Expectations, Capital Gains and Income

Robert J. Hill

and

T. Peter Hill

RRH: HILL & HILL: CAPITAL GAINS AND INCOME

Forthcoming in Economic Inquiry
A theoretical framework for the measurement of income under uncertainty is developed which addresses some long-standing controversies about the treatment of capital gains. The consequences for economic analysis and policy making are potentially serious, since the treatment of capital gains can significantly affect some major macroeconomic aggregates, including national income and savings, balance of payments deficits, government deficits and depreciation.

Robert J. Hill
School of Economics
University of New South Wales
Sydney 2052, Australia

T. Peter Hill
School of Economics & Social Studies
University of East Anglia
Norwich, NR4 7TJ, England
I. INTRODUCTION

This paper argues that the distinction between expected and unexpected capital gains (losses) is fundamental to the measurement of income. The way capital gains are treated can have a significant impact on major macroeconomic statistics such as national income and saving, the balance of payments, government income and saving, and depreciation.\(^1\) The recent sale of spectrum licenses is a case in point. The spectrum, a natural asset over which governments enforce property rights, became unexpectedly valuable as a result of the development of mobile telephones. In 2000 and 2001 some governments realized gains of 30 billion dollars or more in a day by auctioning licenses to use sections of the ‘spectrum’.\(^2\)

The concept of income, although widely used, remains vague. It is necessary to inquire why the concept of income is needed and what use it serves. The main purpose of income is to provide guidance to households or other economic units, including government, on the rate at which they can afford to consume when there is uncertainty about future resources. The more successfully that measured income meets this requirement the greater its power as an explanatory variable for the analysis of consumer behavior.

The treatment of expected and unexpected capital gains in income measurement is a continuing source of controversy.\(^3\) Even in a perfect foresight setting, the concept of income is not straightforward. Hicks (1946) considered a number of definitions. Two definitions in particular command support in the literature. Hicksian income no. 1 is the maximum amount that can be consumed while maintaining wealth intact.\(^4\) Hicksian income no. 2 is the maximum sustainable level of consumption. The general consensus that emerges from the literature is that, with perfect foresight, all capital gains are included in Hicksian income no. 1. However, Asheim (1996) shows that some capital gains are excluded from Hicksian income no. 2 when the interest rate varies over time. Both income concepts coincide when the interest rate is fixed.

When the perfect foresight assumption is relaxed, a distinction must be drawn between expected and unexpected capital gains. It is important that we move beyond a
perfect foresight setting since the concept of income has been developed primarily in order to assist decision taking under uncertainty. If income is meant to act as a budget constraint indicating the resources available for consumption each period, the important question is how should a rational consumer react to unexpected capital gains.

This paper develops a general theoretical framework for the measurement of Hicksian income no. 1 under uncertainty that is capable of handling all kinds of capital gains on all kinds of assets ranging from money market assets to mineral deposits. First, the concept of income with perfect foresight is developed and analyzed. In the following section, uncertainty is introduced. Income depends on expectations of future receipts that are liable to be revised with the passage of time. The time at which income in a particular period is measured is therefore crucial. Income can be measured at any time but attention has tended to focus on the beginning and end of the period. Hicks (1946, 178-9) described income as measured at these times as *ex ante* and *ex post* income. *Ex ante* income, as noted by Eisner (1990, 1180), is essentially the same as Friedman’s (1957) concept of permanent income. *Ex post* income as defined by Hicks – now usually described as Haig-Simons income after two earlier proponents of the concept is a widely used objective measure that is familiar to most economists. However, it is conceptually flawed because it utilizes two different and generally inconsistent set of expectations, those held at the beginning and end of the period. We define a family of conceptually consistent income measures, each based only on the expectations held at the time of measurement, that we describe as *generalized-Hicksian* income. This family nests *ex ante* income but not Haig-Simons income. Generalized-Hicksian income excludes unexpected capital gains whereas Haig-Simons includes them. Given the role that the concept of income is intended to play in demand theory and the analysis of consumption behavior, we conclude that the appropriate concept of income from an economic viewpoint has to be of the generalized-Hicksian type (i.e., unexpected capital gains must be excluded), even though other concepts might be more appropriate for some other purposes such as taxation.
The same conceptual approach is then applied to financial, fixed and natural assets. For financial assets, we focus on fixed-term bonds. There is controversy about the measurement of the interest on bonds after unexpected changes in interest rates occur.\(^6\) We argue that the current treatment implemented by the International Monetary Fund (IMF) and other international and most national agencies is incorrect, thus potentially causing government deficits to be mismeasured.

The measurement of income and capital gains from fixed assets such as buildings and equipment hinges on the treatment of obsolescence. The treatment of obsolescence is a long-standing source of controversy.\(^7\) Rapid technological progress in high technology sectors has recently led to a resurgence of interest in obsolescence and its impact on depreciation and the capital stock.\(^8\) Sometimes all obsolescence is considered to be part of depreciation [e.g., Eisner (1988)] and sometimes none [e.g., Jorgenson (1989)]. In contrast, we argue that the correct treatment depends on the extent to which it is expected. As an expected capital loss, foreseen obsolescence should reduce income while unforeseen obsolescence should not.

For natural assets, our main focus is the huge capital gains or losses experienced by resource-rich countries as a result of price volatility in commodity markets. For example, Aaheim and Nyborg (1995) find that, in certain years, changes in the value of Norway’s stock of oil exceed its GDP. Hence if these capital gains or losses are included in income, a large fall in the price of oil could cause Norway’s national income to become negative in that period. Unless capital gains (losses) are handled correctly, therefore, the national income of resource rich countries can become so distorted and volatile as to be quite useless for analytic and policy purposes.

Our main findings are summarized in the conclusion.

II. ALTERNATIVE CONCEPTS OF INCOME AND THEIR TREATMENT OF CAPITAL GAINS

Income is a fundamental and widely used economic concept. It appears as an argu-
ment in demand functions and as a resource constraint for a utility maximizing consumer. It is closely related to wealth as both concepts depend on the same flows of expected receipts.

**Income Under Perfect Foresight**

Throughout this paper, all prices and values are measured in constant dollars. Let $V_t$ denote the present value at the beginning of period $t$ of the stream of receipts from a stock of assets. For purposes of income measurement, it is necessary to be able to track the value of this stock over subsequent time periods. Let $V_{t,t+k}$ denote the value of the stock at the beginning of period $t+k$, where $k \geq 0$, assuming that no additional investment or disinvestment has taken place up to that point. In other words, $V_{t,t+k}$ is equal to the present value at the beginning of period $t+k$ of the original stream of receipts from period $t+k$ onwards, assuming that none of the receipts from periods $t$ to $t+k-1$ are reinvested. Likewise, let $R_{t,t+k+i}$ denote the receipts earned during period $t+k+i$, where $i, k \geq 0$, on the stock of assets existing at the beginning of period $t$. It is assumed that receipts are paid at the end of each period.

\[
V_{t,t+k} = \sum_{i=0}^{\infty} \left\{ \prod_{j=0}^{i} \left[ \frac{1}{1 + r_{t+k+j}} \right] R_{t,t+k+i} \right\},
\]

where $r_t$ is the real interest rate in period $t$. If the formula for $V_{t,t+k}$ in (1) is written out in full for the cases where $k = 0$ and $k = 1$, it can be seen that $V_t$ and $V_{t,t+1}$ are related as follows:

\[
V_{t,t+1} = (1 + r_t)V_t - R_t.
\]

The value of the actual stock of assets at the start of period $t+k$, namely, $V_{t+k,t+k}$ will not be the same as $V_{t,t+k}$ when some of the receipts earned between periods $t$ and $t+k$ are reinvested, or if some of the assets are sold. For the case where $k = 1$, the amount of this investment is determined by the difference between receipts $R_t$ and consumption $C_t$ in period $t$. Thus,

\[
V_{t+1,t+1} = V_{t,t+1} + R_t - C_t,
\]
\((R_t - C_t)\) being the amount reinvested. Combining (2) and (3), it follows that

\[ V_{t+1, t+1} = (1 + r_t)V_{tt} - C_t. \]  

(4)

To simplify the notation for the remainder of the paper, in cases where \(k = 0\), we will write \(V_t\) and \(R_t\), respectively, in place of \(V_{tt}\) and \(R_{tt}\). Using this notation, rearranging (2), it follows that

\[ r_t V_t = R_t + V_{t,t+1} - V_t. \]  

(5)

Similarly, rearranging (4), it follows that

\[ r_t V_t = C_t + V_{t+1} - V_t. \]  

(6)

As is shown below, these two expressions for the interest earned on a stock of assets provide alternative ways of measuring income.

Following Hicks (1946, 173), income can be defined as either the maximum level of consumption that will maintain wealth intact (Hicksian income no. 1) or as the maximum sustainable level of consumption (Hicksian income no. 2). More formally, Hicksian income no. 1, denoted here by \(Y_t\), is defined as follows:

\[ Y_t \equiv \text{Max}\{C_t : V_{t+1} \geq V_t\}. \]  

(7)

Setting \(V_{t+1} = V_t\) in (6), it follows from (7) that

\[ Y_t = r_t V_t. \]  

(8)

Substituting (5) and (6) into (8), it can be seen that Hicksian income no. 1 can also be written as follows:

\[ Y_t = R_t + V_{t,t+1} - V_t = C_t + V_{t+1} - V_t. \]  

(9)

In other words, Hicksian income no. 1 is equal to total receipts \(R_t\) plus \((V_{t,t+1} - V_t)\), the total capital gains on the stock of assets existing at the beginning of the period. These capital gains are an integral part of income. Equation (9) also shows that income is equal to consumption \(C_t\) plus \((V_{t+1} - V_t)\), the actual change in the value of the stock of
assets. The latter reflects the effects of investment or disinvestment as well as capital gains. This change can be decomposed into the capital gain on the stock held at the start of the period, \((V_{t,t+1} - V_t)\), and net investment during the period, \((V_{t+1} - V_{t,t+1})\): that is,
\[
(V_{t+1} - V_t) = (V_{t,t+1} - V_t) + (V_{t+1} - V_{t,t+1}).
\]
If there is no capital gain, income equals consumption plus net investment.

By contrast, Hicksian income no. 2, denoted by \(\hat{Y}_t\), is defined as follows:
\[
\hat{Y}_t \equiv \text{Max} \left\{ C_t : \sum_{i=0}^{\infty} \left[ \prod_{j=0}^{i} \left[ \frac{1}{1 + r_{t+j}} \right] C_t \right] \leq V_t \right\},
\]
which reduces to
\[
\hat{Y}_t = V_t / \sum_{i=1}^{\infty} \left[ \prod_{j=0}^{i} \left[ \frac{1}{1 + r_{t+j}} \right] \right].
\]
\(\hat{Y}_t\) excludes part of the capital gain (loss) arising from interest rate changes. Hicks’s two concepts of income are equivalent only when the interest rate does not change over time. In this case, both \(Y_t\) and \(\hat{Y}_t\) reduce to \(rV_t\). A rate of consumption equal to \(rV_t\) maintains wealth intact and is sustainable indefinitely if \(r\) does not change.\(^9\)

**Income Under Uncertainty**

An individual’s planned consumption path is constrained by expected wealth. The individual must distinguish between, and react quite differently to, expected and unexpected changes in asset values as the latter require expected wealth, and hence expected consumption possibilities, to be changed whereas the former do not.

In the remainder of this paper we focus on generalizations and different applications of Hicksian income no. 1. It is the simpler concept and is widely used in the national accounting and business accounting literatures.\(^{10}\) The strategy of maintaining wealth intact (i.e., Hicksian income no. 1) in the presence of uncertainty can be interpreted in several different ways, each of which leads to a different concept and measure of income.

When there is uncertainty the estimated value of a person’s wealth at any point of time depends on the time at which the estimate is made. Define \(E_s V_t\) as the expectation at
time $s$ of the value of the stock of assets existing at the beginning of period $t$. Time $s$ can precede, coincide with or follow $t$. Since the stream of receipts ($R_s, R_{s+1}, R_{s+2}, \ldots$) can extend indefinitely into the future, this implies that $V_t$, in general, is never known for certain, even if $s > t$.\textsuperscript{11}

Haig-Simons Income. The Haig (1921)-Simons (1938) definition is widely used by economists.\textsuperscript{12} It is an ex post version of Hicksian income no. 1. Haig-Simons income, denoted here by $Y_t^{HS}$, is defined as follows:

$$Y_t^{HS} \equiv \text{Max}\{C_t : E_{t+1}V_{t+1} \geq E_tV_t\}. \quad (12)$$

The first step to solving the maximization problem in (12) is to generalize equation (6) to allow for uncertainty. Forming expectations at the beginning and end of period $t$, respectively, the following equations are obtained:

$$E_t(r_tV_t) = E_tC_t + E_{t+1}V_t - E_tV_t, \quad (13)$$

$$r_tE_{t+1}V_t = C_t + E_{t+1}V_{t+1} - E_{t+1}V_t. \quad (14)$$

Next, equations (13) and (14) can be combined as follows:

$$E_t(r_tV_t) + r_tE_{t+1}V_t = E_tC_t + C_t + E_{t+1}V_t - E_{t+1}V_t + E_{t+1}V_{t+1} - E_tV_t. \quad (15)$$

Now it follows from (12) that we can set $C_t$ in (15) equal to $Y_t^{HS}$ when $E_{t+1}V_{t+1} = E_tV_t$. Making these substitutions in (15), the following expression is obtained:\textsuperscript{13}

$$Y_t^{HS} = E_t(r_tV_t) + r_tE_{t+1}V_t - E_tC_t - E_tV_{t+1} + E_{t+1}V_t. \quad (16)$$

Finally, using (13) and (14) to substitute for $E_t(r_tV_t)$ and $r_tE_{t+1}V_t$, equation (16) reduces to

$$Y_t^{HS} = C_t + E_{t+1}V_{t+1} - E_tV_t. \quad (17)$$

This is the most familiar representation of Haig-Simons income as the sum of consumption plus the actual change in wealth over the period.\textsuperscript{14} An alternative expression for
\( Y_{t}^{HS} \) can be obtained by generalizing equation (5) to allow for uncertainty as follows:

\[
    r_{t}E_{t+1}V_{t} = R_{t} + E_{t+1}V_{t,t+1} - E_{t+1}V_{t}.
\]

By substituting for \( E_{t}(r_{t}V_{t}) - E_{t}C_{t} \) from (13) and \( r_{t}E_{t+1}V_{t} \) from (18), equation (16) reduces to

\[
    Y_{t}^{HS} = R_{t} + E_{t+1}V_{t,t+1} - E_{t}V_{t},
\]

i.e., the sum of receipts and the actual change in the value of the stock of assets existing at the start of the period.

Haig-Simons income equals the amount a person can consume in period \( t \) and be as well off at the beginning of period \( t+1 \) as they thought they were at the beginning of period \( t \). The basic flaw in the concept is that a prudent consumer has no reason to wish to preserve \( E_{t}V_{t} \) intact as soon as events have shown it to be wrong.

Suppose there is a very large unexpected receipt in period \( t \) such as a lottery win which immediately causes an upward revision in estimated wealth. A household using the Haig-Simons definition of income to guide its consumption decisions would be encouraged to spend all of its lottery winnings in the current period in order to return wealth to its initial level. In addition to being irrational, such a consumption strategy would be quite arbitrary as it would make the rate of consumption depend on the length of the accounting period (of whose existence the household may not even be aware). While these observations may appear trite, they make the Haig-Simons concept completely unsatisfactory and unacceptable as a concept of income.\(^\text{15}\) If a consumer cannot treat the whole of Haig-Simons income as being available for consumption, it cannot be used to explain the behavior of a rational consumer. It cannot be used for purposes of demand analysis or for economic analysis in general. The Haig-Simons concept of income fails when a measure of income is most needed: namely, when there is uncertainty and the consumer has to react to unexpected shocks.\(^\text{16}\)

**Generalized-Hicksian Income.** None of the objections just raised to the Haig-Simons concept of income apply to the ‘generalized-Hicksian’ income concept proposed here.
Generalized-Hicksian income estimates wealth consistently at both the beginning and the end of the period on the basis of the same information and expectations. The expectation at time $s$ of income in period $t$ is

$$E_s Y_t \equiv \text{Max}\{E_s C_t : E_s V_{t+1} \geq E_s V_t\}. \quad (20)$$

Again, generalizing (5) and (6) to allow for uncertainty, we obtain that

$$E_s (r_t V_t) = E_s R_t + E_s V_{t,t+1} - E_s V_t = E_s C_t + E_s V_{t+1} - E_s V_t. \quad (21)$$

Now, setting $E_s V_{t+1} = E_s V_t$ in (21), it follows from (20) that

$$E_s Y_t = E_s (r_t V_t), \quad (22)$$

which using (21) can be rewritten as

$$E_s Y_t = E_s R_t + E_s V_{t,t+1} - E_s V_t = E_s C_t + E_s V_{t+1} - E_s V_t. \quad (23)$$

It should be noted that $R_t$ and $C_t$ in (23) are known if $s > t$. Income as defined in (20) depends on the time, $s$, at which it is estimated. Two cases of particular interest are the start and the end of the accounting period. When $s = t$, $E_t Y_t$ is the *ex ante* definition of income discussed in Hicks (1946, 172). Also, it is worth noting that Friedman’s (1957) concept of permanent income is essentially the same as *ex ante* Hicksian income.

$$E_t Y_t = \text{Max}\{C_t : E_t V_{t+1} \geq E_t V_t\}, \quad (24)$$

so that

$$E_t Y_t = E_t (r_t V_t) = E_t R_t + (E_t V_{t,t+1} - E_t V_t) = E_t C_t + (E_t V_{t+1} - E_t V_t). \quad (25)$$

*Ex ante* income, $E_t Y_t$, is the maximum amount that a person can plan to consume in the period $t$ and expect wealth at the end of the period to be the same as estimated at the start of the period. Hicks argues that this is the concept of income relevant to consumer behavior, as does Friedman (1957) in his permanent income hypothesis.
When $s = t + 1$, the corresponding *ex post* concept of income is obtained.

$$E_{t+1}Y_t = \max\{C_t : E_{t+1}V_{t+1} \geq E_{t+1}V_t\}, \quad (26)$$

so that

$$E_{t+1}Y_t = r_t(E_{t+1}V_t) = R_t + (E_{t+1}V_{t,t+1} - E_{t+1}V_t) = C_t + (E_{t+1}V_{t+1} - E_{t+1}V_t). \quad (27)$$

Measuring income *ex post* does not mean that all uncertainty has been resolved but simply that measurement takes place after the end of the period when actual receipts and consumption in period $t$ are known. *Ex post* income is still a forward looking measure. It depends on expectations of future receipts held at the end of the period. These are the same expectations that determine *ex ante* income for the following period. It should be emphasized that *ex post* income, as defined in (26) and (27), is not the income concept labelled as *ex post* by Hicks in *Value and Capital*. Hicks chose to identify *ex post* income with Haig-Simons, defining it as “consumption plus capital accumulation”. Hicks considered it to have the advantage of being “almost completely objective” but he also dismissed it on the grounds that it must contain unexpected capital gains whereas “The income which is relevant to conduct must always exclude windfall gains” [Hicks (1946, 179)]. It is shown below that, unlike Haig-Simons income, *ex post* income excludes unexpected capital gains.

### Capital Gains

The capital gain on a given stock of assets is defined as the change in its value between two points of time. $G_{(t,t+k)}$ is used to denote the capital gain accruing between the start of period $t$ and the start of period $t + k$ on the stock existing at the start of period $t$.

$$G_{(t,t+k)} \equiv E_{t+k}V_{t,t+k} - E_tV_t \quad (28)$$

Defined this way, capital gains can be divided into expected and unexpected components. The expected capital gain accruing between the start of period $t$ and $t + k$, based on
expectations at time $s$ is

$$E_s G_{(t,t+k)} \equiv E_s V_{t,t+k} - E_t V_t. \quad (29)$$

An unexpected capital gain, denoted here by $UG$, occurs when the estimated value of the stock of assets at some point of time is changed as a result of revised expectations. The unexpected capital gain on the stock at time $t$ resulting from new information gained during the time interval $(h, l)$, where $h \leq l$, is defined as follows:

$$U_{(h,l)} G_t \equiv E_l V_t - E_h V_t. \quad (30)$$

Of particular interest are the cases where $s = t$ or $s = t + k$ in (29), and $h = t$ and $l = t + k$ in (30). For these cases, it is possible to decompose a capital gain into its expected and unexpected components (in two different ways).

$$G_{(t,t+k)} = E_t G_{(t,t+k)} + U_{(t,t+k)} G_{(t+k)} = E_{t+k} G_{(t,t+k)} + U_{(t,t+k)} G_t \quad (31)$$

It can now be seen that Haig-Simons income can be written as the sum of receipts and capital gains, i.e.:

$$Y_t^{HS} = R_t + E_{t+1} V_{t,t+1} - E_t V_t = R_t + G_{(t,t+1)}. \quad (32)$$

Similarly, generalized-Hicksian income can be written as the sum of expected receipts and expected capital gains, i.e.:

$$E_s Y_t = E_s R_t + E_s V_{t,t+1} - E_s V_t = E_s R_t + E_s G_{(t,t+1)}. \quad (33)$$

This in turn implies that ex post income and Haig-Simons income are related as follows:

$$Y_t^{HS} = R_t + E_{t+1} V_{t,t+1} - E_t V_t = (R_t + E_{t+1} V_{t,t+1} - E_{t+1} V_t)$$

$$+ (E_{t+1} V_t - E_t V_t) = E_{t+1} Y_t + U_{(t,t+1)} G_t. \quad (34)$$

Haig-Simons income is equal to ex post income plus the unexpected capital gain or loss in period $t$ resulting from new information that emerged during period $t$. Conversely, (34) shows that the way to estimate ex post income in practice is to start with Haig-Simons
income, which can be measured objectively, and then deduct those gains or losses that, at the end of the period, are known, or deemed, to have been unexpected.

Returning to the example of a lottery, suppose a household wins a lottery of value $X_t$ in period $t$ and that nothing else unexpected happens during this period. It follows that

$$U_{(t,t+1)}G_t = E_{t+1}V_t - E_tV_t = X_t,$$

$$E_{t+1}G_{(t,t+1)} = E_{t+1}V_{t,t+k} - E_{t+1}V_t = r_tX_t.$$

Referring back to equations (32) and (33) it can be seen that Haig-Simons includes the whole of the lottery winnings in income for period $t$, while ex post generalized-Hicksian income includes only the interest earned on the lottery win during period $t$.

### Income and Capital Gains on Individual Assets

The distinction between $V_{t,t+1}$ and $V_{t+1}$ is applicable to a stock of assets, which can be extended by acquisition of new assets or depleted by the sale of existing assets. However, when tracking a single asset through time from $t$ to $t+1$, $V_{t,t+1}$ and $V_{t+1}$ are the same. For single assets, therefore, the formulae for Hicksian income no. 1, Haig-Simons income and generalized Hicksian income simplify as follows:

$$Y_t = r_t V_t = R_t + V_{t+1} - V_t,$$  \hspace{1cm} (35)

$$Y_{t}^{HS} = R_t + E_{t+1}V_{t+1} - E_tV_t,$$  \hspace{1cm} (36)

$$E_sY_t = E_s(r_tV_t) = E_sR_t + E_sV_{t+1} - E_sV_t.$$  \hspace{1cm} (37)

Similarly, the formulae for the total and expected capital gain on an individual asset simplify to

$$G_{(t,t+k)} = E_{t+k}V_{t+k} - E_tV_t,$$  \hspace{1cm} (38)

$$E_sG_{(t,t+k)} = E_sV_{t+k} - E_sV_t.$$  \hspace{1cm} (39)

Since the remaining sections of the paper focus on the measurement of income from individual assets, these are the definitions of income and capital gains that will be used primarily from here on.
III. CAPITAL GAINS AND INCOME FROM FINANCIAL ASSETS: THE CASE OF FIXED-TERM BONDS

A fixed-term bond pays a coupon $R$ for $N - 1$ periods and a lump sum $L$ in period $N$. Hence the only source of uncertainty is the interest rate from period $s$ onward.

$$E_{s}V_{t} = RE_{s} \left\{ \sum_{i=0}^{N-1} \prod_{j=0}^{i} \left[ 1/(1 + r_{t+j}) \right] \right\} + LE_{t} \left\{ \prod_{j=0}^{N} \left[ 1/(1 + r_{t+j}) \right] \right\} \quad (40)$$

At any point of time, there is a firm expectation of a capital gain, or loss, accruing to the holder, or issuer, of the bond over its remaining life equal to the difference between the current market value and the redemption value. This expected gain accrues gradually as the bond approaches maturity. It is a classic example of a gain that constitutes income, the gain being identified with interest on the bond for this reason. The gradual growth in the value of the bond reflects the accumulation of the reinvested interest.

There is some dispute, nevertheless, about the measurement of income from bonds.\textsuperscript{17} This dispute has potentially important implications for the government’s fiscal position, since it is typically a large debtor and interest on the national debt can account for a large part of government expenditures. The government surplus or deficit, as the residual between income and expenditures, is therefore sensitive to any specification errors in the measurement of income from interest payments. Moreover, even very small changes in the government deficit can have significant impacts on both fiscal and monetary policy. Within the European Monetary Union the “Stability Pact” imposes draconian constraints on the economic policies of the governments of its member countries by fixing an upper limit to the government deficit as a percentage of GDP.

The controversy relates to the treatment of an unexpected change in the interest rate, and the resulting unexpected capital gain or loss for the holder and the issuer. The change in the market price of the bond automatically changes the gap between the current price and the fixed redemption value, i.e., the expected return over the remaining life of the bond, which in turn implies that the remaining interest must also change. It is this final implication that is disputed by the IMF and some other international and
national agencies. The IMF argues that the total interest over the life of a bond, and
the rate at which it accrues, cannot change, being inexorably fixed by the price at
which the bond was first issued.\textsuperscript{18} If the same interest continues to be recorded after
the unexpected interest change as before, the total interest after the change diverges
from the sum of the coupon interest and the gain expected by the market. In order to
reconcile a constant rate of interest accrual with the actual current market values of the
bonds, arbitrary adjustment items have to be introduced.\textsuperscript{19}

The interest on bonds can be derived within the theoretical framework presented
above. Consider for simplicity a zero-coupon bond issued at the beginning of period
\(t\). Also, suppose that the bond matures in period \(t + N\). While it is true that the
redemption value is fixed, i.e., \(E_t V_{t+N} = E_{t+1} V_{t+1} = \cdots = E_{t+N} V_{t+N}\), the value of the
bond at any other time, including the time of issue, is not. It depends on when the
valuation is made. The IMF and other agencies with similar views effectively define the
income from the bond in period \(t + i\) (where \(i < N\)) as follows:

\[
Y_{t+i} = E_t V_{t+i+1} - E_t V_{t+i}. \tag{41}
\]

This is a version of generalized-Hicksian income as defined in (37). However, it arbitrar-
ily and irrevocably links estimates of income in all future periods to the expectations
prevailing at the time the bond was issued. Such an estimate of income ceases to be
relevant as soon as events have shown those expectations to be wrong. Both the issuer
and the holder have the option of trading but at prices different from those previously
expected.

Suppose that the interest rate rises unexpectedly in period \(t + i\), i.e., \(r_{t+i} > E_{t+i} r_{t+i}\).
The market value of the bond drops instantaneously and bond holders incur an unex-
pected capital loss. More formally, \(E_{t+i+1} V_{t+i} < E_{t+i} V_{t+i}\) which implies, from equation
(30), that \(U_{(t+i,t+i+1)} G_{t+i} < 0\). Somewhat less intuitive is the fact that \(E_{t+i+1} Y_{t+i} > E_{t+i} Y_{t+i}\) [see equation (37)], i.e., the income earned from the bond rises. However, in the
alternative treatment advocated by the IMF and others the interest (and hence income)
received from the bond is assumed not to rise. It is necessary, therefore, to record part of the subsequent expected rise in the value of the bond as if it were unexpected and not income, even though one point on which there is some consensus in the economics literature is that expected gains of this kind are income. This means that interest payments by governments, and hence government expenditure and the budget deficit (surplus), will be underestimated (overestimated) after a rise in interest rates. Conversely, savings by the sectors holding the government debt will also be underestimated.

IV. CAPITAL GAINS AND INCOME FROM FIXED ASSETS

Depreciation and Income

The concepts of depreciation and income are closely linked. In a perfect foresight setting, Hotelling (1925) defines the depreciation on an asset of vintage \( k \) (i.e., produced in period \( k \)) used in production during period \( t \) as the decrease in its value between the beginning and end of the period.

\[
D_t^k \equiv V_t^k - V_{t+1}^k
\]  

Using equation (35), the Hicksian income no. 1, \( Y_t \), derived from a fixed asset in period \( t \) is linked to depreciation as follows:

\[
Y_t = R_t + V_{t+1}^k - V_t^k = R_t - D_t^k.
\]

\( R_t \) is often described as the gross income from an asset and \( Y_t \) as the net income, the difference between gross and net being identified with depreciation at all levels of aggregation up to GDP and NDP.

When there are unexpected shocks, competing definitions of depreciation exist in the economics literature, each of which is consistent with a different concept of income. One definition of depreciation, \( D_t^{k,HS} \), is consistent with the Haig-Simons concept of income.

\[
D_t^{k,HS} \equiv E_tV_t^k - E_{t+1}V_{t+1}^k
\]
\[ Y_t^{HS} = R_t - (E_tV_t^k - E_{t+1}V_{t+1}^k) = R_t - D_t^{k,HS} \] (44)

Comparing (44) with (38) shows that Haig-Simons depreciation, \( D_t^{k,HS} \), equals \(-G_{(t,t+1)}\), the actual capital loss on the asset in period \( t \) including any unexpected loss due to revised expectations or unexpected events such as wars or natural disasters unconnected with production.

A second definition values \( V_t \) and \( V_{t+1} \) consistently on the basis of the same information and expectations. It is associated with the generalized-Hicksian concept of income.

\[ E_sD_t^k \equiv E_sV_t^k - E_sV_{t+1}^k \]

\[ \Rightarrow E_sY_t = E_sR_t - (E_sV_t^k - E_sV_{t+1}^k) = R_t - E_sD_t^k, \] (45)

where \( E_sD_t^k \) denotes the expectation at time \( s \) of depreciation of an asset of vintage \( k \) in period \( t \). Comparing (45) with (39) shows that \( E_sD_t^k = -E_sG_{(t,t+1)} \), the expected capital loss on the asset in period \( t \). Two important special cases are when \( s = t \) and \( s = t + 1 \), referred to here as \( \text{ex ante} \) and \( \text{ex post} \) depreciation, respectively.\(^{20} \) The relationship between Haig-Simons depreciation and \( \text{ex post} \) depreciation is given in (46).\(^{21} \)

\[ -D_t^{k,HS} = E_{t+1}V_{t+1}^k - E_tV_t^k = (E_{t+1}V_{t+1}^k - E_{t+1}V_t^k) - (E_tV_t^k - E_{t+1}V_t^k) \]

\[ = -E_{t+1}D_t^k + U_{(t,t+1)}G_t. \] (46)

Hence \( \text{ex post} \) depreciation excludes unexpected capital gains or losses, whereas the Haig-Simons definition includes them.

**Unexpected Capital Losses on Fixed Assets**

The main source of unexpected capital losses on fixed assets is unanticipated obsolescence. It can result from unforeseen scientific or technological advances, price shocks (especially to oil) or changes in tastes which reduce the demand for the services of existing assets. Spectacular examples are provided by railways and ocean liners. Even buildings and structures may be eventually retired on grounds of unforeseen obsolescence,
especially as such assets may continue to be used almost indefinitely with suitable main-
tenance. Major unexpected capital losses can also result from the physical damage to
assets caused by wars, terrorist attacks, earthquakes, floods or other natural disasters.

There is still some controversy over the treatment of unexpected capital losses on
fixed assets in national accounts. The international System of National Accounts (SNA)
imply adopts the \textit{ex post} definition of income and depreciation and hence excludes
them. However, Eisner (1988) has argued in favor of a Haig-Simons concept of depreci-
ation that includes unexpected losses such as those due to natural disasters even though
these losses are unconnected with processes of production. The US national accounts
have also used the Haig-Simons concept in the past.\textsuperscript{22} It follows from (46), and the
fact that unexpected capital losses on fixed assets tend to far exceed capital gains, that
Haig-Simons will tend to overestimate depreciation, thus imparting a downward bias to
net national income and net saving. Depreciation, income and saving will also tend to
be unduly volatile.

\underline{Expected Capital Losses on Fixed Assets}

The depreciation on an asset over a period of time can be decomposed into wear
and tear and expected obsolescence. Wear and tear measures the expected decline in
the value of an asset due to the fact that the actual use of the asset reduces the total
quantity of services that it is expected to be capable of delivering over the rest of its life
and possibly also the rate at which they can be delivered. It is essentially a function of
the age of the asset.

Obsolescence, however, is a function of time and independent of the rate at which the
asset is used. An asset will be scrapped if there is no demand for its services even if it still
functions perfectly. This often happens with high technology assets such as computers
whose service lives are determined by obsolescence and not by declining performance.
As obsolescence is a common and familiar phenomenon, it will be built into the present
value of the flow of services expected from the asset at the time it is acquired and thereby
affect the rate at which it subsequently depreciates.

*Ex post* depreciation is decomposed into expected obsolescence, $E_{t+1}\Omega_{t,t+1}^{k,k+1}$, and wear and tear, $E_{t+1}\Lambda_{t+1}^{k,k+1}$, components as follows:

$$E_{t+1}D_t^k = E_{t+1}V_t^k - E_{t+1}V_{t+1}^k$$

$$= [E_{t+1}V_t^k - E_{t+1}V_{t+1}^{k+1}] + [E_{t+1}V_{t+1}^{k+1} - E_{t+1}V_{t+1}^k] = E_{t+1}\Omega_{t,t+1}^{k,k+1} + E_{t+1}\Lambda_{t+1}^{k,k+1} \quad (47)$$

$E_{t+1}\Lambda_{t+1}^{k,k+1}$ (wear and tear) in (47) measures the difference between the values of assets of vintages $k$ and $k+1$ at the end of period $t+1$, while $E_{t+1}\Omega_{t,t+1}^{k,k+1}$ (expected obsolescence) measures the decline in value from period to period of identical assets of the same age, namely the difference between the value of vintage $k$ in period $t$ and vintage $k+1$ in period $t+1$. In practice, the situation may be complicated by the fact that successive vintages may not be homogeneous, each vintage being more efficient than its predecessor. In this case, the different vintages have to be adjusted to convert them into units of constant quality in order to measure wear and tear.

Recently, an old controversy over obsolescence has re-emerged. Within the framework of vintage accounting used for capital stock measurement, Jorgenson (1989) defines depreciation not as the change in the value of an individual asset over time, but as the difference between the values of successive vintages of otherwise identical assets *at the same point of time*. Obsolescence is thereby excluded because it does not affect the relative values of the different vintages. Depreciation is restricted to wear and tear as defined above. This concept is used, for example, by Hulten and Wykoff (1996) and Fraumeni (1997).

In order to measure the income from an individual asset, however, it is necessary to know the expected change in its value over time, which logically must include the effects of expected obsolescence as well as wear and tear. This expected change has long been understood to be the appropriate concept of depreciation for income measurement. For example, in a debate with Pigou (1941), Hayek (1941) strongly advocated the inclusion of expected obsolescence, citing an example in which depreciation is entirely attributable to
obsolescence, a common enough occurrence today with PCs and other hi-tech equipment. He was subsequently backed by Hicks (1942, 178) and Samuelson (1961, 36).

We conclude that to be consistent with the generalized Hicksian concept of income, depreciation must include expected obsolescence and exclude unexpected obsolescence together with other unexpected capital losses. This is the position adopted in national accounts. The fact that there is now some dispute and confusion, with two different concepts of depreciation in circulation, is symptomatic of the fact that there is still no consensus over the underlying concept of income. If expected obsolescence were to be omitted, depreciation would be underestimated thus imparting a systematic upward bias to net national income and saving.

V. CAPITAL GAINS AND INCOME FROM NATURAL CAPITAL ASSETS

Exploration and Capital Gains

Should additions to the natural capital stock due to exploration activities be treated as unexpected capital gains or as produced investment? The answer to this question can have a profound effect on estimates of national income for resource rich countries, since if treated as investment they are part of income, while if treated as unexpected capital gains they are not.

The answer depends on the extent to which the fruits of exploration activities are expected or unexpected. Clearly, they are not completely expected, since exploration involves a significant amount of uncertainty. On the other hand, an oil company would not invest in exploration if it did not expect to find something. Scientific exploration is a productive activity, undertaken by specialist consultants, with an outcome that is predictable over the longer term even though individual exploration projects may be subject to considerable uncertainty. For each exploration project an oil company undertakes or commissions, it must have prior expectations as to how much oil it will find. The expenditures on exploration incurred by an oil or mining company are a form of investment and should provide a lower bound to the estimated value of the expected
find. If the reserves discovered are significantly greater than expected the oil company experiences an unexpected capital gain and if they are smaller it incurs an unexpected capital loss.  

Capital Gains and Spurious Fluctuations in Income and Output

An interesting example where the ‘discovery’ of new stock created a large capital gain occurred in Indonesia in 1974. Changes in US tax law and Indonesian contracts favorable to exploration activities led to a large increase in reported reserves of oil in Indonesia in 1974. As a result, Indonesia experienced a huge unexpected capital gain on its stock of natural capital. Repetto et al. (1989) estimate national income for Indonesia between 1971 and 1984. However, they use the Haig-Simons definition of income, $Y_{t}^{HS}$. As a result, Indonesia’s national income experiences a huge upward spike in 1974, which has nothing whatever to do with the level of productive activity in Indonesia or with the sustainability of its consumption. A similar but even more spectacular example is provided by the exceptional oil finds in Alaska in 1970. According to Nordhaus and Kokkelenberg (1999), these finds augmented US oil assets by nearly 50 per cent, or almost $100 billion at 1987 prices. If the Haig-Simons definition of income were to be used and the additional reserves included in output and income, this would have changed the growth of US GDP between 1969 and 1970 from 0.03 to 3.14 per cent. As Nordhaus and Kokkelenberg observe, p. 81, “the trend in real non-minerals GDP growth would have been seriously distorted, wiping out the 1970 recession and causing an apparent recession in 1971.”

Even larger spurious fluctuations can be caused by including in GDP and national income the capital gains or losses caused by fluctuating commodity prices for small countries with large reserves. For example, suppose the price of oil rises unexpectedly during period $t$, and that this price rise is perceived as reasonably long-term. This implies that, for an oil rich country, $E_{t+1}V_t > E_tV_t$, and hence it experiences an unexpected capital gain, i.e., $U(t,t+1)G_t > 0$. For a country with large oil reserves, this capital gain
could be huge relatively to GDP. In fact, it could exceed GDP. Aaheim and Nyborg (1995) find that, in some years, this is exactly what happened to Norway. If national income is measured using the Haig-Simons definition, $Y_{t}^{HS}$, and full account is taken of capital gains (losses) on natural capital, then the income series for a resource rich country may be so volatile as to be virtually useless for policy purposes, and may indeed occasionally go negative. The ex post income, $E_{t+1}Y_{t}$, of a resource rich country, by contrast, will be relatively insensitive to unexpected changes in the stock or price of commodities since it excludes unexpected capital gains.

VI. CONCLUSION

The concept of income, although fundamental to economics, continues to generate controversy and confusion, especially with regard to the treatment of unexpected capital gains. This paper has developed a general theoretical framework for measuring national and household income, and the income derived from individual assets, in the presence of unexpected capital gains. In particular, our analysis raises a number of important issues for the treatment of financial, fixed, and natural assets. These issues are of more than merely technical interest, since they can significantly affect a number of major macroeconomic aggregates, including national income and saving, balance of payments deficits, government deficits and depreciation.

REFERENCES


FOOTNOTES

*We thanks two anonymous referees for helpful comments.

Hill, R. J.: Associate Professor, School of Economics, University of New South Wales, Sydney 2052, Australia. E-mail r.hill@unsw.edu.au

Hill, T. P.: Formerly Professor, School of Economics & Social Studies, University of East Anglia, Norwich NR4 7TJ, England, and Head of Economic Statistics and National Accounts, OECD, Paris. E-mail Peter.Hill@flordon.freeserve.co.uk

1. See Gale and Sabelhaus (1999) and Joisce and Wright (2001).


3. See for example the debate between Eisner (1990), Scott (1990) and Bradford (1990) and between Laliberté (2002) and Wright (2002).

4. This concept, although often attributed to Hicks (1946), has a long history. For example, in 1832, the German economist Hermann wrote: “Income is that portion of an individual’s receipts which that individual may consume without injury to his capital stock.” [Quoted by Wueller (1938)].

5. Although Hicksian income no. 2 has received more attention in the environmental accounting literature due to its emphasis on sustainability, here we focus on Hicksian income no. 1 since it is the concept more familiar to households, firms and government. Anyway, the choice between the two concepts under perfect foresight is not the issue of this paper. We focus on the fact that economic agents face considerable uncertainty about their future resources and have to revise their expectations as unexpected events occur. The conclusions we reach about the treatment of capital gains would be the same irrespective of which of the two income concepts is used.

7. See, for example, the debate between Pigou (1941), Hayek (1941) and Hicks (1942).


9. Equations (9) and (10) show that if there is no capital gain, Hicksian income no. 1 equals consumption plus net investment. Weitzman’s (1976) seminal paper on national accounting showed that, under the assumption of a constant interest rate in an economy with a single consumption and investment good, national income, defined as the sum of consumption and net investment, can be interpreted as a measure of welfare. In our context, what is interesting is the absence of capital gains in Weitzman’s welfare measure. A natural way of incorporating capital gains into Weitzman’s model is by assuming that the price of capital changes over time in a specified way (as for example might be the case for an open economy exporting a non-renewable natural resource whose price follows the Hotelling rule) rather than being determined endogenously. In this case, the price of capital itself becomes a state variable in the Hamiltonian, and hence the resulting capital gains are part of income (and welfare).

10. See, for example, Meade and Stone (1941) and Solomons (1961).

11. The estimation of \( V_t \) is not considered here. Useful references are Jorgenson (1996) for fixed assets, Miller and Upton (1985) for natural assets, and Deaton (1992) for human capital assets.


13. Equation (16) clearly illustrates that Haig-Simons income cannot be expressed as the return (interest) earned on wealth.

14. Although consumption plus the actual change in wealth is a valid measure of income under perfect foresight, as shown by equation (9) above, the corresponding \( ex \ post \)
version under uncertainty using end of period expectations is not Haig-Simons but equation (27) as explained further below.

15. Notice that when a large receipt is foreseen with certainty during some period, for example an inheritance held in trust, or the redemption of a bond, its discounted value is already included in the household’s wealth at the start of the period. When the receipt occurs, the household’s asset portfolio changes but its total wealth is unaffected.

16. Although Haig-Simons income is, in general, a misleading guide to household consumption decisions, it could nevertheless be useful for determining the tax base. Its chief attraction in this context is that it is relatively easy to measure or assess (a matter of critical importance to tax collectors). However, its use in this context has some unfortunate consequences. In particular, as noted by Kaldor (1955), when the tax system is progressive, it discriminates against households with volatile receipts streams.

17. This dispute has been going on for some years. A comprehensive survey of the issues can be found in Joisce and Wright (2001).

18. More precisely, the IMF in its Balance of Payments Manual (1995) recognizes a change in interest when a bond is traded after a change in the interest rate has occurred. However, in the international System of National Accounts (1993) used by the IMF and national statistical offices, and for which it is jointly responsible with a number of other international agencies, no change of interest is recorded whether the bond is traded or not. A paper posted on the IMF’s website in March 2002 (see Laliberté (2002)) sets out the IMF’s position. A further paper by Wright (2002), which is broadly consistent with our approach, replying to Laliberté was posted on the IMF’s website in July 2002.

19. The same issues are currently under debate in commercial accounting. Under “fair
value accounting”, the interest would be recorded as proposed in this paper and not on the basis of the original issue price. See Joint Working Group of Standard Setters: “Draft Standard and Basis for Conclusion on Financial Instruments and Similar Items”, International Accounting Standards Committee, Dec. 2000, paras. 6.58 to 6.62, quoted in Wright (2002).

20. Hicks (1942, 177) proposed a definition of ex post depreciation which coincides with $E_s D^k_t$ when $s = t + 1$. However, this concept does not figure anywhere in the widely quoted chapter on Income in Value and Capital (1946) and is not consistent with ex post income as defined in that chapter.

21. Since depreciation is defined as a decrease in the value of an asset over time, while an unexpected capital gain is an increase in the value of an asset, it is convenient (following Hotelling) to multiply through by minus 1 in order to reverse the signs in (46).

22. See Katz and Herman (1997).

23. In practice, the potential capital gains or losses are asymmetrical as the possible gains are unlimited while the losses are constrained by the costs of the exploration.

24. See also Aslaksen et al. (1990).

25. To see why this is the case, see equation (34).