Vertical Differentiation in Monetary Search-Theoretic Model: Revisited

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Abstract

Quality levels have been widely discussed in the “second generation” of monetary search-theoretical model. In this article, the discussion on quality levels being produced is extended. We found that whether money holders have quality preferences is crucial to the results. While the effect of the fraction of money on quality levels is consistent with current literature, the effect of the economy’s quality preferences on quality drives the quality levels to opposite directions. The results found in this exercise can be regarded as complementary to the findings in Trejos (1999).

Key words: money search, quality,

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1. Introduction

This paper uses the work of Trejos (1999) as a basis, but extends this work by considering welfare consequences in the decentralized markets, in which both quality and price are determined by the bargaining process, in addition to studying the characteristics of monetary equilibrium. It is argued that when agents can negotiate from a continuum of quality levels and their associated prices, money can still facilitate the exchange, but it may not enhance the quality levels. Therefore, a higher welfare does not result from higher quality levels, but the narrower price differential.

- Literature

Since Williamson and Wright (1994) formalized private information problems in the search theoretical models, it has attracted much attention to how the uncertainty of quality levels affects the role of money. With the presence of lemon problems, Williamson and Wright (1994) find that money can improve welfare when low quality is produced, while Trejos (1997) shows that a larger monetary distribution could worsen the lemon problems and lower welfare. There is no doubt that one way money can increase welfare is by lowering the information cost, either through middlemen [Li (1998, 99)] or through investment in information acquisition [Kim (1996)].

- Gap

Among all the studies stated above, higher welfare comes from more high quality goods in the economy. In other words, when there are only two fixed quality levels, either high or low quality, money would increase welfare by generating more high quality goods. However, would this statement still be true when there is a continuum of quality levels, which can be determined by the agents endogenously? In particular, does money enhance the quality levels while it improves welfare?
In order to gain insight into how money affects the produced quality levels, and hence, welfare, we combine the issues of vertical differentiation and the pure theory of money and we focus on the characteristics of monetary equilibria. In the absence of incentive problems, both divisible goods and indivisible money are assumed. Each agent has its own quality preference, which is uniformly distributed over a range, and agents will meet in pairs to negotiate the quality level to be produced. Each quality level is associated with its price level per unit, which is determined, so that producers would feel indifferent about producing goods at any quality levels. The negotiation is in the form of the Nash bargaining process.

Consequently, this paper is able to show that money can enhance welfare even in the absence of lemons problem if money is not over abundant in the economy. That is because when only two qualities will be endogenously determined from a continuum of quality levels by the agents, who negotiate according to their quality preferences, a larger monetary distribution would lower qualities produced in both high and low levels. The lower qualities produced harm the price of high quality goods more than they do low quality goods; hence the price differential is narrower. However, these lower quality produced levels also induce more high quality goods to be produced, traded, and consumed, so welfare is higher under a larger monetary distribution. This result does not rely on the assumption that money holders have quality preferences.

However, when the range of the economy’s quality preferences shifts upwards, both the quality levels produced and welfare depend crucially on whether money holders have quality preferences. When money holders have quality preferences, the upward shift in the economy’s quality preferences would narrow the quality difference. Specifically, it reduces the high quality produced but increases the low quality produced. The increase in the surplus
of high quality producers is offset by the decrease in the surplus of low quality producers; hence, welfare remains unchanged. However, when money holders have no quality preferences, the upward shift in the economy’s quality preferences reduces both quality levels produced and increases welfare.

The rest of the paper is organized as follows: Section 2 gives a general description of the environment. Section 3 determines the preferred quality allocation, while Section 4 analyzes the cases when only two quality levels are chosen endogenously by the agents, in which both the first best quality allocation and the quality allocation determined by the Nash bargaining process are discussed. Section 5 involves both the conclusion as well as the future work.

2. The Environment

The economy has a continuum of agents, and the mass of agents is normalized to one. The production process is assumed to be instantaneous; hence, there is no production status. Following Kiyotaki and Wright (1993), we assume that each agent has its own specialization in production, which allows it to produce differentiated goods. However, each agent will receive zero utility from consuming its own product; hence, the possibility of autarky is ruled out, and agents have the incentive to search for partners to trade with. To determine the vertical product differentiation, a Mussa-Rosen (1978) type model is adopted. Following their setup, each agent is assumed to have its type \( t \), which identifies the quality preference of the agent, and is strictly positive and normally distributed over the interval \( [a, b] \).

- Monetary distribution

At the beginning of the period, a fraction \( M \) of agents will be chosen randomly to hold money. These agents are called money holders (M), and each of them holds at most one
unit of money. Money is assumed to be indivisible. Thus, the variable M represents the monetary distribution of the economy. The rest of the agents, a fraction (1-M), who don’t hold money, hold commodities. They are called commodity holders (C).

- Production Technology

To simplify the model, commodities are assumed to last for only one period without exception. As a commodity holder, s/he has the technology to produce not only the variety she is specialized in but also any quality level without investment costs. It is assumed that the quality level \( q_t \), at which each agent wishes to produce, follows its quality preference \( \theta \). Each quality level \( q_t \) is associated with a certainty quantity \( q_t^2 \). For example, a commodity holder with a higher \( \theta \) would produce \( q_t \) units of goods at quality level \( q_t^{2} \). Due to the zero utility from self-product consumption, the units produced will all be traded and consumed. The cost function is in the form of \( C(q, q_t) = c q_t^2 \), where \( c \) is the variable cost. Therefore, the marginal cost would be:

\[
MC(q_t) = c q_t^2,
\]

which implies that the increasing marginal cost restricts the quality level being produced.

- Information

It is assumed that each agent knows the variety of goods produced by other agents, but no agent knows the variety of goods preferred by other agents, which generates a double coincidence of wants problem. Let \( x \in (0, 1) \) denote the probability for the variety that one agent produced also preferred by her/his trading partner. When \( x = 0 \), barter trade is ruled out, and only monetary trade exists, which is the focus of this paper.

- Utility

Since the only possibility to trade is through money, let’s assume that the probability that each agent accepts money is one. After trading their one unit of money for their
preferred product, the money holders will receive utility from consumption, which is assumed to have the quasi-concave property:

\[ U(Q, q_i) = \Theta Q_i - P_i q_i, \]  

in which the utility depends positively on both \( Q \) and \( q_i \), but negatively on price \( P_i \). Note that the indivisibility of money simplifies the price level \( P_i \) to be \( 1/q_i \).

• Meeting/Bargaining Process

Since the variety produced by each agent is observable by all agents, each money holder would go straight to the commodity holder who produces her/his preferred variety of goods. Once they meet, they learn the quality preference of their trading partner and start to negotiate the quality level to be produced. The bargaining process follows Trejos (1999); however, the object of this negotiation is \( Q_i \), the quality of the product to be exchanged for one unit of money. Note that once quality is determined, the quantities \( q_i \) associated with it will be produced at that quality level, and the produced quantity will all be traded for one unit of money. After receiving that unit of money, the commodity holder becomes a money holder, and the money holder becomes a commodity holder after consuming the product.

To prevent the high profit of certain \( Q_i \) from attracting all commodity holders to produce at that quality level, a no-arbitrage condition is imposed:

\[ \frac{1}{q_i} - \frac{1}{q_j} = MC(Q_i) - MC(Q_j), \]  

where \( i \neq j \), which implies that the profit a commodity holder receives at all quality levels and their associated quantity is the same.

3. The Preferred Quality Allocation

The simplest case is where the quality level maximizes consumers’ utility and the variety of goods is offered at the price of the marginal cost, called the preferred quality \( Q^{*} \).
This problem can be written as:

\[ Q^0 = \underset{Q}{\arg \max} \{ U(Q) \} \]  \hspace{1cm} (4)

Subject to \[ P \leq MC \], \hspace{1cm} (5)

which will lead to:

\[ Q^0 = \frac{\theta}{2c} \]  \hspace{1cm} (6)

According to the distribution of \( \theta \), a continuum of preferred quality levels will be produced, and they are uniformly distributed over \([a/2c, (a+1)/2c]\).

4. The Quality Allocation—Only two quality levels to be produced

Let’s consider there are only two quality levels produced, as most monetary search-theoretical models with private information have discussed. Given that commodity holders are capable of producing any quality levels, what are the two equilibrium quality levels produced in the economy? Following the traditional vertical differentiation models, we introduce the demand for the two quality levels, and analyze quality allocations in the first best outcome and in the Nash bargaining outcome, in which two cases are discussed: money holders with and without quality preferences.

Let’s assume that the quality levels which commodity holders will produce are either \( Q_H \) or \( Q_L \), in the quantities \( q_H \) and \( q_L \), respectively. Given the distribution of quality preference \( \theta \), there must be a marginal type agent \( \tilde{\theta} \), who feels indifferent to \( Q_H \) and \( Q_L \) at price \( 1/q_H \) and \( 1/q_L \), respectively:

\[ \tilde{\theta} = \frac{1/q_H - 1/q_L}{Q_H - Q_L} \]  \hspace{1cm} (7)

This indicates that the agents with type \( \theta > \tilde{\theta} \) would prefer \( Q_H \), so the demand for \( Q_H \) is \( D_H = b - \tilde{\theta} \) while the agents with type \( \theta < \tilde{\theta} \) would prefer \( Q_L \), so the demand for \( Q_L \) is \( D_L = \tilde{\theta} - a \). Without losing generality, let’s assume \( b = a + 1 \) so that \( a < \tilde{\theta} < (a+1) \), and the
demand for high and low quality levels are: $D_H = a + 1 - \bar{\theta}$, and $D_L = \bar{\theta} - a$.

4.1. The First-Best Quality Allocation (Social Welfare Maximization)

When there are only two quality levels to be produced, the first best quality allocation, called $Q^*_H$ and $Q^*_L$, is defined as the outcome that maximizes social welfare. By solving the following problem:

$$\max_{Q_H, Q_L} \int_0^{a+1} (\theta Q_L - c Q_L^3) d\theta + \int_0^a (\theta Q_H - c Q_H^3) d\theta,$$

the first-best equilibrium quality allocation, $Q^*_H$ and $Q^*_L$, equals $\frac{4a+3}{2c}$ and $\frac{4a+1}{2c}$, respectively [Cremer and Thisse (1994)]. Thus, the equilibrium price differential in the first best quality allocation is:

$$\frac{1}{Q^*_H} - \frac{1}{Q^*_L} = \frac{4a+2}{c}. \hspace{1cm} (8)$$

4.2. Nash Bargaining Quality Allocation

When there are only two quality levels to be made, if the quality levels to be produced are determined by Nash bargaining between money holders and commodity holders, what will the quality allocation be? How would this quality allocation be different from that achieved in the first-best outcome? In this section, two cases will be discussed: the first case is about money holders with quality preferences, and the second case is about money holders without quality preferences.

Recall that the distribution of quality type $\theta$ is public information; individual’s quality preference $\theta$, however, is unobservable publicly. Agents will not learn the quality type $\theta$ of the other agent until the time of meeting. Let CH denote the commodity holders with $\theta > \bar{\theta}$, who prefer to produce products at a high quality level, and let CL denote the commodity
holders with $\theta < \hat{\theta}$, who prefer to produce products at a low quality level.

**Case One: Money holders with quality preferences**

When money holders are distinguished from their quality preference, in the case that only two quality levels will be determined, money holders would prefer either high or low quality goods, but not both. Let MH denote the money holders with $\theta > \hat{\theta}$, who prefer high quality and will trade with $Q_H$ producers only, and let ML denote the money holders with $\theta < \hat{\theta}$, who prefer low quality and will trade with $Q_L$. To be more specific, in the following case, let’s assume that MHs would negotiate with CHs only, while MLs would negotiate with CLs only [Figure 1]. In other words, when MHs meet CLs or MLs meet CHs, they separate right away without a deal.

During the meeting between MH and CH, and the meeting between ML and CL, the quality allocation, $Q_H$ and $Q_L$ and their associated quantities $q_H$ and $q_L$, then will be determined by the bargaining process between the two parties, respectively. After the deal is made, a commodity holder will produce his specialized variety at that quality level in its associated quantity, and trade them with one unit of money held by money holders. Then this commodity holder becomes a money holder, and that initial money holder becomes a commodity holder after consuming the product. Now, our concern is when only two quality levels are distinguished in the economy, what would the quality levels be from negotiation? We start from the value functions, following the description of negotiation process and its equilibrium.

**Value Functions**

Since production is instantaneous, there are four statuses in the economy: CH, CL,
MH, and ML. Let \( V_i \) be the value function of the agent at status \( i \), \( (i=CH, CL, MH, ML) \).

Equations (9)-(12) shows the flow values of agents in different statuses:

\[
\begin{align*}
\text{Equation (9) represents the flow value of a CH. With probability } & \mathcal{M}(a + 1 - \bar{\theta}), \text{ the person who} \\
& \text{prefers the variety produced by this CH would be a MH, who also prefers high quality } (\bar{\theta} > 0). \text{ Then this CH would spend } C(Q_H) \text{ to produce } q_H \text{ units at } Q_H \text{ level and would become} \\
& \text{a MH after the trade. Similarly, a CL would spend } C(Q_L) \text{ on production, and then would} \\
& \text{become a ML after trade, with probability } \mathcal{M}(\bar{\theta} - a) \text{[equation (10)]. Equation (11) shows} \\
& \text{that with probability } (1 - \mathcal{M})(a + 1 - \bar{\theta}), \text{ the producer of the variety preferred by MH would be a} \\
& \text{CH. After trade, this MH would receive } U(Q_H) \text{ from consuming the product and becomes a} \\
& \text{CH. Equation (11) shows the flow value of MH. With probability } (1 - \mathcal{M})(a + 1 - \bar{\theta}), \text{ MH will} \\
& \text{meet a } Q_H \text{ commodity holder. The flow value of ML is similar to that of MH. As presented} \\
& \text{in equation (12), the probability for a ML to trade from a CL is } (1 - \mathcal{M})(\bar{\theta} - a), \text{ then this ML} \\
& \text{will become a CL after consuming the product and receiving utility } U(Q_L). \\
\end{align*}
\]
with the properties: \( \frac{\partial \tilde{q}_i}{\partial V_{i\theta}} > 0 \) and \( \frac{\partial \tilde{q}_i}{\partial V_{c\theta}} < 0 \). The reason is that a higher \( V_{i\theta} \) (i=H, L) implies a higher threat point for money to bargain; consequently, a higher quality level will be produced. A higher \( V_{c\theta} \) (i=H, L), however, indicates a higher threat point for a commodity holder to bargain; hence, results in a lower quality level. Given the equilibrium quality levels \( \tilde{Q}_i \) of this case, let \( \tilde{q}_i \) denote its associated quantity levels. According to equation (13), the price differential of this case can be written as:

\[
\frac{V_{c\theta} + U(\tilde{Q}_i)}{V_{i\theta} - C(\tilde{Q}_i)} = \frac{U'(\tilde{Q}_i)}{C'(\tilde{Q}_i)},
\]

which depends on the equilibrium quality levels.

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**Welfare analysis under Nash Bargaining**

Welfare comes from the two types of agents: money holders and commodity holders. After trade, while money holders receive the utility from consumption, the commodity holders receive the profit from the trade. Therefore, the welfare \( W_1 \) in this case can be written as:

\[
W_1 = M(a+1-\delta) \int_{\delta}^{\delta+1} (P - MC(\tilde{Q}_H)) \tilde{q}_H d\theta + M(\delta - \alpha) \int_{\delta}^{\delta+1} (P - MC(\tilde{Q}_L)) \tilde{q}_L d\theta + (1 - M)(a+1-\delta) \int_{\delta}^{\delta+1} (\tilde{Q}_H - P) \tilde{q}_H d\theta + (1 - M)(\delta - \alpha) \int_{\delta}^{\delta+1} (\tilde{Q}_L - P) \tilde{q}_L d\theta.
\]

The first term is the welfare of commodity holders with higher quality preference. For each commodity holder who prefers high quality, the probability that trade will occur is \( M(a+1-\delta) \). After trade, he will receive \((P-MC)\) from each unit s/he produces. The second term represents the welfare of commodity holders with lower quality preferences. The probability that the trade will occur is \( M(\delta - \alpha) \). The third and the fourth terms are the welfare of money holders with high and low quality preferences, respectively. They will
receive net utility \( (\theta Q - P) \) from consuming each unit of goods. The probability of which the trade will occur for money holders with high quality preferences is \((1-M)(\theta + 1 - \theta)\), and that probability for those who prefer low quality is \((1-M)(\theta - a)\).

**Comparative Statics**

In this section, we focus on the effects of monetary distribution \(M\) and the economy’s quality preference “\(a\)”, on the quality levels and its associated price differential. A higher \(M\) implies a larger fraction of agents holding money; in other words, the amount of money distributed in the economy is larger. Recall that agents’ quality preferences are uniformly distributed in the range of \([a, a+1]\). Thus, any changes of “\(a\)” indicates the shift of the range of quality preferences of the economy.

From equations (9)-(12) and equation (16), we get:

**Proposition 1.1 (Effects of \(M\))**:

1. A larger \(M\) would lower both \(Q_H\) and \(Q_L\), if the following conditions hold:
   1. \(M < \frac{1}{2}\),
   2. \(rC(Q_H) < (a + 1 - \theta)(U(Q_H) - C(Q_H))(1 - 2M) < rU(Q_H)\), and
   3. \(rC(Q_L) < (\theta - a)(U(Q_L) - C(Q_L))(1 - 2M) < rU(Q_L)\).

   If the lower \(Q_H\) and the lower \(Q_L\), resulting from a higher \(M\), leaves \((Q_H - Q_L)\) unchanged, both the price differential \((P_H - P_L)\) and the marginal type \(\theta\) would decrease. Consequently, welfare is higher since more quality goods are produced and consumed.

**PROOF:** see the appendix.

Proposition 1 shows that when less than half of the population holds money, and
when the net utility of consumption (either at high or low quality) is between the discounted utility and the discounted cost, a larger monetary distribution will improve the value of commodity holders but will lower the value of money holders. That is because it is harder for money holders but easier for commodity holders to find a trading partner. Hence, according to equation (16), the produced quality levels \( \bar{Q}_t \) (i=H, L) would clearly be lower. Although a lower \( \bar{Q}_H \) and a lower \( \bar{Q}_L \) drive the price differential in opposite directions, if \( \bar{Q}_H \) and \( \bar{Q}_L \) both decrease the same size, a larger M would result in a smaller price differential. That is because high quality goods were harmed more severely by the reduced quality levels than that of low quality goods. As a consequence, the marginal type \( \delta \) decreases. Given the distribution of the quality preferences, there are more people with higher quality preferences \( (\theta > \bar{\theta}) \), which encourages more higher quality goods to be produced, traded and consumed; which therefore, results in higher welfare.

**Proposition 1.2 (Effects of the economy’s quality preferences)**

*When the range of the economy’s quality preference shifts upwards, a lower \( \bar{Q}_H \) and a higher \( \bar{Q}_L \) will be produced; hence, although the price differential becomes narrower, welfare remains the same as well as the marginal type \( \delta \).*

PROOF: see the appendix.

It is interesting to note that the economy’s preference has a different effect on the agents who prefer high quality and those who prefer low quality. While the variable “a” increases the values of agents preferring high quality (MH and CH), it lowers the values of agents preferring low quality (ML and CL). In other words, the threat point of agents preferring high quality increases, but the threat point of agents preferring low quality
decreases. The positive effect of “a” on both \( V_{MR} \) and \( V_{CH} \) would drive \( Q_H \) in opposite
directions: while a higher \( V_{MR} \) increases \( Q_H \), a higher \( V_{CH} \) decreases \( Q_H \). Although the
variable “a” has a stronger effect on \( V_{MR} \) than on \( V_{CH} \), \( V_{CH} \) has a stronger effect on \( Q_H \)
than on \( V_{MR} \). Overall, it can be shown that the effect of “a” on \( Q_H \) through \( V_{CH} \) dominates,
so an increase in the economy’s quality preference would lower \( Q_H \).

Similarly, the negative effect of “a” on both \( V_{ML} \) and \( V_{CL} \) would also drive \( Q_L \) in
opposite directions. Since the effect of “a” on \( Q_L \) through \( V_{CL} \) is larger than that through
\( V_{ML} \), a higher economy’s quality preference will enhance \( Q_L \). Therefore, we can conclude
that an upward shift in the range of the economy’s quality preference would shrink both the
quality difference to be produced and its associated price differential. This narrower quality
differential and price differential causes higher welfare for each CH while lower welfare for
each CL, but doesn’t change the welfare for each money holder. When higher welfare
received by each CH offsets the lower welfare received by each CL, the total social welfare
remains unchanged.

**Case Two: Money holders without quality preferences**

In this case, we look at identical money holders (M), who would trade for products of
any quality. To begin with, let’s maintain that commodity holders are still distinguished by
their quality preferences, CH and CL. For simplicity, we assume that during the meeting, the
two parties would switch their quality preferences in addition to exchanging the product for
money. That is, after receiving money, the initial commodity holder becomes a money
holder without quality preference, and after consumption, the initial money holder inherits
the initial commodity holder’s quality preference \( \theta \). In other words, M could become either
Ch or CL, depending on whom s/he trades with [Figure 2]. Following the same procedure as
the previous case, the equilibrium quality level will be determined as well as its associated
price differential. Then we can compare the results of both cases.

Value Functions

The value functions of CH, CL, and M, thus can be written as:

\[ rV_{CH} = M[V_M - C(Q_M) - V_{CH}] \]  \hfill (9')
\[ rV_{CL} = M[V_M - C(Q_L) - V_{CL}] \]  \hfill (10')
\[ rV_M = (1-M)[(\theta+1-\theta)[U(Q_M)+V_{CH}-V_{CL}] + (\theta-\theta)[U(Q_L)+V_{CL}-V_{M}]] \]  \hfill (11')

With probability M, the person preferring the variety produced by a CH would be a money
holder so that they can trade. The same probability applies to CL. The flow value of M is a
more complicated one, which is shown in equation (11'). With probability (1-M), the person
who can produce the variety preferred by a money holder would be a commodity holder.

Moreover, the probability that this commodity holder is a CH is \((\theta+1-\theta)\), and the probability
that this commodity is a CL is \((\theta-\theta)\). After trading with CH, this money holder enjoys the
consumption, which offers him/her \(U(Q_M)\), then, this money holder would inherit the CH’s \(\theta\)
and becomes a CH. Similarly, if it is CL whom the money holder trades with, this money
holder will become a CL instead.

Nash Bargaining, Equilibrium, and Welfare

Define \(\hat{Q}_i\) (i = H, L) as the Nash bargaining solution:

\[ \hat{Q}_i = \arg\max \left\{ [V_M - C(Q_M)][V_{CL} + U(Q_L)] \right\}, \]  \hfill (13')

Subject to

\[ V_M - C(Q_M) \geq V_{CL} \]  \hfill (14')
\[ V_{CL} + U(Q_L) \geq V_M \]  \hfill (15')
The equilibrium $\hat{Q}_t$ can be determined by the following first order condition:

$$\frac{V_C + U(\hat{Q}_t)}{V_M - C(\hat{Q}_t)} = \frac{U'(\hat{Q}_t)}{C'(\hat{Q}_t)},$$

(16')

which again implies $\partial Q_t / \partial V_M > 0$, and $\partial Q_t / \partial V_C < 0$. The price differential associated with this equilibrium quality levels is:

$$1/\hat{q}_H - 1/\hat{q}_L = MC(\hat{Q}_H) - MC(\hat{Q}_L).$$

(17')

Total welfare in case two, called $W_2$, is given by:

$$W_2 = \mathcal{M} \left[ \sum_{\theta} \int_{\theta} (P_H - MC(Q_H)) q_H d\theta + \sum_{\theta} \left( P_L - MC(Q_L) \right) q_L d\theta \right]$$

$$+ \left[ 1 - \mathcal{M} \left( a + 1 \right) \sum_{\theta} \left( P_H - P_L \right) q_H d\theta \right] + \left( \theta - a \right) \int_{\theta} (\theta Q_H - P_L) q_L d\theta,$$

(18')

which is the sum of welfare received by agents in different statuses, namely, CH, CL, and M. The first and second terms in equation (18') shows the surplus received by CH and CL, respectively, while the third term represents the surplus of M. It is noted that since M has no quality preferences in case two, the probability for a CH or a CL to trade is M, which is different from case one.

Comparative Statics

Again, in this section, both the effects of M and the economy’s quality preferences, “a”, will be analyzed and compared to the results in case one. Due to the absence of quality preferences, M can either trade with a CH or a CL. These two types of trade opportunities gives a more complicated form of $V_M$ than that in case one. In order to determine the effects of both M and “a” clearly, a reduced form of $V_M$ as a function of M and a, is derived from equations (9')-(11'):
Hence, both $V_{CR}$ and $V_{CS}$ can be written in terms of $V_M$:

$$V_{CR}(M,a;Q_H) = \frac{M[V_t(M,a;Q_H)-C(Q_H)]}{r+M},$$  \hspace{1cm} (20')

$$V_{CS}(M,a;Q_L) = \frac{M[V_t(M,a;Q_L)-C(Q_L)]}{r+M}.$$  \hspace{1cm} (21')

Proposition 2.1 (Effects of Monetary Distribution)

A larger $M$ lowers $\hat{Q}_i$ ($i=H, L$), if the following conditions hold:

(i) $M<1/2$, (ii) $(1-2M)(U(Q_H)-C(Q_H))<rU(Q_H)$, (iii) $(1-2M)(U(Q_L)-C(Q_L))<rU(Q_L)$, and (iv) 

$$(a+1-\delta)(1-M)[rU(Q_H)+M(U(Q_H)-C(Q_H))] - \frac{M(r+M)}{r}[(1-2M)(U(Q_H)-C(Q_H)) - rU(Q_H)]
+ (\delta - a)(1-M)[rU(Q_L)+M(U(Q_L)-C(Q_L))] - \frac{M(r+M)}{r}[(1-2M)(U(Q_L)-C(Q_L)) - rU(Q_L)]
> r(r+1)C(Q_H) > r(r+1)C(Q_L)$$

Moreover, if the size of decrease in both $\hat{Q}_H$ and $\hat{Q}_L$ is the same, a larger $M$ would result in a smaller price differential and higher welfare.

PROOF: see the appendix.

Proposition 2.1 indicates that a larger $M$ would lower both produced qualities $Q_i$ ($i=H, L$), since a larger monetary distribution would lower $V_M$ but increase $V_{CS}$, and hence, clearly reduces both qualities $Q_i$ ($i=H, L$). If both $\hat{Q}_H$ and $\hat{Q}_L$ decrease by the same amount, the price differential would be narrower. Additionally, a lower marginal type indicates more high quality goods will be produced and consumed. Hence, each money holder who trades
with a CH receives higher surplus while the surplus of agents in other statuses remains the same. As a result, the total welfare of the economy increases.

Proposition 1.1 and Proposition 2.1 show that the effect of M on $V_M$ in case one is larger than that in case two. However, the effect of M on $V_C$ in case one is smaller than that in case two. Recall that the effect of $V_C$ on $Q_i$ is larger. Hence, the quality levels $Q_i$ were harmed more severely by a larger monetary distribution in case two, when money holders have no quality preferences than in the case one, when money holders have quality preferences. However, which case has smaller a price differential remains unanswered.

Proposition 2.2 (Effect of the economy’s quality preferences)

The effects of the economy’s quality preference, “a”, on $Q_i$ ($i=H, L$) are not only negative, but also at same size. Therefore, the equilibrium quality differential remains fixed in response to any shift on “a”; as a consequence, the price differential becomes narrower and welfare is higher.

PROOF: see the appendix.

Proposition 2.2 implies that when money holders have no preferences on quality, a higher economy’s quality preference increases the values of all agents, M and Ci ($i=H, L$). Although $V_C$ and $V_M$ affect $Q_i$ in opposite directions, it can be shown that the effect of “a” through $V_C$ dominates, which gives the negative effect of “a” on the overall quality levels $Q_i$. It is interesting that this effect affects both $Q_H$ and $Q_L$ not only in the same direction but also the same size, and leaves the quality differential unchanged, but it would shrink the price differential. This smaller price differential and lower produced quality levels induce more agents produce and consume goods with high quality, and hence, enhance higher welfare.
5. Conclusion

In this paper, we show that money could enhance welfare even in the absence of lemons problem. That is because when only two qualities will be endogenously determined from a continuum of quality levels by the agents according to their quality preferences, a larger monetary distribution would generate lower produced qualities in both high and low levels. The lower produced qualities harm the price of high quality goods more than they do on low quality goods; hence the price differential is narrower. However, these lower produced quality levels also induce more high quality goods to be produced, traded, and consumed, so welfare is higher under a larger monetary distribution. This result does not rely on the assumption whether money holders have quality preferences.

However, when the range of the economy’s quality preferences shifts upwards, both the produced quality levels and welfare depends crucially on whether money holders have quality preferences. When money holders have quality preferences, the upward shift in the economy’s quality preferences reduces the produced high quality but increases the produced low quality. The increase in the surplus of high quality producers is offset by the decrease in the surplus of low quality producers; hence, welfare remains unchanged. However, when money holders have no quality preferences, the upward shift in the economy’s quality preferences reduces both produced quality levels and increases welfare.

So far, we only show whether welfare could be improved under a larger monetary distribution and under a upward shift in the economy’s quality preferences. It would be interesting in comparing welfare as well as the quality and price levels across cases. This would allow more insightful look at how the quality allocations determined by bargaining deviate from the first best quality allocation, and how welfare is affected.
[What else for comparison:

a. Solve the M when $Q=QH(\theta)$ => plug in $M^* \Rightarrow$ solve $QL$ => compare the difference between $QL(\theta)$ and $QL$ => if $QL>QL(\theta)$, the welfare is higher.

b. Play $M$ around the that value and see its effect on $QH$ and $QL$

c. Social welfare: negotiation v.s. preferred quality allocations]
Proof of Proposition 1.1:

1. If all three conditions hold, then \( \frac{\partial V_{Ml}}{\partial M} < 0 \) and \( \frac{\partial V_{Cl}}{\partial M} > 0 \). Additionally, it can be shown that \( |\frac{\partial V_{Cl}}{\partial M}| > |\frac{\partial V_{Ml}}{\partial M}| \), which implies lower \( \bar{Q}_i \).

2. Equation (17) can be expanded to \( \bar{P}_H - \bar{P}_L = c(Q_H^2 - Q_L^2) \). When \( Q_H^2 - Q_L^2 \) is not affected by \( M \), then \( (\bar{P}_H-\bar{P}_L) \) will be lowered by a higher \( M \).

3. Equation (7) can be derived as: \( \bar{Q} = c(Q_H^2 + Q_L^2) \).

4. Equation (18) can be re-written as:

\[
W_1 = \left( a + 1 - \theta \right)^2 \left( M(1-c)Q_H^2q_H + 1/2(1-M)Q_Hq_M(a+\theta) \right) + \left( \theta - a \right)^2 \left( M(1-c)Q_L^2q_L + 1/2(1-M)Q_Lq_M(\theta + a - 1) \right),
\]

where the first two terms represent the welfare of the agents with higher quality preferences, and the last two terms are the welfare of the agents with lower quality preferences. While a larger \( M \) increases the welfare of each MH, it doesn’t not change the welfare of each CH, CL or ML. Given a lower marginal type, there are more agents consuming high quality goods, which increases the total welfare of the society.

Proof of Proposition 1.2:

1. From equations (9)-(12), the following results can be obtained:

(i) \( \frac{\partial V_{MH}}{\partial a} > 0 \), \( \frac{\partial V_{CH}}{\partial a} > 0 \), \( |\frac{\partial V_{CL}}{\partial a}| < |\frac{\partial V_{MH}}{\partial a}| \), (ii) \( \frac{\partial V_{ML}}{\partial a} < 0 \), \( \frac{\partial V_{CL}}{\partial a} < 0 \), \( |\frac{\partial V_{CL}}{\partial a}| < |\frac{\partial V_{MH}}{\partial a}| \),

(iii) Since

\[
\left( \frac{\partial Q_H}{\partial V_{CH}} \right) \left( \frac{\partial V_{CH}}{\partial a} \right) > \left( \frac{\partial Q_H}{\partial V_{MH}} \right) \left( \frac{\partial V_{MH}}{\partial a} \right) \quad \text{and} \quad \left( \frac{\partial Q_L}{\partial V_{CL}} \right) \left( \frac{\partial V_{CL}}{\partial a} \right) > \left( \frac{\partial Q_L}{\partial V_{ML}} \right) \left( \frac{\partial V_{ML}}{\partial a} \right),
\]

it can be shown that \( \frac{\partial Q_H}{\partial a} < 0 \) and \( \frac{\partial Q_L}{\partial a} > 0 \).

2. Equation (17) indicates that if the amount of increase in \( Q_H \) equals to the amount of
decrease in $Q_L$, a higher “a” decreases the price differential $\mathbf{P}_H - \mathbf{P}_L$, but leaves the marginal type $\delta$ unchanged.

3. Equation (18) shows that this upward shift in the range of the economy’s quality preferences increases the welfare of CH, decreases the welfare of CL, but does not change the welfare of both MH and ML.

Proof of Proposition 2.1:

a. Following Proposition 1.2, it can be shown that $\frac{\partial V_\mu}{\partial a} < 0$, if

(i) $M < \frac{1}{2}$, (ii) $r > \frac{M}{r+M}$, and

(iii) $1 - 2M(U(Q_H) - C(Q_H)) > rU(Q_H)$.

b. Given part (a), it can also be shown that $\frac{\partial V_\mu}{\partial a} > 0$, if

$$\begin{align*}
(a+1-\delta)(1-M)[rU(Q_H)+M(U(Q_H)-C(Q_H))] - \frac{M(r+M)}{r} [(1-2M(U(Q_H)-C(Q_H))-rU(Q_H))] \\
+ (\delta - a)[(1-M)[rU(Q_L)+M(U(Q_L)-C(Q_L))] - \frac{M(r+M)}{r} [(1-2M(U(Q_L)-C(Q_L))-rU(Q_L))] \\
> r(r+1)C(Q_H) > r(r+1)C(Q_L)
\end{align*}$$

c. Welfare can be written as:

$$W_2 = M\left[(a+1-\delta)(1-cQ_H^2 Q_H^2) + (0 - a)(1-cQ_L^2 Q_L^2)\right] + (1-M)\left[(a+1-\delta)^2(a+\delta)Q_H^2 Q_H^2 + (\delta - a)^2(\delta + a - 1)Q_L^2 Q_L^2\right].$$

The surplus of a money holder who trades with CH increases when more money is distributed.

Proof of Proposition 2.2:

From the value functions, the following results can be obtained:

$$\frac{\partial V_\mu}{\partial a} \frac{\partial V_{CL}}{\partial a} = \frac{\partial V_{CL}}{\partial \mu} \left( \frac{M}{r+M} \right) \left( \frac{\partial V_\mu}{\partial a} \right) > 0.$$ It is shown that

$$| \left( \frac{\partial V_\mu}{\partial a} \right) \left( \frac{\partial V_{CI}}{\partial a} \right) | > \left| \left( \frac{\partial V_{HI}}{\partial \mu} \right) \left( \frac{\partial V_\mu}{\partial a} \right) \left( \frac{\partial V_{LI}}{\partial a} \right) \right|,$$

where $i = H, L$, so $\frac{\partial V_i}{\partial a} < 0$ (i=H, L), but
\[
\frac{\partial (\hat{Q}_H - \hat{Q}_L)}{\partial a} = 0.
\]
The price differential \(1/\hat{q}_H - 1/\hat{q}_L = MC(\hat{Q}_H) - MC(\hat{Q}_L)\) gives

\[
\frac{\partial (1/\hat{q}_H - 1/\hat{q}_L)}{\partial a} = \sigma(\hat{Q}_H - \hat{Q}_L) \left( \frac{\partial (\hat{Q}_H + \hat{Q}_L)}{\partial a} \right) < 0.
\]
Figure 1: Case 1-M has preferences (MH trades with CH; ML trade with CL)

Figure 2: Case 2-M has no preference (M trades with either CH or CL)
References
