A General Framework to Spatiotemporal Modeling of the Real Estate Market

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Abstract

This paper attempts to define a general modeling framework for spatiotemporal data in the context of the real estate market. By exploring the current spatiotemporal literature for models and filters that can be applied to the real estate market we select the Hierarchical model, the Kriged Kalman Filter (KKF) model, the Spatial-Temporal Linear model (STLM) and the State Space Spatial Error Model (SSSEM) for analysis. In the course of our research we find that the general Hierarchical modeling structure seems to nest all of these models and thus we present a preliminary modeling structure, and specify the restrictions that need to be imposed on this structure to obtain the KKF model, the STLM and the SSSEM. After further development of the preliminary results has been completed, we intend to conduct an empirical comparison of the models with real estate data. The data sets available for this comparison include a data set from Baton Rouge, Louisiana and an Australian data set of the Brisbane Metropolitan Area.

JEL:C13, C31, C32, C33, C52

Keywords: Spatial-Temporal, Real-Estate, State Space Models, Spatial Error Model, Kalman Filter.

Version: 30 July, 2007

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1. Introduction

In our research we aim to present a comprehensive, general framework for models that account for space and time in the context of the real estate market. We begin with an exploration of the Spatial-Temporal (ST) modeling literature to investigate the models and filters that have been used by researchers in the context of the real estate market as well as other areas of study. Consequently, we select a number of models and present them in a general framework which allows us to uncover how these models are related to one another and how their particular specifications are motivated. To compare and contrast the results obtained via different specifications we conduct an empirical comparison using real estate data.

In this paper we present only the preliminary analytical results of our investigation, through further study we hope to achieve a well defined general modeling structure that nests all the models. As the final goal of our research has not yet been reached with certainty, the parameters of comparison in our study have not been fully defined and thus data requirements for the empirical analysis may change. To conduct our empirical analysis we have access to two real estate data sets, a data set from Baton Rouge, Louisiana provided by Pace et al. (2000) and an Australian data set provided by Cominos et al. (2007). The two data sets differ on the information they contain thus the more appropriate set will be chosen once the general framework is finalized.

The real estate market is an integral part of the decisions of the majority of individuals in the economy, thus a clearer understanding of its behavior is of great value to many institutions as well as single economic agents. The intricate interactions between spatial and temporal data exist in many markets, the more effective analysis of which would allow for more reliable information to be obtained from that data.

This document is structured as follows: Section 2 provides a review of the general literature on Spatial-Temporal modeling, Section 3, some of the limitations in the existing literature, Section 4 presents the preliminary results of our analysis, Section 5 discusses the data sets available for empirical comparison and Section 6 presents a summary of our findings as well as the direction and aims of further research.

2. Spatial-Temporal Literature

Spatial-Temporal (ST) models have developed as an extension of time-series techniques to space, spatial modeling techniques to time, or via the interaction of spatial and temporal methods as well as from physical models. ST modeling has been employed in many disciplines such as geology, ecology, hydrology, medicine, social sciences and others. The key issue addressed by the literature is how to correctly model/specify the interactions between time and space in the type of data analyzed and to obtain the desired information/conclusions while minimizing the computational complexity of the model. In ST modeling there does not seem to be a complete general framework thus, most of the presented models are highly tailored to specific types of fields and data. Modeling ST data is complex due the existence of two scales, time and space simultaneously and no simple way to reconcile them. When both scales are considered, the researcher needs to take into account not only temporal and spatial variability separately, but also
the nature of the interaction between space and time as spatial behaviour may change at different points in time and the temporal behaviour may vary at different points in space. The filtering order has been approached differently by various authors, for example, Pace et al. (2000) employ a compound filter approach in their model whereby they account for both combinations, considering time then space and vice versa.

In this paper we briefly discuss the models proposed to deal with various types of data generating processes defined by the specification of the spatial domain. In the ST modeling literature the spatial domain is the dominant determinant of the model choice as it does not possess a natural ordering as the temporal domain. Further, the spatial domain contains more characteristics than the temporal domain that need to be considered thus its analysis is arguably more complex. The temporal domain requires consideration of whether time is continuous or discrete and whether the observations are regularly spaced in time, as well as similar considerations for the spatial domain, the process will also need to dictate the type of spatial data that will be observed and thus the appropriate model.

In ST modeling there are three main spatial data classifications; point process generated data, areal data and point-referenced data. A point process is a process where the spatial location of an event of interest is random so the spatial region of interest is also random. Such a process is usually observed in human/animal epidemiology where a disease breakout may occur at random locations. Areal data are defined by a fixed spatial domain which is partitioned into a finite number of areal units with well defined boundaries, thus each observation is associated with a non-zero volume areal unit instead of a point in the spatial domain. In relation to these data types, the models employed in the literature are Gaussian Markov Random Fields (GMRF) and Conditional Autoregressive (CAR) models. Allcroft and Glasbey (2003) apply this model to rainfall data. GMRF are a subclass of Gaussian fields which have a Markov property; non-adjacent locations are conditionally independent. Point referenced data are defined by a fixed spatial domain (area of study), within which the spatial variable varies continuously. In approaching this type of data, three main modeling methods have been used in the literature; hierarchical type models, the Bayesian Kriged Kalman Filter model and the Kernel Convolution approach.

Real estate data are classified as either areal or point-referenced however, the nature of the real estate market introduces many complications for ST modeling. Banerjee et al. (2004) refer to real estate data as essentially cross-sectional1 and not longitudinal2, as the same locations are not observed in each time period. Subjectivity is introduced into the model selection process via the variable choice, as there is a wide range of neighborhood as well as property (house) characteristics that can be included in the model. Gelfand and Vounatsou (2003) illustrate another issue that may be significant in relation to some real estate data sets, they investigate whether the behaviour of single sale transactions and repeat sales is different, and thus may need to be estimated separately. In their study they conclude that there are significant differences in the two submarkets. This result is driven by the fact that traditional price indices rely on fixed hedonic parameters. Cominos et al. (2007) propose a model that relaxes the assumption of fixed hedonic parameters and propose to compute hedonic imputed price indices instead. The time-varying structure of the hedonic coefficients eliminates the necessity to separate single and repeat sales.

Real estate data have not been considered extensively in the context of ST models, only a few articles have applied some ST methods to this market. Gelfand et al. (2004a) investigate some ST

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1 The spatial locations of observations change with each time period.
2 The same points in space are observed through time.
approaches to real estate data. The focus of the analysis is on the distinction between single as opposed to multiple sales data in relation to a point in the spatial domain. The authors introduce spatial association in the error structure via random effects. Gelfand et al. propose a simple linear ST model with ST errors, where the error structure can be specified in three different ways. Different specifications imply a different approach to the issue of which scale to filter first, one error specification implies that spatial effects are nested in time while another implies that it is the temporal effects that are nested in space. The authors also point out that using asymptotic maximum likelihood standard errors for prediction is flawed as the spatial domain is usually assumed fixed, thus these asymptotic standard errors would be inappropriate to use. Consequently, a Bayesian approach to prediction is adopted. Gelfand et al. (1998) apply a fully Bayesian approach to modeling the real estate market via a hierarchical model. The authors employ a general to specific approach by specifying a large number of models that could be considered and subsequently choosing a few that seem to be the most appropriate. This article highlights the complexities of real estate data via their approach to modeling spatial effects in two different ways, investigating spatial effects that stem from geographical location as well as heterogeneity effects of subdivisions. Gelfand et al. (2004b) emphasize the importance of the specification of a covariance function in models for multivariate spatial data. The issue of stationarity is crucial to developing the correct model for the data. The approach to constructing manageable classes of covariance functions is given by the Linear Model of Coregionalization (LMC) for a stationary process and a Spatially Varying LMC (SVLMC) for a non-stationary process. Their work focuses only on the spatial domain. The authors develop bivariate non-stationary process models for income and selling price of a property and highlight some issues that arise in relation to the real estate market. For instance, although the spatial association is anticipated to diminish, it is not necessarily correct to assume that this is a function of the Euclidean distance. Locations in space will be irregularly spaced out thus it is preferred to model the association directly via multivariate spatial processes as opposed to multivariate random field models.

The model proposed by Pace et al. (2000) is a Spatial Temporal Linear Model (STLM) that combines the Autoregressive Distributed Lag (ARDL) model from time series analysis with Mixed Regressive Spatially Autoregressive model from spatial econometrics which they compare to a traditional indicator based model. Pace et al. highlight the difficulty associated with choosing which scale should be filtered first, the spatial or the temporal. Thus in their model they tackle this problem via a combined filter approach that considers, first, the spatial domain and then the temporal and, second, the temporal scale and then the spatial. Two weight matrices are introduced; the T and S matrices represent the temporal relations among previous observations and the spatial relations among previous observations respectively. These matrices are incorporated into the model to filter spatial and temporal relationships. This model takes advantage of the sparse nature of the real estate data to allow for much faster computational methods.

Some recent work on ST modeling has been conducted by Cominos et al. (2007) in an attempt to cast the Spatial Error Model as a State Space Model in the context of the real estate market. The model is then implemented to obtain a housing price index using hedonic imputation methods. This is the first ST type model applied to Australian data. This study is discussed in greater detail in Section 4.

3 Discussed in Gelfand and Vounatsou (2002).
4 A model containing an extensive set of indicators to account for the spatial and temporal correlation in the data.
3. Limitations in the Existing Literature

One of the main limitations in the ST literature is the absence of a complete general framework for all ST models. The research that has been conducted in the real estate market context exemplifies the divergence of views on the most appropriate modeling structure. However in this study we aim to illustrate that the models can be shown to follow a general structure albeit with different restrictions.

The real estate market presents an analyst with a very large number of variables. The various characteristics of the property and the properties of the geographical location such as the distance from the central business district as well as proximity to other locations of importance such as shopping centers, schools and transport cannot all be included in a parsimonious model. Another issue to be considered in relation to geographical location is the importance of the properties of a particular immediate area/suburb, this is taken into account explicitly in the hierarchical model by Gelfand et al. (1998) however it is only considered implicitly in the STLM by Pace et al. (2000) via the spatial correlation matrix and the principle of ‘neighbours’. The choice of which characteristics are considered insignificant and are thus not included in the model will depend on the researcher and the particular real estate market. For example, the concept of a suburb and the effect on price by the characteristics of the suburb and not only geographical location would be expected to be much more significant in Australia where the concept of a ‘suburb’ is much more widely applied than in Singapore where there are no ‘suburbs’ and the geographical location alone may be assumed to explain the spatial correlation. Therefore it is very difficult to choose a particular modeling structure as superior for all real estate markets.

Although there seems to be a limited degree of agreement on most of the house characteristics that are significant to the determination of the price, the number of significant characteristics is subjective and thus the mean structure specification will differ. In a hierarchical model as the one specified by Gelfand et al. (1998), if one attempts to include too many variables to try to account for all the spatial and temporal relationships it may lead to the destabilization of the sampler for example if both spatial effects, the geographical location as well as the ‘suburb’ terms are included in the model.

Another issue that has not received much consideration in the literature is the choice of a loss function in Bayesian analysis. Most of the current research employs the quadratic loss function without providing a justification for the choice. The choice of a loss function will depend on the decision maker and thus the absolute as well the relative loss functions should also be considered in some cases.

4. Preliminary Findings

The review of ST literature would seem to show that there are many competing modeling approaches in the context of the real estate market. In this section we present in some detail some of the models that have been applied to real estate data and attempt to show they can be cast within a general framework. We consider the Hierarchical model, the State Space Spatial Error Model (SSSEM), The Spatial-Temporal Linear Model (STLM) as well as the Kriged Kalman Filter (KKF) model.

The STLM model proposed by Pace et al. (2000) in the analysis of real estate data is a hybrid between the Autoregressive Distributed Lag model in time series and a mixed regressive Spatially
Autoregressive regression model in spatial econometrics. The model is defined by the following structure;

The general structure of the model is given by

\[ Y = X\alpha + \varepsilon \]  \hspace{1cm} (4.1a)
\[ \varepsilon = W\varepsilon + u \]  \hspace{1cm} (4.1b)

where,

\[ Y^{(n\times T)} \] - Matrix of the observations of the dependent variable (log house prices. \( N \) is the total number of houses in the sample and \( T \) the number of years).
\[ X^{(n\times T\times k)} \] - Matrix of the observations of the independent variables (the characteristics of the house, yearly dummy variables and a constant term)
\[ \alpha^{(1\times k)} \] - Parameter vector
\[ u^{(n\times 1)} \] - Independent and identically distributed (i.i.d) errors that are normally distributed
\[ \varepsilon^{(n\times T)} \] - Spatially Autocorrelated errors
\[ W^{(n\times T\times(n\times T))} \] - Spatial-Temporal weight matrix

It is assumed that the observations are ordered chronologically, the oldest observations in \( X \) are in the first row while the most recent in the nth row.

The matrix \( W \) is partitioned into a spatial component \( S \) and a temporal component \( T \). The \( S \) spatial weight matrix represents the spatial relations among previous observations similarly, the \( T \) temporal weight matrix represents temporal relations among previous observations.

The matrices are weighted in time and space by autoregressive parameters.

\[ W = \phi_s S + \phi_T T + \phi_{ST} ST + \phi_{TS} TS \]  \hspace{1cm} (4.2)
\[ \phi = [\phi_s, \phi_T, \phi_{ST}, \phi_{TS}] \]

In the introduction of the SSSEM by Cominos et al. (2007), the authors postulate that it is highly plausible that the shadow prices of the attributes of a house should evolve over time and thus propose to allow the coefficient vector within the model to vary over time\(^5\). The Spatial Error Model is cast in a state space modeling framework. It is assumed that the vector of parameters (coefficients) is generated by a stochastic process. In the real estate market there is no reason to expect shadow prices to be stationary, the mean value of the parameters may change over time and thus the parameter vector is assumed to follow a random walk.

\(^5\) The coefficients of the hedonic characteristics can be interpreted as shadow prices. See Diewert (2003) and Rosen (1974)
The SSSEM is defined by:

\[ Y_t = X_t \alpha_t + \varepsilon_t \]  \hspace{1cm} (4.3a)
\[ \alpha_t = \alpha_{t-1} + \eta_t \]  \hspace{1cm} (4.3b)
\[ \varepsilon_t = \phi W_t \varepsilon_t + u_t \]  \hspace{1cm} (4.3c)

\[ t = 1, \ldots, T \]

where,

- \( Y_t \): Matrix of the observations of the dependent variable (log house prices). Where \( N_t \) is the number of houses sold in period \( t \).
- \( X_t \): Matrix of the observations of the independent variables, the characteristics of the house and a constant term.
- \( \alpha_t \): Vector of unknown time-varying parameters, (the vector of state variables)
- \( \hat{\alpha}_t \): Estimated vector of parameters.
- \( \varepsilon_t \): Vector of spatially autocorrelated error terms.
- \( \Sigma_t \): Spatial weight matrix.
- \( u_t \): Vector of spatially autocorrelated error terms.

\[ E(\varepsilon_t) = 0 \]
\[ Cov(\varepsilon_t, \varepsilon_t^T) = \Psi_t \]
\[ u_t \sim N(0, \sigma_u^2 I_N) \]
\[ W_t \]: Spatial weight matrix.
\[ \eta_t \sim (0, \sigma_y^2 I_N) \]
\[ E(\varepsilon_t, \eta_t^T) = 0 \]

The specification of the spatial weight matrix defines the elements of \( W_t \) as 1 if the two observations are contiguous and 0 otherwise. The observations are defined as contiguous if they share a border. In their analysis the authors use a Delaunay triangle algorithm to define borders as shown in Figure 1.
The specification of the SSSEM is very closely linked to the STLM. The set of equations that define the SSSEM (4.3) are a more general form of the STLM except for the specification of the weight matrix. The STLM specification is the SSSEM where $\eta_t$ is non-stochastic therefore $\alpha_t = \alpha_{t-1}$, and so the slope parameters in the STLM are constant across time, and only intercept fixed time effects are allowed. The spatial correlation parameter, $\phi$, is fixed across time and space in the SSSEM. The weight matrix is specified as a function of a temporal and a spatial filter in STLM instead of a spatial filter for each time period as in SSSEM.

The KKF model is also a fusion of a state space model from time series analysis and kriging\(^6\) from spatial statistics. The KKF model is outlined by Mardia et al. (1998) in the context of the more general Spatial Temporal General State Space model (STGSSM). In the literature the model has been applied more commonly in climatology as in Sahu and Mardia (2005) in the analysis of pollution levels.

The KKF model is defined by the following expressions; A Spatio-Temporal field $Y(s,t)$ is decomposed into a mean and error components similar to the hierarchical modeling methodology (which is discussed shortly). The mean is then defined as a varying linear combination of $\alpha(t), p$ vector state variable and $X(s)$, the spatial fields\(^7\). The model is defined as follows;

\[
\begin{align*}
Y(s,t) &\in \mathbb{R}^k, t \in T \subset \mathbb{R} \\
Y(s,t) &= \mu(s,t) + \varepsilon(s,t) \quad (4.4) \\
\mu(s,t) &= X_1(s)\alpha_1(t) + X_2(s)\alpha_2(t) + \ldots + X_p(s)\alpha_p(t) = X(s)^T \alpha(t) \quad (4.5) \\
\end{align*}
\]

The substitution of (4.6) into (4.5) at each spatial location $s$ yields the observation equation of the KKF model.

The vector $\alpha(t)$ represents the state of the system of (4.6) for $\mu(s,t)$. The evolution of the state variable is given by the state equation.

\[^6\text{Spatial prediction from autocorrelated spatial models (imputing missing values).}\]

\[^7\text{Also known as the common fields of the STGSS.}\]
\[
\alpha(t) = P\alpha(t-1) + K\eta(t) \\
\eta(t) \sim NID(0, \Sigma_{\eta})
\] (4.7)

In the state equation \(P\) denotes the transition matrix, \(K\) the innovation coefficient matrix and \(\eta(t)\) is a vector of innovations. This component of the model introduces a stochastic component into the trend \(\mu(s,t)\) via the innovation at time \(t\) given by \(K\eta(t)\).

The KKF model assumes that all temporal dependence in \(Y(s,t)\) is expressed by the state vector and state equation thus the error component \(\varepsilon(s,t)\) is a spatially correlated error process.

\[
\text{cov}[\varepsilon(s,t), \varepsilon(s',t')] = 0 \quad \text{for} \ t \neq t' \quad \forall \ s, s'
\] (4.8)

Not all the model parameters \(P, K\) and \(\Sigma_{\eta}\) will be known and so these unknown parameters will need to be estimated.

The choice of the common fields \(X_j(s)\) will be driven by the data as well as a priori modeling considerations. The \(p\) common fields are divided into two sets, \(q\) trend fields and \(r\) principal fields such that \(p = q + r\). The trend fields will account for any discernible trend in the data while the principal fields account for the variation in the detrended data, thus they stem from the covariogram of the detrended data.

Conventional choices of functions employed in spatial kriging are considered for the trend fields. These include: a constant, linear and quadratic functions of coordinate dimensions. The principal fields are selected from a basis of the space of all possible spatial kriging estimates for a given set of \(m\) “normative” sites and for a given second order spatial structure (covariance/variogram). There are up to \(m\) choices of principal fields for the \(m\) normative sites (one for each site). The data are used to choose a set of normative sites as well as a second order structure.

The SSSEM as well as the STLM are contained in the KKF model. It is clear that the SSSEM is a discrete form of KKF, where \(P\) and \(K\) are identities, and the error structure of \(\varepsilon(s,t)\) is given by (4.3c). Similarly, the STLM is obtained via the same restrictions as well as the restrictions specified in the comparison of the STLM and SSSEM however here the error structure is defined by (4.1b).

The hierarchical model provides a relatively simple strategy to incorporating complicated ST interactions in stages. Wikle et al. (1998) outline a general five stage hierarchical model such that the covariance structure is modeled in terms of means at each stage. The hierarchical approach does not dwell on the choice of which scale, the spatial or temporal to consider, it seems to consider both scales simultaneously. An application of this model to economic data (regional GDP of Greece) by Kamarianakis and Prastacos (2001) illustrates some of the difficulties faced when applying a ST model to the economic discipline rather than to a physical process. When modeling spatial interdependencies, the notion of ‘neighbors’ is utilized. When dealing with physical processes a function of the Euclidean distance can be used to determine which points on the spatial plane are considered ‘neighbors’ however in the case of economic sectors the procedure is not as simple. In their study the authors suggest use of the stage of economic advancement of sectors to define neighbours. Fortunately for the real-estate market physical
distance can be used as a measure for neighbours. The first stage outlined by Wikle et al. in the hierarchical structure is the specification of a data model;

\[ Y(s,t) = f_d(\mu(s,t), \theta_1) \]  
(4.9)

\[ Y(s,t) \] denotes the observational data, \( f_d(\cdot) \) is a stochastic function, \( \mu(s,t) \) represents the true data generating process and \( \theta_1 \) is a collection of parameters.

The second stage is the specification of the process model;

\[ \mu(s,t) = f_p(X(s,t), \theta_2) \]  
(4.10)

\( f_p(\cdot) \) is a stochastic function, \( X(s,t) \) is a set of covariates and \( \theta_2 \) is a collection of parameters.

Although in their paper Wikle et al. outline five stages in their model, for our analysis the two stage model is sufficient.

This general structure nests all the previously mentioned models. As we have already shown that the STLM and the SSSEM are contained in the KKF model it only remains to show that the KKF is contained in the general hierarchical modeling structure. The state space structure of the KKF model is defined by the measurement equation given by (4.5) and the state equation (4.7). The equations fit the definition of the two models in the two stage hierarchical model. The measurement equation represents the data model in the first stage of the hierarchical model where \( \theta_1 = 1 \) and \( f_d(\mu(s,t), \theta_1) = \mu(s,t) + \varepsilon(s,t) \). The state equation corresponds to the process model in the second stage of the hierarchical model, the detailed analysis is part of the work in progress.

The results show that we are very close to being able to specify a general framework for the above mentioned models, they seem to be completely contained in the general hierarchical specification however more analytical work is required to complete the specification of the KKF model from the hierarchical model.

5. Data

There are two data sets that are available for the empirical comparison of the studied models. As the complete framework has not been completely established we have not yet chosen which data set is more appropriate for this study. The data set provided by Pace K.R is point referenced data from Baton Rouge, Louisiana between the years 1985 to 1992. This data set has been obtained by Pace et al. (2000) from the Baton Rouge Multiple Listing Service. The observations that have been selected for the sample meet a set of criteria thus the data set has already be cleaned. The observations which could not be geocoded were excluded as a spatial location could not be obtained. The sample observations all have complete information on the age, living area, other area and the number of bathrooms. The sample should represent an average dwelling, thus information on an extremely luxurious house would not be useful. Thus, the houses in the sample have been chosen to be as homogeneous as possible, all have been chosen to have central climate control, are of the most common construction, occupancy and zoning types, all the observations are between 850 and 6000 total square feet in total area, all have more than 500 square feet living area and have a list price above $20,000. The exclusion of outliers as well as missing
observations may bias the results however, as the data set has already been ‘cleaned’ the cleaning technique cannot be chosen and thus limits the results to the cleaning that has already been chosen and performed by Pace et al. (2000). The dependant variable which is the sales price has a range of $13,500 to $421,875 with 5543 observations in total. The geocoding makes use of the 1702 Louisiana South state plane projection based on Clarke 1866 spheroid and the NAD 27 datum. This leads to two coordinate variables, longitude and latitude.

The data set provided by Cominos et al. is much larger however it contains different information and has also been cleaned. The data were obtained from property information services The data were cleaned for errors as well as observations with missing information on property attributes and significant outliers although the data set still contains observations that are located on acreage. This data set includes information on the address, post code, sales price, sales date, map reference, area (sqm), no. of bedrooms, no. of bathrooms and no. of car spaces for the period January 1971 to December 2005. The data have been geocoded using Geodetic Datum of Australia 1994 using MapInfo Professional. The set of considered properties range in price from $1,000 to $30,000,000 (a much larger range than that allowed for in the other data set), have an area of 100 sqm or more, between zero and nine bedrooms and bathrooms, have less than nine car spaces, lie inside the Brisbane Metropolitan Area and are classified under a ‘residential property’.

6. Conclusion

As the research progresses, we hope to identify a comprehensive general framework for a large group of ST models and to show the relationships between the various models. We also aim to conduct an empirical comparison of the models in the context of real-estate market data from one of the two data sets available to us.

Due to time constraints, the aims of our study are limited, it is our hope that we will be able to effectively analyze the selected models within the allocated time frame. However, given more time we would pursue further issues of which modeling specification performs better in forecasting and prediction of prices, as well as investigate the effect of choosing different loss functions in the Bayesian framework estimation analysis.

7. References


