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Abstract

In this paper we propose methodology for the analysis of the relationship between foreign exchange rates and direct central bank intervention. A stochastic volatility model with jumps is employed for the exchange rate, while a threshold model is used for intervention. The jump and latent volatility processes in the stochastic volatility model and latent intervention in the threshold model, are endogenous. To account for this, both models are estimated jointly using Markov chain Monte Carlo (MCMC). The model is applied to the analysis of intervention by the Reserve Bank of Australia (RBA) in the Australian/US dollar exchange rate from 1983 to 2003. We demonstrate a number of key advantages of our approach over existing alternatives. First, employing a stochastic volatility model better captures the spikes in volatility in comparison to an EGARCH alternative. Second, estimating the stochastic volatility model using quasi-maximum likelihood dramatically oversmooths the volatility relative to exact finite sample inference computed using MCMC. This is particularly important in the analysis of central bank activity because intervention is most likely to occur on days with high volatility. Third, accounting for the endogeneity in the exchange rate and intervention models by estimating them jointly improves inference substantially compared to
the two-stage estimation alternative used in previous studies. The empirical work suggests that RBA intervention is partially precipitated by volatility in the foreign exchange rate. However, RBA intervention appears to have exacerbated contemporaneous volatility between 1983 and 1993, but has since avoided having any effect. Analysis of lagged volatility suggests one reason may be improved targeting of intervention to address contemporaneous volatility, as opposed to volatility occurring on previous trading days. The RBA does not appear to respond to jumps identified in the exchange rate.

**Key Words:** Markov chain Monte Carlo, Threshold Model, Reserve Bank of Australia, Quasi-Maximum Likelihood, Stochastic Volatility, Jump Process

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1 Introduction

A floating exchange rate regime is preferred by most developed and emerging economies, and episodes of high volatility are frequently recorded. Central banks use various instruments to influence key exchange rates, with direct intervention (purchase or sale of domestic for foreign currency) being one of these (Neely, 2000; Sarno and Taylor, 2001). While the aims of such direct intervention differ both over time and between central banks, one frequently stated and prominent aim is to reduce exchange rate volatility (King, 2003). The success, or otherwise, of such foreign exchange (FX) intervention is hard to assess using parametric econometric models for several reasons. First, the choice of an appropriate time series model for FX rates is a hotly debated issue (Kim, Shephard and Chib, 1998; Tauchen, 2001). Second, direct intervention is dependent upon FX volatility, which is a process that is not directly observable. Third, the propensity to intervene and volatility are endogenous, in that high volatility results in a greater tendency to intervene and intervention is itself often aimed to reduce volatility.

Most previous research into the effects of central bank intervention on FX volatility has focused on fitting variants of the GARCH model to FX returns, where observed intervention is included as an exogenous variable in the mean and conditional variance equations. This empirical research has consistently indicated that (i) intervention and FX return volatility are either positively related or have no significant relationship at all and (ii) intervention to support a depreciating (resist an appreciating) domestic currency is associated with depreciation (appreciation) in the domestic currency. These results are common to a number of GARCH-based studies of intervention by the Federal Reserve (Doroodian and Caporale, 2001), the European Central Bank (Frenkel et al, 2001), the combined monetary authorities of the G-3 countries (Almekinders and Eijffinger, 1996; Dominguez, 1998; Beine et al, 2002), the Swedish Riksbank (Aguilar and Nyhdahl, 2000), the Bank of Japan (Beine and Szafarz, 2002; Kim and Sheen, 2004) and the Reserve Bank of Australia (Kim et al. 2000; Edison et al, 2003). Conclusion (i) has also been found by a number of recent studies using volatility estimates implied by option prices (Bonser-Neal and Tanner, 1996; Galati and Melick, 2002;
D’Souza, 2002; Rogers and Siklos, 2003), while conclusion (ii) has been supported by event study approaches of Edison et al. (2003) and Fatum and Hutchinson (2003). However, these findings are not without qualifications, as suggested by studies (Dominguez, 1998; Kim et al. 2000; Beine et al, 2003; Beine and Szafarz, 2002; Fatum and Hutchinson, 2003) that observe that the relationship between intervention and the moments of FX returns often differs when large, reported, sustained, and coordinated (between multiple central banks) interventions are distinguished from small, unreported, isolated trades.

Conclusions (i) and (ii) may also be a consequence of the endogeneity between the tendency to intervene and the contemporaneous FX market conditions (in particular volatility) that prompt that decision (Kearns and Rigobon, 2003). Both results are consistent with central banks “leaning against the wind”; that is, a bank intervening to oppose volatile market movements. This insight has led to studies where the central bank’s decision to intervene is modeled as a function of FX market conditions, including the volatility of FX returns and the level of the FX spot rate. In keeping with the latent nature of the tendency to intervene (observed only insofar as it leads to actual intervention), limited dependent variable models are often employed. These studies face the complication that FX volatility is also latent. A common solution is the following - estimate FX return volatility by some variant of the GARCH model or using volatilities implied from option pricing models, and then use the point estimate of volatility as a regressor in the probit (Baillie and Osterberg, 1997; Kim and Sheen, 2002; Frenkel et al, 2003), tobit (Almekinders and Eijffinger, 1994), friction (Almekinders and Eijffinger, 1996; Kim and Sheen, 2002, 2004), or other models (Galati and Melick, 2002; Ramchander and Sant, 2002; Rogers and Siklos, 2003). However, such a two-stage approach does not adequately account for the variation inherent in latent volatility, nor the endogeneity between the tendency to intervene and volatility, so that inference is inaccurate.

To address these problems this paper proposes to employ a stochastic volatility (SV) model for the FX rate, where volatility is a latent stochastic process following an autoregression. Parametric nonlinear models for the effect of direct intervention on both the first and second moments of the FX rate are proposed. A jump process is also included for the mean
of the FX returns, and the model is adjusted to account for missing observations, which is often a feature of data in this area. A threshold model for intervention is employed, where FX volatility and jumps in the mean are related to a partially observed latent intervention process in a parametric nonlinear manner. The latent FX volatility, jump process and partially observed intervention process are endogenous. The entire model is estimated jointly using a Bayesian Markov chain Monte Carlo (MCMC) sampling scheme. The sampling scheme employs steps that are adaptations of those discussed recently in the econometrics and statistics literatures.

The model and estimation methodology proposed in this paper improves upon previous work on central bank intervention and FX rates in several important ways. First, conditional variance in the GARCH family of models is deterministic, which can lead to an overly smooth measure of volatility relative to that obtained from an equivalent SV model. This is particularly important in light of the role of the volatility process in our model for central bank intervention. Second, joint estimation of the threshold and SV models is considered, which is important due to the endogeneity between tendency to intervene and FX volatility. This resolves a longstanding problem in the literature that hitherto inhibited reliable assessment of the effectiveness of direct intervention. Third, recent research points towards the superior properties of exact finite sample inference computed using MCMC for the SV model in comparison to quasi-maximum likelihood (QML) and a number of method of moments based approaches (Kim et al., 1998; Andersen et al., 1999). In particular, in our empirical work QML is shown to dramatically oversmooth the volatility process. Moreover, estimation of the threshold model using MCMC proves to be a reliable and flexible approach (Albert and Chib, 1993).

Our approach is employed to model intervention by the Reserve Bank of Australia (RBA) in the Australia/US dollar exchange rate during the period 1983 to 2003. The analysis is also undertaken for sub-periods where the RBA follows identifiably different intervention strategies. The determinants of this intervention and its effects on the volatility of the FX rate are studied jointly. It is shown how the SV model allows for more accurate estimation of the volatility process than an equivalent EGARCH model, in particular on the higher
volatility days. Similarly, estimation of the SV model using QML is shown to result in severe oversmoothing of the volatility process, compared to exact finite sample inference computed using MCMC. As in the case of the EGARCH model, the highest volatility days are the most dramatically affected, which is crucial here because these are exactly the days the RBA intervenes most frequently and strongly. Joint estimation of both the threshold and SV models is shown to improve inference on the parameters of the threshold model, compared to estimating the parameters conditional on volatility point estimates.

Throughout the period, higher volatility is shown to increase the probability of intervention by the RBA on the opposite side of the market. However, we find that RBA intervention appears to have exacerbated volatility from 1983 to 1993, but has since managed to avoid having any effect. Jumps in the mean of the exchange rate returns were detected in the early period, but surprisingly the RBA did not respond differently to these. One possible explanation is that jumps represent significant agreement amongst market participants, and intervention is likely to be counter-productive in such circumstances. An analysis of the effect of lagged volatility suggests that RBA intervention is partially prompted by volatility on previous trading days. In the early history of intervention by the RBA, market volatility on up to four previous days appears to influence intervention decision-making. However, over the past ten years, intervention appears to have become better targeted at smoothing contemporaneous volatility, as opposed to volatility observed in the market over previous trading days.

The rest of the paper is organized as follows. Section 2 discusses the modeling undertaken in the paper. This includes the SV model with jumps employed for the FX rate, along with its extension to account for missing observations. The threshold model is outlined, along with the issue of endogeneity between FX rate volatility and central bank tendency to intervene. Section 3 discusses approaches available to estimate the SV model, and outlines the Bayesian sampling scheme used to jointly estimate the SV and threshold models. Section 4 employs the estimator to analyze the Australian data on RBA intervention in the Australian/US dollar exchange rate. Section 5 concludes the paper.
2 The Model

2.1 Stochastic volatility with jumps and missing observations

The univariate stochastic volatility model has proven a popular alternative to the GARCH family of models, including for FX returns data (Ruiz, 1994; Kim et al., 1998; Lee, 2000; Morana and Beltratti, 2000). They are more flexible, in that the conditional volatility of the series is stochastic, compared to the deterministic conditional volatility in GARCH (Jacquier et al., 1994; Liesenfeld and Jung, 2000). This results in estimates of the volatility process which better capture any spikes. This may be particularly important in studies of central bank intervention as spikes in the volatility are likely to be associated with intervention. It is also straightforward to extend the SV model to include a wide variety of effects, including various jump processes in the mean, leverage effects and fat tails; see Chib et al. (2002), Jacquier et al. (2003) and references therein. In this study we employ a SV model with jumps in the mean and missing observations as the parametric model for FX returns.

The SV model with jump component (SVJ) is defined as

\[ y_t = k_t q_t + \sigma_t e_t \]
\[ h_t = s'_t \mu + \phi(h_{t-1} - s'_{t-1} \mu) + \sigma_\eta \eta_t, \]  

(2.1)

where \( h_t = \log(\sigma_t^2) \) and \( y_t \) is the continuously compounded return at time \( t \), possibly adjusted for any mean component. The two series \( \{e_t\} \) and \( \{\eta_t\} \) are independent white noise, so that there are no leverage effects. The latent variable \( q_t \) indicates whether a jump in the mean of returns has occurred at time \( t \) and is distributed Bernoulli with the probability of a jump occurring \( \kappa \), while the latent variable \( k_t \) is the size of any jump at time \( t \). The mean of the log-volatility at time \( t \) is \( s'_t \mu \), where \( \mu \) is an \((m_1 \times 1)\) vector of coefficients that require estimation. The vector \( s_t \) is a function of central bank intervention- see section 4.1 for specification in the Australian case. The mean-corrected log-volatilities follow an autoregressive model of lag one with coefficient \( \phi \) and variance \( \sigma_\eta^2 \). Note that the SVJ model is a Euler discretization of a continuous time Ornstein-Uhlenbeck process with Levy component.

The SVJ model can be restated to account for missing observations in a straightforward
manner. Denote the time at which the $i$th observation was made as $t_i$, and set $\Delta t_i = t_i - t_{i-1}$, then equation (2.1) implies

\[
y_{t_i} = k_{t_i}q_{t_i} + \sigma_{t_i}e_{t_i} \\
h_{t_i} = s'_{t_i}\mu + \phi^{\Delta t_i}(h_{t_{i-1}} - s'_{t_{i-1}}\mu) + \epsilon_i,
\]

(2.2)

where $\epsilon_i = \sigma_\eta \sum_{j=0}^{\Delta t_i-1} \phi^j \eta_{t_i-j}$ and $i = 1, \ldots, n$.

Here, there are $n$ observations and the autocovariance between observations $t_i$ and $t_{i-1}$ in the mean-corrected log-volatilities is $\phi^{\Delta t_i}$, which decreases for observations further apart in time, while the variance of the disturbance $\text{Var}(\epsilon_i) = \sigma_\eta^2 \sum_{j=0}^{\Delta t_i-1} \phi^{2j}$ increases. We label this model as the missing observation stochastic volatility model with jumps (MOSVJ).

An alternative method of accounting for missing observations would be to discretize the underlying continuous time Ornstein-Uhlenbeck process over the unequally-spaced grid as in Asai and McAleer (2003). However, we do not do so here because estimation of the MOSVJ model proves to be a straightforward extension of existing procedures. Nevertheless, given the short gaps in the daily data examined in section 4, either approach is likely to lead to similar empirical results.

### 2.2 A threshold model of central bank intervention

Following previous authors (Almekinders and Eijffinger, 1996; Kim and Sheen, 2002; Kearns and Rigobon, 2003) a threshold model for central bank intervention is employed. Actual intervention $z_t$ corresponds to desired intervention $z^*_t$ on day $t$ only if the latter exceeds thresholds $\theta^+$ and $\theta^-$, so that

\[
z_t = \begin{cases} 
0 & \text{if } \theta^- < z^*_t < \theta^+ \\
 \quad z^*_t & \text{otherwise} .
\end{cases}
\]

(2.3)

Latent intervention follows a linear model, so that $z^*_t = w'_t \alpha + u_t$, with $u_t$ distributed independently $N(0, \sigma_u^2)$, for $t = t_1, \ldots, t_n$, and $w_t$ is an $(m_2 \times 1)$ vector of explanatory variables. The thresholds may be interpreted as representing the costs of intervention, or alternatively, the minimum quantity of intervention required to register with other market participants.
The error process \( \{u_t\} \) is assumed independent of the error processes \( \{e_t\} \) and \( \{\eta_t\} \) in equation (2.1).

A central bank intervenes because it seeks to correct sub-optimal market conditions. One key symptom of these conditions is FX rate volatility and another may be jumps in the mean. In this case \( w_t \) will be a function of \( \sigma_t \) and \( k_t q_t \), so that \( \{\sigma_t\} \), \( \{k_t q_t\} \) and \( \{z_{1t}^*\} \) are endogenous processes; for the specification in the Australian case study see section 4.1. Joint estimation of the threshold and MOSVJ models is therefore preferable, and this is discussed further in section 3.

3 Estimation

Estimation of the univariate SV model has been the subject of much discussion over the last decade; see Ruiz (1994), Jacquier et al. (1994), Andersen et al. (1999), Kim et al. (1998) and Chib et al. (2002) and numerous references therein. Method of moments based estimators can prove unstable for this model (Jacquier et al. 1994; Andersen et al. 1999) and quasi-maximum likelihood (QML) is a popular and stable alternative (Ruiz, 1994). However, as we demonstrate in the empirical section, QML can dramatically oversmooth the volatility process and it is difficult to adapt to joint estimation of the threshold model, as well as any extensions of the basic SV model. In comparison, Bayesian finite sample estimation for the SVJ model using MCMC is discussed in detail in Jacquier et al. (1994), Kim et al. (1998) and Chib et al. (2002). We ascribe similar priors and employ a similar sampling scheme to estimate the MOSVJ model as those used by these authors. Bayesian estimation of the threshold model follows the work of Albert and Chib (1993). By combining the sampling steps of both procedures, the threshold and MOSVJ can be estimated jointly, which is difficult to achieve using other methods of estimation. In our empirical work in section 4 we show that this can make a significant difference in the analysis of central bank intervention.
3.1 Priors

For the SV model the priors suggested in Kim et al. (1998) are adopted. The autoregressive coefficient $\phi = 2\phi^P - 1$, where $\phi^P$ is distributed Beta($a_\phi$, $b_\phi$), with $a_\phi = 20, b_\phi = 1.5$. The scale $\sigma^2_\eta \sim IG(a_\sigma/2, b_\sigma/2)$ with $a_\sigma = 5$ and $b_\sigma = 0.01$. Both proper priors are dominated by the likelihood and are uninformative relative to the posterior in most empirical analysis of FX data, including that in section 4. An improper flat prior $p(\mu) \propto$ constant is assumed. For the jump parameters we follow Chib et al. (2002) and reparameterize in terms of $\psi_{t_i} = \ln(k_{t_i} + 1)$, which is assumed to follow the informative prior distribution $N(-0.5\delta, \delta^2)$. The hyperprior is a log normal distribution with $\ln(\delta) \sim N(-3.07, 0.149)$. The parameter $\kappa$ has a conjugate Beta(2, 100) prior, which implies a prior probability of a jump of occurring of 0.0196 with a standard deviation of 0.02. In the threshold model a flat prior on the thresholds $p(\theta^+, \theta^-) \propto I(\theta^+ > 0)I(\theta^- < 0)$ is assumed, where $I(A) = 1$ if $A$ is true and zero otherwise. The priors $p(\alpha) \propto$ constant and $p(\sigma^2_u) \propto 1/\sigma^2_u$ are also assumed.

3.2 Markov chain Monte Carlo

The joint posterior distribution of the model parameters for the MOSVJ and threshold models jointly, or separately, is analytically intractable. Consequently, an MCMC sampling scheme is used to evaluate the posterior. The scheme can be split into three steps: first generate the latent volatilities and parameters in the MOSVJ model, second generate the latent jump variables in the MOSVJ model, and third generate the latent intervention variables and parameters in the threshold model. We outline these three steps below, but refer to previous published work for most of the details.

Step (1): Generate latent volatilities and parameters associated with the SV model

The first step is based on the sampling scheme in Kim et al. (1998; section 3.2), but adjusted to account for missing observations. Let $y = (y_{t1}, \ldots, y_{tn})'$ and $h = (h_{t1}, \ldots, h_{tn})'$ be the data and latent volatilities associated with the MOSVJ model. We follow the popular approach of recasting the model as a conditionally Gaussian linear state space (LSS) model (Harvey,
Ruiz and Shephard, 1994) as follows. Let \( y_i^* = \log(y_i - k_t q_t)^2 - s_i \mu, \) then

\[
\begin{align*}
y_i^* &= x_i + \xi_i \\
x_{i+1} &= F_i x_i + \epsilon_i,
\end{align*}
\]

where \( x_i = h_{t_i} - s_i ' \mu \) and \( F_i = \phi^{\Delta_{t_i}}. \) The observation equation error \( \xi_i \) is distributed log chi-squared with 1 degree of freedom, which can be approximated accurately by a mixture of seven normals– see Kim et al. (1998). Introducing indicators \( \gamma = (\gamma_1, \ldots, \gamma_n)' \), and denoting the means, variances and weights of the seven component approximation by the triplet \( \{m(j), v(j), \omega(j)\} \) for \( j = 1, \ldots, 7 \), the error \( \xi_i | \gamma_i \sim N(m(\gamma_i), v(\gamma_i)) \) so that (3.1) is a conditionally Gaussian LSS model. Let \( q = (q_{t_1}, \ldots, q_{t_n})', \psi = (\psi_{t_1}, \ldots, \psi_{t_n})' \) and \( \Pi = \{\phi, \mu, \sigma, q, \delta, \psi, \kappa\}' \) be the parameters and latent jump variables in the MOSVJ model. If \( \{\Pi \backslash A\} \) is the set \( \Pi \) omitting element \( A \), then the following is step one of the sampling scheme:

(1a) Generate from \( p(h|y^*, \gamma, \Pi) \)

(1b) Generate from \( p(\gamma|h, y^*) \)

(1c) Generate from \( p(\sigma^2|y, h, \{\Pi \backslash \sigma^2\}) \)

(1d) Generate from \( p(\mu|h, \{\Pi \backslash \mu\}) \)

(1e) Generate from \( p(\phi|h, \{\Pi \backslash \phi\}) \)

Step (1a) employs the conditionally Gaussian LSS formulation outlined at (3.1) and generates the states as a block using the approach of Carter and Kohn (1994); see also Shephard (1994). These authors show that generating the states as a block is preferable to generating the states in such Gaussian LSS models one element at a time. We note here that a number of alternative approaches to generate the states exist (de Jong and Shephard 1995; Shephard and Pitt, 1997).

The posterior at step (1b) \( p(\gamma|h, y^*) = \prod_{i=1}^n p(\gamma_i|h, y^*) \), so that each mixture indicator \( \gamma_i \) is generated independently from its seven-point discrete posterior distribution as in Kim et
al. (1998). The posterior for $\sigma^2$ at step (1c) is an inverse gamma density, while the posterior for $\mu$ in step (1d) is a multivariate normal, from which it is straightforward to generate. However, the posterior for $\phi$ is unrecognizable in step (1e), so that a Metropolis-Hastings step is used. We found that the Gaussian proposal centered around the sample autocorrelation employed by Kim et al. (1998) resulted in acceptance rates below 1% in our empirical analysis using unequally spaced data. Instead, we employ a random walk proposal constrained to the stationarity region (-1,1), which in our empirical work results in acceptance rates between 18.3% and 62.7%. Steps (1c) through (1e) differ somewhat to those in Kim et al. (1998) and are therefore detailed in the appendix.

**Step (2): Generate latent variables associated with the jump component**

Step two of the sampling scheme is as follows:

(2a) Generate $p(q|h, \kappa, y)$

(2b) Generate $p(\delta, \psi|h, q, y)$

(2c) Generate $p(\kappa|q, y)$

Generation of the binary jump indicators $q$ in step (2a) can be undertaken through independent sampling of each element because $p(q|h, \kappa, y) = \prod_{i=1}^{n} p(q_{ti}|h_{ti}, \kappa)$. Generation of the block in step (2b) is undertaken using the decomposition $p(\delta, \psi|h, q, y) = p(\delta|h, q, y) \prod_{i=1}^{n} p(\psi_{ti}|h, q, \delta, y)$. First, $\delta$ is generated from $p(\delta|h, q, y)$ using a Metropolis-Hastings step with a Gaussian proposal density. This is followed by generation of each of the elements of $\psi$ from $p(\psi_{ti}|h, q, \delta, y)$ which is Gaussian. Generation of $\kappa$ is from a Beta density in step (2c). For full details of step 2 see Chib et al. (2002).

**Step (3): Generate latent variables and parameters associated with the threshold model**

The last step of the sampling scheme involves generating components of the threshold model conditional on the volatilities $h$. Let $z = (z_{t1}, \ldots, z_{tn})'$ and $z^* = (z^*_{t1}, \ldots, z^*_{tn})'$ be the observed and partially observed latent intervention data. Then append the following steps to the sampling scheme:
(3a) Generate $p(\theta^+, \theta^-|z^*, z)$

(3b) Generate $p(z^*|z, h, \alpha, \sigma_u^2, \theta^+, \theta^-, q, \psi)$

(3c) Generate $p(\alpha, \sigma_u^2|z^*, h, q, \psi)$

In step (3a) the posterior $p(\theta^+, \theta^-|z^*, z) = p(\theta^+|z^*, z)p(\theta^-|z^*, z)$ where both $\theta^+$ and $\theta^-$ are generated independently from uniform distributions. In step (3b) the elements of $z^*$ are conditionally independent, where only $z^*_t|z_t = 0$ needs to be generated from a constrained normal. For step (3c) we first note that $p(\alpha, \sigma_u^2|z^*, h, q, \psi) = p(\sigma_u^2|\alpha, z^*, h, q, \psi)p(\alpha|z^*, h, q, \psi).$ To generate the pair we therefore first generate from $p(\alpha|z^*, h, q, \psi)$ which is a multivariate t-distribution, and then from $p(\sigma_u^2|\alpha, z^*, h, q, \psi)$ which is an inverse gamma. For further details on step (3) see the appendix.

3.3 Inference

The sampling scheme was run for a burnin of 20,000 iterates, followed by a Monte Carlo sampling period of $J = 100,000$ iterates that are assumed to come from the joint posterior distribution of the parameters of the MOSVJ and threshold models. Mixture estimates of posterior means were calculated as point estimates for most parameters, except for $E(\phi|y, z)$ and $E(\delta|y, z)$ where only histogram estimates can be calculated. Marginal posterior 100(1 - $\alpha$)% probability intervals were calculated for parameters by ordering the sample and counting off $J\alpha/2$ of the smallest and largest values. In the empirical analysis in section 4 these intervals are used to identify statistically significant parameters. Posterior mean estimates of the volatilities $E(h|y)$, or ‘smoothed volatilities’, were calculated using a histogram estimate.
4 Intervention by the Reserve Bank of Australia

4.1 The Model

The effect of RBA intervention on FX rate volatility is incorporated through the mean of the log-volatilities in equation (2.2), which we model with the parametric form

\[ s_t' \mu = \mu_0 + (\mu_1 + \mu_2 \text{CUM}_t + \mu_3 \text{SIZ}_t + \mu_4 \text{REP}_t)|z_t| \]

\[ + \mu_5 |\Delta \text{IDIF}_t| + \mu_6 |\text{STATE}_t|. \] (4.1)

The effect of absolute intervention $|z_t|$ on volatility occurs as an interaction with three dummy variables: CUM$_t$, SIZ$_t$ and REP$_t$. Here, CUM$_t$ is equal to 1 if the intervention on day $t$ was proceeded by intervention in the same direction on the previous two trading days, and 0 otherwise. SIZ$_t$ is equal to 1 if the absolute value of intervention on day $t$ was greater than the mean absolute value of all intervention in the sample period, and 0 otherwise. REP$_t$ is equal to 1 if the intervention on day $t$ was reported in the financial press and 0 otherwise. These three dummy variables appear as interaction terms with $z_t$ to isolate the potentially different effects of cumulative, sizable and reported intervention relative to smaller unreported interventions. They represent a nonlinear relationship between intervention and expected log-volatility. Similar interaction terms were used previously by Kim et al (2000) in an EGARCH model for a subset of the data analyzed here.

The specification also incorporates remarks by representatives of the RBA or the Australian Government regarding the desired direction of movement in the exchange rate. The variable STATE$_t$ is equal to 1 if an appreciation was desired, and $-1$ if a depreciation was desired. In addition, changes in the monetary policy of either country, as measured by shifts in the overnight interest rate differential ($\Delta \text{IDIF}_t = \text{IDIF}_t - \text{IDIF}_{t-1}$) between the US and Australia is included. That is, if $i^US_t$ is the Federal Funds Rate for overnight loans set by the US Federal Reserve, and $i^AUS_t$ is the overnight cash rate set by the RBA, $\text{IDIF}_t = i^US_t - i^AUS_t$.

Intervention may also impact on the first moment of FX returns. Therefore, we adjust the continuously compounded returns $\Delta p_t$ for a parametric mean component

\[ x_t' \beta = \beta_0 + (\beta_1 + \beta_2 \text{CUM}_t + \beta_3 \text{SIZ}_t + \beta_4 \text{REP}_t)z_t \]
\[ y_t = \Delta p_t - x_t' \beta. \]  This is a similar parametric model as employed for the mean of the log-volatilities. However, in particular the RBA’s intervention is characterized as ‘leaning against the wind’ when a purchase of US dollars is associated with a depreciation of the Australian dollar, so that \( \beta_1 < 0 \). Introduction of this mean component results in empirical fragility in the SV model and it proves difficult to estimate the coefficients \( \beta \) jointly with the rest of the parameters using any method of estimation. We therefore follow a stream of previous authors (Harvey and Shephard, 1996; Ball and Torous, 1999; Smith, 2002; Kalimpalli and Susmel, 2003) who, when faced with this problem, mean correct the data by estimating \( \beta \) using least squares. While this is not an optimal solution, we note that our focus here is on accounting for the endogeneity between volatility and intervention process, not with the mean of FX returns.

To construct the threshold model, we note that the RBA has stated that defending a particular level of the currency is neither a desirable nor a feasible outcome of intervention operations. Statements by senior RBA officials (Fraser, 1992; Macfarlane, 1993; Rankin, 2002) over its twenty year history of intervention have highlighted the significance of three market conditions to which the Bank responds. The following is drawn from Rankin (2002):

1. Deviations of the spot exchange rate from long term fundamentals that are due to ‘herd behavior, speculative bubbles and fads.’ The RBA claims to respond symmetrically to positive and negative overshotting of the exchange rate. Intervention under these circumstances ‘...could succeed in keeping the exchange rate near its equilibrium level and obviating the need for costly adjustment by the real economy to the incorrect signals which the exchange rate would otherwise give.’

2. Market uncertainty that results from ‘unexpected changes in monetary policy.’ When unexpected changes in interest rates are likely to cause volatile movements in the spot rate, the RBA believes there is a signaling role for intervention to minimize ‘the costs of surprising the market’, thus ‘allowing room for monetary policy to move ahead of market expectations.’
3. Rapid movements in the spot rate in the short term that ‘in the absence of clear 
overshooting ... can at times threaten the orderly functioning of the market, leading to 
a widening of spreads ... and a loss of liquidity.’ On such occasions, RBA intervention 
can ‘restore order by ensuring there is at least one participant - namely the Reserve 
Bank - on the wrong side of the market.’

Based on this information, the following parametric form adapted from that used by Kim 
and Sheen (2002) is employed in the threshold model in our study:

\[ w' \alpha = (\alpha_0 + \alpha_1 \text{SIZDEV}_t) \text{ERDEV}_t + \alpha_2 \text{IDIF}_t + \\
(\alpha_3 + \alpha_4 \text{SIZVOL}_t) \text{RET}_t \sigma_t + \alpha_5 (k_t q_t) \quad (4.3) \]

The variable \( \text{ERDEV}_t \) is a proxy for the deviations of the spot rate from the underlying trend. It is calculated as the difference between the spot rate on day \( t \) and the moving average of the spot price calculated over the previous 150 trading days. Particularly sizable deviations from trend are accounted by the binary variable \( \text{SIZDEV}_t \) which is equal to 1 when the difference between the spot rate and 150 trading day moving average exceeds 5% of the value of the spot rate, and 0 otherwise. It is expected that \( \alpha_0 \) is positive, so that the larger the deviation from trend, the more likely the RBA will intervene. Due to its limited FX reserves, the RBA might be less likely to intervene where particularly sizable trend deviations have taken place, so that \( \alpha_1 \) may be negative. The inclusion of \( \text{IDIF}_t \) in equation (4.3) captures occasions where the RBA intervenes to pre-empt market reactions to changes in either Australian or US monetary policies. If the RBA intervenes to support the domestic currency in light of a positive shift in \( \text{IDIF}_t \) that would cause an immediate depreciation of the Australian dollar, so that \( \alpha_2 \) would be expected to be negative.

The inclusion of \( \sigma_t \) in equation (4.3) is based on the hypothesis that the RBA intervenes to resist short-run volatility in FX markets. \( \text{RET}_t \) is a directional variable set \( \text{RET}_t = 1 \) iff \( \Delta p_t > 0 \), \( \text{RET}_t = -1 \) iff \( \Delta p_t < 0 \) and \( \text{RET}_t = 0 \) in the rare case that \( \Delta p_t = 0 \). \( \text{SIZVOL}_t \) is a binary variable equal to 1 if \( \sigma_t \) exceeds the mean of the volatilities over the estimation period, and 0 otherwise. It is expected that \( \alpha_3 \) would be positive, so that if \( \Delta p_t \) is positive on day \( t \), more volatile trading would be associated with a greater inclination to purchase US dollars.
in exchange for Australian dollars. The parameter $\alpha_4$ captures the different relationship when the market is especially volatile. If the RBA believed that the market was so volatile that intervention could have little practical calming effect, then $\alpha_4$ would be negative. The parametric form at equation (4.3) is therefore a nonlinear function of both the observed spot rates and latent volatilities. The jump term $k_t q_t$ is included to capture RBA reaction to sudden large changes in the FX rate, as distinct from the volatility as captured by the process $\{\sigma_t\}$.

4.2 Data and the Australian Environment

The intervention data $\{z_t\}$ are daily net purchases of US dollars by the Reserve Bank of Australia (RBA) for the 4956 trading days on the Australian Stock Exchange (ASX) between the floating of the Australian dollar on December 12, 1983 and June 30, 2003. The data are expressed in units of $100$ million Australian dollars. Information on intervention operations is released by the RBA with a delay and the present data are the most up to date at the time of writing. The Australian cash rate and US Federal Funds rate were obtained from the RBA and US Federal Reserve’s data depository. The FX returns data $\Delta p_t = 100(\ln(P_t) - \ln(P_{t-1}))$, where $P_t$ denotes the US dollar spot price of an Australian dollar recorded as the daily inter-bank closing mid-rate in Sydney. The $\text{REP}_t$ and $\text{STATE}_t$ variables are those used in Kim et al (2000). An intervention is listed as reported if the *Australian Financial Review* reported the action in the following day’s edition. Statements by Australian Commonwealth Government and RBA officials on the position of the exchange rate were noted from both a record of official statements and the Australian financial press. Data are available on these two variables only between December 12, 1983 and November 30, 1993.

—–Figure 1 about here.—–

Figure 1 plots the daily spot rate, absolute daily continuously compounded returns, daily net purchases of foreign currency by the RBA and the interest rate differential for the period December 1983 to June 2003. In keeping with previous studies of the RBA’s intervention history (Kim et al, 2000; Kim and Sheen, 2002; Edison et al, 2003; Kearns and Rigobon, 2003).
2003; Aruman, 2004) we decompose the data into a number of sub-periods that correspond to different intervention strategies, in addition to studying the post-deregulation dataset as a whole. The following segmentation is employed here, and the descriptive statistics of each period are provided in table 1.

I. 12 December 1983 to 30 June 1986: For the initial 30 month period following deregulation, the RBA maintained a low profile with regular (51% of trading days) but small (average trade of $13.67 million) trades targeting FX market volatility (Rankin, 2002).

II. 1 July 1986 to 30 September 1991: For the five year period leading up to Australia’s recession, the RBA consistently pursued larger (average trade of $63.31 million) and more frequent (68% of trading days) interventions to resist deviations of the spot rate outside perceived acceptable limits (Rankin, 2002).

III. 1 October 1991 to 31 November 1993: For the two year period beginning with the 1991 recession, less frequent (23% of trading days) sizable purchases (average trade: $145 million) of domestic currency were consistently employed by the RBA to prevent the depreciation of the domestic currency that was anticipated to accompany the RBA’s loosening of monetary policy.

IV. 1 December 1993 to 30 June 1995: No intervention took place between December 1993 and June 1995, marking the longest period of absence of the RBA from the FX market since deregulation.

Periods I through IV are identified as separate regimes of intervention by the RBA, and are discussed in greater detail in Kim et al (2000) and Rankin (2002). The RBA claims that the entire period from Jul. 1995 until the end of the currently-available dataset is a single regime of intervention. We disagree with this characterization of a period that has seen significant economic change and clear shifts in RBA intervention strategy, and suggest the following segmentation instead:

V. 1 July 1995 to 25 August 1997: Characterized almost exclusively by regular (51% of trading days) modest purchases of foreign currency (average trade: $39.24 million), this
period saw the RBA accumulate foreign exchange reserves in order to retire the large swap positions built up in sub-period III.

VI. 26 August 1997 to 11 September 2001: This period begins with the RBA’s defense of the domestic currency following the Asian Financial crisis of 1997-1998 and ends with the 11 September 2001 terrorist attacks. Following the concerted defense of the currency in late 1997 / early 1998, the RBA continued to accumulate foreign currency reserves before undertaking another defense of the domestic currency in the first half of 2001 as the currency fell to an all time low, just below US $0.49. The RBA only intervened on 15% of trading days throughout this four year period.

VII. 12 September 2001 to 31 June 2003: Following a strong defense of the domestic currency immediately after the terrorist attacks, the RBA resumed its accrual of foreign currency reserves in late 2002.

4.3 Empirical Results

Section 4.3.1 below examines the impact of the choices of SV model and estimation method on the empirical results. Section 4.3.2 considers the relationship between RBA intervention and volatility over both the whole sample and during the sub-periods identified above, with emphasis on the most recent sub-periods.

4.3.1 Comparing models and methods: 1983 to 2003

Parameter estimates for the entire dataset are presented in table 2 for four different estimators, which are labeled E1 to E4 and described below. First, (E1) QML estimation of the SV model assuming equally-spaced observations. The threshold model is estimated using MCMC, conditional on the smoothed volatility estimates. The second, third and fourth estimators employ joint estimation of the FX and threshold models using MCMC as discussed in
section 3, but where the underlying SV model is assumed (E2) to have equally-spaced data, (E3) to have missing observations but no jumps and (E4) to be the full MOSVJ model, respectively. In addition, table 3 reports the estimates for the β vector for this data separately.

Estimates of σ_η are substantially larger for the Bayesian finite sample inference with the accurate likelihood, compared to the QML based estimator E1. This corresponds to over-smoothing of the volatilities, which is a feature of QML estimation of the SV model. To demonstrate this, figure 2 plots the corresponding smoothed estimates of σ_t for estimators E1, E2 and E4, along with the absolute returns |∆p_t|. For clarity of exposition, the plots cover the period mid-1984 to mid-1989, although identical conclusions can be drawn from plots of the entire period. Comparing the results for the QML estimator E1 to the Bayesian analysis of the equivalent equally-spaced SV model using estimator E2, severe over-smoothing of the volatilities by the QML estimator can be observed. In particular, the Bayesian estimator identifies spikes in the volatility. This is important for central bank intervention because such spikes in volatility are likely to be associated with intervention activity. Other deficiencies in QML estimation of data with mean-reverting volatility and φ close to one are discussed in Kim et al. (1998) and Andersen et al. (1999) and references therein. Choice of estimation method also affects the significance of the intervention-volatility relationship, with QML indicating that the additional effect of sizable interventions, µ_3, is significantly negative, which appears erroneous. In addition, the strength of the positive relationship between intervention and volatility appears exaggerated by the QML estimate of µ_1.

—–Tables 2 and 3 and figure 2 about here.—–

The SV framework exhibits similar advantages over the more traditional GARCH style framework for the analysis of central bank intervention. Figure 3 compares MCMC estimates of σ_t from an MOSVJ model with the equivalent estimates from the EGARCH model of Kim et al. (2000). (For comparability, the SV estimates are based on the same subset of the data considered by these authors.) First, the EGARCH estimates fail to capture spikes in FX rate volatility in comparison to the SV estimates. Second, the assumption of deterministic conditional volatility results in a less flexible fit, with post-spike volatilities likely to be over-
A comparison between panels (b) and (c) in figure 2 demonstrates that accounting for missing observations and jumps in the mean of returns has a minor effect on the majority of volatility estimates. However, for a few observations the effect is substantial. Adjusting for missing observations results in up to a 43.3% difference in the absolute values of $\sigma_t$ and a further difference of up to 39.7% when adjusting for jumps as well. Importantly, the differences tend to occur in periods of higher volatility, which is when intervention is focused. Note also from table 2 that these two model adjustments also result in a substantial change in estimates for $\sigma_\eta$ and $\phi$. Lower persistence and higher volatility is to be expected when equal-spacing of the data is incorrectly assumed because large movements in FX returns which actually take place over a number of non-trading days are attributed to a single day.

Joint estimation using MCMC also appears to impact substantially on the estimates of the threshold model in table 2. Estimates of $\alpha$ obtained conditional on the smoothed volatilities for estimator E1 differ substantially from the joint estimates obtained using estimators E2, E3 and E4. The difference is particularly pronounced for the key parameter $\alpha_3$, with the relationship between volatility and propensity to intervene double the size for estimator E1. Moreover, the marginal posterior distributions tend to be much tighter for the joint estimates, than those obtained conditioned on the volatility estimates.

For estimator E4 only a few jumps in the mean are identified over the entire period, with 14 days having a marginal posterior probability greater than 0.5 of having a jump. Sub-period I contains 7, sub-period II contains 3, sub-period III contains 1 and sub-period VI contains 3. The RBA does not appear to respond differently to these (controlling for other factors, including volatility). One explanation is that jumps in the mean represent significant agreement between market participants, and intervention is likely to be counter-productive in such circumstances.

While this comparison of models and methods is based on empirical results reported for
the whole sample, we note that sub-period based comparisons result in the same conclusions.

4.3.2 Intervention and FX volatility: Analysis by sub-period

Table 4 reports the parameter estimates for the sub-periods using estimator E4, which provides joint Bayesian inference on the MOSVJ and threshold models. The significant positive $\beta_1$ estimate (observed in all sub-periods except III) indicates that intervention to support (resist the rise of) the domestic currency is associated with that currency’s depreciation (appreciation). This result is consistent with the RBA intervening with limited success to oppose the direction of market movements, confirming the results of Kearns and Rigobon (2003) and Edison et al. (2003). The negative sign of $\beta_3$ in some sub-periods indicates that sizable interventions are associated with less significant contrary movements in the FX rate, suggesting sizable interventions are more effective than small trades in shifting the direction of trading. In sub-period III this size effect was large enough to reverse short-term movements in returns.

The periods following the Asian currency crisis and September 11 terrorist attacks support the idea that the size of intervention is important in the Australian setting, reinforcing recent international findings for other central banks.

---Table 4 about here---

The sign of $\mu_1$ is positive for the whole sample and sub-periods I through III. One interpretation is that RBA intervention actually increased volatility in the market during sub-periods I through III. As in the GARCH analysis of Edison et al. (2003), $\mu_1$ is insignificant in the later sub-periods, indicating that the RBA intervention is no longer a source of market volatility.

The effect on volatility of sizable and cumulative interventions are mostly indistinguishable from small trades, except in the post-September 11 sub-period where consecutive interventions are associated with high volatility levels. Interventions that are reported or accompanied by official statements are no different in their association with volatility. One unexpected result in sub-periods V and VII is the significant positive association between
changes in monetary policy reflected by US and Australian interest rates and higher volatility levels. This may result from over-reaction or confusion in FX markets to changes in expectations of monetary policy in either country.

From the threshold model estimates in tables 2 and 4, the tendency to intervene is positively related \( (\alpha_0 > 0) \) to long-run deviations of the spot rate from trend for all sub-periods. Interestingly, while over the whole sample the RBA shrank from intervening in response to sizable trend deviations \( (\alpha_1 < 0) \), in sub-periods V and VII the RBA was likely to intervene in still greater strength when the deviation from trend was sizable. This may be a reflection of the accumulation of foreign currency reserves that is particularly pronounced in these two sub-periods. When restoring reserves in an appreciation phase there are no stock constraints. This is in sharp contrast to the situation of using scarce reserves to defend a currency.

No consistent picture emerges of the role of shifts in the interest rate differential in prompting intervention. While \( \alpha_2 \) has a negative sign for the whole sample (implying that shifts of the interest rate differential in favor of the US are associated with RBA purchases of domestic currency), the parameter’s sign alternates over the sub-periods, having no significant association with intervention in the most recent sub-period. This suggests that the RBA does not consistently use direct intervention to support domestic monetary policy.

For all sub-periods and the entire period, the positive signs of estimates of \( \alpha_3 \) imply that the RBA did intervene in response to volatility in the FX rate. The higher the volatility, the stronger the desire to intervene on the opposite side of the market (that is, ‘leaning against the wind’), although this motivation was insignificant in sub-periods III and VI as measured by the marginal posterior probability interval for \( \alpha_3 \). In sub-period II the RBA was more likely to intervene in response to exceptionally sizable volatility \( (\alpha_4 > 0) \). This reflects the robust intervention policy of the RBA during this sub-period that is unmatched in other periods.

As with regards to the other parameters in the model, the average size of ‘jumps’ in all periods is tightly distributed around a sizable level of 7%. The posteriors for the thresholds \( \theta^+ \) and \( \theta^- \) in all sub-periods are distributed tightly below the minimum positive intervention
and above the smallest negative intervention respectively. The variance of unexplained latent intervention \( \sigma_u^2 \) varies according to the scale of the intervention data in each period.

### 4.3.3 Intervention and Lagged Volatility

In the threshold model at (4.3) the RBA is assumed to respond to volatility in a contemporaneous fashion. However, the RBA’s propensity to intervene may be affected by past volatility, possibly due to institutional decision-making procedures. To investigate this issue we adjust the threshold model at equation (4.3) to the following

\[
\begin{align*}
  w_t' \alpha &= \left( \alpha_0 + \alpha_1 \text{SIZDEV}_t \right) \text{ERDEV}_t + \alpha_2 \text{IDIF}_t + \\
  &\quad + \sum_{j=0}^{4} \alpha_3 j \text{RET}_{t-j} \sigma_{t-j} + \alpha_4 \text{SIZVOL}_t \text{RET}_t \sigma_t.
\end{align*}
\tag{4.4}
\]

We do not include the jump term in (4.4) because, as discussed in section 4.3.2, jumps do not feature in many periods, nor have any significant impact upon propensity to intervene in those sub-periods when jumps do occur. Volatility (multiplied by direction) is lagged by trading day on the ASX because volume of trading in the Australian dollar is very low outside these periods. Table 5 provides the posterior means for \( \alpha \) and \( \sigma_u^2 \) for each of the sub-periods.

Note that the estimates for the FX model will not change and are not reported, and the estimates for \( \theta^+ \) and \( \theta^- \) are identical and are therefore also not reported.

---Table 5 about here---

The parameters \( \alpha_0, \alpha_1, \alpha_2 \) and \( \alpha_4 \), are only marginally perturbed by the inclusion of lagged volatility. The estimates for \( \alpha_3 j, j = 0, \ldots, 4 \) are consistent with the interpretation that the RBA has intervened to smooth contemporaneous volatility. However, over its history, intervention appears prompted to a lesser extent by volatility on previous trading days. For the first ten years of intervention, decisions were significantly related (as measured by posterior probability intervals for the coefficients) to market volatility on up to four previous trading days. Volatility on two previous trading days was significant during sub-periods III and V. In sub-periods VII, volatility on only one previous trading day was significant. There
is little to justify intervening on a particular day to smooth out volatility that occurred on previous days. Therefore, we interpret this result as suggesting an improving ability to intervene contemporaneously in response to volatility in the market (as distinct from intervention directed to affect the level of the currency) by the RBA.

5 Conclusion

In this paper we have proposed an approach for the analysis of central bank intervention in the FX markets. By exploiting recent advances in the financial time series and statistical literatures, we have developed methodology that is substantially more comprehensive than that currently employed in the literature on intervention. We believe that the SV model for FX rates better captures the underlying latent volatility process than previously employed GARCH style alternatives, especially on the highest volatility days that are crucial in the study of central bank intervention. Finite sample estimation using MCMC not only provides substantially more accurate inference compared to quasi-maximum likelihood, it enables joint estimation of both the exchange rate and intervention models. This is particularly important because previous work estimated each model separately, which does not sufficiently account for the strong endogeneity between the latent processes.

We demonstrate the advantage of using our approach by analysing intervention by the RBA in the Australian/US dollar exchange rate. Using the latest data available at the time of writing, we uncover a complex relationship between FX volatility and RBA intervention. In particular, increased volatility increases the probability of intervention by the central bank. However, intervention during the early post-deregulation period appears to have resulted in increased volatility in the markets, although after 1993 this no longer appears to be the case. Analysis of lagged volatility suggests one reason may be improved targeting of intervention to address contemporaneous volatility, as opposed to volatility occurring on previous trading days, over the history of RBA intervention. Once volatility is controlled for, jumps in the exchange rate do not appear to be significantly associated with RBA intervention.

Acknowledgments

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References


A Appendix

In this appendix we compute the posterior densities required to implementing the sampling scheme that are not readily available in existing published work discussed in section 3.

Deriving \( p(h|\phi, \sigma_n^2, \mu) \)

The density is given by:

\[
p(h|\phi, \sigma_n^2, \mu) \propto p(h_t|\phi, \sigma_n^2, \mu) \prod_{i=1}^{n-1} p(h_{t+1}|h_t, \phi, \sigma_n^2, \mu) \propto \left( \frac{\sigma_n^2}{(1-\phi^2)} \right)^{-1/2} \exp \left\{ -\frac{(1-\phi^2)}{2\sigma_n^2} \left( h_{t1} - s_{t1}^\prime \mu \right)^2 \right\} \times \prod_{i=1}^{n-1} \left( \frac{\sigma_n^2}{\Gamma_i} \right)^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2\sigma_n^2 \Gamma_i} \left( h_{t+1} - s_{t+1}^\prime \mu - \phi \Delta t_i (h_t - s_t^\prime \mu) \right)^2 \right\},
\]

where \( \Gamma_i = \sum_{j=0}^{\Delta t_i-1} \phi^{2j} \).

Step (1e): sampling from \( p(\phi|h, \sigma_n^2, \mu) \)

The conditional posterior

\[
p(\phi|h, \sigma_n^2, \mu) \propto p(h|\phi, \sigma_n^2, \mu)p(\phi) \propto \left( \frac{1-\phi^2}{2} \right)^{b_\phi-1} \left( \frac{1+\phi}{2} \right)^{a_\phi-1} \left( 1-\phi^2 \right)^{0.5} \left( \prod_{i=2}^{n} \Gamma_i^{-1/2} \right) \exp \left\{ -\frac{Q_h}{2\sigma_n^2} \right\}
\]

with \( Q_h \equiv \left( 1-\phi^2 \right) \left( h_{t1} - s_{t1}^\prime \mu \right)^2 + \sum_{i=1}^{n-1} \left( \Gamma_i^{-1} \left[ (h_{t+1} - s_{t+1}^\prime \mu) - \phi \Delta t_i (h_t - s_t^\prime \mu) \right]^2 \right) \).

The parameter \( \phi \) is sampled by a random walk Metropolis Hasting step, where a new iterate \( \phi^{\text{new}} \) is generated from a \( N(\phi^{\text{old}}, 0.05^2) \) density constrained to the stationarity region \((-1, 1)\) and centered around the old iterate \( \phi^{\text{old}} \). The new iterate is accepted over the old with probability

\[
\min \left\{ 1, \frac{p(\phi^{\text{new}}|h, \sigma_n^2, \mu)(\Psi(1; \phi^{\text{old}}, 0.05^2) - \Psi(-1; \phi^{\text{old}}, 0.05^2))}{p(\phi^{\text{old}}|h, \sigma_n^2, \mu)(\Psi(1; \phi^{\text{new}}, 0.05^2) - \Psi(-1; \phi^{\text{new}}, 0.05^2))} \right\},
\]

where \( \Psi(x; a, b) \) is a normal CDF with mean \( a \) and variance \( b \).
Step (1c): sampling from $p(\sigma^2_\eta|h, \phi, \mu)$

The conditional posterior is

$$p(\sigma^2_\eta|h, \phi, \mu) \propto p(h|\phi, \sigma^2_\eta, \mu)p(\sigma^2_\eta) \propto (\sigma^2_\eta)^{-(n+a_\sigma)/2} \exp\left\{-\frac{S_\sigma + Q_h}{2\sigma^2_\eta}\right\},$$

so that $\sigma^2_\eta \sim IG((n + a_\sigma)/2, (Q_h + b_\sigma)/2)$.

Step (1d): sampling from $p(\mu|h, \phi, \sigma^2_\eta)$

Under the flat prior for $\mu$, the posterior is multivariate normal with mean $m$ and variance matrix $V$ where

$$m = V(\sigma^2_\eta)^{-1}\left[h_{t_1}z_{t_1} (1 - \phi^2) + \sum_{i=1}^{n-1} \Gamma_i^{-1}(h_{t_{i+1}} - \phi^{\Delta t_i}h_{t_i})(z_{t_{i+1}} - \phi^{\Delta t_i}z_{t_i})\right]$$

$$V = \sigma^2_\eta\left[z_{t_1}z_{t_1}' (1 - \phi^2) + \sum_{i=1}^{n-1} \Gamma_i^{-1}(z_{t_{i+1}} - \phi^{\Delta t_i}z_{t_i})(z_{t_{i+1}} - \phi^{\Delta t_i}z_{t_i})\right]^{-1}.$$

Step (3a) sampling from $p(\theta^+, \theta^-|z^*, z)$

First note that $p(\theta^+, \theta^-|z^*, z) = p(\theta^+|z^*, z)p(\theta^-|z^*, z)$. Then,

$$p(\theta^+|z^*, z) \propto p(z^*, \theta|\theta^+) \propto I(\theta^+ > 0) \prod_{i=1}^{n} \left\{ \begin{array}{l} I(z^*_i > \theta^+) I(z_{t_i} = z^*_i) \\ + I(\theta^- < z^*_i < \theta^+) I(z_{t_i} = 0) \end{array} \right\}.$$

The cutoff $\theta^+$ must lie below $\theta^+_{\text{max}} \equiv \min\{z^*|z^* > \theta^+\}$ otherwise that minimum positive intervention would have been censored. Similarly, it must lie above the maximum of zero (as $\theta^+$ is strictly positive from the model) and $\max\{z^*|z = 0\}$. This implies $\theta^+|z^*, z \sim U[\theta^+_{\text{min}}, \theta^+_{\text{max}}]$, where $\theta^+_{\text{min}} = \max\{\max\{z^*|z = 0\}, 0\}$ and $U[a, b]$ denotes a uniform density with lower bound $a$ and upper bound $b$. Similarly, it can be shown that $\theta^-|z^*, z \sim U[\theta^-_{\text{min}}, \theta^-_{\text{max}}]$, where

$$\theta^+_{\text{max}} = \min\{\min\{z^*|z = 0\}, 0\} \quad \text{and} \quad \theta^-_{\text{max}} = \max\{z^*|z < \theta^-\}.$$
Step (3b): sampling from \( p(z^*|z, h, \alpha, \sigma^2_u, \theta^+, \theta^-, q, \psi) \)

It is straightforward to show the density \( p(z^*|z, h, \alpha, \sigma^2_u, \theta^+, \theta^-, q, \psi) = \prod_t p(z_t^*|z, h, \alpha, \sigma^2_u, \theta^+, \theta^-, q, \psi) \), where \( z_t^* = z_t \) if \( z_t \neq 0 \) and \( z_t^* \sim N(w_t^\prime \alpha, \sigma^2_u) \) constrained to the interval \((\theta^-, \theta^+)\) when \( z_t = 0 \). Note that \( w_t \) is a function of \( \{h_t, q_t, \psi_t\} \).

Step (3c): sampling from \( p(\alpha, \sigma^2_u|z^*, h, q, \psi) \)

We note that \( p(\alpha, \sigma^2_u|z^*, h, q, \psi) \propto p(\sigma^2_u|\alpha, h, z^*, q, \psi)p(\alpha|z^*, h, q, \psi) \). We first generate \( \alpha \) from its conditional posterior, which is multivariate t with \( n-m_2 \) degrees of freedom, location \((W'W)^{-1}W'z^*\) and scale \( \frac{n-m_2}{S_w}W'W \), where \( W = [w_{t1}|...|w_{tn}] \) and \( S_w = z^*z^* - z^*W(W'W)^{-1}W'z^* \). Second, we generate \( \sigma^2_u \) from \( \text{IG}((n-m_2)/2, S_w/2) \) distribution.
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<th>Purchases Max</th>
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Table 1: Summary of Intervention by the Reserve Bank of Australia during the entire period and the six sub-periods in the empirical study. The mean amount of an intervention, minimum and maximum sales and purchases are reported in millions of Australian dollars. The number of trading days in each period is recorded, along with the number of days on which direct intervention was undertaken.
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<th>E1 (SV QML)</th>
<th>E2 (SV MCMC)</th>
<th>E3 (MOSV MCMC)</th>
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<tr>
<td>$\phi$</td>
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<tr>
<td></td>
<td>[0.976, 0.990]</td>
<td>[0.954, 0.980]</td>
<td>[0.967, 0.986]</td>
<td>[0.960, 0.984]</td>
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<tr>
<td>$\sigma^2_\eta$</td>
<td>0.019</td>
<td>0.041</td>
<td>0.029</td>
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<tr>
<td></td>
<td>[0.012, 0.030]</td>
<td>[0.027, 0.061]</td>
<td>[0.019, 0.043]</td>
<td>[0.022, 0.057]</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>-1.494**</td>
<td>-1.343**</td>
<td>-1.346**</td>
<td>-1.359**</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>0.819**</td>
<td>0.404**</td>
<td>0.400**</td>
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<tr>
<td>$\mu_2$</td>
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<td>0.025</td>
<td>0.026</td>
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</tr>
<tr>
<td>$\mu_3$</td>
<td>-0.452**</td>
<td>-0.135</td>
<td>-0.132</td>
<td>-0.114</td>
</tr>
<tr>
<td>$\mu_5$</td>
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<td>0.020</td>
<td>0.018</td>
<td>0.021</td>
</tr>
<tr>
<td>Jump Component</td>
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<tr>
<td>$\delta$</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>0.067</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.043, 0.137]</td>
</tr>
<tr>
<td>$\kappa$</td>
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<td>n/a</td>
<td>n/a</td>
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<tr>
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<td></td>
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<td>[0.008, 0.026]</td>
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<tr>
<td>Intervention Equation</td>
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<tr>
<td>$\alpha_0$</td>
<td>5.224**</td>
<td>5.240**</td>
<td>5.240**</td>
<td>5.232**</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-1.453*</td>
<td>-1.229*</td>
<td>-1.228*</td>
<td>-1.216*</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-0.016**</td>
<td>-0.015**</td>
<td>-0.015**</td>
<td>-0.015**</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.255**</td>
<td>0.138**</td>
<td>0.137**</td>
<td>0.145**</td>
</tr>
<tr>
<td></td>
<td>[0.141, 0.369]</td>
<td>[0.075, 0.202]</td>
<td>[0.074, 0.202]</td>
<td>[0.080, 0.202]</td>
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<tr>
<td>$\alpha_4$</td>
<td>0.064</td>
<td>0.068</td>
<td>0.068</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>[-0.052, 0.180]</td>
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<td>[-0.037, 0.179]</td>
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<td>n/a</td>
<td>n/a</td>
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<tr>
<td>$\theta^+$</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>$\theta^-$</td>
<td>-0.010</td>
<td>-0.010</td>
<td>-0.010</td>
<td>-0.010</td>
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<tr>
<td>$\sigma^2_u$</td>
<td>0.420</td>
<td>0.465</td>
<td>0.465</td>
<td>0.465</td>
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</tbody>
</table>

Table 2: Parameter estimates for the entire dataset using the four different estimators. Point estimates for the parameters correspond to posterior means, except in the volatility equation for the QML estimator (E1). Single stars correspond to significance at the 90% level and double stars at the 95% level, as measured by the marginal posterior probability intervals for most parameters. For a few key parameters the posterior probability intervals are also reported. For the QML estimator, asymptotic standard errors were used for the significance tests and confidence intervals of the SV model parameters.
Figure 1: Plots of the data employed in the Australian case study. Panel (a): Daily AUS$/US$ exchange rate in US$. Panel (b): Absolute value of the continuously compounded daily exchange rates $\Delta p_t$. Panel (c): Daily net purchases of US$ by the RBA denominated in AUS$. Panel (d): The Australian US interest rate differential $\text{IDIF}_t$ in percent. The different sub-periods are delimited by vertical dotted lines and dated.
Figure 2: Each panel contains smoothed estimates for $\sigma_t$ (bold line) and absolute FX returns $|\Delta p_t|$ (thin line) for the period mid-1984 to mid-1989. Panels (a), (b) and (c) correspond to estimators E1, E2 and E4, respectively, as discussed in the text. For estimator E4 the observations identified as having a posterior probability greater than 0.5 of being classified as a jump are marked with a hollow circle.
Figure 3: Comparison of conditional volatility estimates from the EGARCH model of Kim et al. (2000) (bold line) and the MOSVJ model of estimator E4 proposed in this paper (thin line).
Table 3: Estimates of the $\beta$ vector for the entire sample period. Single stars correspond to significance at the 90% level and double stars significance at the 95% level as measured using White-corrected standard errors.

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_5$</th>
<th>$\beta_7$</th>
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<tbody>
<tr>
<td></td>
<td>0.028*</td>
<td>0.739**</td>
<td>-0.074*</td>
<td>-0.567**</td>
<td>-0.023</td>
<td>-0.008</td>
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</table>
Table 4: Parameter estimates for the six sub-periods in which the RBA intervened in the FX market. Single and double stars correspond to significance at the 90% and 95% levels, respectively. This is measured by the marginal posterior probability intervals for all parameters, except the $\beta$ estimates, where White-corrected standard errors are employed. Marginal posterior probability intervals are reported in parentheses for key parameters.
<table>
<thead>
<tr>
<th></th>
<th>I ('83-'86)</th>
<th>II ('86-'91)</th>
<th>III ('91-'93)</th>
<th>V ('95-'97)</th>
<th>VI ('97-'01)</th>
<th>VII ('01-'03)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>0.331</td>
<td>4.066**</td>
<td>14.275**</td>
<td>3.510**</td>
<td>3.227**</td>
<td>3.034**</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.033</td>
<td>0.816</td>
<td>-3.811</td>
<td>10.805**</td>
<td>2.239</td>
<td>3.257**</td>
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<tr>
<td>$\alpha_2$</td>
<td>0.003**</td>
<td>-0.027**</td>
<td>0.046**</td>
<td>-0.094**</td>
<td>0.102**</td>
<td>-0.004</td>
</tr>
<tr>
<td>$\alpha_30$</td>
<td>0.076**</td>
<td>0.267**</td>
<td>0.236</td>
<td>0.281**</td>
<td>0.053</td>
<td>0.081**</td>
</tr>
<tr>
<td>$\alpha_31$</td>
<td>0.015**</td>
<td>0.178**</td>
<td>0.457**</td>
<td>0.083**</td>
<td>0.029</td>
<td>0.037*</td>
</tr>
<tr>
<td>$\alpha_32$</td>
<td>0.019**</td>
<td>0.129**</td>
<td>0.216**</td>
<td>0.064*</td>
<td>0.020</td>
<td>0.031</td>
</tr>
<tr>
<td>$\alpha_33$</td>
<td>0.014*</td>
<td>0.026</td>
<td>0.037</td>
<td>0.036</td>
<td>0.028</td>
<td>0.017</td>
</tr>
<tr>
<td>$\alpha_34$</td>
<td>0.012</td>
<td>0.076**</td>
<td>0.004</td>
<td>0.059</td>
<td>0.003</td>
<td>-0.041</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>-0.013</td>
<td>0.241</td>
<td>0.196</td>
<td>-0.190*</td>
<td>0.014</td>
<td>-0.050</td>
</tr>
<tr>
<td>$\sigma_u^2$</td>
<td>0.016</td>
<td>0.636</td>
<td>1.332</td>
<td>0.107</td>
<td>0.334</td>
<td>0.063</td>
</tr>
</tbody>
</table>

Table 5: Posterior means of coefficients in the intervention equation for each sub-period when lagged volatility is incorporated. Single and double stars correspond to significance at the 90% and 95% levels, respectively, as measured by the posterior probability intervals.