An Analysis of Means Tested Social Pensions*

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10th June 2009

Abstract

The complex matrix of retirement policy trade-offs – encompassing elements of paternalism, market failure, and overlaying incentives in a life-cycle context – have received much attention in the literature. But the issue of whether publicly funded retirement provision should be means tested, and if so how, has received limited attention, although it has been highlighted from time to time. This paper examines the economic welfare effects of means testing using a stochastic overlapping generations model calibrated to the UK economy. A labour leisure choice is incorporated, with multiple individuals with different endowments of effective labour. Our results indicate that a change in the taper rate has implications on both welfare and economic aggregates. In particular, in contrast to much received wisdom, higher taper rates increase social welfare.

**JEL Classification:** E21, H55

**Keywords:** Means-Tested Pensions, Welfare, Social Security

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*We are grateful to the Australian Research Council for financial support.

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1 Introduction

Many countries run social insurance programs that provide protection to their citizens for possible income fluctuations that they may face when they are old, unemployed, or disabled. Costs of social insurance programs in the developed countries, in particular costs of age pension programs, are rapidly increasing as a consequence of their ageing populations. Although age pension programs vary in terms of their benefit, financing, and coverage structures, their overall benefits and costs do not qualitatively differ: Age pension programs provide longevity insurance and can be welfare improving when private annuities markets are missing or when individuals do not save enough for retirement because of myopia. On the other hand, they negatively affect individuals' labour supply and savings decisions.

Most developed countries run two-tier age pension programs. The first tier may be universal or means-tested (or some combination), and in most cases is financed from general revenue. While universal age pension programs guarantee a certain minimum benefit for all retired individuals, means-tested age pension program payments are subject to income and/or asset tests, and are structured to deliver income contingent on inadequate private resources. We will refer to these as "social pensions." Second tier pension programs can be either defined benefit (DB) or defined contribution (DC) and enrolment to these plans is often compulsory. While unfunded [Pay As You Go (PAYG)] versions of DB and DC plans are administered by governments, funded DC pension plans can also be administered by non governmental entities.

In PAYG financed DB age pension programs current retirees' benefits are paid by taxes borne by current employees. In other words, the government taxes individuals who have higher marginal propensity to save in order to transfer resources to individuals who have high propensity to consume. This, in turn, negatively affects the level of savings. In addition, individuals reduce their labor supply especially towards retirement because of the availability of age pensions. Using large scale overlapping generations (OLG) models and calibrating them to US data, Auerbach & Kotlikoff (1987), Imrohoroglu et al. (1995), and Hugget & Ventura (1999) show that US social security reduces social welfare because the benefits of a PAYG program in terms of providing insurance against income fluctuations and longevity risk are more than offset by its negative effects on savings and labor supply.

Although the effects of PAYG financed DB age pension programs on individuals' labor supply and savings decisions have been extensively analyzed, analysis of social pensions has been very limited. Nonetheless there are a few empirical and computational studies that try to assess behavioral implications of social pensions. Neumark & Powers (1998) and Neumark & Powers (2000) empirically analyze the effects of the means-tested US Supplemental Security

1 Although private annuities markets exist they are thin (see Diamond et al. (2005) for the discussion about it). For further discussion of the consequences of myopia in an individual's life-time savings, we refer Fuster et al. (2005).

2 While defined benefit (DB) pension benefits depend on employees' tenure and past earnings, defined contribution (DC) pension benefits depend on contributions made by employees and their employers and interest earnings on those contributions.

3 Note that social pension programs can also be financed PAYG fashion separate from the government's general budget.
Income (SSI) program, which is the social pension program of the US, on individuals’ labor supply and savings decisions. They show that an increase in SSI benefits results in a reduction in aggregate labor supply and savings because potential participants of the programme, in particular those who are close to retirement age, reduce their savings and labour supply. In a similar vein, Disney & Smith (2002) analyze the behavioral implications of the UK’s social pension programme.

Hugget & Ventura (1999) compare the steady states of the current US system with a reform proposal (replacing the current system with a two tier structure) using a large scale stochastic OLG model. While the first tier is a compulsory DC plan, the second tier guarantees a minimum income. Both first and second tier are PAYG financed. The reform plan is similar to policies already in place in some OECD countries such as Australia and the UK. Their results did not favor the implementation of the reform. In a recent paper, Sefton et al. (2008) assess the quantitative implications of the UK’s means-tested social pension program and evaluate alternatives. In particular, they consider the effects of a recent policy reform applied in the UK that reduced the marginal tax rates on private income of means tested retirement benefits from 100% to 40%. They conclude that the policy reform encourages poor people to save more and delay retirement but have opposite effects on richer households. The results clearly show that means-tested pension programs influence individuals’ labor supply and savings decisions. In a similar vein, Kudrna & Woodland (2008) analyze the welfare effects of a means tested pension system using a deterministic small open economy OLG model of the Australian economy.

In this paper we offer an analysis of a means tested social pension program that is similar to that of the UK to quantize the effects of means testing on economic aggregates and social welfare. In order to do so, we develop a closed economy overlapping generations (OLG) model and calibrate our model economy to that of the UK. The UK is of special interest in this context because its retirement policies embrace both significant means tested pension and earnings dependent PAYG social security. Following the common practice in the literature we also present the results of the small open economy version of our model (see for example Fehr et al. (2008) and Diaz-Gimenez & Diaz-Saavedra (2008)). Our model consists of 65-period lived individuals who were born with different skills. Individuals face mortality and idiosyncratic income risks and they make decisions on their consumption, saving, and labor supply. We mimic institutional features of the UK’s social security system following Sefton et al. (2008) and calibrate our economy to UK data.

Our paper can be thought as an extension of Sefton et al. (2008) in the Auerbach & Kotlikoff (1987) tradition and an extension of Hugget & Ventura (1999) in terms of providing comparisons between various pension programs that are not considered in their paper.

To isolate the effects of the taper rate on economic aggregates we first assume that a PAYG financed means tested program [it resembles a combination of the UK’s basic state pension (BSP) and the means-tested program] is the only public pension program in the economy and vary taper rates. We show that higher taper rates lead individuals to save more by reducing their payroll tax burden. Yet, especially lower income individuals draw down their assets.
faster to get more public pension in their old age. Later we add an earnings dependent PAYG pension program [similar to the UK’s State Second Pension (S2P)] to the model and precisely calibrate it to the UK data. Here we compare aggregate and welfare effects of six pension arrangements. In the benchmark, we model the UK’s pension program in 2003, which consists of BSP with 40% taper rate and S2P. Because the S2P already provides some pension benefit, we do not observe that individuals decumulate their assets faster in their middle and old ages. Hence, an increase in the taper rate makes more positive impact on economic aggregates. We also show that, in contrast to Sefton et al. (2008), lowering the taper rate from 100% to 40% reduces saving by all individuals including those on lower income. Having only S2P or universal pension system does not generate much improvement compared with the benchmark case. In line with previous literature (see Imrohoroglu et al. (1995) and Hugget & Ventura (1999)), we find that the elimination of both BSP and S2P creates the largest welfare gain. In our model specification, we find that a 100% taper rate is optimal. But non-linear taper rates may in fact be optimal – the question of an optimal means test remains for further investigation.

2 Overview of the UK’s Pension Systems

In this section we summarize the main features of the UK’s pension and income tax systems. Further details can be found in Sefton et al. (2008) and The Pension Service (2008).

The UK’s pension system consists of two tiers: the Basic State Pension (BSP) and the earnings-dependent State Second Pension (S2P). In addition, the government provides support to low income pensioners through a means-tested program.

- Basic State Pension: The basic state pension is received by individuals who are at the state pension age (SPA) [currently 60 for a woman and 65 for a man]. It is based on the National Insurance contributions (NICs) an individual has paid or been credited with during his/her working life. The basic state pension is a flat rate benefit but the amount of benefit depends on the number of qualifying years. If an individual earns annual income that exceeds the Lower Earnings Limit (LET) in a particular year, that year is considered as a qualifying year for the individual. In order to receive the full benefit an individual’s qualifying years should be equal to 90% of his/her working life (between 16 and SPA).

- State Second Pension: Participation in the state second pension scheme is mandatory unless an individual opts out and participates in a private pension scheme. In contrast to the first tier, a state second pension benefit is based on an individual’s earning history. An individual’s S2P entitlement is calculated as follows: Individuals who earn at, or above, the Lower Earnings Limit (LEL), but below the Lower Earnings Threshold (LET) are treated as if their earnings were at the LET. Earnings above the LEL are called as surplus earnings. Surplus earnings up to the LET are multiplied by 46%; surplus earnings between the LET and the Upper Earnings Threshold (UET) are multiplied by 11.5%; surplus earnings between UET and the Upper Earnings Limit (UEL) are multiplied by
Any individual who earns above the LEL in a given year is credited with the amount calculated by the procedure above. At retirement each year’s credit is re-scaled by average earnings growth and averaged over the working lifetime. An individual’s state second pension is equal to the sum of the average values.

UK’s pension system is administered by the HM Revenue and Customs (HMRC) and it is self-financing. More precisely, the pension system is a part of the larger National Insurance (NI) scheme that is financed through the NICs (taxes paid by employers and employees on earnings). NIC liability commences when an employee’s total earnings exceed the LEL. If an employee earns between the LEL and the Earnings Threshold (ET) then both employee and employers’ contributions are 0% of earnings; if an employee earns more than the ET but less than or equal the UEL then the employee’s contribution is 11% and the employer’s contribution is 12.8% ; if an employee earns more than the UEL than the employee’s contribution is 1% and the employer’s contribution is 12.8%.

If employee opts out for a private pension scheme his/her tax rate on earnings is reduced by 1.6% (contracting-out rebate).

The government supports low income pensioners through the means tested program. Anyone aged 60 or over gets a minimum income (including the BSP and the S2P). Benefits received between ages 60 and 64 years old are subject to a 100% taper rate. Prior to the reform in October 2003, benefits received starting from age 65 were subject to a 100% taper rate. After the reform benefits from age 65 become subject to a 40% taper rate. The means-tested program was called as the Minimum Income Guarantee (MIG) program prior to the reform and it is called as the Pension Credit (PC) program after the reform.

3 The Model Economy

We use a general equilibrium OLG model economy that consists of heterogenous individuals, a public sector, and a private sector.

3.1 The Public Sector

The government runs a public pension system that consists of defined benefit and a means-tested programs and makes consumption expenditures. The public pension system’s expenses are financed by taxes collected from labor earnings in a self-financed manner. In other words, the system is financed in a pay as you go (PAYG) fashion and it is treated as independent from the general budget. The government collects income and consumption taxes and confiscate accidental bequests to finance its consumption expenditures.

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4Note that the percentage rates are equal to the rates used for an individual who reaches state pension age in the 2003/2004 tax year. The LEL is £4004; the LET is £11200; the UET is £25600; and the UEL is £30900 in the same tax year.

5Note that these rates were in use in the 2003/2004 tax year. The annual ET was equal to £4615 in the same tax year.

6In 2003/2004 tax year the government provided the minimum income of at least £102.10 per week for a single and £155.80 for a couple (including the state pension benefits).
3.1.1 The Pension System

The pension program of our benchmark model is similar to that of the UK. All individuals are required to participate to the program through their tax payments on their gross labor earnings (similar to NICs). The government provides earnings-dependent pension benefits to all individuals when they are retired. In addition, the government guarantees at least a minimum pension income to each individual through a means-tested program. Both programs are financed by tax payments on gross labor earnings. The pension system in our model economy slightly differs from the actual pension system of the UK. The main difference is that our model does not include the almost universal basic state pension program. The reason is as follows. In the UK if employees earn less than the LEL in a given year, they do not contribute a NIC in that year. Individuals with insufficient NICs receive their basic state pensions at a reduced rate or they do not receive the basic state pension at all. There are many reasons such as immigration that can affect individuals NICs. However, in our model, all individuals participate to the system through the compulsory payroll tax payments regardless of their earnings levels. As a result, every individual is entitled to a basic state pension. Since there is no opt-out option in the model, every individual receives a state second pension benefit too. This results in almost all individuals having more pension income than the minimum pension income that is determined by the government. If we incorporate the almost universal basic state pension to model then the means-tested program becomes redundant. Hence, we choose model the means-tested program only. Note that when the taper rate is zero, the program becomes universal. Since the UK’s BSP and the means-tested program complements each other, we choose to call our first-tier pension program in the model as the BSP.

We calculate individuals’ earnings-dependent pension benefits by using a formula that is similar to the one used in the UK to calculate S2Ps. First, we calculate individuals’ average growth-adjusted earnings over their working lifetime. Then, we use the similar threshold values and rates used in the 2003/2004 tax year in the UK to calculate state second pension benefits. Hence, an individual who is at age $j$ and whose state variable is denoted by $x$ receives the following earnings-dependent retirement benefit after he reaches the exogenously determined retirement age, $j^*$:

$$b_j(x) = \begin{cases} 
0.46 \times (LET - LEL) & \text{if } \hat{w} \leq LET, \\
0.46 \times (LET - LEL) + 0.115 \times (\hat{w} - LET) & \text{if } LET < \hat{w} \leq UET, \\
0.46 \times (LET - LEL) + 0.115 \times (UET - LET) + 0.235 \times (\hat{w} - UET) & \text{if } UET < \hat{w} \leq UEL,
\end{cases}$$

(1)

where $\hat{w}(x)$ denotes the individual’s growth adjusted average life-time earnings.
Individuals who reach retirement age might be entitled to benefits that are subject to the income test.\footnote{In our model individuals can receive the means-tested benefits only after they reach the exogenously determined retirement age (it is the equivalent of the state pension age). However, in the UK, individuals might be entitled to means-tested benefits before they reach to the state pension age. The actual means-tested benefits are also subject to asset tests. Individuals receive the minimum of the retirement benefits that are determined by asset and income tests.} Means-tested benefits are determined as follows:

\[ b'_j(x) = \max(b^* - \phi y_j(x), 0), \quad (2) \]

where \( b'_j(x) \) is the means-tested benefit received by an \( j \) year old individual; \( b^* \) is the minimum pension income guaranteed by the government; \( \phi \) is the taper rate; and \( y_j(x) \) is the individual’s gross income. If we set the taper rate, \( \phi \) equals to 0 then the means-tested pension program becomes an universal pension program that provides benefits to all retirees.

Both earnings-dependent and means-tested pension programs are financed through the payroll taxes collected from labor earnings. In order to disentangle the tax burdens of DB pension and means-tested programs we assume that both programs are managed separately and financed through separate payroll taxes. In our model payroll tax rates, \( \tau_s \) and \( \tau_p \) will be determined endogenously.

### 3.1.2 Income Taxation

The UK has a progressive income tax system. Individuals who are at or above the state pension age have higher personal allowances from their total incomes than younger individuals. There are three different taxable bands and rates.\footnote{In 2003 – 2004 tax year an individual who has a taxable income (income after any personal allowances and reliefs are deducted) between 0 – £1,960 faces 10% tax rate; an individual who has a taxable income between £1,961 – £30,500 faces 22% tax rate; and an individual who has a taxable income over £30,500 face 40% tax rate.} We use a quadratic function \( \tau_i(y_j) \) that passes through origin to approximate average income tax rates in the UK before and after the retirement.

### 3.2 Technology

Output \( Y \) at time \( t \) is produced by an aggregate technology that uses labor \( (L) \) and capital \( (K) \) inputs. The technology is represented by a Cobb-Douglas constant returns production function,

\[ Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}. \quad (3) \]

Output shares of capital and labor are given by \( \alpha \) and \( 1 - \alpha \) respectively. The exogenously given technology level \( A \) grows at a constant rate \( g \) and capital depreciates at a constant rate \( \delta \).
\( \delta \in (0, 1) \). Firms maximize their profits by setting wage and rental rates equal to marginal products of labor and capital respectively:

\[
\begin{align*}
    w_t &= (1 - \alpha)A_t \left( \frac{K_t}{L_t} \right)^{\alpha}, \\
    r_t &= \alpha A_t \left( \frac{K_t}{L_t} \right)^{\alpha-1}.
\end{align*}
\]

### 3.3 Demographics and Endowments

We consider an economy populated by overlapping generations. Every period \( t \) a generation of individuals is born. Individuals face random lives and live maximum of \( J \) periods. The population grows at a constant rate \( n \). An individual’s probability of surviving up to age \( j \) conditional on surviving up to age \( j-1 \) is denoted by \( \nu_j \). Demographic patterns are stable and the constant cohort share of the generation \( j \) can be written as follows:

\[
\mu_j = \frac{\nu_j \mu_{j-1}}{1 + n} \quad \text{for} \quad j = 2, 3, ..., J,
\]

where \( \sum_{j=1}^{J} \mu_j = 1 \).

An individual’s productivity in a given period depends on his age \( j \) and an idiosyncratic period shock \( s_j \) to his productivity. In particular, we use the following functional form: \( \epsilon_j s_j \), where \( \epsilon_j \in \epsilon \), the set of age-dependent mean efficiency profiles. We analyze both the case in which an individual faces a shock at his birth and live with that shock forever (permanent shock case) and the case in which an individual faces a new shock during each period of his life time (temporary shock case). Since the shocks are independently distributed there is no aggregate uncertainty in the model. Following Huggett & Parra (2006), we assume that shocks are log-normally distributed i.e. \( \log(s_j) \sim N(\mu, \sigma^2) \) and the log-normal distribution is approximated by 5 evenly-spaced discrete values in logs on the interval \([-\frac{\sigma^2}{2}, -3\sigma, -\frac{\sigma^2}{2} + 3\sigma]\). In both cases, probabilities are found by calculating the area under the normal distribution conditional on the value of \( s_j \).

### 3.4 Preferences

All individuals have identical preferences over consumption and leisure denoted by the following life-time utility function:

\footnote{Because we are only interested in steady state values, time subscripts will be dropped from the equations during the rest of the analysis.}
\[
E \left[ \sum_{j=1}^{J} \beta^{j} \prod_{i=1}^{j} u(c_{j}, l_{j}) \right]
\]

where \( E \) is the expectation operator and \( \beta \) is the time-discounting factor.

Each individual is endowed with 1 unit of labor. The amounts of labor and leisure chosen at age \( j \) are given by \( l_{j} \) and \( (1 - l_{j}) \) respectively. The following is the period utility function:

\[
u(c, l) = \frac{c^{1-\rho}}{1-\rho} + \kappa \frac{(1-l)^{1-\varphi}}{1-\varphi}, \]

where \( \rho, \varphi \in [0, +\infty) \). The coefficient of relative risk aversion is given by \( \rho \). While the inter-temporal elasticity of substitution for consumption is \( \frac{1}{\rho} \), the Frisch elasticity of leisure is \( -\frac{1}{\varphi} \). The parameter \( \kappa \) measures the dislike for work relative to the enjoyment of consumption.

King et al. (2002) show that the above period utility function is compatible with the balanced growth path if \( \rho = 1 \) i.e. \( u(c, l) = \log(c) + \kappa \frac{(1-l)^{1-\varphi}}{1-\varphi} \). Hence, we use the log period utility function in our analysis. Without loss of generality, the value of the parameter \( \kappa \) is normalized to unity.

### 3.5 Budget Constraint

An individual’s gross income at period \( j \) as follows:

\[
y_{j} = \begin{cases} 
(1+r)a_{j} + l_{j} \epsilon_{j} s_{j} w & \text{if } j < j^{*}, \\
(1+r)a_{j} + b(x, j) & \text{if } j \geq j^{*}.
\end{cases}
\]

Hence, the individual’s growth-adjusted budget constraint can be written as

\[
\begin{align*}
c_{j} + (1+g)a_{j+1} & \leq (1+r)a_{j} + (1-\tau_{s})l_{j} \epsilon_{j} s_{j} w - \tau_{i}(y_{j}) \text{ when } j < j^{*}, \\
 c_{j} + (1+g)a_{j+1} & \leq (1+r)a_{j} + (1-\tau_{s})b(x, j) + b'(x, j) - \tau_{i}(y_{j}) \text{ when } j^{*} \leq j \leq J - 1, \\
 c_{J} & = (1+r)a_{j} + (1-\tau_{s})b(x, j) + b'(x, j) - \tau_{i}(y_{j}) \text{ when } j = J.
\end{align*}
\]

We further assume that individuals cannot borrow against their future income at any age:

\[
a_{j} \geq 0, \forall j.
\]

### 3.6 An Individual’s Decision Problem

An individual’s decision problem in our model economy can be written as a dynamic programming problem. Denoting the value function of the agent at age \( j \) by \( V_{j} \), the decision problem
is represented by the following form:

\[ V_j(x_j) = \max_{c_j, l_j} \{ u(c_j, (l_j)) + \beta \nu_{j+1} E V_{j+1}(x_{j+1}) \} \]  

(11)

subject to equations 9 and 10. Note that \( x \) denotes a state vector of the form \((a, s, \nu)\).

### 3.7 Equilibrium

Our equilibrium definition follows Auerbach & Kotlikoff (1987), Imrohoroglu et al. (1995), Imrohoroglu et al. (2003), and Hugget & Ventura (1999). We suppose that pension system is self-financing and the government runs the balanced budget. The pension programs’ tax rates, \( \tau_s \) and \( \tau_p \), and the consumption tax rate, \( \tau_c \), are endogenously determined in order to satisfy the balanced budget conditions. We represent the individual state space by \( X \), where \( x \in X \).

Given a set of time-invariant government fiscal policy instruments \( \{LET, LEL, UET, UEL, b, \phi, \tau_i(.)\} \), a stationary equilibrium is defined as a set of value functions \( \{V_j(a_j, s_j, v_j)\}_{j=1}^J \), individuals’ decision rules \( \{c_j(,), a_j(,), l_j(,)\}_{j=1}^J \), relative prices of labor and capital \( \{w, r\} \), pension program tax rates \( \tau_s \) and \( \tau_p \), consumption tax rate \( \tau_c \), and age dependent distributions of individuals \( \Lambda_j(x) \) that they must satisfy the following conditions.

1. Given fiscal policy and prices individuals’ decision rules \( \{c_j(,), a_j(,), l_j(,)\}_{j=1}^J \) solve individuals’ decision problem 11 subject to the constraints 9 and 10.

2. Firms maximize profits by choosing relative prices \( \{w, r\} \) according to the equations 4 and 5.

3. The age dependent and time-invariant measure of individuals is computed as follows:

\[ \Lambda_{j+1}(x) = \sum_s \Pi(s_{j+1}, s_j) \int_X d\Lambda_j, \]  

where \( \Pi(s_{j+1}, s_j) \) is the transition matrix for the shocks. \( \Lambda_1(x) \) is given.

4. Aggregate variables are derived from individuals’ behaviors:\(^{14}\)

\[ K = \sum_{j=1}^J \mu_j \int_X a_j(x) d\Lambda_j, \]

\[ L = \sum_{j=1}^J \mu_j \int_X [1 - l_j(x)] d\Lambda_j, \]

\[ C = \sum_{j=1}^J \mu_j \int_X c_j(x) d\Lambda_j. \]

5. Age pension programs are self-financing:

\(^{14}\)Note that in the small open economy version of our model the aggregate capital is derived from the equation 5 by taking the interest rate as given.
The sum of accidental bequests satisfy the following equation:

\[ \eta = \sum_{j=1}^{J} \mu_j \int \left(1 - \nu_{j+1}(z)\right)a_j(x) d\Lambda_j. \]

7. The government’s income tax revenue is given by:

\[ T_i = \sum_{j=1}^{J} \mu_j \int t_i(y_j(x)) d\Lambda_j. \]

8. The government runs a balanced budget:

\[ G = T_i + \tau_C C + \eta. \]

6. The goods market clears:\textsuperscript{15}

\[ C + (1 + g)(1 + n)K + G = Y + (1 - \delta)K. \]

4 Calibration

This section defines the parameter values of our model. The limit and threshold values used for the calculation of earnings dependent pension benefits are set equal to their actual values in the 2003/2004 tax year (LEL = £4004; LET = £11200; UET = £25600; UEL = £30940). Similarly, we set the value of guaranteed minimum pension income, \( b^* \) to its actual yearly value for single individuals in the same tax year (\( b^* = £5309 \)). In our benchmark calibration we set the value of taper rate, \( \phi \) to the pre-reform rate of 100%. Later, we set it to 0% (universal flat rate) and 40% (post-reform rate).

We estimate pre- and post- retirement income tax rates by using a quadratic function that passes through the origin. In particular we use the following functional form:

\[ \tau_j = \begin{cases} 
[2.698755 \frac{y_j(x)}{10000} - 0.0423237(\frac{y_j(x)}{10000})^2]/100 & \text{if } j < j^*, \\
[1.508241 \frac{y_j(x)}{10000} - 0.0222008(\frac{y_j(x)}{10000})^2]/100 & \text{if } j \geq j^*. 
\end{cases} \]

Batini \textit{et al.} (2000) reports the values of labor’s share of income \((1 - \alpha)\) in the UK between 1970 and 1995. The values fluctuate between 68% and 74% and their average is approximately 70%. Hence, we set the value of labor income share to 0.70. The growth rate of the technological

\textsuperscript{15}In the open economy version, the goods market clears if the following holds:

\[ C + (1 + g)(1 + n)K + G + NX = Y + (1 - \delta)K, \]

where \( NX \) represents net exports.
progress is set to the average growth rate of the UK’s GDP per capita between 1999 and 2007 i.e. \( g = 2.8\% \). Weale (2004) estimates the capital depreciation rate, \( \delta \) as 4.82% in the UK in 2002. We use the same value. The technology level \( A \) can be chosen freely and we set it to 1.

Each model period corresponds to a year. Individuals are born at a real life age of 21 (model age of 1) and they can live up to a maximum real life age of 85 (model age of 65). The population growth rate is assumed to be equal to the long-term average growth rate of the UK’s population i.e. \( n = 0.5\% \) (National Statistics (2009a)). The sequence of conditional survival probabilities in the model, \( v_j \) is set equal to the sequence of conditional survival probabilities of men in the UK that is calculated by using 2002 – 2004 data (National Statistics (2009b)). The mandatory retirement age is 65 (model age of 45), which is equal to the UK’s state pension age for men. The age dependent efficiency index, \( \epsilon_j \) is set as follows: Robinson (2003) estimates age-earnings profiles across educational groups by using various specifications. We take her estimates of weekly earnings at different amounts of experience, normalize the data by setting the value of weakly earning of a 1-year experienced man to 1 and interpolate the normalized data by using the spline method for missing values. Blundell & Etheridge (2008) calculate the variance of permanent and temporary shocks to earnings in the UK as approximately 0.8 and 0.5 in 2003. We use the average of two values as the variance of the shock in the model i.e. \( \sigma^2 = 0.065 \).

Following Heathcote et al. (2008), without loss of generality, we set the value of \( \kappa \) to 1. There is no consensus on the values of the Frisch elasticities of labor supply and leisure. Domeij & Flodén (2006) state that the Frisch elasticity of labor supply takes values between 0.1 and 0.3. However, they show that these values are downward-biased and claim that unbiased estimates are larger. Since elasticity estimates vary at a large margin, we, instead of using the value of one of these estimates, set the value of leisure parameter, \( \varphi \) to 2 so that we can closely replicate the labor-supply profile across ages. Although there is no restriction for the value of the time discount factor (\( \beta \)) in the OLG models, it is generally taken as smaller than one. For instance, Auerbach & Kotlikoff (1987) and Hubbard & Judd (1987) set \( \beta = 0.9852 \). Hurd (1989) estimates \( \beta \) as 1.011. Imrohoroglu et al. (1995), Hugget & Ventura (1999), and Storesletten et al. (1999) use Hurd’s estimate. Since conditional survival probabilities enter into their model, the effective time discount factor (\( \beta v_j \)) is smaller than one during the most of an individual’s lifetime. In our choice of \( \beta \) we follow Auerbach & Kotlikoff (1987) and Hubbard & Judd (1987) and set \( \beta < 1 \) in order to make sure that there is no artificial welfare gain because of choice of the time-discount factor. As in Auerbach and Kotlikoff we adjust the value of \( \beta \) to calibrate to UK’s capital-output ratio of 2.26.19,20

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16GDP per capita growth rates are taken from Eurostat (2009).
17It is the average annual population growth rate between 2001 and 2007.
18Robinson (2003) estimates weakly earnings at different amounts of experience for both men and women who belong to a low, medium, or high educational group. She uses quadratic, cubic, and quartic specifications. We use the values of her estimates for men who belong to the low educational group which is conducted by using a quadratic specification.
20In the open economy version of our model we set the value of net exports equal to the ratio of value of net exports in 2003 to the GDP in the same year.
<table>
<thead>
<tr>
<th>Public Sector</th>
<th>2003 – 2004 tax year values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limit and threshold values</td>
<td>Minimum guaranteed pension income $b^*$</td>
</tr>
<tr>
<td>Minimum guaranteed pension income $b^*$</td>
<td>2003 – 2004 tax year value for a single individual</td>
</tr>
<tr>
<td>Taper rate $\phi$</td>
<td>40%</td>
</tr>
<tr>
<td>Income tax rate $\tau^i$</td>
<td>Estimated by a quadratic function</td>
</tr>
<tr>
<td>Government expenditures, $G$</td>
<td>0.227898</td>
</tr>
</tbody>
</table>

| Production                                         |                                                              |
|---------------------------------------------------|                                                              |
| Capital share of the GDP $\alpha$                  | 0.30                                                          |
| Annual depreciation of capital stock $\delta$      | 4.82%                                                         |
| Annual per capita output growth rate $g$           | 2.8%                                                          |

| Demographics and Endowments                        |                                                              |
|---------------------------------------------------|                                                              |
| Maximum possible life span $J$                     | 65                                                            |
| Obligatory retirement age $j^*$                    | 45                                                            |
| Growth rate of population $n$                      | 0.5%                                                          |
| Conditional survival probabilities $v_j$          | UK 2002 – 2004                                                 |
| Age efficiency profile $\epsilon_j$              | Robinson (2003)                                                |
| The variance of the employment shock $\sigma^2$   | 0.065                                                         |

\[
\text{Markov transition matrix II for skills} =
\begin{bmatrix}
0.012224 & 0.2144 & 0.54675 & 0.2144 & 0.012224 \\
0.012224 & 0.2144 & 0.54675 & 0.2144 & 0.012224 \\
0.012224 & 0.2144 & 0.54675 & 0.2144 & 0.012224 \\
0.012224 & 0.2144 & 0.54675 & 0.2144 & 0.012224 \\
0.012224 & 0.2144 & 0.54675 & 0.2144 & 0.012224 \\
\end{bmatrix}
\]

| Preferences                                        |                                                              |
|---------------------------------------------------|                                                              |
| Annual discount factor of utility $\beta$         | 0.948                                                         |
| Leisure parameter $\varphi$                        | 2                                                             |

Table 1: Parameter Values of The Benchmark Calibration
5 Results

In this section we first present the results of the permanent shock case version of our model. Later, we present the results of the temporary shock case.\textsuperscript{21}

5.1 A Counter-factual Example

To isolate the effects the means tested program on economic aggregates and welfare we first shut down the earnings-dependent PAYG program (the state second pension, S2P) and vary taper rates of the BSP. In this example, we do not worry about hitting the UK’s capital-output ratio and hence, we call it the counter-factual. We then add an earnings-dependent PAYG system that is very similar to the S2P into the model and calibrate the model to the UK data in 2003, setting the taper rate equal to the pre-reform rate of 1. We call this case the benchmark. We try to be as precise as possible to hit the UK’s capital-output ratio in 2003 in the benchmark case. Later we conduct policy analyses by assuming that the taper rate is equal to the reform rate of 0\textsuperscript{4}.\textsuperscript{14} and the taper rate is equal to 0. We also compare the cases in which the means tested pension program (BSP) is removed (only S2P exists); the earnings dependent PAYG program is removed (only a universal pension program exists); and both the means tested and earnings dependent PAYG pension programs are removed (privatization).

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\tau_p$ (%)</th>
<th>$L$</th>
<th>$K$</th>
<th>$C$</th>
<th>$Y$</th>
<th>$\eta$</th>
<th>$K/L$</th>
<th>$K/Y$</th>
<th>$r$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>24.28</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>2.2572</td>
<td>8.4707</td>
</tr>
<tr>
<td>0.1</td>
<td>22.56</td>
<td>100.0792</td>
<td>99.8705</td>
<td>101.2226</td>
<td>100.0156</td>
<td>95.6259</td>
<td>99.7738</td>
<td>2.2539</td>
<td>8.4838</td>
</tr>
<tr>
<td>0.2</td>
<td>20.85</td>
<td>100.0582</td>
<td>99.7607</td>
<td>102.4288</td>
<td>99.9949</td>
<td>91.5780</td>
<td>99.6512</td>
<td>2.2519</td>
<td>8.5058</td>
</tr>
<tr>
<td>0.3</td>
<td>19.23</td>
<td>100.0309</td>
<td>99.3827</td>
<td>103.5684</td>
<td>99.8584</td>
<td>86.9674</td>
<td>99.3143</td>
<td>2.2464</td>
<td>8.5352</td>
</tr>
<tr>
<td>0.4</td>
<td>17.70</td>
<td>100.0422</td>
<td>99.3859</td>
<td>104.7860</td>
<td>99.8585</td>
<td>83.2879</td>
<td>99.2771</td>
<td>2.2465</td>
<td>8.5407</td>
</tr>
<tr>
<td>0.5</td>
<td>16.18</td>
<td>100.0640</td>
<td>101.0009</td>
<td>106.6803</td>
<td>100.7639</td>
<td>84.7290</td>
<td>100.2765</td>
<td>2.2625</td>
<td>8.4415</td>
</tr>
<tr>
<td>0.6</td>
<td>15.23</td>
<td>100.9340</td>
<td>102.1594</td>
<td>107.8865</td>
<td>101.3629</td>
<td>85.9636</td>
<td>101.0723</td>
<td>2.2749</td>
<td>8.3724</td>
</tr>
<tr>
<td>0.7</td>
<td>14.59</td>
<td>101.0883</td>
<td>102.7742</td>
<td>108.6301</td>
<td>101.7032</td>
<td>86.5905</td>
<td>101.5083</td>
<td>2.2810</td>
<td>8.3303</td>
</tr>
<tr>
<td>0.8</td>
<td>13.89</td>
<td>101.3067</td>
<td>103.4474</td>
<td>109.4859</td>
<td>102.0482</td>
<td>87.7951</td>
<td>101.9321</td>
<td>2.2881</td>
<td>8.2925</td>
</tr>
<tr>
<td>0.9</td>
<td>13.32</td>
<td>101.4914</td>
<td>104.1359</td>
<td>110.2267</td>
<td>102.4199</td>
<td>88.7276</td>
<td>102.4320</td>
<td>2.2950</td>
<td>8.2463</td>
</tr>
<tr>
<td>1.0</td>
<td>12.80</td>
<td>101.6589</td>
<td>104.7504</td>
<td>110.8794</td>
<td>102.7107</td>
<td>89.9869</td>
<td>102.8599</td>
<td>2.3020</td>
<td>8.2071</td>
</tr>
</tbody>
</table>

Table 2: Effects of Taper Rates on Economic Aggregates

Table 2 reports equilibrium outcomes for range of taper rates. Not suprisingly, an increase in the taper rate decreases young individuals’ payroll tax burdens. As a result, they have larger net wage income from which to save for their future consumption in each period. However, additional saving at young ages reduces the amount of state pension benefit received later. In other words, each £1 increase in saving creates $\phi$ cents reduction in state pension benefits without taking interest earnings into account. This implies that retired individuals accelerate their decumulation of assets to be entitled to larger state pension benefits during their remaining

\textsuperscript{21}We also conducted the same analysis by using a small open economy version of our model.
life-time. Initial increases in taper rates (up to 40%) do not generate higher capital stock. In fact, the capital stock decreases slightly with each increase in the taper over this range. This is because individuals’ desires to receive larger state pension benefits outweigh the income effect of a higher taper rate (a lower payroll tax). Yet, because each increase in the taper rate causes higher net wage earnings, the amount of labor supplied (see the third column) increases slightly with an increase in the taper rate. Up to 40% taper rate, each 10% increase in the taper rate decreases the capital stock but increases the amount of labor supplied. Because the capital stock decreases relatively more than the amount of labor supply, we observe that output slightly decreases with each 10% increase in the taper rate.

Yet, we still observe an increase in average consumption levels. The intuition is as follows. An increase in the taper rate up to 40% does not increase the level of savings. Although young individuals increase their savings slightly, old and middle age individuals decumulate their assets substantially. This, in turn causes a reduction in the overall level of savings. As a result, both the capital stock and the level of accidental bequests decrease. However, the sharp decrease in the accidental bequests creates a moderate increase in the average consumption level although output level has slightly decreased because of the lower capital stock. Note that in our model accidental bequests are confiscated by the government to finance its non-productive expenditures. Hence, any reduction in them positively affects the level of average consumption. Above 40%, we observe that an increase in the taper rate increases the capital stock in the economy: the income effect of higher taper rates outweigh individuals’ desire to receive higher pension benefits. As expected, \( Y \) increases as a consequence of increases in both \( K \) and \( L \). This in turn results in higher average consumption levels. Over this range, an increase in \( C \) is generated by an increase in \( Y \), not a decrease in \( \eta \) (in fact, \( \eta \) starts to increase). Thus, in this range, a higher taper rate’s positive effect on individuals’ savings outweigh its negative effect on asset decumulation.

To demonstrate the effects of taper rates on consumption and saving levels at each age we present average life-cycle profiles of asset holdings and consumption. Figure 1 depicts asset holdings. An increase in the taper rate (a decrease in the payroll tax rate) does not create a substantial increase in younger individuals’ savings. Yet, after retirement, an increase in the taper rate causes an abrupt decrease in older individuals’ asset holdings. Furthermore, an increase in the taper rate extends the duration of the dependence on pension benefits as the only source of the retirement income.
Figure 1: Average asset holdings across the life-cycle

Figure 2 depicts average consumption levels. Figures 1 and 2 reveal together that most of the increase in a young individual’s net income, which is generated by a higher taper rate, is spent on consumption in younger ages. Since old individuals decumulate their assets rapidly when the taper rate is high, their consumption levels fall behind those in lower taper rates sometime after retirement. As a result, individuals’ consumption levels show greater discrepancy when the taper rate is high.

Figure 2: Average consumption profile across the life-cycle

As we explained above, in our model, there are five types of individuals who differ in terms of their skill levels. Higher skill individuals are more productive and hence, their wage earnings are larger. This, in turn, leads them to hold more assets against future income fluctuations and longevity risk. To investigate the effects of taper rates further we direct our attention to the cross sectional asset holdings and consumption profiles of each type of individuals. By doing so we can analyze the reactions of different income groups to a change in the taper rate. We report the figures in the appendix.

Figures 5 – 9 show the cross sectional asset holdings of different types of individuals. Not surprisingly, the poorer individuals’ asset holdings are lower than those of richer individuals at every age. Poorer individuals’ asset holdings reach the peak level far before the compulsory retirement age but richer individuals’ asset holdings reach the peak level at around the
retirement age. The poorest type (type I) saves at the lower rate before the peak point and decumulates its asset faster after the peak point. The poorest individuals decumulate their assets completely at around age 73 when the taper rate is 1 but they decumulate their assets completely at age 81 and 85 when the taper rate is 0.4 and 0 respectively (see figure 5). In contrast, middle-income individuals (types II and III) save more when taper rate is higher. Yet, the decumulation pattern does not change: assets are decumulated faster when the taper is high. It looks like richer individuals’ accumulation decisions are not affected by change in taper rates. Although richer individuals (types IV and V) decumulate their assets faster when taper rate is higher, the pace of decumulation is slower than the poorer income groups.

Figures 10 – 14 demonstrate the cross-sectional consumption profiles of different types. It looks like while lower taper rates provide better consumption smoothing for poorer individuals higher taper rates provide better consumption smoothing for richer individuals. Higher taper rates cause poorer individuals to decumulate their savings abruptly. This, in turn causes big distortions in their consumption levels after retirement. It is also evident from the figures that higher taper rates make poorer individuals depend only on pension benefits most of their remaining life-time.

We now turn to the welfare effects of a decrease in the taper rate. To measure the welfare cost of a decrease in the taper rate we use the compensating variation (CV) measure, which is defined as the consumption supplement needed at each age in order to equate the welfare of a new born individual without knowledge of labor endowment in an economy with a lower taper rate to the welfare of a newborn individual in an economy with 100% taper rate. Table 3 reveals that higher taper rates increase social welfare in our example. For example, an individual requires 5.82% increase in his annual consumption to be compensated for being in an economy with 0% taper rate. Our result confirms the findings of Feldstein (1987) who shows that a means tested program is generally superior.

<table>
<thead>
<tr>
<th>Taper rate</th>
<th>Pension tax rate (%)</th>
<th>CV(% of consumption)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>12.80</td>
<td>0</td>
</tr>
<tr>
<td>0.9</td>
<td>13.32</td>
<td>0.29</td>
</tr>
<tr>
<td>0.8</td>
<td>13.89</td>
<td>0.62</td>
</tr>
<tr>
<td>0.7</td>
<td>14.59</td>
<td>1.0</td>
</tr>
<tr>
<td>0.6</td>
<td>15.23</td>
<td>1.33</td>
</tr>
<tr>
<td>0.5</td>
<td>16.18</td>
<td>1.85</td>
</tr>
<tr>
<td>0.4</td>
<td>17.70</td>
<td>2.70</td>
</tr>
<tr>
<td>0.3</td>
<td>19.23</td>
<td>3.40</td>
</tr>
<tr>
<td>0.2</td>
<td>20.85</td>
<td>4.14</td>
</tr>
<tr>
<td>0.1</td>
<td>22.56</td>
<td>4.96</td>
</tr>
<tr>
<td>0.0</td>
<td>24.28</td>
<td>5.82</td>
</tr>
</tbody>
</table>

Table 3: Effects of Taper Rates on Welfare
5.2 Full Policy Results

Next we compare aggregate and welfare effects of six different social security arrangements assuming that productivity shocks are permanent. In the benchmark case we mimic the UK’s pre-reform social security comprising basic state pension (BSP) with 100% taper rate and the state second pension (S2P). Our benchmark calibration is able to generate the capital-output ratio of the UK in 2003. Then we decrease the taper rate to 40% (reform rate), and 0% respectively. We also analyze the cases in which only the S2P exists; only the universal pension program (BSP with 0% taper rate) exists; and both the BSP and the S2P are removed (privatization).

The first three rows of Table 4 shows that employees’ payroll tax rates ($\tau_p$) increase significantly with a decrease in the taper rate. This in turn reduces individuals’ net wage incomes and hence, negatively affects their saving decisions. Because individuals’ old age consumption is also supported by the S2P, they do not decumulate their assets as fast as in the previous counter-factual example (compare figures 15 – 19 to figures 5 – 9). We also observe that lower taper rates cause much lower aggregate capital and aggregate labor supply when the S2P exists (see tables 2 and 4). This in turn creates much lower aggregate consumption. Closer look at the figures 15-19 reveals that 100% taper rate generates higher saving rates for almost all income groups. In contrast to the results of Sefton et al. (2008), the reduction in means testing taper rate does not create higher saving rates for poor households. Yet, our results regarding middle and high income individuals are compatible with those of Sefton et al. (2008): while reducing the taper rate from 100% to 40% reduces savings of middle-income individuals, it does not affect higher-income individuals. Hence, negative effects of reducing taper rate on aggregate capital stock, labor supply, and consumption are larger in our environment. In contrast to Sefton et al. (2008) our model incorporates the S2P that has created distortions on individuals’ saving and labor supply decisions. Taking the S2P as given and changing the taper rate creates more distortions than an environment where there is no S2P. Removing the BSP from the system creates a slight increase on aggregate variables because of the reduction in individuals’ tax burdens. The universal pension system’s tax burden is almost same as that of the benchmark case and hence we do not observe much change on economic aggregates. Table 4 reveals that the largest gain occurs when both the BSP and the S2P are removed as it is well established in the literature (see for instance, Imrohoroglu et al. (1995) and Hugget & Ventura (1999)).

Figure 3 shows average asset holdings across the life-cycle. Although individuals’ asset holdings are slightly lower in earlier ages under 100% taper rate, they become higher by age 30. It is also interesting to see that individuals do not decumulate their assets faster when the taper rate is 1.0. A comparison of figures 1 and 3 reveals that the existence of the S2P prevents faster decumulation.
It looks like higher taper rate provides more consumption smoothing. This result is also in contrast to the one we observed in the counter-factual example (compare figures 2 and 4). In the absence of the S2P, although higher taper rates boost savings before in earlier ages, they hasten decumulations in middle and old ages. Hence, we observed that less consumption smoothing with higher taper rates. Yet, when the S2P exists, although higher taper rates increase savings do not hasten decumulations so that we observe better consumption smoothing.

The last three rows of table 5 demonstrate that reducing the taper rate causes an abrupt
decrease in welfare because of the compounded labor tax distortions. Although individuals in the economy with the pre-reform UK pension system need 13.97% consumption increase to be compensated for not being in an economy without BSP and S2P (privatization), individuals in the economy with the post-reform UK pension system need 22.32% increase in their consumption to be compensated for not being in the privatized economy. Welfare cost of universal pension system and the pre-reform UK pension system is almost same. The second best alternative is removing the BSP and having only S2P. Our welfare results in line with the previous studies that state that the privatization of social security system creates the highest welfare.

<table>
<thead>
<tr>
<th></th>
<th>$\phi$</th>
<th>$\tau_p$(%)</th>
<th>$\tau_s$(%)</th>
<th>$CV$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Privatization</td>
<td>–</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>S2P</td>
<td>–</td>
<td>0.00</td>
<td>22.88</td>
<td>12.49</td>
</tr>
<tr>
<td>Universal BSP</td>
<td>0.0</td>
<td>24.29</td>
<td>0.00</td>
<td>13.37</td>
</tr>
<tr>
<td>BSP&amp;S2P</td>
<td>1</td>
<td>2.17</td>
<td>22.88</td>
<td>13.97</td>
</tr>
<tr>
<td>BSP&amp;S2P</td>
<td>0.4</td>
<td>12.97</td>
<td>22.81</td>
<td>22.32</td>
</tr>
<tr>
<td>BSP&amp;S2P</td>
<td>0.0</td>
<td>24.18</td>
<td>22.77</td>
<td>31.59</td>
</tr>
</tbody>
</table>

Table 5: Welfare Effects of Reforms

Tables 6 and 7 present aggregate and welfare effects of various social security arrangements when productivity shocks are temporary. A comparison of tables 4 and 6 shows that direction and magnitude of effects of various programs on economic aggregates do not change.

<table>
<thead>
<tr>
<th></th>
<th>$\phi$</th>
<th>$\tau_p$(%)</th>
<th>$\tau_s$(%)</th>
<th>L</th>
<th>K</th>
<th>C</th>
<th>Y</th>
<th>K/L</th>
<th>K/Y</th>
<th>r(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSP&amp;S2P</td>
<td>1.0</td>
<td>5.84</td>
<td>22.76</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>2.26</td>
<td>8.50</td>
</tr>
<tr>
<td>BSP&amp;S2P</td>
<td>0.4</td>
<td>14.62</td>
<td>22.66</td>
<td>96.06</td>
<td>87.80</td>
<td>87.38</td>
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<td>92.34</td>
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<td>9.29</td>
</tr>
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<td>BSP&amp;S2P</td>
<td>0.0</td>
<td>24.24</td>
<td>22.52</td>
<td>93.42</td>
<td>81.17</td>
<td>77.60</td>
<td>88.38</td>
<td>88.45</td>
<td>2.08</td>
<td>9.60</td>
</tr>
<tr>
<td>S2P</td>
<td>–</td>
<td>0.00</td>
<td>22.79</td>
<td>101.49</td>
<td>103.96</td>
<td>108.21</td>
<td>102.63</td>
<td>102.26</td>
<td>2.29</td>
<td>8.29</td>
</tr>
<tr>
<td>Universal BSP</td>
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<td>0.00</td>
<td>100.23</td>
<td>100.74</td>
<td>105.80</td>
<td>99.23</td>
<td>100.42</td>
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</tr>
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<td>0.00</td>
<td>104.39</td>
<td>117.98</td>
<td>134.70</td>
<td>108.85</td>
<td>112.36</td>
<td>2.45</td>
<td>7.48</td>
</tr>
</tbody>
</table>

Table 6: Effects of Reforms on Economic Aggregates (Transitionary)

Not surprisingly, welfare implications of various programs do not change under the temporary shock case too.22

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22Robustness tests of our results have been successfully performed and available upon request.
6 Conclusion

The issue of whether publicly funded retirement provision should be means tested, and if so how, has received limited attention, although it has been highlighted from time to time (e.g. Feldstein (1987)). This paper examines the economic welfare effects of means testing using a stochastic overlapping generations model loosely calibrated to the UK economy. A labor-leisure choice is incorporated, with multiple individuals with different endowments of effective labor.

To isolate the effects of taper rate on economic aggregates we first assume that the BSP is the only public pension program in the economy and vary taper rates. We show that higher taper rates lead individuals to save more by reducing their payroll tax burden. Yet, especially lower income individuals draw down their assets faster to get more public pension in their old ages.

Later we add the S2P to the model and precisely calibrate to the UK data. This dramatically changes our results. Here we compared aggregate and welfare effects of six pension arrangements. In the benchmark, we model the UK’s pension program in 2003, which consists of BSP with 40% taper rate and S2P. Because the S2P already provides some pension benefit, we do not observe that individuals decumulate their assets faster in their middle and old ages. Hence, an increase in the taper rate make more positive impact on economic aggregates. We also show that, in contrast to the results of Sefton et al. (2008), lowering the taper rate from 100% to 40% do not make any positive impact on individuals’ saving decisions including lower income individuals. Having only S2P or universal pension system do not generate much improvement compared to the benchmark case. Not surprisingly, as in the previous studies, our results show that the elimination of both BSP and S2P creates the largest welfare. In our model specification, we find that a 100% taper rate is optimal. But non-linear taper rates may in fact be optimal – the question of an optimal means test remains for further investigation.

References


Appendix A: Solution Method

Our solution procedure closely follows that of Hugget & Ventura (1999) and Imrohoroglu et al. (2003). In particular, we use the following algorithm:
1. We guess values for capital \((K)\), labor \((L)\), and tax rates \((\tau_s, \tau_p, \text{and} \tau_c)\).

2. Calculate the corresponding factor prices.

3. Calculate the optimal decision rules: \(a(x,j), l(x,j), \text{and } c(x,j)\)

   - Create discrete sets \(A = \{0, ..., a_m\}\) for asset values, \(S = \{s_1, ..., s_5\}\) for productivity shocks, and \(H = \{h_1, ..., h_{20}\}\) for average past earnings. Choose the upper bound \(a_m\) such that it never binds. Solve for the control variables \(y = (c,l)\) at each grid point \(x = \{a,s,h\}\) by using the simplex method starting from the last period. Use linear interpolation to calculate off-grid points decision rules and the value function.

4. Calculate new values of \(K, L, \text{and tax rates } (\tau_s, \tau_p, \text{and } \tau_c)\) implied by the optimal decision rules and the balanced budget conditions.

   - Simulate time paths of consumption, labor, asset holdings, means-tested and earnings dependent pension benefits for many individuals: Start with a newly born individual and randomly draw the individual’s productivity shock and the survival outcome; use the optimal policy functions to construct next period’s state variables; recursively follow this process until the individual dies. Repeat the same process 10000 times. Compute the aggregates as averages across the 10000 simulations.

5. If the guess values in the first step close enough to the computed values in the fourth step, this is a stationary recursive equilibrium. Otherwise iterations continue until the convergence realizes.

**Appendix B: Figures I**

![Figure 5: Type I's asset holdings](image-url)
Figure 6: Type II’s asset holdings

Figure 7: Type III’s asset holdings

Figure 8: Type IV’s asset holdings

Figure 9: Type V’s asset holdings
Figure 10: Type I’s consumption profile

Figure 11: Type II’s consumption profile

Figure 12: Type III’s consumption profile

Figure 13: Type IV’s consumption profile
Appendix C: Figures II

Figure 14: Type V’s consumption profile

Figure 15: Type I’s asset holdings

Figure 16: Type II’s asset holdings

Figure 17: Type III’s asset holdings
Figure 18: Type IV's asset holdings

Figure 19: Type V's asset holdings

Figure 20: Type I's consumption profile

Figure 21: Type II's consumption profile
Figure 22: Type III’s consumption profile

Figure 23: Type IV’s consumption profile

Figure 24: Type V’s consumption profile