Can hyperbolic discounting explain the gym-pass puzzle?

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Abstract

Malmendier and Vigna (2006) in the journal article ”Paying not to go to the Gym” found that empirical studies about consumers’ behaviour are difficult to reconcile with standard preferences theory. For example, standard preferences are unable to explain why people who buy a gym pass will not go to the gym very often later on. This study aims to explore whether hyperbolic discounting can explain this kind of behaviour. In this paper, I propose a simple behavioural model of intertemporal choices using present-biased preferences and explore the assumptions that are necessary to explain the observed behaviour of the gym-pass puzzle.

1 Introduction

1.1 Intertemporal choice

Everyday people make different kinds of decisions, and most of these decisions involve consequences that occur at different points in time. For example, deciding whether to clean the garage today or tomorrow involves tradeoffs among costs and benefits that do not happen at the same time. These types of decisions are called intertemporal choices. They do not only affect people's current levels of utility, i.e. their current level of happiness or satisfaction, but also have an impact on people's future utility levels.

In order to explain intertemporal choices, two important concepts have been developed: time discounting and time preference. Time discounting refers to people caring less about the future, including factors that diminish the expected utility generated by future consequences, such as uncertainty or change in tastes [3]. For instance, when given the choice of receiving $\$100$ either today or tomorrow, most people choose to receive $\$100$ today. This is because the uncertainty associated with future outcomes makes people assign a higher utility for receiving $\$100$ today. This is why the term "time preference" describes the preference for immediate utility over delayed utility.

1.2 Literature Review

Different types of models have been proposed to understand intertemporal choices. In 1937, Paul Samuelson [19] first proposed the discounted-utility (DU) model which includes both a discount factor and an instantaneous utility function. The central assumption of this model is that all of the motives underlying intertemporal choices can be condensed into a single constant parameter: the discount rate [3]. However, the DU model has some limitations. Many studies [22] [17] [7] [6] show that the discount function cannot be fully explained by a single constant discount rate. That's why numerous alternative models have been developed. Some of them modify the discount function, like the hyperbolic discounting model, while others modify the instantaneous utility function of the original DU model.

Beginning with David Laibson [8] [9], the implications of declining discount rates was analysed. This study has been followed by E.S. Phelps [14] and Pollak [15] who introduced a functional form of hyperbolic discounting with time-inconsistent preferences and a discount rate which decreases over time. However, the hyperbolic discounting model did not take into account some factors like self awareness for changing preferences. As a result, another alternative model of discount function called self awareness was proposed by Strotz [21] and Pollak [15]. In this model, a person can be divided into 2 extremes of either complete "naive" (can not predict future performances accurately) or complete "sophisticated" (fully aware of self-control problems). O'Donoghue and Rabin [13] examine how people's behaviors depend on their level of sophistication about their own time-inconsistency.

Other studies proposed alternative models which focus on the possible enrichment of the instantaneous utility function of the DU model. James Duesenberry [2] first proposed the habit-formation model for which the level of current consumption can be affected by the utility from past consumptions. This idea was then more formally developed by Pollak [16] and Harl Ryder and Geoffrey Heal [18]. Closely related to the habit-formation models, the reference-point model incorporates some ideas from prospect theory [4] [5]. In this model, the value function used to evaluate the outcomes is defined as the utility level difference between today and some reference point. Loewenstein and Prelec [10] applied the value function to intertemporal choice to explain the magnitude effect. Furthermore, some alternative models include some "anticipation" into the instantaneous utility model, like Elder and Jevons. These models show that utility is not only affected by the current consumption, but also the anticipation of future consumptions. A final alternative model of the utility function incorporates "visceral" influences which show that instantaneous utility function can also be affected by visceral influences such as hunger, sexual desire and physical pain. Finally, Loewenstein [11] [12] argues that economics should take more seriously the implications of such influences in tastes.

In this paper, I adopt the framework of the hyperbolic discounting model, and modify its instantaneous utility function in order to capture a phenomenon known as the "Gym Pass Puzzle".

1.3 Motivation

The motivation for this study resides in the findings of the paper entitled "Paying not to go to the Gym" by S.D.Vigna and U.Malmendier [20]. In their paper, Vigna and Malmendier analyse a dataset from three U.S health clubs with information on both the contractual choice and the day-to-day attendance decisions of $7,752$ members over three years. The contractual choice includes monthly and yearly memberships of the health club, whereas members can cancel their membership at the end of each month or each year given some cancellation fee. The membership is automatic renewed if there is no cancellation.

In this paper, I focus on one of the main findings of the "Paying not to go to the Gym" paper.

This finding shows that the actual per visit payment of people who join the club membership is much higher than the per visit fee charged for people without membership. This is due to a low monthly average number of visits of the people who join the membership, around 3-4 times only.

In order to study why some people would pay more than they should to go to the gym, I propose the following utility model. The utility function in each time period is a function of the instantaneous utility in that period and the sum of discounted utility in future periods, where the discount rate is adopted from the hyperbolic discounting model. The instantaneous utility function in each time period is affected by three parameters: a commitment device (the membership) real value, people's health and the number of times people visit the gym per period. Whereas the commitment device value and people's health are positively related with the instantaneous utility function, the number of visits per period is negatively related to the instantaneous utility function. In order to explain why the majority people still join the membership even though they know that the actual per-visit fee is higher than the per-visit fee charged without membership, I maximise the utility functions in each time period with respect to the number of visits per period. It is expected that the optimal number of visits to the gym should be positive in order to maximize people's utility level. This result can then be used to explain why the majority of gym users buy the membership as a commitment device to improve their utility.

Section 2 presents the functional form of the traditional DU model and the hyperbolic discounting model. Section 3 shows the details of the proposed utility model with three time periods, and presents the assumptions and expected results. Section 4 concludes.

2 Discounted Utility Model and Hyperbolic Discounting Model

2.1 Discounted Utility Model

In 1937, Paul Samuelson [19] first proposed the DU model. It expresses the utility at time period t as a function of the sum of the discount function times the instantaneous utility at that time. This model specifies a person's intertemporal preferences over consumption profiles $(c_t, \ldots, c_T)$ [3].

$$U^t(c_t, \ldots, c_T) = \sum_{k=0}^{T-t} D(k) u(c_{t+k}) \quad \text{where} \quad D(k) = \left(\frac{1}{1+\rho}\right)^k \tag{1}$$

In this formulation, $u(c_{t+k})$ is interpreted as the instantaneous utility function in period $t+k$, whereas $D(k)$ is the discount function in period t to in period $t+k$. The variable ρ

represents the individual's pure rate of time preference which is actually the discount rate.

2.2 Hyperbolic Discounting Model

The hyperbolic discounting model represents time-inconsistent preferences. This means that a plan which has been decided for some periods $t+n$ and beyond, and that is considered as being optimal at period t, is not regarded as optimal when time $t+n$ arrives. The functional form of the hyperbolic discounting model is represented as follows:

$$D(k) = \begin{cases} 1 & \text{if } k = 0 \\ \beta\delta^k & \text{if } k > 0 \end{cases} \tag{2}$$

In this formulation, β represents the time-inconsistent preferences and δ is the usual discount factor between the present and future payoffs. The variable k is simply the time period. If tomorrow's return is discounted at a rate $\delta \in (0, 1)$, then the day after tomorrow's return is discounted at a rate δ^2, and so on. Therefore, the variable $\beta \in (0, 1)$ is a second discount parameter which incorporates time-inconsistency into the formula. As a result, time-inconsistent agents discount tomorrow at $\beta\delta$, and the next day at $\beta\delta^2$. See Bhattacharya and Lakdawalla [1].

The following is a simplified example that shows how hyperbolic discounting captures a decreasing discount rate over time. Therefore, we limit this model to three periods, the minimum length necessary to illustrate time-inconsistency.

Table 1: Example

Time period	T_0	T_1	T_2
Discount factor	1	$\beta\delta$	$\beta\delta^2$

Table 2: Discount rate

Discount rate b/w T_0 and T_1	$(1-\beta\delta)/\beta\delta$
Discount rate b/w T_1 and T_2	$(\beta\delta-\beta\delta^2)/\beta\delta^2 = (1-\delta)/\delta$

The above example shows that given 3 time periods: T_0, T_1 and T_2, the discount factors corresponding to each time period are 1, $\beta\delta$ and $\beta\delta^2$ based on the hyperbolic discounting formula. Therefore, the discount rate between T_0 and T_1 is $(1-\beta\delta)/\beta\delta$, and the discount rate between T_1 and T_2 is $(\beta\delta-\beta\delta^2)/\beta\delta^2 = (1-\delta)/\delta$. Since both β and δ are between 0 to 1 (hence, $\beta\delta < \delta$),

$1/\beta\delta$ should be greater than $1/\delta$. In addition, we also can derive $(1-\beta\delta) > (1-\delta)$ because of $\beta\delta < \delta$. As a result, it is clear that the discount rate between T_1 and T_2 is less than the discount rate between T_0 and T_1 (i.e., $(1-\delta)/\delta < (1-\beta\delta)/\beta\delta$) which shows the decreasing (hyperbolic) discount rate over time. From this example, we know that an individual who has hyperbolic preferences prefers current utility rather than delayed utility. In order to make this individual indifferent between his current payoff and his delayed payoff, a very high discount rate would have to be assigned.

3 The Proposed Utility Model

3.1 Assumptions

Two assumptions that are used in this paper. The first assumption stipulates that people have time-inconsistent preferences, which is represented by the discount factor β in the hyperbolic discounting model. These time-inconsistent preferences are the origin of the self-control problem.

The second assumption is about the degree of sophistication of the gym users. As previously said, people can be divided into 2 extremes: completely naive and completely sophisticated. A partial naive is considered as being in between these two extremes. In this paper, the completely naive individual refers to someone who cannot predict his future attendance to the gym at all and overestimates his number of visits to the gym. On the other hand, the completely sophisticated individual refers to someone with time-inconsistent preferences who uses the membership as a commitment device.

I focus my interest on sophisticated people because time-inconsistent individuals who do not understand their self-control problem never have positive demand for self-control devices (commitment device). Therefore, their level of utility can not be improved or maximised. However, sophisticated people who understand their self-control problem will take steps to combat it, so that their utility will improve by the provision of the commitment device. See Bhattacharya and Lakdawalla for more details [1].

3.2 The details of the proposed utility model

Based on the hyperbolic discounting (time-inconsistent preference), the next period's utility is discounted by the factor $\beta\delta$, and the following period's utility is discounted by the factor $\beta\delta^2$ ($\beta \in (0,1)$ and $\delta \in (0,1)$). Let the time-inconsistent instantaneous utility denoted as u_t . The utility in each period is denoted as U_t for the 3 time periods. As a result, the functional form of the utility in each period is the following:

$$
\begin{aligned}
U_3 &= u_3 \\
U_2 &= u_2 + \beta\delta u_3 \\
U_1 &= u_1 + \beta\delta u_2 + \beta\delta^2 u_3
\end{aligned} \tag{3}
$$

The instantaneous utility, u_t, depends on: the commitment device value C_t, the health condition H_{t-1} (which depends on the number of visits in the previous periods), and the number of visits per period n_t. The general form of u_t is shown as $u_t(C_t, H_{t-1}, n_t)$. It is assumed that the commitment device value increases the level of utility, so that $\frac{\partial u_t}{\partial C} > 0$. In addition, the health parameter H_t is also considered positively related to the level of utility, $\frac{\partial u_t}{\partial H} > 0$ since people's utility level will be increasing if they become healthier. Finally, the number of visits lowers the utility, $\frac{\partial u_t}{\partial n} < 0$, due to the effort associated with going to the gym. The utility function in each time period is therefore as follows:

$$
\begin{aligned}
U_3 &= u_3(C_3(n_3), H_2(n_0, n_1, n_2), n_3) \\
U_2 &= u_2(C_2(n_2), H_1(n_0, n_1), n_2) + \beta\delta u_3(C_3(H_2, n_2), H_2(n_0, n_1, n_2), n_3(H_2, n_2)) \\
U_1 &= u_1(C_1(n_1) + H_0(n_0), n_1) + \beta\delta u_2(C_2(H_1, n_1), H_1(n_0, n_1), n_2(H_1, n_1)) \\
&+ \beta\delta^2 u_3(C_3(H_1, n_1), H_2(n_0, n_1, n_2), n_3(H_1, n_1))
\end{aligned} \tag{4}
$$

by assuming $H_0(n_0) = 0$.

The health condition in period t is a function of the number of visits and represents a constraint in the maximization problem.

$$
\begin{aligned}
H_1(n_0, n_1) &= \ln n_0 + \ln n_1 \\
H_2(n_0, n_1, n_2) &= \ln n_0 + \ln n_1 + \ln n_2
\end{aligned} \tag{5}
$$

The logarithm function is used to encompass the fact that the marginal benefit of going to the gym on someone's health decreases as the number of visits increases.

In addition, we assume that the commitment device value, C_t is a function of the number of visits at period t, and is expressed as follows:

$$
C_t = -p + n_t^2 \tag{6}
$$

where the number of visits n_t increases the value of the commitment device C_t, so that $\frac{\partial C_t}{\partial n_t} > 0$, and p, the membership fee, decreases the value of the device.

The purpose of a commitment device is to force people to go to the gym and become healthier. The more people go to the gym, the more valuable this commitment device is. For example, if you pay for a gym membership but your attendence to the gym is null this commitment device is not valuable at all. That is why the real value of the commitment device positively depends on the number of visits n_t and is negatively related to the membership cost p. Furthermore, the value of the commitment device C_t is not linearly related to n_t because the value of C_t will rise faster as the number of visits increases.

Sophisticated people with time-inconsistent preference understand their self-control problem, so they buy the gym membership as a commitment device to improve their utility. Optimal decision for sophisticated people represents a subgame -perfect equilibrium, which can be derived by backwards induction [1]. Therefore, I start to analyse from period 3 and maximize each period's utility with respect to n_t. In the second period, n_3 depends on the health condition H_2 and the numbers of visits n_2 which is taken as given. In period 1, both n_2 and n_3 depends on H_1 and n_1 which are all taken as given. The functional form is shown below:

$$\begin{aligned}
&\max_{n_3} u_3(C_3, H_2(n_0, n_1, n_2), n_3) \\
&\max_{n_2} u_2(C_2, H_1(n_0, n_1), n_2) + \beta\delta u_3(C_3(H_2, n_2), H_2(n_0, n_1, n_2), n_3(H_2, n_2)) \\
&\max_{n_1} u_1(C_1 + H_0(n_0), n_1) + \beta\delta u_2(C_2(H_1, n_1), H_1(n_0, n_1), n_2(H_1, n_1)) \\
&\quad +\beta\delta^2 u_3(C_3(H_1, n_1), H_2(n_0, n_1, n_2), n_3(H_1, n_1))
\end{aligned} \tag{7}$$

The first order conditions in each time period are therefore:

$$\begin{aligned}
&\frac{\partial u_3}{\partial C_3}\frac{\partial C_3}{\partial n_3} + \frac{\partial u_3}{\partial n_3} = 0 \\
&\frac{\partial u_2}{\partial C_2}\frac{\partial C_2}{\partial n_2} + \frac{\partial u_2}{\partial n_2} + \beta\delta\left[\frac{\partial u_3}{\partial C_3}\frac{\partial C_3}{\partial n_2} + \frac{\partial u_3}{\partial H_2}\frac{\partial H_2}{\partial n_2} + \frac{\partial u_3}{\partial n_3}\frac{\partial n_3}{\partial n_2}\right] = 0 \\
&\frac{\partial u_1}{\partial C_1}\frac{\partial C_1}{\partial n_1} + \frac{\partial u_1}{\partial n_1} + \beta\delta\left[\frac{\partial u_2}{\partial C_2}\frac{\partial C_2}{\partial n_1} + \frac{\partial u_2}{\partial H_1}\frac{\partial H_1}{\partial n_1} + \frac{\partial u_2}{\partial n_2}\frac{\partial n_2}{\partial n_1}\right] + \\
&\beta\delta^2\left[\frac{\partial u_3}{\partial C_3}\frac{\partial C_3}{\partial n_1} + \frac{\partial u_3}{\partial H_2}\frac{\partial H_2}{\partial n_1} + \frac{\partial u_3}{\partial n_3}\frac{\partial n_3}{\partial n_1}\right] = 0
\end{aligned} \tag{8}$$

After that, by plugging Eq.5 and Eq.6 into Eq.9, we can obtain the following result as

$$\frac{\partial u_3}{\partial C_3}(2n_3) + \frac{\partial u_3}{\partial n_3} = 0$$

$$\frac{\partial u_2}{\partial C_2}(2n_2) + \frac{\partial u_2}{\partial n_2} + \beta\delta\left[\frac{\partial u_3}{\partial C_3}\frac{\partial C_3}{\partial n_2} + \frac{\partial u_3}{\partial H_2}\left(\frac{1}{n_2}\right) + \frac{\partial u_3}{\partial n_3}\frac{\partial n_3}{\partial n_2}\right] = 0$$

$$\frac{\partial u_1}{\partial C_1}(2n_1) + \frac{\partial u_1}{\partial n_1} + \beta\delta\left[\frac{\partial u_2}{\partial C_2}\frac{\partial C_2}{\partial n_1} + \frac{\partial u_2}{\partial H_1}\left(\frac{1}{n_1}\right) + \frac{\partial u_2}{\partial n_2}\frac{\partial n_2}{\partial n_1}\right] + \beta\delta^2\left[\frac{\partial u_3}{\partial C_3}\frac{\partial C_3}{\partial n_1} + \frac{\partial u_3}{\partial H_2}\left(\frac{1}{n_1}\right) + \frac{\partial u_3}{\partial n_3}\frac{\partial n_3}{\partial n_1}\right] = 0 \tag{9}$$

3.3 Expected result

I did not have time yet to solve the model entirely, but my conjecture is that the optimal number of visits in each time period should be positive, which would explain the major finding in the ”Paying not to go to the Gym” paper. Although the actual per visit fee for people who join the club membership is much higher than the per visit fee charged for people without membership, people with time-inconsistent preference will still buy the gym membership. This is because they know they have self-control problems and time-inconsistent preferences (see the assumption in section 3). Therefore, they use the gym membership as a commitment device, and this helps them maximize their utility level in each time period. In this way, people with time-inconsistent preferences are better off since they go to the gym and exercise, which increases their utility level.

4 Conclusion

I proposed a three periods utility model, based on hyperbolic discounting model which is used for time-inconsistent preferences, in order to explain the gym pass puzzle problem. Under the assumptions that people with time-inconsistent preferences are sophisticated, the proposed instantaneous utility model includes a commitment device value, the health of the individual and the numbers of times this individual visits the gym. Because sophisticated people with time-inconsistent preferences are fully aware their self-control problem, they use the gym membership as a commitment device in order to improve their utility level. The optimal numbers of visits in each time period is derived using backwards induction, i.e. by maximizing the utility from period three to period one with respect to the number of visits in each time period.

In the proposed model, people with time-inconsistent preferences understand their self-control problem, and therefore know that if they do not join the gym membership they may not go to the gym at all even though the per-visit fee is relatively lower. As a result, they choose to buy the gym-pass as a commitment device.

References

[1] Bhattacharya J. and D. Lakdawalla (2004): "Time Inconsistency and Welfare," *NBER Working Paper Series*, no. 10345.

[2] Duesenberry, J. (1952): *Income, Saving, and the Theory of Consumer Behavior.* Cambridge, MA: Harvard U. Press.

[3] Frederick S., G. Loewenstein, and T. O'Donoghue (2002): "Time Discounting and Time Preference: A Critical Review," *J. Econ. Literature* vol. xl, pp. 351-401.

[4] Kahneman D. and A. Tversky (1979): "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica* 47, pp. 263-92.

[5] Kahneman D. and A. Tversky (1991): "Loss Aversion in Riskless Choice: A Reference Dependent Model," *Quart. J. Econ.* 106, pp. 1039-61.

[6] Kirby K. N. (1997): "Bidding on the Future: Evidence Against Normative Discounting of Delayed Rewards," *J. Experiment. Psych.:General* 126, pp. 54-70.

[7] Kirby K. N. and N. N. Marakovic (1995): " Modeling Myopic Decisions: Evidence for Hyperbolic Delay-Discounting with Subjects and Amounts," *Org. Behav. Human Decision Proc.* 64, pp.22-30.

[8] Laibson D. (1994): "Essays in Hyperbolic Discounting," Ph.D. Dissertation, MIT.

[9] Laibson D. (1997): "Golden Eggs and Hyperbolic Discounting," *Quart. J. Econ.* 112, pp. 443-77.

[10] Loewenstein G. and D. Prelec (1992): "Anomalies in Interemporal Choice: Evidence and an Interpretation," *Quart. J. Econ.* 107:2, pp. 573-97.

[11] Loewenstein G. (1996): "Out of Control: visceral Influences on Behavior," *Org. Behav. Human Decision Proc.* 65, pp. 272-92.

[12] Loewenstein G. (2000): "Emotions In Economic Theory and Economic Behavior," *Amer. Econ. Rev. Pap. Proceed.* 90, pp. 426-32.

[13] O'Donoghue T. and M. Rabin (1999): "Doing It Now or Later," *Amer. Econ. Rev.* 89:1, pp. 103-24.

[14] Phelps E. S. and R. Pollak (1968): "On Second-Best National Saving and Game-Equiliibrium Growth," *Rev. Econ. Stud.* 35, pp. 185-99.

[15] Pollak R. A. (1968): "Consistent Planning," *Rev. Econ. Stud.* 35, pp. 201-208.

[16] Pollak R. A. (1970): "Habbit Formation and Dynamic Demand Functions," *J. Polit. Econ.* 78:4, pp. 745-63.

[17] Rachlin H., A. Raineri, and D. Cross (1991): "Subjective Probability and Delay," *J. Exper. Anal. Behav.* 55:2, pp. 233-44.

[18] Ryder H. E. and M. H. Geoffrey (1973): "Optimal Growth with Intertemporally Dependnet Preference," *Rev. Econ. Stud.* 40, pp. 1-33.

[19] Samuelson P. (1937): "A Note on Measurement of Utility," *Rev. Econ. Stud.* 4, pp. 155-61.

[20] Stefano D. V. and U. Malmendier (2006): "Paying Not to Go to the Gym," *Amer. Econ. Rev.* 96, pp. 694-719.

[21] Strotz R. H. (1955-56): "Myopia and Inconsistency in Dynamic Utility Maximization," *Rev. Econ. Stud.* 23:3, pp. 165-80.

[22] Thaler R. H. (1981): "Some Empirical Evidence on Dynamic Inconsistencey," *Econ. Letters* 8, pp. 201-07.