LIFECYCLE INVESTING: DOES IT MAKE SENSE TO REDUCE RISK AS RETIREMENT APPROACHES?

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“Lifecycle Investing”, or the gradual reduction in the investment risk assumed by investors as they approach retirement, has been a marketing success. This paper reviews the theory behind lifecycle investing, including commenting on Samuelson “Risk and Uncertainty: A Fallacy of large numbers”. The paper then develops a model using Monte Carlo simulation, to evaluate risk in aggregate across the entire investment horizon. The model identifies the proportions of risk assumed versus the timeframe of investment and comments on methods for Lifecycle Investing.

INTRODUCTION

Lifecycle investing, or varying the risk profile of an investor’s portfolio over the life of the investor, has been a common practice amongst product manufacturers, and is becoming increasingly popular with the shift to defined contribution retirement funding. Underlying this concept, is the notion that as people get closer to retirement, and particularly as they enter the ‘decumulation’ phase, the portfolio risk ought to be reduced so as to lessen the variability of the portfolio return, and reduce the risk of loss.

Lifecycle investing has had extensive literature. Samuelson, in his paper “Lifetime Portfolio Selection by Dynamic Stochastic Programming” argued that risk tolerance in itself can be constructed independent of the stage of life, and outlined an important approach to resolving portfolio selection. Zvi Bodie in numerous papers furthered the debate in Lifecycle investing. Campbell and Viceira presented an approach that took into account investor time horizon, and Brandt et al, develop a simulated approach to determining the optimum portfolio.

Most models are concerned with optimizing an individual investor utility. However in practice, Funds cannot readily accommodate investor specific utilities in developing products purchased en masse. This paper employs Monte Carlo simulation of a typical investor model, to assess the variation in risk and return over the lifetime of investors.

An important variation of this approach, is that this model investigates the lifecycle of a typical investor during both the accumulation and decumulation phases.
Section 1 outlines the model assumptions. Section 2 details the findings and potential for developing the model further in cases of asymmetric risk tolerance and longevity risk. Section 3 concludes. Appendix 1 presents the data from the model.

I: THE MODEL ASSUMPTIONS

The model is set up to accommodate an investor who contributes a set proportion of their income in a fund providing a stochastic risky asset up until retirement, at which point the investor draws down from the fund indefinitely or until the money runs out.

The purpose of the model is not to determine individual maximization of utility (for example the Ramsey model), but to proposition policy.

Let an individual in the Accumulation phase be defined as:

\[ \sum_{t=0}^{T} (1 + g)^t Y_t = W_i^R \]  

(1)

Where \( W_i^R \) is the value on Retirement of savings by individual \( i \), experiencing a return \( g \), over savings duration \( T \), on net investments, \( Y_t \).

Let an individual in the Decumulation phase be defined as:

\[ W_i^D R - ((Z_i X \in \mathbb{E} (1 + g) Z_i | Z_i > 0)) = \delta \]  

(2)

Assuming a zero bequest motive, \( \delta = 0 \)

The model is then set up to define the return \( g \) as stochastic, as follows:

\[ g \in \mathbb{E} (Z_i X \in \mathbb{E}) \]  

(3)

In order to add reality to the model, the various parameters are calibrated against current norms as follows:

\[ Y_{12} = 9\% \times A W O T E \]  

(4)

\[ \text{Calibrate } Z_i, \text{such that}: \ \{(2) = 0 | \sigma = 0 \text{ and Life Expectancy} = 83\} \]  

(5)

Constants are calibrated as follows:

\(^1\) Full Time Adult Average Weekly Ordinary Time Earnings in Australia
Age at: Starting Savings = 24; Retirement = 65; X=100 (as a notional cut off) (6)

Inflation is a constant over the period, set at 3% (7)

is set at the average standard deviation of balanced funds, at 5.20% (8)

Baseline investment return is set at 7.00% (9)

The model is set up as 120 scenarios, with each scenario drawing that scenarios specific return randomly from the set defined in (3). The scenarios are run simultaneously, and a number of iterations are attempted in order to establish the model stability with regard to the average mean and standard deviation of the scenarios. An iteration that delivers averages that correspond closely with the constants for return, standard deviation and the average age at which \( \mathbb{E}_t = 0 \) occurs, is chosen for analysis.

The model baseline parameters and chosen iteration results are set out in Table 1 below:

<table>
<thead>
<tr>
<th>Table 1: Summary Results of chosen iteration of Monte Carlo Simulation of 120 Scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Age at Retirement:</td>
</tr>
<tr>
<td>2. Achieving the fund mean return of:</td>
</tr>
<tr>
<td>3. Expected Bequest, ( \mathbb{E} ), (Terminal Value at Death) :</td>
</tr>
<tr>
<td>4. Expected Age at death of :</td>
</tr>
<tr>
<td>5. No capacity to rely on age pension or additional income</td>
</tr>
<tr>
<td>6. Actual Terminal value at age 83 is :</td>
</tr>
<tr>
<td>6. Actual Terminal value at age 83 (in today’s $) is</td>
</tr>
<tr>
<td>7. Economic Deflator (Inflation Proxy) of :</td>
</tr>
<tr>
<td>8. Annualised Standard Deviation (( \sigma )) of :</td>
</tr>
<tr>
<td>9. Day 0 required consumption ( \mathbb{E}_t ) to achieve $0 bequest motive</td>
</tr>
<tr>
<td>10 Age at which money runs out:</td>
</tr>
<tr>
<td>12. Risk Free Rate</td>
</tr>
</tbody>
</table>

Even limited to 120 runs, the model achieves the baseline constants readily.

**II: MODEL FINDINGS**

The analysis is done at an aggregate (or Fund\(^2\)) level, without considering the individual utilities. A refinement of this model that allows for asymmetrical loss/gain utility, and of hyperbolic time discounting, will produce outcomes that identify the behavioral implications of

\(^2\)A Fund can be defined as a Superannuation Fund, or Mutual Fund
constructing a portfolio based on the results of this model. However enticing this analysis would be, it is outside the scope of this paper, but could be readily accommodated at some future time.

Furthermore, the model has been constructed to develop policy parameters in an Australian Superannuation context, assuming a fund with many members, each of whom are compulsorily required to contribute a set percentage of earnings.

Under this construction, the model allows policy makers to debate the construction of a single lifecycle set of factors that on average meets the constraints set out.

The results of the chosen iteration set out a number of important principles:

**Principle 1:** Survivor Bias means that the Fund benefits from those individuals that leave the fund early.

The model identifies a residual average value of $>0$ as shown in Figure 1 below. This is due to the model being calibrated such that the average of all bequests at age 83 is zero. This implies that, for particular scenarios where the money runs out early, the contribution of that scenario to the average is as a negative value.

This is equivalent to assuming borrowing by the individual in that scenario. In the model however, negative values are excluded; individuals cannot borrow to supplement their income, and effectively are removed as a call on the aggregate funds.

This is tantamount to survivor bias in the model, and may be thought of as a call option on social security.

In terms of the Fund itself, it presents the interesting possibility of accessing part of this ‘unintended bequest’ as source of insurance.

![Incidence of Unintended Bequests](image)

*Figure 1: Incidence of Unintended Bequests by Scenario*
**Principle 2:** The percentage of total wealth contributed at each age varies, but peaks around the time of retirement.

Over the lifecycle of an individual, the total wealth contributed is defined as being the investment returns generated by investing in the risky assets at each point in time. The value of contributions by the individual as a saving, are specifically excluded.

As such the contribution to aggregate investment income at time \( t \) must assume that this investment income is invested for the duration of the time horizon of the investor. This approach allows the compounding effect of the investment return to be fully accommodated.

Consequently the value of investment return \( g \) at time \( t \) is defined as:

\[
V_{cr} = v_{tr} \int_{t-d}^{t} e^{g \tau} d\tau
\]

(10)

Where:

- \( d \) is the time period during which the specific investment return \( v_{tr} \) is generated.

This overstates the contribution of investment returns as it does not specifically allow for money running out early. Later models can be tailored to accommodate this deficiency.

Assuming then that the aggregate return experienced by investor \( i \) is:

\[
I_t = \sum_{r} V_{ir}
\]

(11)

Then the contribution to the lifecycle wealth accumulation through investment returns for investor \( i \) can be thought of as:

\[
\left( P - \frac{V_{ir}}{I_t} \right) \left( 1 - \delta \right)
\]

(12)

This is shown in Figure 2 below.
Clearly the percentage contributed 10 years before and 5 years after retirement is the most important component of the aggregated investment generated wealth. This is merely due to the value of the accumulated assets, and has direct bearing to the risk assumed at each lifecycle stage.

**Principle 3: Risk as a proportion of the total accumulated wealth varies even though lifetime risk stays constant.**

The risk experienced at each time $t$ is the variation of investment returns, and corresponds to the standard deviation of the investment. The model assumes a constant standard deviation, and hence the standard deviation of the average returns of all scenarios returns to this constant. However the standard deviation at time $t$, as a proportion of total returns experienced by the individual varies over time $t$ as a result of the accumulation and then decumulation of assets.

The risk at time $t$ is defined as:

$$\sigma_{td} = \left( \text{std dev} \sum_{i=0}^{t} I_{id} \right) \sigma_0$$  \hfill (13)

And the dollar value of $\sigma_{td}$ is expressed as a percentage of the total investment return as follows:

$$\text{Risk} = \frac{\sigma_{td}}{\sum_{i=1}^{t} I_{it}}$$  \hfill (14)

The proportion of single period risk as a percentage of return generated by the investment over the aggregate of investment returns can then be shown in Figure 3 below:

Figure 2 Percentage of total wealth accumulated, contributed at each age.
Figure 3 Risk assumed in each period as a proportion of aggregate return
**Principle 4: Risk as a proportion of investment return at each period varies.**

Following from (10), where it is shown that the proportion of contribution to total investment generated wealth varies by age, it can readily be shown that the single period investment return will vary in inverse proportion to this contribution.

Assuming (10), we define the standard deviation as:

\[ \text{Standardised Risk} = \frac{\text{std dev} (V_{t|T})}{\text{Mean}(V_{t|T})} \]

(15)

This highlights the impact of single period risk (as defined by standard deviation) to the impact of the investment return where this investment return is reinvested at the mean return over the remaining time horizon as set out in (10).

Figure 4 below highlights this effect:

![Graph showing Standardised Std Dev as Percentage of Investment Returns Reinvested](image)

**Figure 4** Standardised Single Period risk as a percentage of Investment Returns assumed reinvested for the remaining time horizon

The implication of risk implicitly reducing as a percentage of aggregated investment returns reflects the changing nature of the investment horizon and assets.
III. SUMMARY

The paper presents a Monte Carlo simulation of the lifecycle of investment during both the accumulation and decumulation phases, and identifies a number of principles that arise from the modelling including:

1. Unintended bequests creating a survivorship benefit
2. The percentage of total wealth contributed through investment returns concentrated about the point of retirement
3. The single period risk taken on by individual varying as a proportion of total wealth contributed
4. The single period risk varying inversely to percentage of total wealth contributed.

The policy implications are that efficient products are more likely to arise through considering the entire lifecycle model.
REFERENCES:

Bodie, Z. “What Everyone Should Know About Life-Cycle Saving and Investing”, Boston University, 27 October 2006


Campbell, John Y. and Luis M. Viceira, Strategic Asset Allocation: Portfolio Choice for Long-Term Investors, Oxford University Press, New York, N.Y., 2002,


<table>
<thead>
<tr>
<th>Baseline</th>
<th>AVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Age at Retirement</td>
<td>65</td>
</tr>
<tr>
<td>2. Achieving the fund mean return of 7.00%</td>
<td>7.02%</td>
</tr>
<tr>
<td>3. Expected Bequest (Terminal Value at Death)</td>
<td>$0</td>
</tr>
<tr>
<td>4. Expected Age at death of 83</td>
<td>83</td>
</tr>
<tr>
<td>5. No capacity to rely on age pension or additional income</td>
<td>0</td>
</tr>
<tr>
<td>6. Actual Terminal value at age 83</td>
<td>$0</td>
</tr>
<tr>
<td>7. Economic Deflator of 3%</td>
<td>3%</td>
</tr>
<tr>
<td>8. Annualised Standard Deviation of 5.20%</td>
<td>5.20%</td>
</tr>
<tr>
<td>9. Day 0 required Consumption to achieve $0 bequest motive</td>
<td>-$41,139</td>
</tr>
<tr>
<td>10. Age at which money runs out</td>
<td>83</td>
</tr>
<tr>
<td>11. Risk Free Rate</td>
<td>5.50%</td>
</tr>
</tbody>
</table>

**APPENDIX 1:**
|   | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| **1. Age at Retirement** | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 | 65 |
| **2. Achieving the fund mean return of 7.36%** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| **3. Expected Bequest (Terminal Value at Death)** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| **4. Expected Age at death** | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 | 83 |
| **5. No capacity to rely on age pension or additional retirement income** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| **6. Actual Terminal value at age 83 (in today’s $)** | $2,736,638 | $3,734 | $0 | $0 | $2,338,906 | $338,116 | $0 | $605,643 | $2,230,650 | $3,163,478 | $2,407,148 | $0 | $2,394,303 | $0 | $0 | $0 | $3,581 | $94,148 | $0 | $1,017,469 |
| **7. Actual Terminal value at age 83 (in today’s $)** | $476,614 | $403 | $0 | $0 | $443,889 | $62,289 | $0 | $181,822 | $807,386 | $225,683 | $439,547 | $0 | $3,359,850 | $0 | $0 | $0 | $1,215 | $0 | $177,879 |
| **8. Economic Deflator** | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% | 3% |
| **9. Annualised Standard Deviation of 5.96%** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |
| **10. Risk Free Rate** | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $0 |

**1. Age at Retirement**

**2. Achieving the fund mean return of 7.36%**

**3. Expected Bequest (Terminal Value at Death)**

**4. Expected Age at death**

**5. No capacity to rely on age pension or additional retirement income**

**6. Actual Terminal value at age 83 (in today’s $)**

**7. Actual Terminal value at age 83 (in today’s $)**

**8. Economic Deflator**

**9. Annualised Standard Deviation of 5.96%**

**10. Risk Free Rate**