planners typically set a ratio of 70% of post-retirement income to maintain a comparable standard of living experienced before retirement.

Table 5 also indicates that when the monthly payout is increased to $1700 to attain a replacement ratio of 46% of the pre-retirement average income, the probability of loss to the supplier is higher at 0.8024. This is in stark contrast to the negligible probability of loss when the monthly annuity payout is $1400. Hence, we may interpret the difference of $300 as the loading factor levied by the RM supplier. The loading factor can be as high as 21% ($300/$1400). Our findings are consistent with Caplin (2002) and others who accord the incompleteness of the reverse mortgage market to high loading factors.

We next evaluate the RM market when the supplier is a non-profit motivated supplier, for example the government. As profit is not a major consideration for the public provider, in our computation of the accumulated loan value (L_t) in equation (1), risk-free interest rate r is used instead of the risk-embedded rate y which incorporates both the risk premium and profit margin. For easy comparison, we define the *breakeven annuity* as the annuity which yields zero *MPVP* for the private supplier. We repeat the simulation process to obtain the new breakeven month for the public provider while setting the payout at the breakeven annuity.

Table 6 tabulates simulation results which compare the adequacy of the RM instruments provided by profit or non-profit motivated suppliers of RM. These include the mean monthly payout at the breakeven annuity, its standard deviation, the 5th and 95th percentiles, the probability of loss, the first breakeven month, and replacement ratio based on the mean and median monthly income of the household. We repeat the simulations using different initial property values assumption, ranging from $220,000 to $300,000.

== Insert Table 6 ==

As can be observed from Table 6, for a given property value of $220,000, the mean breakeven monthly payout is $1475. This level of the breakeven annuity represents a replacement ratio of 49% of the average household income. However, there may be no private supplier as the probability of loss is close to 0.581, implying that it is highly probable that the accumulated loan balance is higher than the net value of the property at the time of repayment. As such, it is interesting to know the possible effect of government intervention in the market. Table 6 indicates that at the same average breakeven annuity, the probability of loss for the public provider is now almost negligible as the public provider has lower cost of fund and is able to charge the loan balance at the risk free interest rate.

In addition, at a higher initial property value of $300,000, the homeowner can expect to unlock the housing equity to yield an income which is almost 67% of the pre-retirement average income. But again the market may be missing as the probability of loss for the private supplier is still high at 0.586. However, the probability of loss is only 0.003 for a public provider. Furthermore, compared to the private supplier, the first breakeven month for the public provider occurs 176 months later.

We concur that profitability is vital for the private RM suppliers but not so for the public suppliers. Figure 2 displays the probability of loss for both the public and private supplier using different levels of monthly annuity payouts. It also compares the probability of loss for various property appreciation rates. As can be gleaned from Figure 2, the probability of loss is smaller at higher appreciation rates. For example, when the annuity level is at $1600, the probability of loss for the private provider is 0.586 when α is 5%. But at a higher appreciation rate at 6%, the probability of loss is lower at 0.476. In addition, the probability of loss for a private supplier always lies to the left of that for a public provider, implying that the public provider is able to support a reasonably higher level of life annuity compared to the private supplier. Hence the market is more efficient under a public than a private supplier.

== Insert Table 7 and Figure 2 ==

Both Figure 2 and Table 7 show that the completeness of the RM market depends on the appreciation rates of the property. Table 7 shows that at α = 6%, the mean monthly breakeven payout is $2154, hence implying a high replacement ratio of 72%. This no doubt will attract a demand for RM. However, there may be no private supply as the probability of loss is 0.476. This is due to the non-discourse nature of RM. However at a lower appreciation rate of 3%, the mean monthly breakeven payout is $879, representing a low replacement ratio of 30%. The probability of loss is 0.5 for the private supplier and 0.296 for a public provider. This is clearly undesirable as even the public provider will suffer a loss.

== Insert Figure 3 ==

Figure 3 plots the simulated first breakeven month m^* at different monthly annuity levels for both the public provider and the private supplier. As can be gleaned, the higher is the appreciation rate, the higher the breakeven month. For example, using the benchmark parameters, when α =5%, the average first breakeven month for the public provider is at the 338th month compared to 511th month for the private supplier. With α = 6%, the first breakeven month for the public provider and the private supplier is at the 490th and 364th month, respectively.

Table 8 presents simulation results using different risk-free interest rates at 3%, 4% and 5%, respectively. As the RM supplier charges a higher interest rate, the lower is the mean breakeven payout annuity and RM becomes less adequate to finance retirement. At the benchmark risk-free interest rate of 3%, the monthly annuity payout is $1609 which implies a replacement ratio of 54%. However at a higher interest rate of 5%, the monthly payout is $1067, thereby implying a mere replacement ratio of 36%.

== Insert Table 8 ==

Moreover, values of the breakeven annuity are affected by various yield curves used in the simulation exercises. Table 9 tabulates the simulated mean breakeven monthly annuity payout with the yield curves generated by the Cox, Ingersoll and Ross (CIR) stochastic model, fixed yield rate (FYC) and the constant yield curve (CYC) models, respectively. In general both the CIR and FYC models give consistent values of probability of loss and the first breakeven month. But values generated from the CYC model deviate substantially from those generated from the CIR and FYC models. This may due to using flat rates to discount cash flows.

== Insert Table 9 ==

5. Conclusion

We have examined the monthly annuity payouts in terms of replacement ratios and drawn implications on the adequacy of RM in financing retirement needs. Using Monte Carlo simulations, we compute the mean present value of profit for different levels of annuity, the probability of loss and the first breakeven month. We compare the viability of the RM under a private supplier with a public provider based on different costs of capital. Such a comparison yields important policy implications on the need for a low cost supplier.

Our simulation findings indicate that although most public housing homeowners have their wealth tied up in their flats, property values are inadequate to support retirement expenditure at the 70% replacement ratio. If these home owners are willing to lower their expectation to attain at about 54% of the retirement income, then it is possible to convert their house into a stream of future income by borrowing from the public supplier. However, a replacement ratio at 54% may be moderately low compared to the recommended ratio of 70% in most of the developed countries. But there are alternative sources of retirement income in Singapore. For example, an important source is family support, particularly from children.[25] The living arrangement among the elderly testifies to strong family support. As can be observed from Appendix 1, less than 13% of the elderly are not living with spouse or children. Only 6.6% of the elderly live alone. In 2000, about 60% of the elderly live with their working children. This is especially so for the elderly aged 75 and over, as about 50% of them live with their children.

Our study is not without caveats. For instance, we have not factored in the tenure of HDB flats. As most HDB flats are on lease for 99 years, finance companies may not be willing to take on a flat with less than 60 years left on the tenure. Another possible financing option is for homeowners to down-grade to smaller flats and to convert the financial gains from the sales of flats into some forms of annuity instruments. In fact, in March 1998, HDB introduced the Studio Apartment Scheme, whereby the elderly flat owners could sell off their flats in the resale market and use part of the proceeds to buy a smaller studio apartment from HDB.[26] The remaining fund may be used to top up their medical savings account to ensure that the elderly have adequate funds to meet their medical needs or it may be invested in annuities which yield regular monthly income. Furthermore, since October 2003, many HDB-imposed restrictions on public housing were lifted, thereby allowing flat owners to monetise their assets by subletting the entire flats.

[25] Singapore has a Parents Maintenance Bill, which stipulates that children are legally obliged to care for their elderly parents.
[26] The studio apartments are smaller, which come in two sizes: 35 to 45 sq m. They are specially equipped with elderly-friendly features and are sold on a 30-year lease.

Another caveat is that we have excluded other social dimensions which could possibly affect the success of reverse mortgages. These factors include bequest motives and the psychological reluctance among the elderly to mortgage their homes to financial institutions. Such social norms are likely to prevail so that even with a public provider, the reverse mortgage market may still remain thin. However, these issues are left for future investigation.

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Appendix 1
Living arrangements of elderly Singaporeans in 2000

	Total	65-74	75 & over
Living with spouse	**50.4**	**58.5**	**35.0**
No children in household	13.9	15.8	10.3
With working children in household	33.1	38.9	22.2
With non-working children in household	3.4	3.9	2.4
Living with Children only	**37.2**	**29.8**	**51.4**
With working children in household	33.2	27.6	43.8
With non-working children in household	4.1	2.2	7.6
Not living with spouse or children	**12.3**	**11.7**	**13.6**
Alone	6.6	6.5	6.9
With other elderly persons only	1.2	1.3	1.1
Others	4.5	3.9	5.7

Source: Singapore, Department of Statistics (2002), Singapore Census of Population, 2000, Advance Data Release No. 6, Households and Housing.

Table 1
Household residential property asset ratios in 2000

	Housing Assets/ Total Assets (%)	Housing Assets/ Personal Disposable Income (%)	Housing Assets/ GDP (%)
Singapore	51	452	230
United States	28	155	113
Japan	40	294	198
France	47	271	176
United Kingdom	39	292	197

Source: Singapore, Department of Statistics (2003). *Wealth and Liabilities of Singapore Household,* Occasional Paper on Economic Statistics.

Table 2
Sample of price range of flats offered by HDB and Average valuation of resale flat in 2000

Town	HDB list price ($)	Resale price($)
Sembawang	98,000 – 162,000	204,000
Jurong West	99,000 – 56,000	186,100
Woodlands	104,000 – 151,000	192,500
Choa Chu Kang	110,000 – 162,000	217,200
Bukit Panjang	110,000 – 166,000	187,400
Seng Kang	120,000 – 186,000	228,100

Source: Singapore, Housing Development Board, Annual Report. 2000/01 and HDB Average Valuation by town and flat types, available at: http://www.hdb.gov.sg

Table 3
Profile of household by housing types in Singapore

	3-room HDB	4-room HDB	5-room HDB	Executive HDB	Private
Average Floor Area (sq ft)	550-900	750-1100	1200-1500	1500-1900	1300-2000
Average Household Income per annum ($)	42,200	52,900	77,200	95,100	149,700

Source: Adapted from: http://www.mof.gov.sg/budget/budget_2002/budget2002_p4_19_a10.html

Note: Household incomes are estimated based on the 1997/98 Department of Statistics (DOS) Household Expenditure Survey (HES) and adjusted to 2001 levels.

Table 4
Values of parameters for the benchmark scenario

(i) Age of mortgagor at the first monthly annuity payout = 62

(ii) Typical mortgagor is a 4-room HDB dweller
Average pre-retirement household income = $3,719
Median pre-retirement household income = $3,000

(iii) Initial appraised property value of the 4-room flat (P_0) = $240,000
Annual growth rate of the property (α) = 5%
Initiation fee (λ) = 1% of the appraised value of the property
Closing fee (τ) = 3.5% of the selling price of the property

(iv) Risk-free interest rate or cost of capital (r) = 3 %
Cost of capital to RM private supplier (y) = r + 1%
Cost of capital to RM buyer (i) = r + 2%

(v) Cox-Ingersoll-Ross model of stochastic interest rates:
Initial interest rate = 3%
Average interest rate (r_a) = 3%

Table 5
Present value of profit, probability of loss for private RM suppliers using benchmark parameters

Level of Annuity	Breakeven month (*m**)	Probability of loss	Present value of profit (*PVP*)			
			Mean	Standard deviation	5th percentile	95th percentile
1000	528	0.0000	62899	961	61189	64460
1200	528	0.0000	60245	4451	54238	72215
1300	528	0.0000	48439	5173	41585	62930
1400	528	0.0000	33835	5386	27037	48980
1500	478	0.0367	18061	5429	11473	33240
1520	455	0.0871	14783	5423	8279	29965
1540	429	0.1742	11507	5414	5064	26660
1560	403	0.2911	8209	5403	1828	23332
1580	375	0.4203	4892	5391	-1431	19986
1600	350	0.5374	1557	5378	-4709	16620
1650	301	0.7226	-6850	5339	-12949	8115
1700	273	0.8024	-15338	5295	-21242	-498
1800	233	0.8747	-32486	5192	-37970	-17962
1900	206	0.9121	-49818	5100	-54859	-35638
2000	186	0.9350	-67265	5004	-71854	-53447
2200	157	0.9595	-102377	4833	-106052	-89302
2400	136	0.9716	-137664	4702	-140397	-125352

Table 6
Probability of loss and first breakeven month under private and public RM suppliers using different initial property values

	Initial property value				
	\$ 220K	**\$ 240K**	**\$ 260K**	**\$ 280K**	**\$ 300K**
Mean payout of breakeven annuity	1475	1609	1743	1876	2011
Standard deviation	5010	5472	5894	6260	6675
5th percentile	-8106	-9091	-9752	-10233	-11274
95th percentile	8368	8913	9448	9791	10887
Probability of loss					
Private supplier	0.581	0.586	0.585	0.584	0.586
Public provider	0.004	0.004	0.004	0.004	0.003
Breakeven month					
Private supplier	339	338	338	338	338
Public provider	511	511	511	511	514
Replacement ratio[a] (3000 per month)	49%	54%	58%	63%	67%
Replacement ratio[b] (\$3719 per month)	40%	43%	47%	50%	54%

Note:
a. Replacement ratio based on the median monthly income of HDB households.
b. Replacement ratio based on the average monthly income of HDB households.

Table 7
Probability of loss and first breakeven month under private and public RM suppliers using different property appreciation values

	Rates of property appreciation (α)			
	3%	**4%**	**5%**	**6%**
Mean payout of breakeven annuity	879	1194	1609	2154
Standard deviation	2714	3814	5472	7321
5^{th} percentile	-4595	-6066	-9091	-12202
95^{th} percentile	4534	6087	8913	11919
Probability of loss				
Private supplier	0.500	0.535	0.586	0.476
Public provider	0.296	0.154	0.004	0.019
Breakeven month				
Private supplier	358	350	338	364
Public provider	401	435	511	490
Replacement ratio[a] ($3000 per month)	30%	40%	54%	72%
Replacement ratio[b] ($3719 per month)	24%	32%	43%	58%

Note:
a. Replacement ratio based on the median monthly income of HDB households.
b. Replacement ratio based on the average monthly income of HDB households.

Table 8
Sensitivity of simulation results to alternative risk-free interest rates

	Interest rate (*r*)		
	3%	**4%**	**5%**
Mean breakeven annuity	1609	1316	1067
Standard deviation	5472	4615	3714
5^{th} percentile	-9091	-7356	-5841
95^{th} percentile	8913	7670	6275
Probability of loss			
Private supplier	0.586	0.545	0.512
Public provider	0.004	0.048	0.149
Breakeven month			
Private supplier	338	348	355
Public provider	511	471	436
Replacement ratio[a] (Y=$3000 per month)	54%	44%	36%
Replacement ratio[b] (Y=$3719 per month)	43%	35%	29%

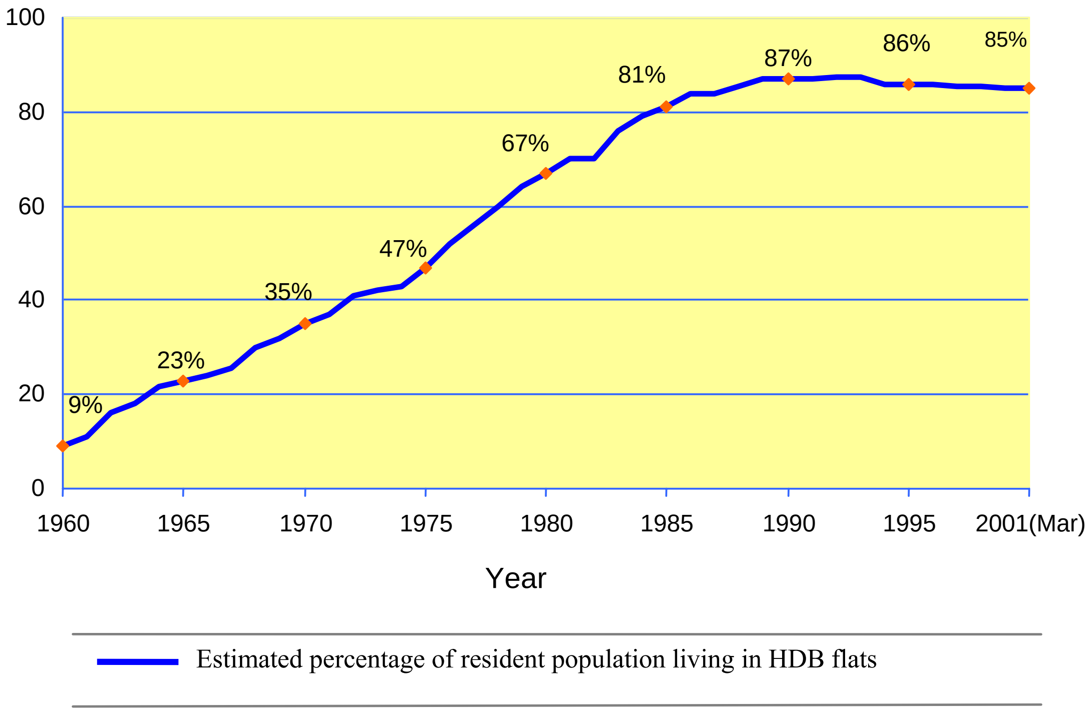
Note:
a. Replacement ratio based on the median monthly income of HDB households.
b. Replacement ratio based on the average monthly income of HDB households

Table 9
Sensitivity of simulation results to different interest rate models

Cases	Interest rate Models	Private supplier m^*	Private supplier Prob (loss)	Public provider m^*	Public provider Prob (loss)
$\alpha = 3\%$,	CIR	358	0.500	401	0.296
A = $879	FYC	360	0.492	405	0.280
	CYC	297	0.735	320	0.657
$\alpha = 4\%$,	CIR	350	0.535	435	0.154
A = $1194	FYC	352	0.527	440	0.135
	CYC	272	0.801	296	0.738
$\alpha = 5\%$,	CIR	338	0.586	511	0.004
A = $1609	FYC	330	0.619	526	0.000
	CYC	233	0.874	254	0.838
$\alpha = 6\%$,	CIR	364	0.476	490	0.019
A = $2154	FYC	367	0.458	528	0.000
	CYC	181	0.940	193	0.927

Note:
The mean first breakeven month at the corresponding breakeven annuity are computed using three different yield curve models. CIR represents the yield curve generated by the Cox, Ingersoll and Ross model; CYC is the flat 4% yield curve and FYC is the fixed yield curve based on the mean spot rates of government bonds with various durations. A is the mean level monthly payout, α is the property appreciation rates.

Figure 1: Percentage of population housed in HDB flats



Note: Data from 1960 to 1979 refer to Total Population (including both residents and non-residents. From 1980 onwards, the data refer to Resident Population only (i.e. Singaporeans and permanent residents) and exclude non-residents.

Source: Singapore, HDB Research & Planning Department.

Figure 2: Probability of Loss for Providers at Various Property Growth Rates

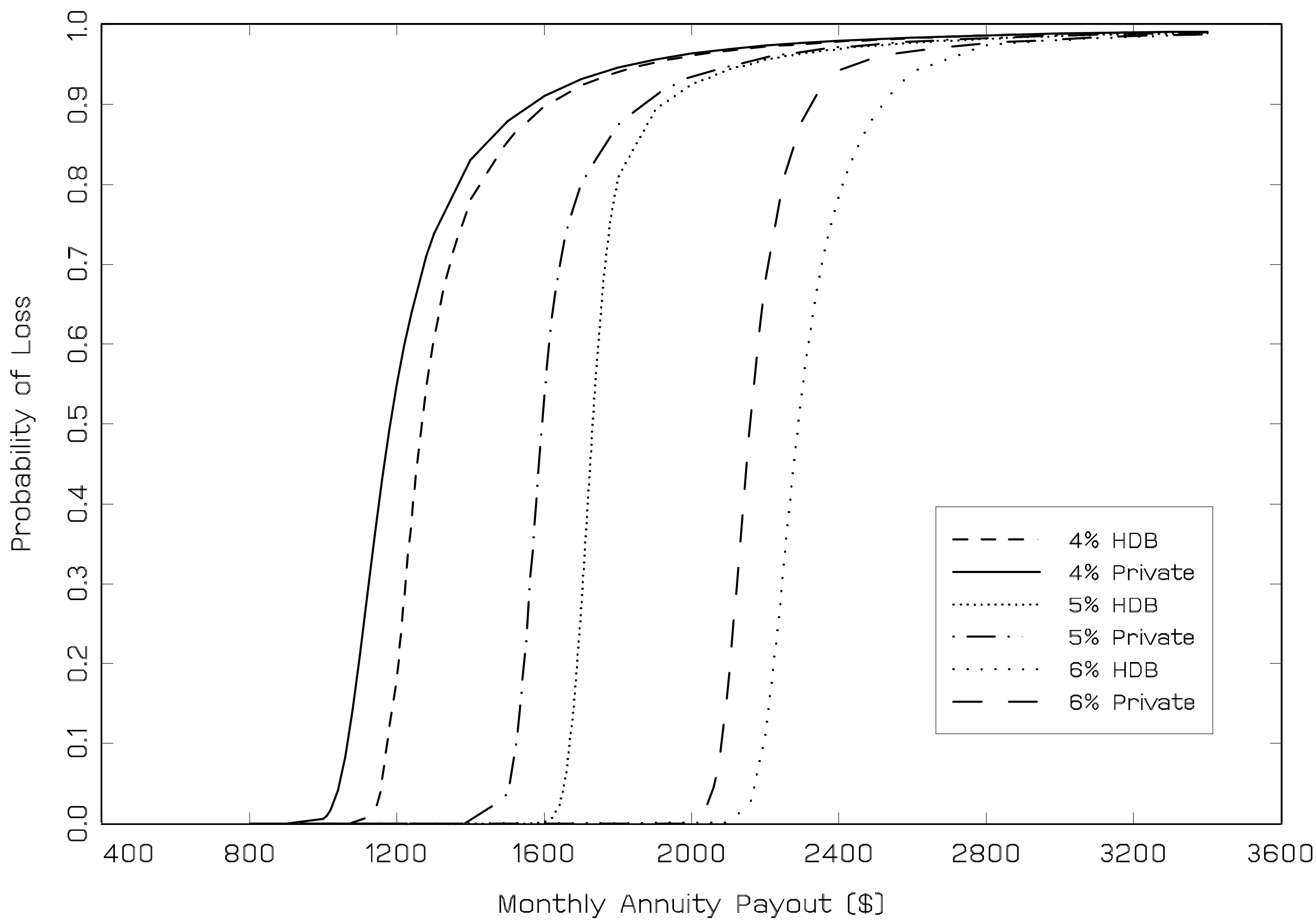


Figure 3: Breakeven Month for Providers at Various Property Growth Rates