Towards a Dynamic Asset Allocation Framework for Target Retirement Funds: Getting Rid of the Dogma in Lifecycle Investing

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ABSTRACT

Lifecycle funds offered to retirement plan participants gradually roll down their exposure to stocks while approaching the target date of retirement. This movement away from equities and towards less volatile assets like bonds and cash is done following an asset allocation principle which emphasizes growth of portfolio in the initial years and preservation in the later years. We show that such deterministic switching rule produce inferior wealth outcomes for the investor compared to strategies that dynamically alter allocation between growth and conservative asset classes at different stages based on cumulative portfolio performance relative to a set target. The dynamic allocation strategies exhibit clear second-degree stochastic dominance and almost first-degree stochastic dominance over strategies which switch assets unidirectionally without cognizance to the portfolio performance.
Lifecycle or target retirement funds have gained favour with retirement plan investors in recent years. As a rule of thumb, these funds initially have a high allocation to stocks but with time gradually move towards less volatile assets like bonds and cash. Thus, we are told, they offer the best of both worlds – robust portfolio growth in the early years and preservation of the accumulated wealth as the investor approach the golden years of retirement. And the best part of all is that once enrolled there is no need for the investors to keep constant vigil over their investment strategy. Ameriks and Zeldes (2004) and several others have highlighted the problem of inertia bias among retirement plan participants which is often manifested in the reluctance to change allocation of their plan assets with time. Since lifecycle funds switch assets automatically following a preset glide path laid down by the plan provider, they are thought to be an effective antidote to this apparent flaw in investor behavior.

But does the predisposition of lifecycle funds to switch out of equities benefit the investors of target retirement funds? Empirical research in the past has generally found that a switch to low-risk assets prior to retirement can reduce the risk of confronting the most extreme negative outcomes. Lifecycle strategies are also said to reduce the volatility of wealth outcomes making them desirable to investors who seek a reliable estimate of final pension a few years before retirement\(^1\). On the other hand, there is near unanimity among these researchers that these benefits come at a substantial cost to the investor - giving up significant upside potential of wealth accumulation offered by more aggressive strategies.

This paper questions the rationale of the deterministic nature of switching from stocks to fixed income assets as is the prevalent practice among most lifecycle funds. The most common argument cited by the proponents of the lifecycle strategy in retirement plans is apparently straightforward – the probability that returns from stocks would outperform (underperform) those from bonds and cash increases (decreases) with the length of the investment horizon. If this is true, then long horizon investors may prefer to have a higher allocation to stocks in their portfolio

\(^1\) For example, see Ludvik (1994) or Blake, Dowd, and Cairns (2001).
compared to investors with shorter investment horizons\textsuperscript{2}. It is also argued that younger investors in retirement plans should heavily invest in stocks not only because of the prospect of higher returns but also for the reason that investors have enough time to recover from stock market downturn(s) should that happen. On the other hand, for older investors with few years to retire, holding such aggressive portfolio can spell disaster. A major slump in stock market just before retirement can potentially wipe away years of investment gains with little time to salvage the situation. But would this imply that investors should automatically reduce the proportion of stocks in their retirement portfolio as years go by? The following example would explain why the answer is not always yes.

Suppose an investor has a horizon of 40 years. Following popular lifecycle strategies, she decides to puts most of her money in stocks for the initial 20 years and then gradually switch to bonds and cash over the last 20 years. Once this allocation decision is made, she puts it on an autopilot (like most lifecycle funds) and goes off to sleep. However, the stock market returns following the investment decision do not augur well for the investor. Due to a prolonged bear market there are several years of negative returns eroding the value of her portfolio. After 20 years, the balance in her account is next to nothing and this gets gradually switched to bonds and cash. Subsequent returns in the account are stable but low. When our Rip Van Winkle investor wakes up after 40 years, she finds herself in a financial situation quite different from what was anticipated while setting the investment strategy. She may even find herself poorer in real terms than what she was 40 years ago.

Undoubtedly the above example is an extreme one and describes only one of the several possibilities that an investor can expect to encounter over long horizon. Yet it reveals the Achilles’ heel of the lifecycle funds currently in market. These funds follow a pre-determined ‘Rip Van Winkle’ asset allocation strategy where not only the switching of assets is always unidirectional – from stocks to fixed income – but it is also done in proportions that are pre-specified during the inception of the fund. In our example, had the stock market offered very high

\textsuperscript{2}This is sometimes referred to as ‘time diversification’. Samuelson (1989, 1994) shows that if returns are independently and identically distributed such long horizon effect cannot exist. While Samuelson’s argument is mathematically sound, mean reversion in stock returns is a well documented empirical phenomenon. For example, Poterba and Summers (1988), provides evidence from the US market.
returns during the last 20 years, the investor would stand to gain very little because her investments were automatically switched from stocks to bonds and cash during that period following the allocation strategy she had set on autopilot. The pre-programmed lifecycle strategy was blind to the fact that she had accumulated too little wealth in the initial years to necessitate switching to conservative assets. The asset switching in that case virtually ensures that she miss the only realistic chance she had to reverse her bad fortune.

The problem for the retirement plan members enrolled in lifecycle fund goes even deeper than our hapless investor. Typically the plan members make regular contributions to the retirement account as opposed to a single investment made at the beginning of the 40 year period in our example. As contributions are normally fixed percentage of the members’ salaries, they are expected would grow larger over time with growth in earnings. Therefore, as Shiller (2005) points out, the lifecycle strategy is heavily in stock market in the early years when the contribution size is relatively small and switches out of it when earnings and contributions grow larger in later years. This can be counterproductive as by moving away from stocks to low return assets just when the size of their contributions (and accumulation fund) are growing larger, the investor may be foregoing the opportunity to earn higher returns on a larger sum of money invested.

One cannot help wonder at the fact that why lifecycle funds need to have their benchmark asset allocation policy cast in stone. A possible alternative approach would be to switch to conservative assets a few years before retirement when a plan member feels that the wealth in the retirement account adequately meets his or her accumulation objective and therefore needs to be preserved. By the same token, the plan member not to switch to less volatile but low return assets in case the past performance of the portfolio has been unsatisfactory leaving him or her with inadequate wealth relative to the accumulation target. Thus, the decision to switch or not to switch and even how much to switch at any stage largely depends (or should depend) upon the cumulative performance of the retirement portfolio in the preceding years.

In this paper, we extend this alternative approach by proposing a dynamic lifecycle strategy which is flexible in adjusting its allocation between growth and conservative assets while
approaching retirement depending on to what extent the plan member’s wealth accumulation objective has been achieved at that time. In other words, this strategy is responsive to past performance of the portfolio relative to the investor’s target return in determining the right mix of assets in future periods. While initially it invests heavily in equities just as any other lifecycle strategy, the switching to fixed income is not automatic. It only takes place if the investor has accumulated wealth in excess of the target accumulation at the point of switch. Also, after switching to conservative assets, if the accumulation falls below the target in any period, the direction of switch is reversed by moving away from fixed income and towards stocks. But does this strategy result in improved outcomes for the retirement plan member? To find out we compare and contrast the outcomes of such dynamic strategy with those achieved following a regular lifecycle strategy.

Blake et al. (2001) test a similar lifecycle strategy with performance feedback although their benchmark is set in terms of a replacement ratio i.e. ratio of pension to final salary. The similarity between their threshold strategy and the dynamic lifecycle strategy proposed in this paper is that both resort to an aggressive asset allocation strategy if the portfolio underperforms the set benchmark (or lower threshold in their case) and vice versa. However, their strategy switches assets based on performance feedback right after the member joins the retirement plan while in our case the asset switching starts a few years before retirement which is more akin to the conventional lifecycle model. They also set two distinct thresholds (upper and lower) to determine the direction and extent of asset switching.

Conventional Versus Dynamic Lifecycle Strategy

In comparing conventional lifecycle and dynamic lifecycle strategies, we consider the case of a hypothetical individual who joins the plan with starting salary of $25,000. The earnings grow linearly at the rate of 4% per year over next 41 years, the duration of the individual’s employment life. Throughout this period, the member makes regular annual contributions amounting to 4% of earnings in the retirement plan account. We assume that the contributions are credited annually
to the member’s accounts at the end of every year. This means that the first contribution by the member is made at the end of first year followed by 39 more contributions in as many number of years. No contribution is made in the final year of employment.

Our hypothetical plan member can choose between a conventional lifecycle strategy and a dynamic strategy to invest the contributions. We consider two variations of the conventional lifecycle strategy, namely $LC_{20,20}$ and $LC_{30,10}$, which invest in a 100% stocks portfolio for 20 years and 30 years respectively following the first contribution. Thereafter both of them linearly switches from stocks to bonds and cash over the remaining 20 (and 10) years in such a manner that at the point of retirement all assets are held in bonds and cash. This type of allocation is akin to that of a typical lifecycle or target retirement funds which invest heavily in equities in the initial years and gradually switch to fixed income instruments as they approach maturity. Similarly the dynamic strategy has two variations, namely $DLC_{20,20}$ and $DLC_{30,10}$, corresponding to the above lifecycle strategies. They invest in the same 100% stocks portfolio as the two lifecycle strategies during the first 20 (and 30) years. Thereafter every year the strategies review how the portfolio has performed relative to the investor’s accumulation objective. If the value of the portfolio at any point is found to equal or exceed the investor’s target, the portfolio partially switches to conservative assets. Otherwise, it remains invested 100% in stocks. From our formulation of the strategies, it is clear that while $DLC_{20,20}$ and $DLC_{30,10}$ uses performance feedback control in switching assets, $LC_{20,20}$ and $LC_{30,10}$ do not.

Although people may have different accumulation objective on retirement, we need to make a plausible assumption about the accumulation target set by the hypothetical individual employing the dynamic allocation strategies in this paper. Dimson, Marsh, and Staunton (2002) have compiled returns for US stocks, bonds, and bills from 1900. We use an updated version of their dataset and find the geometric mean return offered by US stocks between 1900 and 2004 is 9.69%. We assume that the second individual sets a target of achieving a return close to this rate, say 9%, on the retirement plan investments. In other words, the retirement portfolio under the dynamic strategy aims to closely match the compounded accumulation of a fund where contributions are annually reinvested at 9% nominal rate of return.
For \( DLC_{20,20} \) which invests in 100% stocks portfolio for 20 years, we assume that the individual sets a target of 9% compounded annual rate of return on investment for the initial 20 year period. At the end of 20 years, if the actual accumulation in the retirement account exceeds the accumulation target, the assets are switched to a relatively conservative portfolio comprising 80% stocks and 20% fixed income (equally split between bonds and cash). However, if the actual accumulation in the account is found to fall below the target, the portfolio remains invested in 100% stocks. This performance review process is carried out annually for the next 10 years and assets allocation is adjusted depending on whether the holding period return outperforms or underperforms the target. In the final 10 years the same allocation principle is applied with only one difference. If the value of the portfolio in any year during this period matches or exceeds the investor’s target accumulation at that point, 60% of assets are invested in equities and 40% in fixed income (equally split between bonds and cash). Failing to achieve the target return for the holding period, results in all assets being invested in a 100% stocks portfolio.

For \( DLC_{30,10} \), which invests in 100% stocks for the 30 years after making the first contribution, the investor has the same target of return of 10% compounded annually. After 31 years, if the portfolio value in any year matches or exceeds the target accumulation, 20% assets are switched to fixed income (equally split between bonds and cash). A failure to achieve the target performance results in the portfolio being invested in 100% equities. The performance of the portfolio relative to the target is monitored annually and the asset allocation is adjusted accordingly. In the final 5 years before retirement, if the portfolio performance at any point matches or exceeds the target accumulation at that point, 40% of assets are switched to fixed income (equally split between bonds and cash).

**Simulating Wealth Outcomes**

To generate simulated investment returns under the two conventional lifecycle strategies (say \( LC_{20,20} \) and \( LC_{30,10} \)) and their corresponding dynamic lifecycle strategies (\( DLC_{20,20} \) and \( DLC_{30,10} \))
we use an updated version of the dataset of annual nominal returns for US stocks, bonds, and bills originally compiled by Dimson et al. (2002) and commercially available through Ibbotson Associates. The dataset spans a considerably long period of 105 years between 1900 and 2004 and thus capture both favorable and unfavorable returns on the individual asset classes over the entire twentieth century within our simulation trials. However, to examine of holding period returns of assets over horizons as long as 40 years, 105 years worth of returns data may not be sufficient. There are only two independent, non-overlapping 40-year holding period observations within our dataset. Any conclusion based on a sample of two observations cannot be deemed reliable.

To get around the problem of insufficient data, we use bootstrap resampling method. The empirical annual return vectors for the three asset classes in the dataset is randomly resampled with replacement to generate asset class return vectors for each year of the 40 year investment horizon confronting the two hypothetical retirement plan investors. Since we randomly draw rows (representing years) from the matrix of asset class returns, we are able to retain the cross-correlation between the asset class returns as given by the historical data series while assuming that returns for individual asset classes are independently distributed over time.

As the resampling is done with replacement, a particular data point from the original data set could appear multiple times in a given bootstrap sample. This is particularly important while examining probability distribution of future outcomes. For example, 1931 is the worst year for stock market in our 105 year long dataset. In that year return from stocks was -44% while bonds and bills offered returns of 1% and -5% respectively. Although this is only one observation in the century long data, a bootstrap sample of 40 yearly returns can include this return observation for 1931 many times in any sequence. Similarly, return observations for other years, good or bad, can also be repeated a number of times within a bootstrap sample. Since this method allows for inclusion of such extreme possibilities (like a -44% return occurring many number of times in a particular 40-year long return path), by obtaining a large number of bootstrap samples from the observed historical data, one can capture a much wider range of future possibilities.
The asset class return vectors obtained by bootstrap resampling are combined with their respective weightings under each asset allocation strategy to generate portfolio returns for each year in the 40 year horizon. The simulation trial is iterated 10,000 times for lifecycle strategy $LC_{20,20}$ and its corresponding dynamic strategy $DLC_{20,20}$ thereby generating 10,000 independent 40 year return paths that would govern the possible wealth outcomes for the individuals following them. A separate set of experiment (comprising of another 10,000 trials) is conducted for the other pair of lifecycle and dynamic strategies, $LC_{30,10}$ and $DLC_{30,10}$. For the purpose of doing a comparative analysis, we include two other allocation strategies – (i) a 100% stocks strategy and (ii) a balanced strategy which allocates in the ratio of 60:30:10 between stocks, bonds, and cash in both sets of experiments and provide the results in next section.

**Simulation Results**

The resampling method described above generates a range of terminal wealth outcomes under the conventional lifecycle strategies and their corresponding dynamic strategies. The parameter estimates for the wealth distribution under the different strategies are presented in Exhibit 1. From panel A, which provides the results for the conventional lifecycle and dynamic lifecycle strategies (both of which always remain invested in 100% stocks for the first 20 years), the difference is stark. The mean and the median outcome for the dynamic lifecycle strategy $DLC_{20,20}$ exceeds those for the conventional lifecycle strategy $LC_{20,20}$. The gap between the outcomes in this case, however, is lower than what it was between $DLC_{20,20}$ and $LC_{20,20}$. This is expected as $DLC_{30,10}$ and $LC_{30,10}$ strategies invest in the same portfolio (100% stocks) for ten more years.
In addition to the conventional and the dynamic lifecycle strategies, which are of primary interest in this paper, we also simulate wealth outcomes for the 100% stocks and the balanced strategy. The mean outcomes for the 100% stocks strategy are higher than both the conventional and dynamic strategy pairs. Given the existence of large positive equity premium in our data, this result is unsurprising. While the median and the first quartile outcomes for the 100% stocks strategy is higher than those of \( LC_{20,20} \) and \( LC_{30,10} \) they fall short of both \( DLC_{20,20} \) and \( DLC_{30,10} \). This suggests that dynamic strategies are superior in protecting the investors from the risk of adverse wealth outcomes than both the aggressive 100% stocks strategy and the conventional lifecycle strategy which adopts a pre-determined conservative allocation principle in later years.

The ineffectiveness of lifecycle switching in protecting investors from the risk of confronting adverse wealth outcomes on retirement is clear when we look at the balanced fund simulation results. The balanced fund, whose mean and median outcomes are inferior to all the other three strategies, outperforms \( LC_{20,20} \) in terms of the first quartile estimate. This apparently puts a huge question mark on the efficacy of the conventional lifecycle strategies. Dynamic lifecycle strategies, again, seem to produce better results in this respect. But we take up this issue later in this paper.

Despite the dynamic strategies (\( DLC_{20,20} \) and \( DLC_{30,10} \)) outperforming their conventional lifecycle counterparts (\( LC_{20,20} \) and \( LC_{30,10} \)) in terms of the mean, median, and the lower and upper quartile outcomes, can we conclude they are superior investment vehicles for the retirement plan members? This cannot be answered with certainty without comparing the entire range of outcomes under the two approaches. Stochastic dominance is a well known approach used in this type of situation because it relies on the entire distribution of outcomes.\(^3\) It also places minimal restrictions on the investors’ utility functions and makes no assumption (like

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\(^3\) Since the distribution of wealth outcomes get increasingly asymmetric over long horizons, the mean-variance framework is not useful in this situation. We also refrain from making any strong assumption on the utility function (like quadratic) of the retirement plan members.
normality) about the distributions. We use this approach here to find out whether investors would prefer terminal wealth distribution under one asset allocation strategy over that of the other.

Formally, given utility of wealth is a non-decreasing function i.e. $U'(W) \geq 0$, if $F$ and $G$ represents respectively the cumulative distributions of terminal wealth outcomes under the dynamic lifecycle strategy and the conventional lifecycle strategy, the former dominates the latter under the first degree stochastic dominance (FSD) rule if and only if:

$$F(W) \leq G(W) \quad \forall W$$

In plain words, this means that the dynamic lifecycle strategy would dominate the corresponding conventional lifecycle strategy by the FSD criterion if the cumulative distribution of terminal wealth outcomes under it always remains below the cumulative wealth distribution of the conventional lifecycle strategy.

Due to the strong conditionality it imposes regarding the cumulative distributions not intersecting each other even once, FSD cannot be applied in ordering distributions in many cases. The second degree stochastic dominance criterion (SSD) which is a weaker condition than FSD can be useful in these situations. SSD can be applied to a large class of problems because it works within the framework of risk aversion, an assumption widely used in finance literature (Hadar and Russell, 1969). Formally, given $U'(W) \geq 0$ and $U'(W) \leq 0$, the dynamic lifecycle strategy dominates the conventional lifecycle strategy under the SSD criterion if and only if:

$$\int_0^{\infty} F(W)dW \leq \int_0^{\infty} G(W)dW$$

This implies that the area under $F$ has to be equal or less than the area under $G$ for the dynamic strategy to dominate the conventional strategy by the SSD rule. Unlike FSD, SSD allows for $F$ and $G$ to cross each other as long as the above condition is met.

Exhibit 2 demonstrates the cumulative distributions of terminal wealth achieved under $LC_{20,20}$ and $DLC_{20,20}$ strategies. Again for the purpose of comparison, we also show cumulative wealth distributions for the 100% stocks and the balanced strategies. The horizontal axis of the graph represents the nominal dollar value of the portfolio at the point of retirement. As explained above,
if the CDF for one strategy lies under (or to the right of) other CDFs, it is likely to result in a superior outcome relative to other strategies. Also, if CDF for a strategy is generally steeper than the others, the strategy can be considered to result in less variable outcomes.

It is clear that except for a very small part on the left of the point X, the cumulative distribution plot of $DLC_{20,20}$ remains much under that of $LC_{20,20}$. Therefore, although the dynamic lifecycle strategy does not dominate the conventional lifecycle strategy by the strict FSD criterion, it does dominate under SSD because the area under cumulative distribution $F$ of the dynamic strategy is clearly far less than that under cumulative distribution $G$ of the conventional lifecycle strategy. Except for a very small section on the left of point X representing wealth outcomes of about $500,000 or less after 41 years, we can infer from the cumulative distributions that the investor employing $DLC_{20,20}$ has higher chance of achieving any particular accumulation outcome than the investor employing $LC_{20,20}$. For example, the former has about 75% probability of accumulating more than one million dollars at retirement whereas the later has got only a 60% chance of crossing that milestone. If investors set a target of achieving a compounded return of 9% minimum on their investments, which amounts to an accumulated wealth of at least $1.69 million at retirement, our results indicate that the $DLC_{20,20}$ strategy would achieve this goal with almost 50% certainty. With $LC_{20,20}$ strategy, this probability drops to only 25%. The gap between the cumulative distribution functions for the two strategies widens as we move up further towards higher accumulation figures although after a point (roughly around two million dollars) its starts diminishing gradually.

A comparison of the cumulative distributions of the lifecycle strategies $LC_{20,20}$ and $DLC_{20,20}$ with that of 100% stocks strategy reveals two important results. First, we find that the distribution of conventional lifecycle strategy $LC_{20,20}$ always remains above that of 100% stocks strategy except for the small section on the left of point X (representing only about the worst 5% of outcomes). This undermines the effectiveness of conventional lifecycle strategies in protecting the wealth investors from the vagaries of stock market downturns. Had it been the case, we would have found X much to the right of its current location i.e. $LC_{20,20}$ would have dominated the
100% stocks strategy for a much larger percentage of outcomes in the lower end of the distribution. In contrast, we find the cumulative distribution of $DLC_{20,20}$ remains below that of the 100% stocks strategy for a much longer section (the left side of Y). This clearly suggests its effectiveness in reducing the risk of investor’s wealth breaching any floor level of wealth to the left of Y. Although it does not dominate the 100% stocks strategy under SSD criterion, it does much better in terms of producing superior outcomes in the below median range, which is likely to be viewed as the zone of risk for most investors. Remarkably it is obvious from the diagram that our hypothetical investor has a slightly higher chance of achieving the target wealth outcome of $1.69$ million by employing the $DLC_{20,20}$ instead of the 100% stocks strategy.

Now we turn our attention to the cumulative wealth distribution functions for the other lifecycle and dynamic strategy pair - $LC_{30,10}$ and $DLC_{30,10}$. This is presented in Exhibit 3. As before, we also show cumulative wealth distributions for the 100% stocks and the balanced strategies. Apart from a small part in the extreme lower tail of the distributions representing terminal wealth outcomes below $500,000, the cumulative wealth distribution function of $DLC_{30,10}$ (F) always remains below that of $LC_{30,10}$ (G). Thus, there is clear second degree stochastic dominance of $DLC_{30,10}$ over $LC_{30,10}$ indicating that any risk averse investor would find the former more appealing to the latter. As is the case with $LC_{20,20}$ and $DLC_{20,20}$ pair, the distance between the CDF plots is larger in the middle than in the extremes. In other words, the dynamic strategy dominates the conventional strategy over for a vast range of outcomes by a staggering margin.

In relation to the target accumulation outcome of $1.69$ million at retirement, Exhibit 3 indicates that the $LC_{30,10}$ strategy would achieve this goal with about 40% certainty. Although this is significant improvement compared to the performance of $LC_{20,20}$, it still falls short of the corresponding dynamic strategy, $DLC_{30,10}$, which surpasses the target on more than 50% of occasions. The cause for $LC_{30,10}$ putting up a superior performance relative to $LC_{20,20}$ strategy in attaining the target may be attributed mainly to the fact that the former invests in a 100% stocks portfolio for a longer duration (30 years) compared to that of the latter (20 years). However to
apply the same argument to explain the dominance of dynamic strategies over corresponding lifecycle strategies appears too simplistic. Had this been the only reason, 100% stocks strategy would have outperformed other strategies in terms exceeding the target accumulation. But as is evident from Exhibit 3, the probability of achieving the target wealth outcome with $DLC_{30,10}$ strategy is clearly higher than that with 100% stocks strategy. Also, the median outcome for $DLC_{30,10}$ strategy is larger than that of 100% stocks strategy.

But what is the success (or failure) rate of the dynamic strategy over other strategies in different possible future states of the world? This knowledge is important to the investor yet comparing probability distributions of terminal wealth under different competing strategies would not provide a clear answer. This is because in doing so we are comparing the $n$-th percentile outcome of one strategy with the $n$-th percentile outcome of the other. In plain words, the good scenarios under one strategy are compared to the good scenarios under another and likewise the bad outcomes are pitted against the bad outcomes. But for any particular future state of the world (with a particular asset return path over the investment horizon), this comparison may not be very useful. For example, if stock returns turn out to be very poor compared to other assets in a particular state of the world, the 100% stocks strategy would produce inferior outcome relative to a balanced strategy no matter how attractive or dominating the wealth distribution of the former appears compared to the latter.

Recall that the asset class return path over the 41 year horizon is unique for each trial in our simulation experiment. Each of those 10,000 trials represents a different possible future state of the world. Therefore, for each trial, we compare the wealth outcomes under all four strategies, the main point of interest being how the dynamic strategy perform vis-à-vis other strategies. To be specific, we compute the shortfall probability as of $DLC_{20,20}$ and $DLC_{30,10}$ as well their average size of shortfall compared to the other 3 strategies. These shortfall measures are likely to constitute an important part of what the investors view as downside risk of adopting the dynamic allocation strategy. The results, provided in Exhibit 4, show that the dynamic strategy has small chance of underperforming the conventional lifecycle strategy. Only for 19% of trials the wealth

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4 The actual number of possibilities is obviously infinite.
outcome of the dynamic strategy $DLC_{20,20}$ falls short of that of the corresponding lifecycle strategy $LC_{20,20}$. For $DLC_{30,10}$, the chance of it underperforming the corresponding lifecycle strategy $LC_{30,10}$ however increases to 26% i.e. one in four. However, the average size of the shortfall in both the cases is quite miniscule ($34462$ and $50273$) compared to the average size of terminal wealth outcomes which run into millions.

Further comparing individual trial outcomes, we find that the $DLC_{20,20}$ strategy gives the 100% stocks strategy a close run. Chance of doing better with either strategy is almost even with the 100% stocks strategy emerging the winner in 51% of the trials. But when compared with $DLC_{30,10}$, the 100% stocks strategy fares better only in 43% of trials i.e. the dynamic strategy emerges winner in a majority of cases. The average size of the shortfall for the dynamic strategy in both cases, however, is quite high at $582,815$ and $343,890$ respectively. This is not unexpected with the 100% stocks strategy producing several spectacularly large wealth outcomes in the above median and particularly in the upper quartile range. Relative to the balanced strategy, the chance of underperformance of the dynamic strategy is minimal. $DLC_{20,20}$ and $DLC_{30,10}$ strategy underperforms the balanced strategy only in 10% and 11% of the trials respectively. The average size of shortfall in both cases is extremely small at $6,110$ and $6,907$ respectively.

While our evidence so far overwhelming suggests superiority of dynamic strategies over conventional lifecycle strategies, the saving grace for the latter may lie in the zone of most adverse outcomes. This is represented by the left portion of X in the CDF plots in Exhibits 2 and 3 where the lifecycle strategies actually dominate corresponding dynamic strategies. It is also apparent from the diagrams that this zone is constituted by outcomes that are below the 10th percentile mark for every strategy. To have some idea about how large the differences actually are between the adverse outcomes under different strategies, we report the VaR estimates at confidence levels of 99%, 95%, and 90% for both sets of simulation trials in Exhibit 5. We also estimate the Expected tail loss (ETL) at 95% confidence level which is essentially a probability weighted average of all below-VaR outcomes at that specified level of confidence.
As is evident in the CDF plots, both the lifecycle strategies $LC_{20,20}$ and $LC_{30,10}$ produce 95% and 99% VaR estimates compared to their dynamic counterparts (and 100% stocks strategy). The differences between the 95% VaR estimates (less than $25,000) do not appear to be large enough to ring any alarm bell. But when one compares the 99% VaR estimates, the differences between the lifecycle and dynamic strategy grow considerably larger. The estimated 99% VaR estimate for $LC_{20,20}$ strategy is almost $100,000 more than that of the corresponding dynamic strategy $DLC_{20,20}$. Between $LC_{30,30}$ and $DLC_{30,10}$, the corresponding difference, however, is smaller than $50,000.

Yet one would be reluctant to declare lifecycle funds to be the preferred investment strategy even under the unreasonable assumption that investors care only about the zone of extremely adverse wealth outcomes (below 10th percentile in this case). This is because the balanced fund produces better 95% VaR estimate than both $LC_{20,20}$ and $LC_{30,10}$. In terms of 99% VaR estimates, the balanced fund outperforms $LC_{30,30}$ but underperforms $LC_{20,20}$. When we consider the average for all outcomes below 95% VaR estimates, the balanced fund produces ETL estimates that are higher than both $LC_{20,20}$ and $LC_{30,10}$. These results suggest that if the retirement plan investors are concerned about improving the floor level of possible wealth outcomes or protection from extreme downside risk, they are more likely to be better off by investing in a static balanced fund rather than a conventional lifecycle fund.

**Conclusion**

The evidence presented in this paper exposes the inherent weakness of traditional lifecycle investing for members of retirement plans. By blindly rolling over to conservative assets in later part of the investment horizon the lifecycle funds clearly seem to be missing a trick. While pulling out of volatile assets like stocks while the plan member nears retirement is generally accepted as sensible investment advice, traditional lifecycle funds appear to implement this
strategy in a dogmatic manner that completely disregards the investors’ wealth accumulation objectives.

As we have demonstrated in this paper, the mechanistic switching strategy from growth to conservative assets following any age based rule of thumb is clearly sub-optimal to a dynamic strategy that considers the actual accumulation in the retirement account before switching assets. This paper proposes a specific dynamic asset allocation strategy where the switching of assets at any stage is based on cumulative investment performance of the portfolio relative to the investors’ set expectations at that stage. Unlike conventional lifecycle asset allocation rules where the switching of assets is preordained to be unidirectional, this dynamic strategy can switch assets in both directions: from aggressive to conservative and vice versa. Using simple rules of stochastic dominance, we show that such a dynamic lifecycle strategy vastly outperforms a conventional lifecycle strategy in terms of accumulation outcomes over long horizon.

When comparing percentile outcomes in our trials, the only occasion when we find lifecycle strategies to do better than the dynamic strategies is outcomes at below 5\textsuperscript{th} percentile range. However, the differences do not appear to be large enough to negate the appeal of dynamic strategies to the average investor in view of their overall dominance over lifecycle strategies. Even for these extremely adverse wealth outcomes in our trials, we find that the static balanced asset allocation strategy generally does better than lifecycle strategy. Therefore an investor whose sole concern is improving the floor level of the extremely adverse wealth outcomes is likely to prefer investing in a balanced fund rather than a lifecycle fund.

We have conducted a large number of trials in this paper to capture different possibilities about future asset class returns over the investment horizon of the retirement plan investor. According to our results, the chance of the dynamic strategy underperforming the lifecycle strategy at the end of such long horizon is small (although not insignificant). Not only does the dynamic strategy produce superior terminal wealth outcomes compared to the lifecycle strategy in a vast majority (about 75 -80\%) of cases, it appears to have a fair chance of outperforming an all equity strategy. In fact, the dynamic lifecycle strategy \textit{DLC}_{30.10} in this paper which invests in an all equity portfolio for the first 30 years and then adjusts asset allocation on an annual basis, seems to have
more than even chance of beating the strategy which invests in an all equity portfolio for the entire horizon.

It is hard to imagine that most human beings as unduly optimistic or pessimistic that they would care only about the extreme outcomes. Very few among us are seriously positive about the chance of winning the lotto at any point in our life. Nor does the idea of being hit by a car is something that we regularly consider when stepping out of home. For the vast majority of the population, decisions in life including investment are generally governed by the vast middle range of possibilities. It is precisely because of this reason that the dynamic strategy looks appealing in the context of our problem. Ignoring the extremities, it invariably results in much higher wealth accumulation potential compared to the conventional lifecycle strategy. Remarkably this is achieved while reducing downside risk compared to an all equity strategy as evidenced from the dominance of the dynamic strategy in the below median range of wealth outcomes.

Apparent from the relative superiority over other allocation strategies from a risk-return perspective, the dynamic strategy proposed in this paper has another distinct appeal for the retirement plan investors. Since the allocation is responsive to performance feedback, it provides them some sense of control over their investment decision on a continual basis. Without giving up the basic tenet of lifecycle investing - seeking reduced volatility in the value of the accumulation fund as one gets closer to retirement – the dynamic strategy overcomes its limitations to a large extent. It is important to clarify here that we are not suggesting in any manner that the dynamic allocation rule proposed in this paper is the optimal strategy for all or even most retirement investors. Our evidence only points towards the general approach practitioners certainly need to consider in designing lifecycle funds for retirement plans.
References


Ludvik, Peter, 1994, Investment strategy for defined contribution plans, 4th AFIR International Colloquium (Orlando).


EXHIBIT 1

TERMINAL VALUE OF RETIREMENT PORTFOLIO IN NOMINAL DOLLARS

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Mean</th>
<th>Median</th>
<th>25th Percentile</th>
<th>75th Percentile</th>
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<tr>
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<td>Dynamic</td>
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<tr>
<td>Lifecycle</td>
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<td>1163836</td>
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<td>100% Stocks</td>
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EXHIBIT 2
CUMULATIVE DISTRIBUTION PLOTS FOR FIRST PAIR OF LIFECYCLE AND DYNAMIC STRATEGIES ($LC_{20,20}$ and $DLC_{20,20}$)

Cumulative Distribution of Terminal Wealth for Various Asset Allocation Strategies

- Dynamic
- Lifecycle
- 100% Stocks
- Balanced
- Target

Target Accumulation at Retirement

Terminal Wealth ($\times 10^6$)
EXHIBIT 3

CUMULATIVE DISTRIBUTION PLOTS FOR FIRST PAIR OF LIFECYCLE AND DYNAMIC STRATEGIES ($LC_{30,10}$ and $DLC_{30,10}$)
EXHIBIT 4

Shortfall Measures of Dynamic Strategies Relative to other Asset Allocation Strategies

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Shortfall Probability</th>
<th>Average Shortfall ($)</th>
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<td>$DLC_{20,20}$</td>
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<tr>
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EXHIBIT 5
VaR and ETL Estimates for Different Asset Allocation Strategies

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<th>VaR at Different Confidence Levels</th>
<th>ETL at 95% Confidence Level</th>
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