Delegation and Commitment in Durable Goods Monopolies*

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Abstract

This paper studies a simultaneous-move infinite-horizon delegation game in which the principal of a durable goods monopoly entrusts pricing decisions to a manager who enjoys monetary rewards but dislikes production effort. The delegation contract allows for continual interference with managerial incentives: in each period the principal rewards the manager according to her performance. We show that when the cost of delegation is low relative to profits, the principal can attain the precommitment price plan in a time consistent Markov-perfect equilibrium. The paper analyzes the robustness of this result under alternative specifications of timing and objectives. We also provide a numerical characterization of the Markov-perfect equilibrium pricing and remuneration strategies for the case of linear-quadratic payoffs.

KEYWORDS: durable goods monopoly; delegation; Markov perfect equilibria

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1 Introduction

In many dynamic decision problems, economic behavior is determined by the availability of commitment technologies. The ability to commit credibly to future policies can influence the expectations of forward-looking agents, which in turn affects perceived intertemporal trade-offs. The issue of internal dynamic consistency of economic decisions has gained particular prominence in the context of durable goods monopolies. In his pioneering work, Coase (1972) studies the implications of rational expectations for market power. He argues that sales of durable goods provide a rationale for expectations of subsequent price reductions, motivating consumers to postpone purchases, thus depressing current market prices. Therefore, commitment mechanisms which give credibility to future pricing targets allow the monopoly to increase profits by maintaining higher prices.

Our paper studies the separation of ownership from day-to-day management in durable goods monopolies and demonstrates its effectiveness as an intertemporal commitment tool in a model of rational expectations. We analyze an infinite-horizon game between the owner and the manager of a durable goods monopoly. We treat the owner as the principal, and the manager as her agent. The manager dislikes production effort but enjoys monetary rewards. Delegation is modelled not as a one-shot event, but rather as a continual process developing over time: in each period the manager and the principal interact by simultaneously choosing respectively the current market price and the managerial compensation. In a dynamic setup with interdependent payoffs, commitment through delegation is non-trivial because both parties will have the opportunity to engage in future incentive adjustments. However, the delegation contract still enables the durable good monopoly to resolve its time-inconsistency problem. The reason is that it decouples the principal's instantaneous payoff from her future strategies.

Commitment through delegation of management offers important advantages over alternative commitment tools proposed previously: i) it does not require legal enforceability of contracts and the existence of a secondary market; and ii) it circumvents moral hazard problems and commodity abuse that renting may create. Thus, long-term management contracting can mitigate existing internal inconsistencies even when other commitment instruments are infeasible or costly. Moreover, delegation is a common feature of corporate hierarchy: the overwhelming majority of medium and large firms are structured in a way that establishes a clear-cut boundary between management and ownership. Thus, durable goods producers already have easy access to this commitment technology.

Our main result states that if the principal ignores the cost of delegation (e.g. managerial wages are negligible relative to monopoly profits), she can motivate the manager to choose the profit maximizing precommitment price path in a perfect rational expectations equilibrium. The adoption of Markov-perfect equilibrium as a solution concept ensures that pricing and remuneration strategies will be supported by rational expectations, and therefore dynamically consistent. It is in the market participants' self-interest to adhere to the precommitment price sequence *in all periods and for all states.* Thus, the durable goods monopolist can implement precommitment pricing without requiring enforceability of any legal contracts that she might enter in order to precommit her future self. This outcome does not depend on the manager's utility and is therefore robust to random preference shocks. Furthermore, since the implementation of the equilibrium does involve trigger strategies, the decision makers only need to know the current state of the world, and they can be arbitrarily impatient.

We also consider the implications of delegation costs and alternative timing:

- While the principal's concern with management costs will distort the price path away from the precommitment optimum, delegation maintains its precommitment function. We provide conditions under which equilibrium managerial compensation is low, enabling the principal to improve over the no-delegation time consistent equilibrium.

- Sequential-move costless delegation fails to decouple current profits from the principal's future choices, and is therefore unable to resolve her time inconsistency problem. The equilibrium price path is identical to the time-consistent plan of a monopolist who does not resort to delegation.

Finally, the adoption of linear-quadratic payoffs enables us to numerically characterize the Markov-perfect equilibrium price and remuneration strategies.

There exists a substantial body of literature on durable goods monopolies originating from the seminal work of Coase (1972). He conjectures that rational expectations will force the seller to saturate the market at all dates, and thus earn zero profits. Some subsequent research, which includes Stokey (1982), Bond and Samuelson (1984), Gul, Sonneschein and Wilson (1986), provides conditions for the validity of this hypothesis (such as infinite horizon, patience and negligible delay between trading periods). Another strand of literature explores the adoption of commitment technologies and their effect on market conduct. Bulow (1982) shows that renting can eliminate the monopolist's time consistency problem by severing the intertemporal linkage between periods. Furthermore, Bulow (1986) argues that planned product obsolescence can be used to weaken future incentives to lower market prices. Other commitment tools available to a durable goods monopolist include, among other things, guaranteed buy back of the product at the original price, destroying production capacity, and building a reputation for maintaining high prices.

While delegation has been overlooked in the context of durable goods monopolies, its commitment value has been recognized in the dynamic oligopoly literature. Sklivas (1987), Fershtman and Judd (1987) analyze a duopoly game in which principals entrust output or price decisions to managers whose compensation is tied to both sales and profits. They show that: i) the separation of ownership from management increases profits relative to an opponent firm which does not resort to delegation; and ii) in equilibrium principals will design contracts that strategically distort managerial incentives away from profit maximization. Competition-driven delegation is further studied by, among others, Miller and Pazgal (2001), Basu (1994), Baye, Crocker and Ju (1996).

The rationale for delegation in oligopolistic interactions is based on the strategic nature of market competition: contract design is used to obtain an "instantaneous" first-mover advantage over the opponent firm. We assign a somewhat different role to this instrument. In our model delegation is being used as an intertemporal commitment device, which allows current decision makers to attain desired future outcomes.

Furthermore, the present paper may also shed light on the time consistency of economic decisions and policies in the macroeconomics literature. Kydland and Prescott (1977) first recognized that central banks conducting monetary policy have a commitment problem which gives rise to an inflationary bias. They show that welfare can be improved if the social planner foregoes discretion and adopts rules that limit her freedom of choice. Rogers (1987) analyzes this issue in the context of fiscal policy. Rogoff (1985) focuses on delegation as an institutional remedy to the time consistency problem outlined by Kydland and Prescott. He demonstrates that the appointment of an independent central banker whose preferences differ from government's (e.g. she places "too large" a weight on inflation-rate stabilization), will mitigate the existing commitment issues.

We study a delegation model which differs from Rogoff (1985) in several key aspects:

- **dynamic delegation**: unless completely isolated from the decision making process, a time-inconsistent principal will have an incentive to continually interfere with post-delegation management. We account for this by examining a dynamic principal-agent relationship involving repeated interactions. Unlike Rogoff's central banker, in our model the manager is not independent: when determining managerial compensation the principal takes into account past pricing decisions.

- **irrelevance of managerial preferences**: Rogoff's one-shot delegation model requires identifying and employing an agent with specific "socially optimal" preferences, which may present significant difficulties. However, we show that if the principal is not concerned with the cost of delegation, she can attain the optimal precommitment policy path in a rational expectations equilibrium irrespective of the manager's utility.

The remainder of the paper is organized as follows: Section 2 defines the industry structure, technology and preferences; it also describes the delegation game. In Section 3 we characterize the two important benchmark policy paths: the precommitment price path and the time consistent price path in the absence of commitment technologies. The MPE of the costless delegation game is derived in Section 4. Its properties are illustrated with a numerical example. In Section 5 we analyze the robustness of the results to changes in the payoffs and the timing of activities. Section 6 concludes.

2 Setup

2.1 Demand and Industry Structure

The industry structure adopted here is an infinite-horizon analogue of Bulow (1982). There is a mass M of heterogeneous consumers who participate in the market for an infinitely durable commodity: a purchase decision yields a perpetual stream of benefits over time. Each consumer can buy at most one unit, and after the purchase she leaves the market. Let v denote the monetary value of the instantaneous benefit generated by the durable good and suppose that future utility is discounted by a common factor β. Consumers differ in their perception of the benefits they derive from the commodity. We assume that the preference parameter v is distributed according to a cdf $\Phi(v)$ with support $[0,1]$.

Consider any equilibrium price path $\{p^t\}_{t=0}^{\infty}$ that is monotonically decreasing and dynamically stable: $p^{t-1} - p^t \geqslant p^t - p^{t+1} \geqslant 0$. Since $\beta < 1$, we have that $p^{t-1} - p^t > \beta(p^t - p^{t+1})$. Therefore, the following property will hold in all periods:

$$p^{t-1} - \beta p^t > p^t - \beta p^{t+1} \text{ for all } t > 0 \tag{1}$$

All market participants are fully rational and have correct expectations regarding future prices. When choosing the date of purchase, consumers weigh foregone benefits against expected price reductions. They would delay the purchase from period $t-1$ to period t if:

$$v + \frac{\beta v}{1-\beta} - p^{t-1} < \frac{\beta v}{1-\beta} - \beta p^t \Leftrightarrow v < p^{t-1} - \beta p^t$$

Furthermore, given the consumers' expectations regarding the next period's market price p_e^{t+1}, in period $t \geq 0$ they will choose not to delay the purchase to period $t+1$ if:

$$v + \frac{\beta v}{1-\beta} - p^t \geqslant \frac{\beta v}{1-\beta} - \beta p_e^{t+1} \Leftrightarrow v \geqslant p^t - \beta p_e^{t+1}$$

Thus, current purchases depend not only on past and current market prices, but also on expectations regarding future pricing policies: if buyers anticipate a bigger price cut in the subsequent period, more of them will choose to postpone consumption. In the remainder of the paper we impose rational expectations: $p_e^{t+1} \equiv p^{t+1}$.

Note that if property (1) holds, then prices $p^{t+2}, p^{t+3}, ...$ are irrelevant for the period-t buyers, and thus have no effect on period-t demand. If a consumer prefers not to delay the purchase from period t to period $t+1$, she would also prefer not to delay it until any later period T, since $v \geqslant p^t - \beta p^{t+1} > p^{t+1} - \beta p^{t+2} > ... > p^{T-1} - \beta p^T$.

The above assumptions imply that period-t demand $(t > 0)$[1] for the durable good is:

$$x^t = x(p^{t-1}, p^t, p^{t+1}) = M\big(\Phi(p^{t-1} - \beta p^t) - \Phi(p^t - \beta p^{t+1})\big) \tag{2}$$

Property (1) ensures that demand will be positive in all periods.

Assumption A 1 *The cdf of the benefit evaluation satisfies* $\Phi'' \geqslant 0, \Phi''' \leqslant 0$.

Assumption A1 implies that $\partial^2 x^t / \partial (p^t)^2 \leqslant 0, \partial^2 x^t / \partial (p^{t+1})^2 \leqslant 0$.

On the production side, in each period the market is served by a single producer with a cost function $C(x^t)$ and discount factor δ. The monopolist's period-t profit is given by:

$$\pi^t = p^t x(p^{t-1}, p^t, p^{t+1}) - C\big(x(p^{t-1}, p^t, p^{t+1})\big) = \pi(p^{t-1}, p^t, p^{t+1}) \tag{3}$$

Assumption A 2 *The monopolist's cost function satisfies* $C''(x^t) > 0$.

Demand concavity and cost convexity guarantee that $\partial^2 \pi^t / \partial (p^t)^2 < 0, \partial^2 \pi^t / \partial (p^{t+1})^2 < 0$.

[1] In period 0 demand is $x^0 = M(1 - \Phi(p^0 - \beta p^1))$.

2.2 Delegation of Management

Now suppose that for a compensation w^t per period the principal can entrust the pricing decisions to a manager who experiences disutility from the effort associated with production, but enjoys income. Thus, her period-t payoff is $u^t = u(x^t, w^t)$.

Assumption A 3 *Managerial instantaneous utility satisfies* $\partial u^t/\partial w^t > 0$, $\partial u^t/\partial x^t < 0$, $\partial^2 u^t/\partial (x^t)^2 < 0$.

With some abuse of notation, managerial utility can be written as $u^t = u(x^t, w^t) = u(p^{t-1}, p^t, p^{t+1}, w^t)$. Note that assumption A3 and demand concavity imply $\partial^2 u^t/\partial (p^t)^2 < 0$.

The delegation contract takes effect in period 1, with the principal setting both the starting wage w^1 and the starting price p^1. In each of the subsequent periods the principal and the manager simultaneously and non-cooperatively choose the current compensation w^t and the market price p^t, respectively. After the announcement of the price for that period, consumers make their purchase decisions. The management contract is of infinite duration, and also specifies severance payments that are high enough to eliminate future incentives to fire the manager (or shut down).

The manager's objective is maximization of lifetime utility $U^\tau = \sum_{t=\tau}^{\infty} \delta^{t-1} u^t$. In order to focus on the commitment value of delegation in a durable goods monopoly, Section 4 ignores the cost of delegation by assuming that managerial remuneration is small relative to profits: the principal simply maximizes the discounted stream of future gross profits $\Pi^\tau = \sum_{t=\tau}^{\infty} \delta^{t-1} \pi^t$. The assumptions of costless delegation and simultaneous strategy selection are relaxed in Section 5.

3 Direct Pricing

First consider the benchmark problem of a durable goods monopoly that cannot resort to delegation. Thus, all pricing decisions are made directly by the principal. The necessary conditions characterizing the profit maximizing price sequences are derived in Appendix A.

3.1 Precommitment Price Path

Suppose that in period 1 the monopolist can precommit to an entire sequence of future prices. It is well known that when unit costs are constant and consumers are patient, the firm will choose to shut down after the first period. However, we are interested in a dynamic delegation relationship, where the principal can continually interfere with managerial incentives. To ensure that a precommitting monopolist will want to supply positive quantities in all periods, we analyze a case where consumers are sufficiently impatient and production costs are convex. A low β diminishes the negative impact of future prices on current demand, while cost convexity motivates the monopolist to smooth production over time. Stokey (1979) provides general conditions under which precommitment may imply intertemporal price discrimination, giving rise to a monotonically decreasing price path.

If a precommitting monopolist chooses to operate in all periods, her optimal precommitment price sequence $\{p^t\}_{t=1}^{\infty}$ will satisfy

$$\pi_2^t + \delta\pi_1^{t+1} = 0, \ t = 1 \tag{4}$$

$$\pi_3^{t-1} + \delta\pi_2^t + \delta^2\pi_1^{t+1} = 0, \ t \geqslant 2 \tag{5}$$

where the subscript i denotes the partial derivative with respect to the i-th argument (e.g. $\pi_i^t = \partial\pi^t(p_1, .., p_i, .., p_n)/\partial p_i$).

Since (4), (5) are obtained through unconstrained maximization, this price plan attains the highest possible lifetime profit. However, a policy which follows (5) cannot be time consistent. If the monopolist reoptimizes in a later period $\tau \geqslant 2$, the recalculated profit maximizing price p^{τ} will solve (4) instead of (5), thus diverging from the earlier precommitment plan. The underlying reason for this dynamic inconsistency of the above policy is that the instantaneous period-t profit π^t depends on, among others, the next period's price p^{t+1}. When the period-$t+1$ pricing decision is made, period t is already sunk. Since the future decision maker does not internalize the effect of her decisions on past profits, she will make a downward revision of the prices associated with previous precommitment plans.

3.2 Time-Consistent Price Path

Whenever precommitment is not feasible, sophisticated decision makers will have to account for future temptations to deviate from the currently optimal price sequence. The discrepancy between current and future objectives suggests that decision making should be modelled as a game between a sequence of players representing the "selves" of the monopolist associated with each period: the subgame-perfect equilibrium of this intrapersonal game generates a time consistent decision stream.

We focus on the Markov-perfect equilibrium (or the perfect rational expectations equilibrium) of this pricing game, where strategies are restricted to depend only on the current state of the industry: $p^t = f(p^{t-1})$ for all t. Furthermore, we restrict the analysis to MPE in differentiable strategies. The differentiability requirement is useful computationally and helps eliminate potential indeterminacy of MPE. Stokey (1981) demonstrates that if the strategy set is extended to include discontinuous functions, there exists an infinite number of Markov-perfect equilibria. However, she argues that these equilibria are difficult to accept from an economic point of view, because discontinuous expectations seem unrealistic. Klein, Krussel and Rios-Rull (2002) note that differentiability enables us to obtain a set of necessary conditions with a simple economic interpretation.

In period 1 a sophisticated monopolist expects that future price choices will adhere to a strategy function (or "expectations function") $f_e(p)$. Thus, optimality requires that the choice of current prices satisfy the Bellman equation:

$$V(p^{t-1}) = \max_{p^t} \left\{ \pi\left(p^{t-1}, p^t, f_e(p^t)\right) + \delta V(p^t) \right\} \text{ for all } t \geqslant 1 \tag{6}$$

Let $f(p)$ be the optimal current pricing strategy:

$$f(p^{t-1}) = \arg\max_{p^t} \left\{ \pi\left(p^{t-1}, p^t, f_e(p^t)\right) + \delta V(p^t) \right\} + \delta V(p^t) \} \tag{7}$$

Expectations are fulfilled along the equilibrium price path, therefore:

$$f_e(p^t) \equiv f(p^t) \text{ for all } t \geqslant 1 \tag{8}$$

The recursive formulation of the problem ensures the time consistency of the pricing policy.

Definition 4 *The Markov perfect equilibrium of the durable goods monopoly pricing game is characterized by a value function $V : \mathcal{R}_+ \rightarrow \mathcal{R}$ that solves (6) and a strategy function $f : \mathcal{R}_+ \rightarrow \mathcal{R}_+$ that is a fixed point of the mapping defined by (7), (8).*

Dynamic programming yields a necessary condition for the MPE pricing strategy.

Proposition 5 *Suppose that assumptions A1 and A2 are satisfied. The MPE strategy $f(p)$*

of the durable-goods monopoly pricing game satisfies the generalized Euler equation:

$$\pi_2^t + f_1(p^t)\pi_3^t + \delta\pi_1^{t+1} = 0 \text{ for all } t \geqslant 1 \tag{9}$$

Proof. See Appendix A ■

The term $f_1(p^t)\pi_3^t$ incorporates the "internal strategic effect": when the monopolist chooses the current price, she also takes into account its effect on current demand and profit through the next period's pricing decision.

When the period-$t+1$ decision maker recalculates her optimal price sequence, she ignores the negative effect of a reduction in the period-$t+1$ price on the previous profits π^t. Thus, from the period-t viewpoint, the next period's price will be set suboptimally low. The expectations of low future prices induce the monopolist to compensate by reducing current prices in order to boost demand. Consequently, the time consistent price sequence is typically below the precommitment price sequence, thus generating lower lifetime profits. The implication for a dynamically inconsistent decision maker is that she would benefit from any intertemporal commitment device which would subsequently enable her to attain the price path specified by (5).

4 Delegated Pricing

This section analyzes intertemporal commitment through the separation of ownership and control within durable goods monopolies. In particular, we focus on the simultaneous-move costless delegation game Γ described above, in which the principal entrusts pricing decisions to a manager who receives a monetary compensation in exchange for her effort. The simultaneous choice of prices and wages captures the idea that when setting the period's wage, the principal cannot directly observe the current managerial effort.

Since both decision makers are time inconsistent, we model delegation as a game between sequences of their "agents" associated with each period. Again, we restrict the analysis to strategies that are differentiable functions of the current industry state. Markov perfection ensures the dynamic consistency of the pricing and remuneration strategies: no player will want to unilaterally deviate at any point in the game for all states. Furthermore, differentiability allows a natural comparison to the time consistent no-delegation equilibrium.

Note that w^{t-1} does not directly affect period-t payoffs. However, if the players believe that current remuneration will affect future prices, they will treat the previous period's wage as an element of the industry state. In equilibrium these beliefs will be self-fulfilling: in any given period $t \geqslant 2$ the state can be summarized by p^{t-1}, w^{t-1}. When instantaneous profits are given by (3), the principal's problem is well-defined: if the manager's Markov-perfect pricing strategy depends on past wages, the current wage choice will affect the next period's price, and through that current profits.

Let $p^t = f(p^{t-1}, w^{t-1})$ be the manager's period-t pricing strategy and let $w^t = g(p^{t-1}, w^{t-1})$ be the principal's remuneration strategy. Optimality and rational expectations imply that in equilibrium these strategies will solve:

$$\Pi(p^{t-1}, w^{t-1}) = \max_{w^t} \left\{ \pi\left(p^{t-1}, f(p^{t-1}, w^{t-1}), f\big(f(p^{t-1}, w^{t-1}), w^t\big)\right) + \delta\Pi\big(f(p^{t-1}, w^{t-1}), w^t\big) \right\} \tag{10}$$

$$V(p^{t-1}, w^{t-1}) = \max_{p^t} \left\{ u\left(p^{t-1}, p^t, f\big(p^t, g(p^{t-1}, w^{t-1})\big), g(p^{t-1}, w^{t-1})\right) + \delta V\big(p^t, g(p^{t-1}, w^{t-1})\big) \right\}, \tag{11}$$

where

$$g(p^{t-1},w^{t-1}) = \arg\max_{w^t}\left\{\pi\left(p^{t-1}, f(p^{t-1},w^{t-1}), f\big(f(p^{t-1},w^{t-1}),w\big)\right) + \delta\Pi\big(f(p^{t-1},w^{t-1}),w^t\big)\right\} \tag{12}$$

$$f(p^{t-1},w^{t-1}) = \arg\max_{p^t}\left\{u\left(p^{t-1},p^t, f\big(p^t, g(p^{t-1},w^{t-1})\big), g(p^{t-1},w^{t-1})\right) + \delta V\big(p^t, g(p^{t-1},w^{t-1})\big)\right\}. \tag{13}$$

Definition 6 *The Markov-perfect equilibrium of the durable goods monopoly delegation game consists of value functions* $\Pi(p,w)$, $V(p,w)$ *that solve Bellman equations (10), (11) and strategy functions* $g(p,w)$, $f(p,w)$ *that are a fixed point of the mapping defined by (12), (13).*

Consider the principal's Bellman equation (10). The simultaneous choice of prices and wages implies that the principal's period-t payoff π^t now depends only on her contemporaneous remuneration strategy w^t. Subsequent decisions regarding future wages no longer have any repercussions for current profits: when the manager chooses the period-$t+1$ pricing strategy $p^{t+1} \equiv f(p^t, w^t)$, she is still unaware of the period-$t+1$ wage w^{t+1}. Thus, delegation resolves the dynamic inconsistency problem of the durable goods monopoly by decoupling current profits from the principal's future decisions.

Next, we show that if the cost of delegation is ignored, the principal can fine-tune managerial monetary incentives to obtain her unconstrained optimum: the precommitment price path. It is worth noting that this result is quite general and robust to changes in the assumptions regarding demand.

Proposition 7 *Suppose that assumptions A1 through A3 are satisfied. The MPE strategies of the durable goods monopoly simultaneous-move costless delegation game* Γ *beginning in* *period 2 satisfy the necessary conditions:*

$$\pi_3^t + \delta\pi_2^{t+1} + \delta^2\pi_1^{t+2} = 0 \text{ for all } t \geqslant 1 \tag{14}$$

$$\begin{aligned} &u_2^t + f_1(p^t, w^t)u_3^t + \delta u_1^{t+1} + \delta g_1(p^t, w^t)\left(f_2(p^{t+1}, w^{t+1})u_3^{t+1} + u_4^{t+1}\right) \\ -\delta &\frac{g_1(p^t, w^t)g_2(p^{t+1}, w^{t+1})}{g_1(p^{t+1}, w^{t+1})}\left(u_2^{t+1} + f_1(p^{t+1}, w^{t+1})u_3^{t+1} + \delta u_1^{t+2}\right) = 0 \text{ for all } t \geqslant 1. \end{aligned} \tag{15}$$

Proof. See Appendix B ■

Condition (14) represents the principal's Euler equation and characterizes the equilibrium remuneration choice. Given initial prices, this equation is sufficient to pin down the MPE price path of the delegation game. Note that (14) is the same as the precommitment condition (5) of a durable goods monopolist who does not engage in delegation. Therefore, beginning in period 2, it will generate an identical price sequence[2]. The important distinction is that now this price plan emerges from the interactions of sophisticated players who use time consistent strategies. Furthermore, the contract between the principal and the manager is self-enforcing: rational players will follow through on their pricing and remuneration strategies in all periods and states of the world.

The above result does not depend on managerial preferences. In a setup where strategies are chosen simultaneously and delegation is costless, the principal will attain the precommitment optimum even if the manager undergoes unanticipated preference shocks.

Equation (15) describes the intertemporal trade-off of the manager: she is willing to incur effort disutility today if she expects to be rewarded for that in future periods. The

[2] To obtain her precommitment optimum, the period-1 principal chooses her preferred price p^1 and a wage w^1 that would motivate the manager to choose the precommitment price p^2 in the following period, i.e. w^1 solves $f(p^1, w^1) = p^2$. Subsequent interactions will yield a price sequence that follows (14).

LHS incorporates the payoff effects of a deviation from the equilibrium price path.

- A marginal change in the current pricing strategy will affect current and future demand. Time consistency and rational expectations imply that the resulting effort disutility effect can be further broken down into: i) direct effect, captured by the term $u_2^t + \delta u_1^{t+1}$; and ii) internal strategic effect, embodied in the term $f_1(p^t, w^t)u_3^t$.

- Furthermore, a change in the current price will have repercussions for future monetary rewards. The wage adjustment will affect utility directly and through the internal strategic effect. This is accounted for by the term $\delta g_1(p^t, w^t)\big(f_2(p^{t+1}, w^{t+1})u_3^{t+1} + u_4^{t+1}\big)$.

- Rational expectations imply that the manager will react concurrently to the anticipated wage adjustment. These secondary price corrections will affect effort disutility directly and through the internal strategic effect. The payoff consequences are reflected by the term $\delta \frac{g_1(p^t, w^t)g_2(p^{t+1}, w^{t+1})}{g_1(p^{t+1}, w^{t+1})}\big(u_2^{t+1} + f_1(p^{t+1}, w^{t+1})u_3^{t+1} + \delta u_1^{t+2}\big)$.

Along the equilibrium path prices are determined optimally, so all effects sum up to 0.

4.1 Numerical Simulations

In this subsection we use numerical simulations to quantify the properties of the delegation equilibrium studied above. We adopt a linear-quadratic payoff specification, which yields a computable Markov perfect equilibrium in linear remuneration and pricing strategies.

4.1.1 Linear-Quadratic Payoff Specification

Assume that the consumers' benefit evaluation v is uniformly distributed. Thus, any monotonically decreasing price sequence which satisfies (1) would yield a linear instantaneous demand:

$$x^t = M\big((p^{t-1} - \beta p^t) - (p^t - \beta p^{t+1})\big) \text{ for all } t > 0 \tag{16}$$

Furthermore, suppose that the monopolist's cost function is quadratic:

$$C(x^t) = \frac{\psi}{2}(x^t)^2 \tag{17}$$

The above assumptions imply a linear-quadratic instantaneous profit that is given by

$$\pi^t = Mp^t\big((p^{t-1} - \beta p^t) - (p^t - \beta p^{t+1})\big) - \frac{M^2\psi}{2}\big((p^{t-1} - \beta p^t) - (p^t - \beta p^{t+1})\big)^2 \tag{18}$$

Finally, suppose that the manager is endowed with preferences that are represented by a linear-quadratic utility function:

$$u^t = Pw^t - \frac{Q}{2}(w^t)^2 - Rx^t - \frac{S}{2}(x^t)^2 \tag{19}$$

We focus the analysis on equilibrium paths with positive marginal utility of income ($P - Qw^t > 0$ for all t) and negative marginal utility of effort ($-R - Sx^t < 0$ for all t).

4.1.2 Equilibrium Characterization

Under the linear-quadratic payoff specification defined above, the precommitment Euler equation (5) and the time-consistent Euler equation (9) become respectively

$$M\beta(p^t - \psi x^t) - M\delta(\beta + 1)(p^{t+1} - \psi x^{t+1}) + \delta x^{t+1} + M\delta^2(p^{t+2} - \psi x^{t+2}) = 0 \tag{20}$$

and

$$-M(\beta+1)(p^t - \psi x^t) + x^t + f_1(p^t)M(p^t - \psi x^t) + M\delta\beta(p^{t+1} - \psi x^{t+1}) = 0 \qquad (21)$$

If β is sufficiently low, (20) will generate a price sequence that is monotonically decreasing.

Now consider the MPE of the simultaneous-move costless delegation game. We conjecture that the equilibrium pricing and remuneration strategies are given by:

$$p^t = a + b_1 p^{t-1} + b_2 w^{t-1},\ w^t = m + n_1 p^{t-1} + n_2 w^{t-1}$$

Note that the linearity of the manager's pricing strategy and the quadratic cost function ensure that the principal's instantaneous payoff (18) is concave in her choice variable w_t.

Substitution of the payoff definitions and strategy conjectures in (15) yields:

$$\begin{aligned}&-R + SMx^t(1+\beta) - b_1(R + SMx^t\beta) - \delta(R + SMx^{t+1}) - \delta n_1\big(b_2(R + SMx^{t+1}\beta) - (P - Qw^{t+1})\big)\\ &\quad -\delta n_2\big(-R + SMx^{t+1}(1+\beta) - b_1(R + SMx^{t+1}\beta) - \delta(R + SMx^{t+2})\big) = 0 \qquad (22)\end{aligned}$$

Applying coefficient matching to (20) and (22) gives us equations for the parameters of the equilibrium pricing and remuneration strategies. We focus on solutions that are dynamically stable: the eigenvalues of the matrix:

$$\begin{bmatrix} b_1 & b_2 \\ n_1 & n_2 \end{bmatrix}$$

are restricted to be within the unit circle.

4.1.3 Remuneration and Managerial Utility

Although managerial preferences do not affect prices, they have important repercussions for equilibrium wages. Consider the case of quadratic utility as specified by (19). The parameter Q determines the sensitivity of the manager's marginal utility of income to changes in remuneration. A high value of Q implies that the principal can easily affect the manager's intertemporal trade-off.

Next, we show that equilibrium wages will be low if Q is high enough. We construct a modified game $\hat{\Gamma}(n)$, in which the principal chooses $\hat{w}^t$, while the manager receives compensation $\omega^t = n\hat{w}^t$. Delegation is costless, therefore in equilibrium managerial remuneration will be identical to that in Γ: $\omega^t = w^t$, $\forall t$. Since ω^t does not depend on n, it follows that $\hat{w}^t$ and n are inversely related. Finally, condition (22) implies that $\{\hat{w}^t\}_{t=1}^{\infty}$ will be the equilibrium wage sequence in the costless delegation game Γ, where the manager's payoff is $u^t = Pw^t - \frac{nQ}{2}(w^t)^2 - Rx^t - \frac{S}{2}(x^t)^2$. A big n translates into more sensitive marginal utility of income in Γ.

4.1.4 Numerical Example

Now we use a base scenario parameter set to compute the MPE of the delegation game. The parametric specification of the numerical example and the equilibrium strategy parameter values are presented in Table 1.

β	δ	M	ψ	P	Q	R	S
.4	.7	200	.003	500	1	.005	.001

a	b_1	b_2	m	n_1	n_2
-.2964	.8453	.0006	424.10	-1.5699	.1513

Table 1: Numerical Example and Equilibrium Strategy Parameter Values

Figure 1 illustrates the precommitment price path, as well as the time-consistent price plan of a durable goods monopolist who does not resort to delegation for an initial condition

$p^0 = 1$. As expected, the precommitment prices are strictly above the time-consistent prices in all periods. Figure 2 depicts the wage plan that supports the precommitment prices in a time consistent equilibrium. It also demonstrates that an increase in Q reduces equilibrium wages.

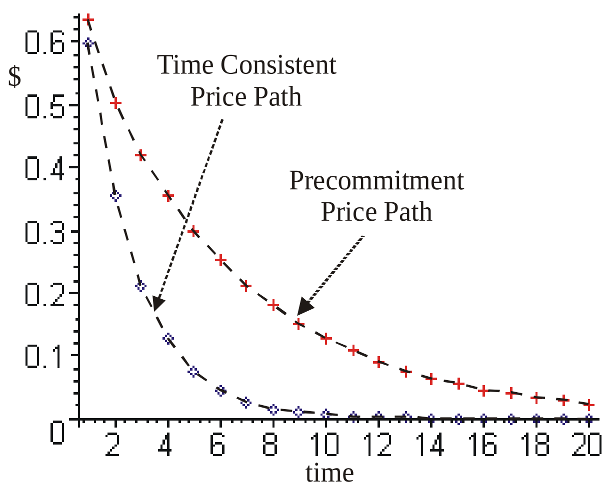

Figure 1

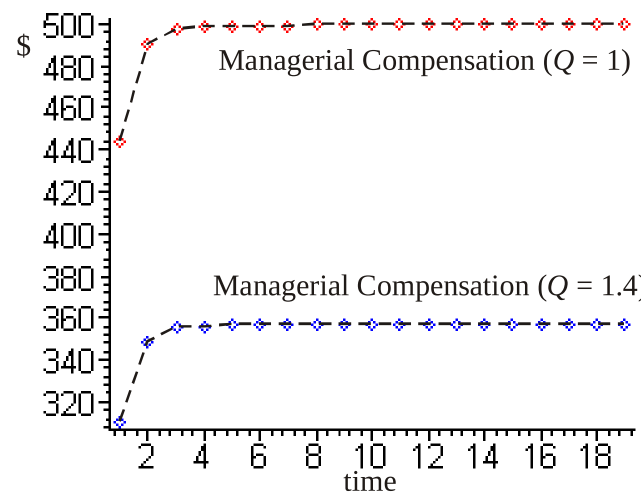

Figure 2

5 Extensions

This section analyzes the sensitivity of our delegation equilibrium to departures from the assumptions underlying the costless delegation game. In particular, we explore the impact of cost considerations and alternative timing on the commitment properties of delegation.

5.1 Costly Delegation

First, suppose that the monetary rewards needed to motivate the manager to choose the precommitment price path are non-negligible relative to monopoly profits. In this environment the commitment value of delegation will be weighed against its cost.

In order to study the effect of cost considerations, we now assume that the principal's objective is maximization of lifetime profit net of managerial compensation:

$$\tilde{\Pi}^{\tau} = \sum_{t=\tau}^{\infty} \delta^{t-1}(\pi^t - w^t)$$

Thus, managerial remuneration will affect the principal's payoff directly, as well as through its intertemporal incentive effect on the manager's pricing decisions.

Under this payoff specification, the principal's Bellman equation can be written as:

$$\Pi(p^{t-1}, w^{t-1}) = \max_{w^t} \left\{ \pi\left(p^{t-1}, f(p^{t-1}, w^{t-1}), f\big(f(p^{t-1}, w^{t-1}), w^t\big)\right) - w^t + \delta \Pi\big(f(p^{t-1}, w^{t-1}), w^t\big) \right\} \tag{23}$$

The manager's objective is unchanged. Her equilibrium strategy solves (11).

A brief inspection of equation (23) shows that costly delegation still eliminates the link between the principal's current payoffs and her future remuneration strategies, thus preserving its intertemporal commitment value. However, cost considerations will prevent the principal from precisely attaining the precommitment price path.

Proposition 8 *Suppose that assumptions A1 through A3 are satisfied. The MPE strategies of the durable goods monopoly costly delegation game Υ satisfy the necessary conditions:*

$$-\frac{1}{f_2(p^t, w^t)} + \frac{\delta f_1(p^{t+1}, w^{t+1})}{f_2(p^{t+1}, w^{t+1})} + (\pi_3^t + \delta \pi_2^{t+1} + \delta^2 \pi_1^{t+2}) = 0 \text{ for all } t \geqslant 1 \tag{24}$$

$$\begin{aligned} & u_2^t + f_1(p^t, w^t) u_3^t + \delta u_1^{t+1} + \delta g_1(p^t, w^t)\big(f_2(p^{t+1}, w^{t+1}) u_3^{t+1} + u_4^{t+1}\big) \\ & -\delta \frac{g_1(p^t, w^t) g_2(p^{t+1}, w^{t+1})}{g_1(p^{t+1}, w^{t+1})}\big(u_2^{t+1} + f_1(p^{t+1}, w^{t+1}) u_3^{t+1} + \delta u_1^{t+2}\big) = 0 \text{ for all } t \geqslant 1 \end{aligned} \tag{25}$$

Proof. See Appendix C. ■

The principal's new equilibrium condition (24) has an additional term $-\frac{1}{f_2(p^t, w^t)} + \frac{\delta f_1(p^{t+1}, w^{t+1})}{f_2(p^{t+1}, w^{t+1})}$ that accounts for current and future cost considerations. This term will distort the equilibrium price path away from the precommitment optimum.

The value of delegation as a commitment instrument now depends on managerial preferences. When the game fundamentals translate into an equilibrium wage profile that is insignificant relative to profits, the equilibrium price plan will be close to (14).

Under the linear-quadratic managerial utility specification (19), the distortion term will be small when the value of Q is high enough, which implies a sensitive marginal utility of income. To see this, consider a new game $\hat{\Upsilon}(n)$, where the principal's instantaneous payoff is defined as $\pi^t - n\tilde{w}^t$. It is easy to show that in this game the equilibrium strategies satisfy (25) and:

$$-\frac{n}{f_2(p^t, \hat{w}^t)} + \frac{\delta n f_1(p^{t+1}, \hat{w}^{t+1})}{f_2(p^{t+1}, \hat{w}^{t+1})} + (\pi_3^t + \delta\pi_2^{t+1} + \delta^2\pi_1^{t+2}) = 0 \qquad (26)$$

If the parameter n goes to zero, the game $\hat{\Upsilon}(n)$ converges to the costless delegation game Γ. By the lower hemicontinuity of MPE, the equilibrium price sequence will converge to the precommitment price plan generated by (5). Finally, note that the equilibrium of $\hat{\Upsilon}(n)$ is the same as the equilibrium of a costly delegation game Υ, in which the manager's instantaneous payoff is $u^t = Pw^t - \frac{Q}{2n}(w^t)^2 - Rx^t - \frac{S}{2}(x^t)^2$.

The above argument can be generalized for any utility function of the type $u(x^t, w^t) = \eta(x^t) + \varphi(w^t)$, provided that there exists $r < 0$ such that $\varphi'(nw^t) = n^r\varphi'(w^t)$.[3] A higher absolute value of $\varphi''(w^t)$ would imply lower equilibrium compensation.

[3] An example of such function would be $u(x, w) = \eta(x) + w^\sigma$, where $\sigma < 1$.

5.2 Alternative Timing

Now we investigate the sensitivity of the delegation equilibrium to changes in the timing of strategy selection. In particular, we analyze a costless delegation game in which the principal chooses the current compensation before the manager's pricing decision.

The sequential strategy choice implies an asymmetry of the players' perceptions regarding the contemporary industry state. From the principal's viewpoint, the period-t industry state can be summarized only by the previous price p^{t-1}. Let her Markov-perfect remuneration strategy be $w^t \equiv g(p^{t-1})$. The manager is the second mover, thus her perceived industry state is now characterized by (p^{t-1}, w^t) and her Markov-perfect strategy is $p^t \equiv f(p^{t-1}, w^t)$.

The MPE strategies of the sequential-move costless delegation game solve:

$$\Pi(p^{t-1}) = \max_{w^t} \left\{ \pi\left(p^{t-1}, f(p^{t-1}, w^t), f\big(f(p^{t-1}, w^t), g(f(p^{t-1}, w^t))\big)\right) + \delta\Pi\big(f(p^{t-1}, w^t)\big)\right\} \quad (27)$$

$$V(p^{t-1}, w^t) = \max_{p^t} \left\{ u\left(p^{t-1}, p^t, f\big(p^t, g(p^t)\big), w^t\right) + \delta V\big(p^t, g(p^t)\big)\right\}, \quad (28)$$

where rational expectations imply that:

$$g(p^{t-1}) = \arg\max_{w^t} \left\{ \pi\left(p^{t-1}, f(p^{t-1}, w^t), f\big(f(p^{t-1}, w^t), g(f(p^{t-1}, w^t))\big)\right) + \delta\Pi\big(f(p^{t-1}, w^t)\big)\right\} \quad (29)$$

$$f(p^{t-1}, w^t) = \arg\max_{p^t} \left\{ u\left(p^{t-1}, p^t, f\big(p^t, g(p^t)\big), w^t\right) + \delta V\big(p^t, g(p^t)\big)\right\} \quad (30)$$

The principal's Bellman equation (27) shows that sequential-move delegation preserves the link between current monopoly profits π^t and her future remuneration strategy $w^{t+1} \equiv g(p^t)$. The period-$t+1$ wage constitutes an element of the state space of the period-$t+1$ manager. Thus, w^{t+1} affects the period-$t+1$ pricing strategy $p^{t+1} \equiv f(p^t, w^{t+1})$, and through it period-$t$ profits. This suggests that sequential-move costless delegation cannot resolve the principal's time inconsistency problem.

Proposition 9 *The MPE strategies of the durable goods monopoly sequential-move delegation game satisfy necessary conditions:*

$$\pi_2^t + \big(f_1(p^t, w^{t+1}) + f_2(p^t, w^{t+1})g_1(p^t)\big)\pi_3^t + \delta\pi_1^{t+1} = 0 \textit{ for all } t \geqslant 2 \tag{31}$$

$$u_2^t + f_1(p^t, w^{t+1})u_3^t + \delta u_1^{t+1} + g_1(p^t)\big(f_2(p^t, w^{t+1})u_3^t + \delta u_4^{t+1}\big) = 0 \textit{ for all } t \geqslant 2 \tag{32}$$

Proof. See Appendix C. ∎

It is easy to demonstrate that costless sequential-move delegation has no commitment power: it generates a price sequence identical to the time-consistent plan of a durable goods monopolist who does not resort to delegation. Consider the principal's necessary condition (31). Let $p^{t+1} = \tilde{f}(p^t)$ denote the equilibrium law-of-motion of market prices under sequential-move delegation. Since the MPE pricing and remuneration strategies are respectively $f(p,w)$ and $g(p)$, this implies that $\tilde{f}(p) = f(p, g(p))$. Thus, we can rewrite (31) as:

$$\pi_2^t + \tilde{f}_1(p^t)\pi_3^t + \delta\pi_1^{t+1} = 0 \tag{33}$$

Any law-of-motion function $\tilde{f}(p)$ that solves (33) would also solve (9). Similarly, if $f(p)$ solves (9), it would also solve (33).

6 Conclusion

This paper studies intertemporal commitment through delegation of management in a durable goods monopoly setup. We explore a simultaneous-move infinite-horizon game in which the principal entrusts pricing decisions to a manager who dislikes production effort but enjoys monetary rewards. The separation of ownership from day-to-day pricing decisions eliminates the dependence of current profits on the principal's future policies, thus alleviating the monopolist's dynamic consistency problem.

The analysis demonstrates that when the cost of delegation is low relative to instantaneous profits, the principal can attain the optimal precommitment price plan in a perfect rational expectations equilibrium. The management contract is time-consistent: no player has an incentive to deviate from her equilibrium strategy in any period. For the case of linear quadratic payoffs we provide a numerical characterization of the delegation equilibrium.

We also explore the sensitivity of this result to changes in the payoff structure and the timing of activities: i) costly delegation has commitment power, but the principal's cost considerations distort the price path away from the precommitment optimum; and ii) sequential-move delegation has no commitment power and yields the time-consistent price path of a durable goods monopolist who does not engage in delegation.

Appendix A. Pricing Without Delegation

Precommitment Price Path

Suppose that in period 1 the monopolist can precommit to an entire sequence $\{p^t\}_{t=1}^{\infty}$ of market prices. The decision maker in that period maximizes remaining lifetime profit:

$$\max_{\{p^t\}_{t=1}^{\infty}} \Pi^1 = \sum_{t=\tau}^{\infty} \delta^{t-1} \pi^t(p^{t-1}, p^t, p^{t+1}).$$

Differentiation with respect to p^1 yields the first-order condition (4). Similarly, differentiation with respect to an arbitrary p^t (where $t \geqslant 2$) gives us condition (5).

Time Consistent Price Path

Now consider the case with no intertemporal precommitment. Suppose that the stationary Markov-perfect strategy is given by $p^t = f(p^{t-1}), \forall t$. Assumptions A1, A2 guarantee the concavity of π^t in the current price p^t. Differentiating the current decision maker's Bellman equation (6) with respect to p^t yields the first-order condition:

$$\pi_2^t + f_1(p^t)\pi_3^t + \delta V_1(p^t) = 0. \tag{34}$$

Thus, we have that:

$$V_1(p^t) = -\frac{\pi_2^t + f_1(p^t)\pi_3^t}{\delta}. \tag{35}$$

By assumption $f(p)$ is the Markov perfect equilibrium strategy. Therefore,

$$V(p^{t-1}) = \pi(p^{t-1}, f(p^{t-1}), f(f(p^{t-1})) + \delta V(f(p^{t-1})). \tag{36}$$

Differentiating (36) with respect to p^{t-1} yields:

$$V_1(p^{t-1}) = \pi_1^t + f_1(p^{t-1})\pi_2^t + f_1(p^t)f_1(p^{t-1})\pi_3^t + \delta f_1(p^{t-1})V_1(p^t). \tag{37}$$

Substituting the derivative of the value function $V_1(p)$ from (35) into (37) gives:

$$-\frac{\pi_2^{t-1} + f_1(p^{t-1})\pi_3^{t-1}}{\delta} = \pi_1^t + f_1(p^{t-1})\pi_2^t + f_1(p^t)f_1(p^{t-1})\pi_3^t - f_1(p^{t-1})\pi_2^t - f_1(p^{t-1})f_1(p^t)\pi_3^t. \tag{38}$$

After rearranging (38) and shifting it one period ahead we get (9).

Appendix B. MPE Of The Simultaneous-Move Costless Delegation Game

Suppose that in each period t the Markov-perfect equilibrium strategies of the principal and the manager are respectively $w^t = g(p^{t-1}, w^{t-1})$ and $p^t = f(w^{t-1}, p^{t-1})$.

1. The Principal's Necessary Condition

First consider the Principal's Bellman equation (10). Provided that the manager's pricing strategy is not too convex, assumptions A1, A2 will ensure that π^t is concave in the current wage w^t. Differentiation with respect to w^t yields the first-order condition:

$$f_2(p^t, w^t)\pi_3^t + \delta\Pi_2(p^t, w^t) = 0. \tag{39}$$

Solving for $\Pi_2(p^t, w^t)$ gives us:

$$\Pi_2(p^t, w^t) = -\frac{f_2(p^t, w^t)\pi_3^t}{\delta}. \tag{40}$$

By assumption $g(p, w)$ is the principal's Markov-perfect equilibrium strategy. Therefore, it satisfies the recursive equation:

$$\begin{aligned}\Pi(p^{t-1}, w^{t-1}) &= \pi\left(p^{t-1}, f(p^{t-1}, w^{t-1}), f\big(f(p^{t-1}, w^{t-1}), g(p^{t-1}, w^{t-1})\big)\right) \\ &\quad +\delta\Pi\big(f(p^{t-1}, w^{t-1}), g(p^{t-1}, w^{t-1})\big).\end{aligned} \tag{41}$$

Differentiating (41) with respect to w^{t-1} yields:

$$\begin{aligned}\Pi_2(p^{t-1}, w^{t-1}) &= \big(f_1(p^t, w^t)f_2(p^{t-1}, w^{t-1})f_2(p^t, w^t)g_2(p^{t-1}, w^{t-1})\big)\pi_3^t \\ +f_2(p^{t-1}, w^{t-1})\pi_2^t &+ \delta f_2(p^{t-1}, w^{t-1})\Pi_1(p^t, w^t) + \delta g_2(p^{t-1}, w^{t-1})\Pi_2(p^t, w^t).\end{aligned} \tag{42}$$

Substitution of $\Pi_2(p, w)$ from (40) into (42) gives us an expression for $\Pi_1(p, w)$:

$$\Pi_1(p^t, w^t) = -\frac{\pi_3^{t-1}}{\delta^2} - \frac{\pi_2^t}{\delta} - \frac{f_1(p^t, w^t)\pi_3^t}{\delta}. \tag{43}$$

Similarly, differentiating (41) with respect to p^{t-1} yields:

$$\begin{aligned}\Pi_1(p^{t-1}, w^{t-1}) &= \big(f_1(p^t, w^t)f_1(p^{t-1}, w^{t-1}) + f_2(p^t, w^t)g_1(p^{t-1}, w^{t-1})\big)\pi_3^t \\ +\pi_1^t + f_1(p^{t-1}, w^{t-1})\pi_2^t &+ \delta f_1(p^{t-1}, w^{t-1})\Pi_1(p^t, w^t) + \delta g_1(p^{t-1}, w^{t-1})\Pi_2(p^t, w^t).\end{aligned} \tag{44}$$

After substituting $\Pi_1(p,w)$ from (43) and $\Pi_2(p,w)$ from (40) into (44) and shifting the expression two periods ahead we obtain (14).

2. The Manager's Necessary Condition

Now consider the problem of the agent. Assumptions A1 through A3 imply that u^t is concave in the current price p^t. Differentiating Bellman equation (11) with respect to p^t yields the first-order condition:

$$u_2^t + f_1(p^t, w^t)u_3^t + \delta V_1(p^t, w^t) = 0. \tag{45}$$

From this equation we obtain an expression for $V_1(p,w)$:

$$V_1(p^t, w^t) = -\frac{u_2^t + f_1(p^t, w^t)u_3^t}{\delta}. \tag{46}$$

By assumption, the Markov perfect equilibrium strategies of the principal and the manager are respectively $g(p,w)$ and $f(p,w)$. Therefore, they satisfy the manager's recursive equation

$$\begin{aligned} V(p^{t-1}, w^{t-1}) &= u\left(p^{t-1}, f(p^{t-1}, w^{t-1}), f\big(f(p^{t-1}, w^{t-1}), g(p^{t-1}, w^{t-1})\big), g(p^{t-1}, w^{t-1})\right) \\ &\quad + \delta V\big(f(p^{t-1}, w^{t-1}), g(p^{t-1}, w^{t-1})\big). \end{aligned} \tag{47}$$

After differentiating (47) with respect to p^{t-1} we get:

$$\begin{aligned} V_1(p^{t-1}, w^{t-1}) &= u_1^t + f_1(p^{t-1}, w^{t-1})u_2^t + \big(f_1(p^t, w^t)f_1(p^{t-1}, w^{t-1}) + f_2(p^t, w^t)g_1(p^{t-1}, w^{t-1})\big)u_3^t \\ &\quad + g_1(p^{t-1}, w^{t-1})u_4^t + \delta f_1(p^{t-1}, w^{t-1})V_1(p^t, w^t) + \delta g_1(p^{t-1}, w^{t-1})V_2(p^t, w^t). \end{aligned} \tag{48}$$

Substitution of $V_1(p,w)$ from (46) in (48) gives us an equation for $V_2(p,w)$:

$$V_2(p^t,w^t) = -\frac{u_2^{t-1} + f_1(p^{t-1},w^{t-1})u_3^{t-1} + \delta u_1^t}{\delta^2 g_1(p^{t-1},w^{t-1})} - \frac{f_2(p^t,w^t)u_3^t + u_4^t}{\delta}. \tag{49}$$

Differentiating (47) with respect to w^{t-1} yields:

$$V_2(p^{t-1},w^{t-1}) = f_2(p^{t-1},w^{t-1})u_2^t + \left(f_1(p^t,w^t)f_2(p^{t-1},w^{t-1}) + f_2(p^t,w^t)g_2(p^{t-1},w^{t-1})\right)u_3^t \tag{50}$$

$$+g_2(p^{t-1},w^{t-1})u_4^t + \delta f_2(p^{t-1},w^{t-1})V_1(p^t,w^t) + \delta g_2(p^{t-1},w^{t-1})V_2(p^t,w^t).$$

Finally, after substituting $V_1(p,w)$ from (46) and $V_2(p,w)$ from (49) in (50) and shifting it two periods ahead we obtain (15).

Appendix C. Extensions

Just as before, assumptions A1 though A3 ensure that the players's instantaneous payoffs are concave in their choice variables.

1. The Costly Delegation Game

First, consider the principal's problem. Differentiating Bellman equation (23) with respect to w_t yields the first-order condition

$$f_2(p^t,w^t)\pi_3^t - 1 + \delta\Pi_2(p^t,w^t) = 0. \tag{51}$$

Solving for $\Pi_2(p^t,w^t)$ gives us

$$\Pi_2(p^t, w^t) = -\frac{f_2(p^t, w^t)\pi_3^t - 1}{\delta}. \tag{52}$$

By assumption $g(p, w)$ is the principal's Markov-perfect equilibrium strategy. Thus,

$$\begin{aligned}\Pi(p^{t-1}, w^{t-1}) &= \pi\left(p^{t-1}, f(p^{t-1}, w^{t-1}), f\big(f(p^{t-1}, w^{t-1}), g(p^{t-1}, w^{t-1})\big)\right) \\ &\quad -g(p^{t-1}, w^{t-1}) + \delta\Pi\big(f(p^{t-1}, w^{t-1}), g(p^{t-1}, w^{t-1})\big).\end{aligned} \tag{53}$$

Differentiating (53) with respect to with respect to w^{t-1} yields

$$\begin{aligned}\Pi_2(p^{t-1}, w^{t-1}) &= \big(f_1(p^t, w^t)f_2(p^{t-1}, w^{t-1}) + f_2(p^t, w^t)g_2(p^{t-1}, w^{t-1})\big)\pi_3^t \\ -g_2(p^{t-1}, w^{t-1}) + f_2(p^{t-1}, w^{t-1})\pi_2^t &+ \delta f_2(p^{t-1}, w^{t-1})\Pi_1(p^t, w^t) + \delta g_2(p^{t-1}, w^{t-1})\Pi_2(p^t, w^t).\end{aligned} \tag{54}$$

Substituting $\Pi_2(p, w)$ from (52) into (54) gives us an equation for $\Pi_1(p, w)$:

$$\Pi_1(p^t, w^t) = -\frac{\pi_3^{t-1}}{\delta^2} - \frac{\pi_2^t}{\delta} - \frac{f_1(p^t, w^t)\pi_3^t}{\delta} + \frac{1}{\delta^2 f_2(p^{t-1}, w^{t-1})}. \tag{55}$$

Next, differentiating (53) with respect to with respect to p^{t-1} yields

$$\begin{aligned}\Pi_1(p^{t-1}, w^{t-1}) &= \big(f_1(p^t, w^t)f_1(p^{t-1}, w^{t-1}) + f_2(p^t, w^t)g_1(p^{t-1}, w^{t-1})\big)\pi_3^t \\ -g_1(p^{t-1}, w^{t-1}) + \pi_1^t + f_1(p^{t-1}, w^{t-1})\pi_2^t &+ \delta f_1(p^{t-1}, w^{t-1})\Pi_1(p^t, w^t) + \delta g_1(p^{t-1}, w^{t-1})\Pi_2(p^t, w^t).\end{aligned} \tag{56}$$

Substituting $\Pi_2(p, w)$ from (52) and $\Pi_1(p, w)$ from (55) into (56) obtains (24).

The manager's Bellman equation remains unchanged. Thus, her Markov-perfect equilibrium necessary condition is still given by (25).

2. Sequential-Move Delegation

Differentiating the principal's Bellman equation yields the first-order condition

$$f_2(p^{t-1}, w^t)\pi_t^2 + \big(f_1(p^t, w^{t+1}) + f_2(p^t, w^{t+1})g_1(w^{t+1})\big) f_2(p^{t-1}, w^t)\pi_t^3 + \delta f_2(p^{t-1}, w^t)\Pi_1(p^t) = 0. \tag{57}$$

Solving (57) for $\Pi_1(p)$ yields

$$\Pi_1(p^t) = -\frac{1}{\delta}\left(\pi_t^2 + \big(f_1(p^t, w^{t+1}) + f_2(p^t, w^{t+1})g_1(w^{t+1})\big)\pi_t^3\right). \tag{58}$$

By assumption the principal's equilibrium remuneration strategy is $g(p)$. Thus, the following recursive equation must hold:

$$\Pi(p^{t-1}) = \pi\left(p^{t-1}, f\left(p^{t-1}, g(p^{t-1})\right), f\big(f(p^{t-1}, g(p^{t-1})), g(f(p^{t-1}, g(p^{t-1})))\big)\right) + \delta\Pi\left(f\left(p^{t-1}, g(p^{t-1})\right)\right). \tag{59}$$

Differentiating (59) with respect to p^{t-1} gives us

$$\begin{aligned}\Pi_1(p^{t-1}) &= \big(f_1(p^t, w^{t+1}) + f_2(p^t, w^{t+1})g_1(p^t)\big)\big(f_1(p^{t-1}, w^t) + f_2(p^{t-1}, w^t)g_1(p^{t-1})\big)\pi_3^t \qquad (60)\\ &+\pi_1^t + \big(f_1(p^{t-1}, w^t) + f_2(p^{t-1}, w^t)g_1(p^{t-1})\big)\pi_2^t + \delta\big(f_1(p^{t-1}, w^t) + f_2(p^{t-1}, w^t)g_1(p^{t-1})\big)\Pi_1(p^t).\end{aligned}$$

Substituting (58) into (60) obtains (31).

Now consider the problem of the manager. Differentiating Bellman equation (28) yields the first-order condition

$$u_2^t + \big(f_1(p^t, w^{t+1}) + f_2(p^t, w^{t+1})g_1(p^t)\big)u_3^t + \delta\big(V_1(p^t, w^{t+1}) + g_1(p^t)V_2(p^t, w^{t+1})\big) = 0. \tag{61}$$

By assumption the equilibrium pricing strategy is $f(p, w)$. Thus, the following recursive equation must hold:

$$V(p^{t-1}, w^t) = u\left(p^{t-1}, f(p^{t-1}, w^t), f\big(f(p^{t-1}, w^t), g(f(p^{t-1}, w^t))\big), w^t\right) + \delta V\left(f(p^{t-1}, w^t), g\big(f(p^{t-1}, w^t)\big)\right). \tag{62}$$

Differentiating (62) with respect to p^{t-1} yields

$$\begin{aligned} V_1(p^{t-1}, w^t) &= u_1^t + f_1(p^{t-1}, w^t)u_2^t + \big(f_1(p^t, w^{t+1}) + f_2(p^t, w^{t+1})g_1(p^t)\big) f_1(p^{t-1}, w^t)u_3^t \\ &\quad + \delta f_1(p^{t-1}, w^t)\big(V_1(p^t, w^{t+1}) + g_1(p^t)V_2(p^t, w^{t+1})\big). \end{aligned} \tag{63}$$

Substituting $V_1(p^t, w^{t+1}) + g_1(p^t)V_2(p^t, w^{t+1})$ from (61) into (63) gives us

$$V_1(p^{t-1}, w^t) = u_1^t. \tag{64}$$

Next, differentiate (62) with respect to w^t:

$$\begin{aligned} V_2(p^{t-1}, w^t) &= f_2(p^{t-1}, w^t)u_2^t + \big(f_1(p^t, w^{t+1}) + f_2(p^t, w^{t+1})g_1(p^t)\big) f_2(p^{t-1}, w^t)u_3^t + u_4^t \\ &\quad + \delta f_2(p^{t-1}, w^t)\big(V_1(p^t, w^{t+1}) + g_1(p^t)V_2(p^t, w^{t+1})\big). \end{aligned} \tag{65}$$

Again, substitute $V_1(p^t, w^{t+1}) + g_1(p^t)V_2(p^t, w^{t+1})$ from (61) into (65):

$$V_2(p^{t-1}, w^t) = u_4^t. \tag{66}$$

Finally, substituting (64) and (66) into the first-order condition (61) gives us (32).

BIBLIOGRAPHY

[1] Baye, M., K. Crocker, and J. Ju, "Divisionalization, Franchising, and Divestiture Incentives in Oligopoly", *The American Economic Review*, 1996, pp. 223-236

[2] Basu, K., *Lectures in Industrial Organization Theory*, Basil Blackwell, Oxford, 1993

[3] Basu, K., "Stackelberg Equilibrium in Oligopoly: An Explanation Based on Managerial Incentives", *Journal of Economic Letters*, 1995, pp. 459-464.

[4] Bond, E., and L., Samuelson, "Durable Good Monopolies with Rational Expectations and Replacement Sales", *RAND Journal of Economics*, 1984, pp. 336-345

[5] Bulow, J., "Durable Goods Monopolist", *Journal of Political Economy*, 1982, pp. 314-322

[6] Bulow, J., "An Economic Theory of Planned Obsolescence", *Quarterly Journal of Economics*, 1986, pp. 729-749

[7] Coase, R., "Durability and Monopoly," Journal of Law & Economics, 1972, pp. 143-149

[8] Fershtman, J., and K. Judd, "Equilibrium Incentives in Oligopoly", *American Economic Review*, 1987, pp. 927-940

[9] Fudenberg, D., and J. Tirole, *Game Theory*, Cambridge, Mass.: MIT Press, 1991

[10] Gul, F., H. Sonnenschein, and R. Wilson, "Foundations of Dynamic Monopoly and the Coase Conjecture", *Journal of Economic Theory*, 1986, pp. 155-190

[11] Klein, P., P. Krusell, and J. Rios-Rull, "Time Consistent Public Expenditure", 2002, working paper

[12] Kydland, F., and E. Prescott, "Rules rather than Discretion: The Inconsistency of Optimal Plans", *Journal of Political Economy*, 1977, pp. 473 - 492

[13] Miller, N., and A. Pazgal, "The Equivalence of Price and Quantity Competition with Delegation", *RAND Journal of Economics*, 2001, pp. 284-301

[14] Rogers, C. A., Expenditure Taxes, "Income Taxes, and Time Inconsistency", *Journal of Public Economics*, 1987, pp. 215-230

[15] Rogoff, K., "The Optimal Degree of Commitment to an Intermediate Monetary Target", *The Quarterly Journal of Economics*, 1985, pp. 1169-1189

[16] Sklivas, S., "The Strategic Choice of Managerial Incentives", *RAND Journal of Economics*, 1987, pp. 452-458

[17] Stokey, N., "Intertemporal Price Discrimination", *Quarterly Journal of Economics*, 1979, pp. 355-371

[18] Stokey, N., "Rational Expectations and Durable Goods Pricing", *Bell Journal of Economics*, 1981, pp. 112-128