Integrating Financial and Demographic Longevity Risk Models: An Australian Model for Financial Applications

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Background

1. Longevity risk and financing
   a. Ageing populations, improving mortality and decreasing fertility
   b. Funding of retirement through DC funds and lack of longevity insurance
   c. Trends and volatility in longevity (“Toxic” for reinsurers)

2. Modelling and risk management
   a. Product design and costing, risk management, securitization
   b. Longevity guarantee products (life annuities, lifetime withdrawal guarantees)

3. Mortality models
   a. Actuarial life table models (deterministic, projections, participating products)
   b. Demographic (stochastic projections, age parameters, stochastic trend)
   c. Financial (trend and volatility, flexibility for pricing – price of risk and dependence)

Research Aims

1. Review demographic and financial models for longevity risk
   a. Demographic models – e.g. Lee-Carter (1992) and extensions

2. Develop a model for Australian data
   a. Australian population data
   b. Lives over age 60
   c. Financial framework to calibrate price of mortality risk
   d. Dependence between ages

3. Demonstrate application of the model
   a. Securitization of longevity risk
   b. Multiple age portfolio and dependence
   c. Calibration of price of risk to Insurance Linked Security Market

Mortality Models

- Demographic models – Lee-Carter (1992) and Extensions:
  \[ m_{x+t} = a + b \cdot e^{k \cdot x} + \varepsilon \]

- Financial models – Dahl (2004) and Extensions:
  - Derived from financial models for interest rate risk (Vasicek, 1977; Cox et al., 1985)
  - Model trend and volatility
  - Incorporate risk neutral pricing
  - Extensive research and applications of term structure interest rate models

Mortality Model Structure

Mortality rate – general structure
\[ p(x,t) = p(x) f(t) + p(f)^{T} f(x) + D^{T} D f(x) \]
\[ \Phi(x,t) = (x + f) + (f(t) + \Phi(t) + \Phi(t)^{T}) \Phi(x,t) \]

Inhomogeneity with age and time
Volatility from dependent shocks
Fitted Model
\[ dp(x,t) = (x + f) + (f(t) + \Phi(t) + \Phi(t)^{T}) \Phi(x,t) \text{ for all } x.\]

Cohort effect in trend
Mortality changes depend on level, age and time

Dependence and Principal Components Analysis (PCA)

- Remove trends and analyse standardised residuals
- Analysis of covariance matrix of stochastic mortality factors - \( \text{Cov}(x,t) - \Sigma \)

Using PCA, decompose \( \Sigma \) into its eigenvectors \((V_i)\), and eigenvalues (diagonal matrix \(T\))

Cholesky decomposition of \( \Sigma \)

Using PCA, decompose \( \Sigma \) into its eigenvectors \((V_i)\), and eigenvalues (diagonal matrix \(T\))

- 15 factors explain 92% of mortality changes

The Mortality Model

Analysis of fitted model

Fitted residuals normally distributed, mean zero, standard error 1, without trends across age or time

Line asymptotic correlation values provide high confidence in each parameter estimate

Parameter | Mean-Square | Std. Error | Mean-Square | Std. Error | Mean-Square | Std. Error | Mean-Square | Std. Error
--- | --- | --- | --- | --- | --- | --- | --- | ---
| V1 | 0.9487 | 0.0005 | 0.9487 | 0.0005 | 0.9487 | 0.0005 | 0.9487 | 0.0005
| V2 | 0.0657 | 0.0008 | 0.0657 | 0.0008 | 0.0657 | 0.0008 | 0.0657 | 0.0008

Pearson's chi-square indicates model fits the observed data well
The Mortality Model - Projections

- Mortality expected to continue improving over the next 20 years (except ages 95-100)
- Passage of cohort through time can be noted
- Volatility highest under perfect dependence, except at the oldest ages

Model is calibrated using Lane (2000) risk premium model and 2007 mortality bond issues using non-linear least squares:

\[ \lambda^* \text{ is chosen so that:} \]

Demonstration of Application of Model - Calibration for Pricing

Model is calibrated using Lane (2000) risk premium model and 2007 mortality bond issues using non-linear least squares:

\[ \sum_j \lambda_j \mathcal{N}(X_j) = \sum_{i=1}^{\infty} \mathcal{N}(x_i) \]

So that for each \( x \) and \( z \):

\[ \lambda = \sum_j \lambda_j \mathcal{N}(X_j) \]

Application to Securitized Longevity Risk

- Tranche premiums are calibrated using the Lane model and model 'prices of risk' \( \lambda \) are implied from the model

Summary and Further Research

Financial model framework implemented
- Model estimated based on Australian population data for ages over 60
- Model allows for age dependence and models all ages as a system
- Age-dependence modelled through Principal Components. Important for modelling mortality-linked risk for multi-age portfolios.
- Mortality model allows the ‘price of risk’ to vary by age and time.
- Price of risk readily calibrated to traded risk linked securities and applied to securitization of longevity risk

Further research
- Extension to wider range of ages, difference versus trend stationary model, analysis of factors and number of factors for model parsimony
### Selected References


### Questions and Comments