Hierarchical Package Bidding: A Paper & Pencil Combinatorial Auction

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Abstract

We introduce a new combinatorial auction format based on a simple, transparent pricing mechanism tailored for the hierarchical package structure proposed by Rothkopf, Pekeč, and Harstad (1998) to avoid computational complexity. This combination provides the feedback necessary for bidders in multi-round auctions to discern winning bidding strategies for subsequent rounds and to coordinate responses to aggressive package bids. The resulting mechanism is compared to two leading alternatives in a series of laboratory experiments involving varying degrees of value synergies. Based on these "wind tunnel" tests the FCC has decided to use hierarchical package bidding in the major upcoming 700MHz auction.

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1. Introduction

Auctions with multiple items are typically conducted in an environment in which bidders' values depend on acquiring combinations, e.g. networks of broadcast licenses or timber rights for adjacent tracts of land. Concerns for economic efficiency and revenue enhancement have led the Federal Communications Commission (FCC) to run auctions simultaneously for large numbers of licenses in a series of bidding rounds, with provisional winners being announced after each round. Under the simultaneous multi-round auction format (SMR), the highest bid on each license becomes the provisional price that must be topped in a subsequent round. This approach has been copied in other countries with considerable success, but experimental evidence indicates that efficiency and revenue may be reduced when bidders hesitate to incorporate synergy values into their bids for fear that they will end up winning only part of a desired package (see, for instance, the references in Brunner et al., 2007).

The "exposure problem" is a major concern in what is arguably the auction of a lifetime, i.e. the upcoming FCC 700MHz spectrum auction.[1] This spectrum has better propagation and penetration properties than any spectrum sold before and is extremely valuable for wireless applications (the FCC has set *minimum* prices at over 10 billion). More importantly, the wireless industry is concentrated and the 700MHz auction provides the last opportunity for a new firm to enter the market. For an entrant to be successful in the wireless market, however, it has to acquire a nationwide footprint, which is virtually impossible with the current SMR format because of exposure risk.

Some pre-packaging of licenses into larger groups may help solve the exposure problem. In FCC Auction 65, for example, bids were proportionally higher for large blocks of bandwidth for air-to-ground communications; a block with 3 times as much bandwidth as a smaller block sold for about 4.5 times as much. In Auction 66 that closed in the fall of 2006, a 20MHz band divided into over 700 local areas sold for 2.27 billion dollars, but the same amounts of bandwidth sold for 2.45 billion when divided into 176 regions, and for 4.17 billion when divided into 12 regions. Of course, pre-packaging generally disadvantages small bidders who might be part of an optimal allocation due to incumbency or efficient local operations. Instead of dividing licenses into large or small groups, there may be considerable gains in allowing competition that determines the packages, e.g. between bidders on individual licenses, regional groups, and a single national license. A combinatorial auction solves this problem by allowing bids on packages of various sizes

[1] The 700MHz spectrum has been coined the "FCC's crown jewels" by FCC commissioner Adelstein and is more generally referred to as "beach front property." According to former FCC chief of staff Blair Levin, "...the 700MHz auction will be the biggest spectrum auction ever held."

and letting the bidding competition determine the market structure.

There are two steps to be done after each round of a combinatorial auction: (i) the determination of provisional winners or "assignment" part, (ii) and the information provision or "pricing" part. The assignment part, which involves finding the non-overlapping bids that maximize seller revenue, is easy to explain but NP-hard (non-deterministic polynomial-time hard) to do. The number of possible allocations grows exponentially, and with many objects for sale there is no guarantee the best allocation is found in a reasonable amount of time – this is commonly referred to as the computational complexity problem. The problem is manageable in the sense that the computer can run for a fixed amount of time and the best solution at that point can be chosen. Then this solution can be used as a lower bound from where next round's solution can be found ("branch-and-bound"). However, some experts claim they can "wreck any combinatorial auction with enough licenses" by abusing computational issues. In any case, bidders will not be able to reproduce the outcome of a round to understand why their bids did not win, unless they solve an NP-hard problem quickly. Approximations must be particularly worrisome for public officials who anticipate that losing bidders may later complain about the assignments and prices in a particular round.

Rothkopf, Pekeč, and Harstad (1998) propose a type of "hierarchical" pre-packaging to avoid computational issues in combinatorial auctions. In this approach, there are several hierarchy levels of varying package sizes. For instance, if there are only three levels then the lowest could contain individual licenses, the middle level could contain non-overlapping regional packages, and the highest level could contain the national package. With this tree structure, the revenue maximization problem is recursive and can be solved in a linear manner, since revenue-maximizing "winners" at one level can be compared with those at the next level up in the hierarchy.

This paper proposes and tests a simple pricing formula for hierarchical combinatorial auctions. If a bid on an individual license is provisionally winning, then that bid would become the price for the license, as is the case for SMR. The idea underlying the pricing is that prices for individual licenses would have to be scaled up by lump-sum "taxes" to share the burden of unseating a provisionally winning package bid. For example, if a bid on a regional package is provisionally winning, prices for individual licenses would be increased so that bidders on individual licenses in that region would know how high they have to bid to unseat the provisional regional winner. In this sense, prices help these bidders solve a coordination or "threshold problem," since each would prefer that someone else bear the cost of unseating the package bid.

The key feature of the proposed mechanism is that the resulting license prices are composed of intuitive and easy to compute components that match the transparency of the recursive revenue-maximizing allocation rule. These simple hierarchical allocation and pricing rules are the basis for the "hierarchical package bidding" (HPB) format that the FCC will use for the upcoming 700MHz auction. In the Procedures Public Notice (October 5, 2007) the FCC motivates their choice as follows *"... we will use HPB in part because the mechanism for calculating prices is significantly simpler than other package bidding formats..."*[2]

Even though a simple tree structure with hierarchical package bidding provides straightforward price indicators for how high bids on individual licenses and packages must be to "get into the action," there is a concern that the definitions of non-overlapping packages at each level may not match the interests of particular bidders. For example, suppose that a bidder has super-additive values for multiple licenses that are not spanned by a particular regional package definition. In this case, package bidding does not fully protect from exposure risk, and the "second-best" nature of the constraints added by hierarchical package bidding may result in lower rather than higher revenues and efficiencies. This possibility motivated the laboratory experiments to be discussed below. For purposes of comparison, we also consider the SMR format currently used by the FCC, and a fully flexible auction format that does not restrict package bids that can be submitted. This fully flexible format is a version of the FCC's Modified Package Bidding (MPB) that uses the Resource Allocation Design (RAD) pricing mechanism proposed by Kwasnica et al. (2005).[3] RAD prices are essentially approximations to shadow prices determined by the dual of a constrained revenue-maximization problem, as explained in more detail below.

In the next section we provide a general description of the assignment and pricing rules for auctions with hierarchically pre-defined packages. Section 3 discusses the experiment design and procedures. Section 4 provides aggregate statistics on efficiency, revenue, and bidder profits for the three auction formats. Section 5 contains a discussion of the exposure problem we observe in the SMR format and the threshold problem that occurs with flexible package bidding. In spectrum auctions, it is often the case that licenses that cover different blocks of spectrum within the same geographic region are viewed as substitutes while licenses that cover different regions are viewed as complements. In section 6, we present results for two additional experiments based on related designs with a mix of substitutes and complements. Section 7 concludes. A summary data table can be found in the Appendix.

[2]See `http://fjallfoss.fcc.gov/edocs_public/attachmatch/DA-07-4171A1.pdf`. The specific pricing and assignment rules are described in Appendix H. An earlier FCC Public Notice that invited feedback about our mechanism mentions the importance of the experiments and the relative success of HPB, see `http://fjallfoss.fcc.gov/edocs_public/attachmatch/DA-07-3415A1.pdf`. We would like to stress that the HPB format resulted from independent research. We designed the auction prior to proposing it to the FCC who responded by asking us to test it in the lab.

[3]The MPB experiments reported below employ the RAD design described by Kwasnica et al. (2005).

2. Simple Combinatorial Assignment and Pricing

The hierarchical structures considered here can be formed by repeatedly breaking up larger packages into smaller ones. For example, the top level could consist of a single nationwide package, which is divided into smaller regional packages at the second level, which in turn contain many individual licenses belonging to the first level. More generally, let there be $H \geq 1$ hierarchy levels, labeled by $h = 1, \cdots, H$, where level h contains I_h packages. The lowest level contains the smallest units, e.g. individual licenses, and higher levels consist of bigger packages. Level-h packages are denoted $P^h_{i_h}$ for $i_h = 1, \cdots, I_h$ and cover $\alpha^h_{i_h}$ bidding units (e.g. population times bandwidth). The number of packages within a hierarchy falls as we go up in the hierarchy tree, i.e. $I_{h'} \leq I_h$ for $h' \geq h$. Packages within a hierarchy are non-overlapping, i.e. for level-h packages $P^h_{i_h} \neq P^h_{j_h}$ we have $P^h_{i_h} \cap P^h_{j_h} = \emptyset$. Furthermore, a package from a lower hierarchy level is contained in exactly 1 package from each of the higher levels, i.e. for each $P^h_{i_h}$ and for each $h' > h$ there is a unique level-h' package $P^{h'}_{j_{h'}}$ such that $P^{h'}_{j_{h'}} \supset P^h_{i_h}$. The number of bidding units covered by a package equals the number of bidding units covered by the level-1 items it contains

$$\sum_{P^1_{i_1} \subset P^h_{i_h}} \alpha^1_{i_1} = \alpha^h_{i_h}. \tag{1}$$

In particular, each hierarchy level covers the entire nation: $\sum_{i_h=1}^{I_h} \alpha^h_{i_h} = \alpha$ for each level $h = 1, \cdots, H$, with α the total number of bidding units in the nation.

The combinatorial auction with hierarchically structured packages is "simple" in the sense that the assignment problem can be solved in a linear manner. First, for each hierarchy level h, we find the highest bids on the packages within the hierarchy. Let these best bids be denoted by $b^{\max}(P^h_{i_h})$. To find the optimal assignment and revenue we follow a recursive algorithm, defining revenues $R(P^h_{i_h})$ for packages of all levels.

1. Set $h = 1$ and define revenues for packages in this level to be $R(P^1_{i_1}) = b^{\max}(P^1_{i_1})$ for $i_1 = 1, \cdots, I_1$. Furthermore, label these high bids "provisionally winning."

2. If $h < H$, increase h by 1 and do step 3, otherwise quit.

3. If $b^{\max}(P^h_{i_h}) > \sum_{P^{h-1}_{i_{h-1}} \subset P^h_{i_h}} R(P^{h-1}_{i_{h-1}})$, where the sum is over all level-$(h-1)$ packages contained in $P^h_{i_h}$, then $R(P^h_{i_h}) = b^{\max}(P^h_{i_h})$ and $b^{\max}(P^h_{i_h})$ is labeled provisionally winning and bids from all lower levels on packages that overlap with $P^h_{i_h}$ are unmarked. Otherwise, $R(P^h_{i_h}) = \sum_{P^{h-1}_{i_{h-1}} \subset P^h_{i_h}} R(P^{h-1}_{i_{h-1}})$. Return to step 2.

By construction, the maximum revenue is the sum of revenues for items in the top level, $R = \sum_{i_H=1}^{I_H} R(P^H_{i_H})$, and the provisionally winning bids are those that are still marked after the algorithm finishes. Note that the total number of comparisons is linear in the number of predefined packages.

Prices are assigned to the smallest possible objects, i.e. objects in the lowest hierarchy level. The main idea is to match the recursive approach of the assignment part and add a "tax" to level-1 prices (proportional to the number of bidding units covered) if the revenue of a lower level falls short of that of the next level up. This tax is such that for every hierarchy level, h, the best revenue $R(P^h_{i_h})$ associated with a level-h package $P^h_{i_h}$ can be obtained by summing the prices of the level-1 packages contained in $P^h_{i_h}$. *A fortiori*, summing the prices of all level-1 packages yields the best possible revenue. Let $p(P^1_{i_1})$ denote the price of level-1 package $P^1_{i_1}$.

1. Set $h = 1$ and define prices for packages in this level to be $p(P^1_{i_1}) = b^{\max}(P^1_{i_1})$ for $i_1 = 1, \cdots, I_1$.

2. If $h < H$, increase h by 1 and do step 3, otherwise quit.

3. For each level-h package $P^h_{i_h}$, add $\tau^h(P^1_{i_1}) = \frac{\alpha^1_{i_1}}{\alpha^h_{i_h}}\big(R(P^h_{i_h}) - \sum_{P^{h-1}_{i_{h-1}} \subset P^h_{i_h}} R(P^{h-1}_{i_{h-1}})\big) \geq 0$ to the price $p(P^1_{i_1})$ of each level-1 package $P^1_{i_1}$ contained in $P^h_{i_h}$. Return to step 2.

The recursive algorithms used to determine prices and allocations are similar and could be combined to run simultaneously after each round of the auction. Furthermore, they are trivial from a computational viewpoint.

The level-1 prices that result from this recursive approach can be neatly summarized as:

$$p(P^1_{i_1}) \;=\; b^{\max}(P^1_{i_1}) \;+ \sum_{P^h_{i_h} \supset P^1_{i_1}} \frac{\alpha^1_{i_1}}{\alpha^h_{i_h}} \Big(R(P^h_{i_h}) \;\; - \sum_{P^{h-1}_{i_{h-1}} \subset P^h_{i_h}} R(P^{h-1}_{i_{h-1}}) \Big), \tag{2}$$

where the (outer) sum is over all higher-level packages that contain $P^1_{i_1}$. There is exactly one such package at each higher level, so the sum contains a single term for each $h > 1$, which is the level-h tax $\tau^h(P^1_{i_1})$, defined above. The right-side of (2) is therefore equal to a "base price" augmented with non-negative taxes: $p(P^1_{i_1}) = b^{\max}(P^1_{i_1}) + \sum_{h>1} \tau^h(P^1_{i_1})$. This interpretation was used in the instructions phase of the experiments reported below.

Proposition. *The prices defined in (2) have the following properties:*

(i) *Prices reduce to standard SMR prices in the absence of package bids. Taxes, if any, are proportional to the number of bidding units covered.*

(ii) *Prices signal how high bids must be to unseat current winners at any hierarchy level.*

(iii) *Prices are "market clearing," i.e. the sum of individual license prices within a package exceeds (equals) the amount of a losing (winning) bid for the package.*

Proof. Without package bids $R(P^h_{i_h}) = \sum_{P^1_{i_1} \subset P^h_{i_h}} b^{\max}(P^1_{i_1})$ for all packages $P^h_{i_h}$ so (2) reduces to $p(P^1_{i_1}) = b^{\max}(P^1_{i_1})$. To show (ii), define the level-h revenue $R(h) = \sum_{i_h=1}^{I_h} R(P^h_{i_h})$. Using (1), it is readily verified that the linear prices in (2) sum up to level-H revenues:

$$\sum_{i_1=1}^{I_1} p(P^1_{i_1}) \;=\; R(1) + \sum_{h>1}(R(h) - R(h-1)) \;=\; R(H),$$

which is the maximum revenue by construction. Hence, prices of the level-1 items sum to the winning bids, irrespective of the levels in which the winning bids occur. Property (iii) follows by construction and reflects the fact that the assignment problem has an integer solution, so dual prices exist. □

The proportional tax property in (i) ensures that small licenses do not get overpriced.[4] The second property shows how the prices help smaller bidders avoid the "threshold" problem that potentially occurs when they have to coordinate their bids in response to an aggressive package bid. The third property does not generally hold with flexible (non-hierarchical) package bidding.[5]

3. Experimental Design

3.1. Auction Formats

Our main design involves 7 bidders and 18 licenses, with three alternative auction formats. All experiments were conducted using *j*Auctions, which enables bidders to create "custom" packages in order to see the value complementarities associated with winning combinations of licenses.[6] Under flexible package bidding (MPB), bidders could bid on these self-created

[4]Consider a two-level hierarchy with 1 national package divided into three licenses, labeled A, B, and C, which cover different bidding units: A and C are small ($\alpha_A = \alpha_C = 1$) while B is large ($\alpha_B = 10$). Suppose that values for A and C are somewhere in the [5,20] range while the value for B is somewhere in the [50,100] range. Furthermore, suppose there are three small bidders and one large bidder: bidder 1 wants A, bidder 2 wants B, and bidder 3 wants C, while bidder 4 values only the package ABC. If opening bids of the small bidders are 4 on licenses A–C while bidder 4 places a bid of 60 on ABC then without correcting for bidding units, prices are $p_A = p_B = p_C = 20$. This would cause bidders 1 and 3 to drop out since their values are below 20. In other words, small licenses get over-priced, thereby eliminating small bidders from the auction. In contrast, the pricing formula (2) yields prices of $p_A = p_C = 4 + \frac{1}{12}(60-12) = 8$ and $p_B = 4 + \frac{10}{12}(60-12) = 44$, giving small bidders a chance to compete.

[5]Consider, for example, the following four bids: $b_{ABC} = 30$ and $b_{AB} = b_{AC} = b_{BC} = 24$. No market clearing prices exist in this case.

[6]*j*Auctions is a JAVA-based suite of auction programs developed by Jacob Goeree under NWO-VICI grant 453.03.606.

packages. Under the baseline simultaneous multi-round auction (SMR), these custom packages would be shown but could not be placed into the bidding basket. Finally, under hierarchical package bidding (HPB) bidders are permitted to submit bids for pre-defined packages but not for custom packages.

Without package bidding (SMR), the highest bids submitted for each license in a round become the provisionally winning bids. With package bidding (either MPB or HPB), the provisionally winning bids for licenses or packages are those that maximize seller revenue. At the end of each round, bidders receive information on all provisionally winning bids (for licenses and packages) and the corresponding ID numbers. Bidders also see the prices for all licenses, the sum of their own values for the licenses and packages that they are provisionally winning, and the sum of prices that would be paid for those licenses and packages if the auction had ended. Bidding continues from round to round until no new bids are submitted (or withdrawn under SMR), at which time the provisionally winning bids become the final bids that determine allocations and prices paid. Under SMR, bidders have limited opportunities to withdraw provisionally winning bids: in at most two rounds of the auction, bidders can withdraw as many provisionally winning bids as they wish.[7] Withdrawals are subject to penalties that compensate the seller for lower prices obtained: if the license is sold, the penalty is equal to the difference between the withdrawn bid and the final sales price (if this difference is positive, otherwise the penalty is 0). If the license goes unsold, the penalty is 25% of the withdrawn bid.

The bids in each round are used to construct prices that place lower bounds on the bids that can be submitted in the subsequent round. Under SMR, prices are simply equal to the highest bids for the licenses. Under MPB, the prices for licenses are set so that the losing bids on licenses (or packages) are less than or equal to the corresponding license prices (or sum of prices for individual licenses in that package) and the winning bids are equal to the corresponding prices. Although the intuition behind these "Walrasian" constraints is clear, the presence of complementarities in license values may preclude the existence of such dual prices. In this case, prices are approximated in a manner that minimizes some measure of the extent to which the constraints are not satisfied.[8] As shown in the previous section, HPB prices are Walrasian prices (not approximations). These prices are computed in a straightforward recursive manner by scaling up high bids at each level so that they sum to the high bid at the next level of the hierarchy.[9]

[7]Since package bidding protects bidders from the exposure problem, withdrawals were not permitted under MPB and HPB.

[8]In addition, there may be multiple solutions to the constraints, and a second constrained optimization problem is run to resolve the indeterminacy, e.g. maximizing the minimum price, or minimizing the sum of squared deviations from previous round prices (Kwasnica et al., 2005).

[9]Under SMR, new and current provisionally winning bids are considered to determine provisional winners for the next round. Under MPB and HPB all bids received are considered (to prevent cycles),

New bids at the start of a round must exceed provisionally winning bids under SMR by at least one bid increment (3 points in the experiment), whereas new bids under the two package bidding formats must exceed the price of a license or sum of prices for licenses in a package by at least one bid increment for each license in the package. For example, the next bid on a package of 2 licenses with prices 5 and 10 would have to be 21 (= 5 + 10 + 3 + 3), but higher bids of 24, 27, etc. would also be allowed.[10]

In the FCC auctions, financial pre-qualifications determine initial bidding activities for each participant, where activity is measured in terms of population and bandwidth ("MHz-pop" units). Each license in the experiment has one activity unit to avoid the extra complexity. Activity has a "use it or lose it" feature, except that a provisionally winning bid does not need to be resubmitted to maintain activity. Therefore, activity *in this round* is defined in terms of the number of different licenses for which a bidder is the provisional winner or for which a bid is submitted *in the previous round*, either individually or as part of a package. Under MPB and HPB, a bid from a previous round which was not then a winning bid might become a winning bid later as a result of others' bidding behavior. However, this does not raise a bidder's activity. In other words, bidders' activities can only decrease over the course of the auction. If a person's bidding activity is below the permitted level, then the permitted activity falls to that lower level in the subsequent round.

3.2. Sessions and Bidder Interests

Each auction involved 7 participants: six "regional" bidders (labeled 1 through 6) and one "national" bidder (labeled 7). A graphical representation of bidders' interests is shown in Figure 1. The large circle (licenses A through L) on the left contains licenses of interest to the national and regional bidders. For example, license A is of interest to regional bidders 1 and 6 and to the national bidder 7. Note that each regional bidder has an interest in 4 adjacent licenses, with partial overlap in these interests. The smaller circle on the right (licenses M through R) contains licenses only of interest to regional bidders, e.g. license P is of interest to regional bidders 3 and 4. Activity and purchase limits were such that regional bidders can acquire at most four licenses, and the national bidder can acquire

which implies that bids from previous rounds may become winning. The FCC is considering allowing bidders to "drop" non-provisionally winning bids in a single round.

[10]Under MPB, after a round in which news bids were submitted but revenue did not increase, the bid increment needs to be raised to avoid cycling. Consider the following scenario where the bid increment is assumed to be 3: suppose there is a winning bid of 30 on the ABC package and three other bids of 29 on AB, AC, and BC. The computed prices for A, B, and C are 10 each. In the next round the minimum acceptable bids are 26 (=10+10+3+3) for each package of two licenses. So the losing bidders could resubmit their bids of 29, and minimum acceptable bids would again be 26 for each license etc. This cycling behavior can occur whenever a package bid is winning and losing bidders are bidding on sub-packages. The solution is to increase the bid increment (from 3 to 6 to 9, etc.).

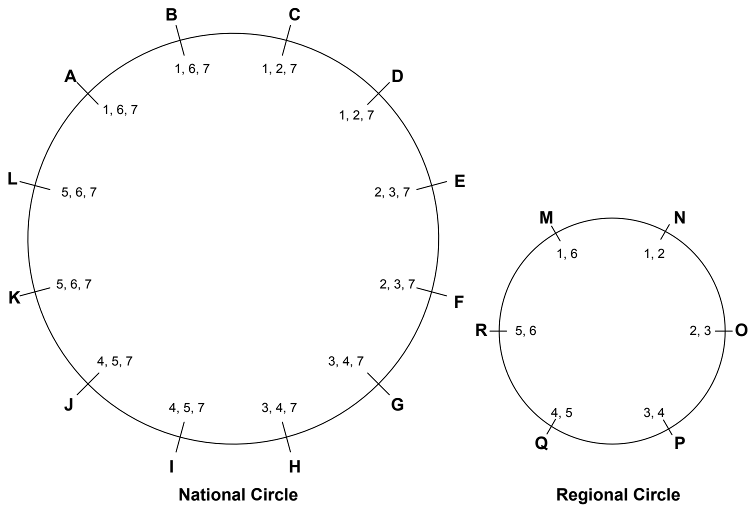

Figure 1. *Regional bidders 1-6 are interested in 4 licenses from the national circle and 2 licenses from the regional circle. National bidder 7 is interested in all 12 licenses from the national circle.*

up to twelve licenses on the larger circle. One useful feature of the smaller circle was to reduce earnings inequities in cases where the national bidder managed to win a national license or a large share of licenses, which was expected to happen more frequently with package bidding.

The license values for each bidder were randomly determined, and values for combinations were determined by scaling up the values of the individual licenses in the combination. For the national bidder, the baseline draw distributions are uniform on the range [0, 10] for licenses A-D and I-L and uniform on the range [0, 20] for licenses E-H. For regional bidders, the baseline draw distributions are uniform on the range [0, 20] for licenses A-D and I-L and uniform on the range [0, 40] for licenses E-H. Finally, for licenses M-R the baseline draw distributions are uniform on the range [0, 20]. These value distributions (not the actual draws) were common knowledge among the bidders. Note that the E-H region of the national circle is, on average, worth more to the national and regional bidders. This asymmetry allows us to measure the impact of "pre-packaging" on regional bidders of different strengths. In particular, the regional bidder with an interest in the high-value licenses E-H would often be a strong competitor with the national bidder, and a threshold problem could arise if the other regional bidders drop out of the bidding without coordinating a strong response to an aggressive national bid.

In all formats, bidders could bid on individual licenses. Recall that bidders cannot bid on packages under SMR, and they have full flexibility in bidding on packages under MPB, subject to activity constraints. Under HPB, the admissible packages have a hierarchical structure consisting of a single national package ABCDEFGHIJKL in our HPB_2 design, where the subscript indicates the number of levels in the hierarchy. We also implemented a three-level hierarchy, with an additional middle level consisting of three non-overlapping regional packages: ABCD, EFGH, and IJKL in our HPB_3-odd design and ABKL, CDEF, and GHIJ in our HPB_3-even design, see Table 1 (licenses E, F, G, and H are shown in bold to indicate their higher values). In other words, in HPB_3-odd the odd numbered regional bidders can bid on their preferred packages but not the even numbered bidders, while in HPB_3-even the reverse is true. In HPB_2, regional bidders could only submit bids for individual licenses, while the national bidder could bid on individual licenses and/or on the national package.[11] Each of these 5 treatments was replicated 5 times as indicated in Table 1, for a total of 25 sessions (using different groups of 7 participants in each session). For each replication or "wave," we generated new sets of random values for all the bidders; we used the same sets of values across auction formats to reduce performance differences due to the random draws.

Value complementarities were chosen so that the optimal allocation would involve a single national package on the large circle for some sets of value draws and a combination of regional package awards and individual license awards in others. For both national and regional bidders, each license acquired goes up in value by 20% (with two licenses), by 40% (with three licenses), by 60% (with four licenses), etc., and by 220% if the national bidder wins all twelve licenses A-L. In waves 1-4, complementarities occur among all licenses while in wave 5 they occur only among licenses from the national circle. For example, if bidder 1 wins the combination ABM, the value synergies apply to licenses A and B and

[11]The hierarchy structures employed in the experiment simplified the explanation of the HPB pricing rule. Consider, for example, the three-level hierarchy used in HPB_3-odd and HPB_3-even. Let the 12 items on the national circle be indexed by $r = 1, \cdots, 3$ to denote the region and by $i = 1, \cdots, 4$ to denote a license within the region. Let $b_{ir}^{\max}$ denote the best bid for license i in region r, $b_r^{\max}$ the best package bid for region r, and $b^{\max}$ the best package bid for the nation-wide license. Define the revenue for region r as $\text{Rev}_r = \max(b_r^{\max}, \sum_{i=1}^4 b_{ir}^{\max})$ and the national revenue as $\text{Rev} = \max(b^{\max}, \sum_{r=1}^3 \text{Rev}_r)$. The price p_{ir} of license i in region r is simply the maximum bid for the license plus possibly a "regional tax" (if the sum of individual bids falls short of the regional profit) plus possibly a "national tax" (if the sum of regional profits falls short of the national profit):

$$p_{ir} = b_{ir}^{\max} + \frac{\alpha_{ir}}{\alpha_r}\big(\text{Rev}_r - \sum_{i'=1}^{4} b_{i'r}^{\max}\big) + \frac{\alpha_{ir}}{\alpha}\big(\text{Rev} - \sum_{r'=1}^{3} \text{Rev}_{r'}\big),$$

with $\alpha_r = \sum_{i=1}^4 \alpha_{ir}$ the number of bidding units in region r and $\alpha = \sum_{r=1}^3 \alpha_r$ the number of bidding units nationwide. (For simplicity, each license covers $\alpha_{ir} = 1$ bidding unit in the experiment.) As an example, suppose the high bids are 3 on each of the 12 items, the regional bids are 24, and the national bid is 84. In this case, the national bid is winning and prices for each of the items are the high bids plus a regional and a national tax: $3 + \frac{1}{4}(24 - 4 \times 3) + \frac{1}{12}(84 - 3 \times 24) = 7$, which determines bids on each item needed to unseat the winning national bid. Finally, for the two-level hierarchy used in HPB_2 the above pricing rule further simplifies to $p_i = b_i^{\max} + \frac{\alpha_i}{\alpha}\big(\text{Rev} - \sum_{i'=1}^{12} b_{i'}^{\max}\big)$, where $i = 1, \cdots, 12$ now labels the twelve licenses on the national circle. In other words, the price p_i of license i is now simply the maximum bid for the license plus possibly a "national tax."

		# Bidders (Activity)		Licenses		Available	License
	# Groups	Regional	National	Regional	National	Packages	Synergy
SMR	5	6 (4)	1 (12)	A-L, M-R	A-L	None	20%
HPB_2	5	6 (4)	1 (12)	A-L, M-R	A-L	ABCD**EFGH**IJKL	20%
HPB_3-odd	5	6 (4)	1 (12)	A-L, M-R	A-L	ABCD, **EFGH**, IJKL, ABCD**EFGH**IJKL	20%
HPB_3-even	5	6 (4)	1 (12)	A-L, M-R	A-L	CD**EF**, **GH**IJ, ABKL, ABCD**EFGH**IJKL	20%
MPB	5	6 (4)	1 (12)	A-L, M-R	A-L	All	20%

Table 1. *Complementarities Design*

M in waves 1-4 and only to A and B in wave 5. The national bidder can acquire up to twelve licenses and has value complementarities for all licenses in all five waves.

Participants were recruited from the Caltech student population for sessions that lasted about an hour and a half. Including the experiments reported in section 6, we conducted 58 sessions using 340 subjects.[12] Each session began with an instruction period and 3 practice auctions, followed by 6 auctions used to determine their earnings. The number of auctions was announced in advance. Bidder roles were reassigned randomly after each auction in order to attenuate earnings differences across national and regional bidders and to help bidders understand the strategic considerations faced by both types of bidders. Earnings were calculated by converting points to dollars at a rate of 2 points per dollar. Average earnings were about \$40 per person, including a \$5 show-up fee, and payments were made in cash immediately after the final auction.

4. Results: Aggregate Data

In this section we report the main indicators for auction performance: efficiency, revenues, and bidders' profits.

4.1. Efficiency

Market efficiency is defined as the value of the allocation obtained in the auction (the actual surplus, S_{actual}) divided by the value of the best possible allocation (the maximum possible surplus, $S_{optimal}$):[13]

$$efficiency = \frac{S_{actual}}{S_{optimal}} \times 100\%. \tag{3}$$

[12] The results for 20 of these sessions were submitted to the FCC in a consulting report that is cited in their decision to adopt HPB for the package bidding segment of the upcoming 700MHz auction (FCC Public Notice, August 2007).

[13] The total number of possible allocations with this setup is 27,433,982.

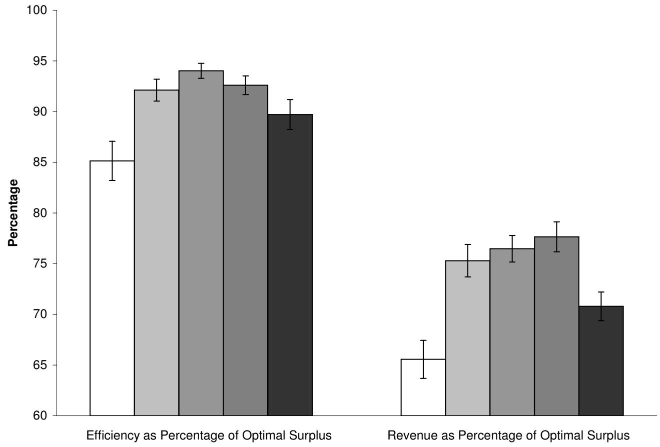

Figure 2. *Efficiencies and Revenues by Auction Format*

SMR (white), HPB_2 (light gray), HPB_3-odd (medium gray), HPB_3-even (dark gray), MPB (black)

The summary Table A1 in the Appendix lists average efficiencies across sessions and auction formats.

Package bidding is designed to help bidders avoid the "exposure problem" of bidding high for licenses with high complementarities. As expected, switching from SMR to the package auction formats raises efficiency: from 85.1% to 89.7% for MPB, to 92.1% for HPB_2, and to 92.6% and 94.0% for HPB_3-even and HPB_3-odd respectively. The left part of Figure 2 shows average efficiencies across formats: the white bar corresponds to SMR, the light-gray bar to HPB_2, the medium-gray bar to HPB_3-odd, the dark-gray bar to HPB_3-even, and the black bar to MPB. The standard deviations are indicated by the bracketed intervals at the top of each bar. Notice that efficiencies are higher and less variable under the HPB formats as compared to SMR and MPB.

The performance differences suggested by Figure 2 are supported by a statistical analysis. As illustrated by Figure 2, the HPB environments yield very similar results (in terms of efficiency and revenue) and, hence, we will pool the observations from the HPB environments. Below $\sim$ indicates a pair-wise ordering that is not significant, $\succ^{*}$ indicates significance at the 10% level, $\succ^{**}$ indicates significance at the 5% level, and $\succ^{***}$ indicates significance at the 1% level.

Result 1. *Efficiencies are ranked* HPB $\succ^{***}$ SMR *and* HPB $\succ^{*}$ MPB.

Support. See the Appendix for an overview of session averages across auction formats. There are five averages for SMR and MPB, corresponding to the five value waves, and fifteen averages for HPB after pooling the HPB environments. The non-parametric test employed is a Wilcoxon matched-pairs signed-rank test. The difference in ranks between

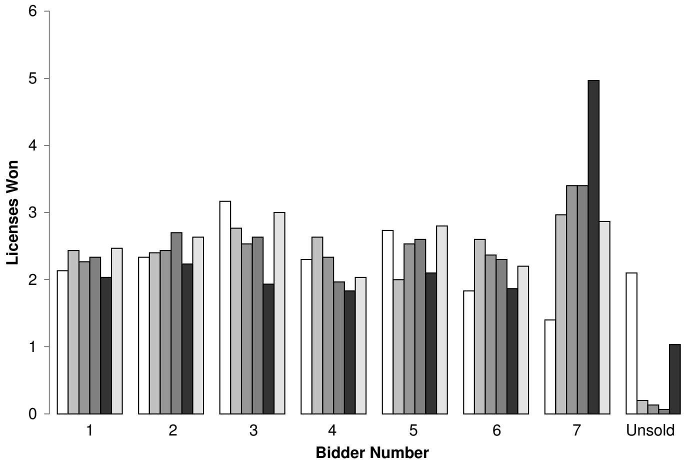

Figure 3. *Licenses Acquired by Regional Bidders (1-6), the National Bidder (7), and the Number of Unsold Licenses. Optimal Awards of Licenses Shown in Yellow.*
SMR (white), HPB_2 (light gray), HPB_3-odd (medium gray), HPB_3-even (dark gray), MPB (black)

MPB and SMR is 9 (five observations), which is not significant. The difference in ranks between HPB and MPB is 90 (fifteen observations), which is significant ($p = 0.09$). Finally, the difference in ranks between HPB and SMR is 120 (fifteen observations), which is significant at the one-percent level ($p < 0.001$). □

One cause of efficiency reductions with SMR is the incidence of unsold licenses, which happens at a rate of 2.1 licenses (out of 18) when averaged over all sessions. Likewise, on average 1.0 license is unsold under MPB while there are virtually no unsold licenses with HPB, see Figure 3.

Result 2. *The numbers of unsold licenses are ranked* SMR $\sim$ MPB $\succ^{***}$ HPB.

Support. The higher rate at which licenses are awarded under HPB is clear from the right-most set of bars in Figure 3. The difference between HPB and MPB in terms of license sales rates is significant with a Wilcoxon matched-pairs signed-rank test ($p < 0.001$ with fifteen observations) while the difference between MPB and SMR is (borderline) insignificant ($p = 0.13$ with five observations). □

4.2. Revenues

Revenues are also normalized by the value of the best possible allocation:

$$revenue = \frac{R_{actual}}{S_{optimal}} \times 100\%. \tag{4}$$

The introduction of package bidding enhances revenues as shown in the right part of Figure 2. Switching from SMR to the package auction formats raises revenue from 65.6% to 70.8% for MPB, to 75.3% for HPB_2, and to 76.5% and 77.9% for HPB_3-odd and HPB_3-even respectively. As before, these comparisons can be evaluated with a Wilcoxon test based on the session averages reported in the Appendix.

Result 3. *Revenues are ranked* HPB $\succ^{***}$ MPB $\succ^{**}$ SMR.

Support. The difference in ranks between MPB and SMR is 15 (five observations), which is significant ($p = 0.04$). The difference in ranks between HPB and MPB is 120 (fifteen observations), which is significant at the one-percent level ($p < 0.001$). □

The brackets on each of the bars on the right side of Figure 2 indicate the standard deviations of the revenues across formats. Note that SMR and MPB result in lower and more variable revenues as compared to HPB. Also, it is apparent from Figure 2 that the average efficiencies and revenues with only two tiers are slightly lower than for the three-tier hierarchies. Even though this difference is not significant, it suggests that in some situations it is important to have enough hierarchy levels to allow small bidders to compete effectively when they are part of the efficient allocation.

4.3. Bidders' Profits

Consistent with the definitions of revenue and efficiency, bidders' profits are normalized by the value of the best possible allocation:

$$profits = \frac{\sum_i \pi^i_{actual}}{S_{optimal}} \times 100\%. \tag{5}$$

This profit is the difference between actual surplus and seller's revenue, except for SMR where possible penalties from withdrawing winning bids are recorded separately (see the Appendix). Rather than simply reporting the profits for the bidders as a group it is useful to show them by bidder type since this highlights the impact that package bidding and/or pre-packaging has on different types of bidders. Figure 4 displays bidders' profits by treatment and bidder number, using the same color-coding as in Figure 2. Again, the standard deviations are indicated by the brackets at the top of each bar.

The ability to bid for combinations allows national bidders to bid high on large packages and avoid the exposure problem, resulting in positive profits for the national bidder (7) in the MPB and HPB auctions. In contrast, the national bidder loses money (in all waves, see the Appendix) when the SMR format is used. These losses are not surprising given prior results on SMR experiments; they result from the value complementarities

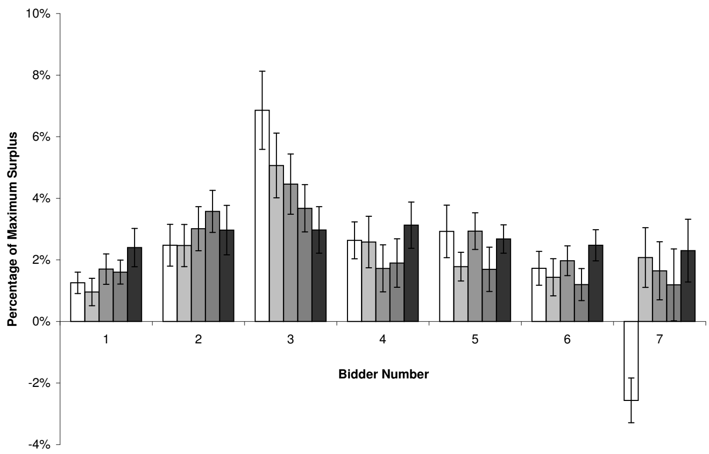

Figure 4. *Profits for Regional Bidders (1-6) and the National Bidder (7)*

SMR (white), HPB_2 (light gray), HPB_3-odd (medium gray), HPB_3-even (dark gray), MPB (black)

that create an exposure problem for the national bidder. In the experiment, the effects of negative earnings on individual behavior were mitigated by the fact that bidder roles were randomly assigned in each auction. The differences in profits for the national bidder are corroborated by non-parametric tests. In contrast, the differences in profits for regional bidders (slightly higher under SMR than MPB and HPB) are not significant.

Result 4. *The national bidder's profit is ranked* MPB $\sim$ HPB $\succ^{***}$ SMR. *The regional bidders' profits (as a group) are ranked* SMR $\sim$ MPB $\sim$ HPB.

Support. For the national bidder's profit the difference in ranks between HPB and SMR is 110 (fifteen observations), which is significant ($p = 0.004$). The difference in ranks between MPB and HPB is 62 (fifteen observations), which is not significant. For the regional bidders' profits the difference in ranks between SMR and MPB is 88 (fifteen observations), which is (borderline) insignificant ($p = 0.11$). The difference in ranks between SMR and MPB is 7 (five observations), which is not significant. □

5. Discussion

5.1. The Exposure Problem in SMR

One reason for the efficiency and revenue advantages conferred by package bidding vis-à-vis SMR is apparent from looking at the outcomes of the first four auctions of wave 5 in which the optimal allocation frequently involved awarding large-circle licenses to the national bidder: 12, 12, 12, and 11 licenses in auctions 1-4 respectively.[14] Under SMR, the numbers of licenses actually obtained by the national bidder in these four auctions were 7, 2, 5, and 1, which shows that the national bidder was unable to overcome the exposure problem and obtain large networks even when it was optimal to do so. The national bidder was much more successful for the package bidding auctions with the same value draws; the numbers of licenses obtained by the national bidder were 12, 12, 12, and 12 licenses under MPB and HPB_3-odd, 12, 12, 12, and 11 under HPB_2, and 12, 7, 12, 12 under HPB_3-even.[15]

The effects of exposure are apparent in the first SMR auction of wave 1. The optimal allocation involved awarding all 12 licenses on the large circle to the national bidder whose values for the licenses are listed in Table 2 together with the final prices. For all licenses, the final price exceeded the national bidder's stand-alone value for the license, and the total cost (249) was much higher than the sum of the individual values (102). Suppose the national bidder had conserved all activity up to the final round and was willing to top the current prices of Table 2, which would involve a minimum expenditure of 285 ($= 249 + 36$) for a national package worth 326 ($= 3.2 \times 102$). If the other bidders (who all have sufficient activity to respond) came back with increases of one additional bid increment, the cost of acquiring the national package would exceed its value. In this case, the national bidder could not profitably win any of the licenses. An even worse outcome would result when only some of the regional bidders respond and the national bidder would win a less valuable subset. For example, suppose the national bidder would win only the high-value EFGH combination, which, including synergies, is worth $102 = (19 + 17 + 10 + 18) \times 1.6$, but would require an expenditure of at least 177 to top the prices listed in Table 2.

The national bidder has to evaluate the risk of being "exposed" during the course of the auction, not just at the end, since decisions to maintain activity are made on a round-by-round basis. In the experiment, the national bidder gradually lost activity and was the provisional winner for fewer licenses as the auction proceeded. At the start of round 8, for example, the national bidder was winning licenses A, C, E, and G with a combined value of 55 (including synergies) and prices of 9, 3, 24, and 9 respectively for a total cost of 45. At the start of round 9, the national bidder was only winning license

[14] Recall that in wave 5, small bidders enjoy synergies only for licenses acquired from the national circle.

[15] Performance differences are also apparent in the awards of blocks of licenses to regional bidders. There were 13 cases where the optimal allocation provided at least 3 of the 4 high-value licenses (E-H) to bidder 3, the only bidder who had an interest in all of these and could bid on them as a package in HPB_3-odd. Package bidding generally does better in these cases, with overall average efficiency of 85% for SMR, as compared with 88% for MPB and 93% for HPB. In both package bidding formats, the EFGH package was sometimes awarded to bidder 3 when it should not have been, but the efficiency consequences were small and certainly smaller than the consequences of not awarding one or more of these high-value licenses at all as happened several times with SMR (a total of six cases out of 13, with three of the four high-value licenses unsold in auction 5 of wave 3).

License	A	B	C	D	E	F	G	H	I	J	K	L	Total
Value	9	5	2	5	19	17	10	18	8	3	3	3	102
Final Price	12	18	3	9	42	60	15	48	12	12	12	6	249

Table 2. *National Bidders' Values and Final Prices for Licenses A–L.*

SMR Auction 1 in Wave 1

C because regional bidders raised their bids to 12, 27, and 12 on licenses A, E, and G. The total cost of acquiring the ACEG combination became 63 and exceeded the value of 55. At this point, the national bidder (rationally) decided to let the activity drop and to withdraw the provisionally winning bid of 3 on license C since it was worth only 2 to this bidder.

In this auction, license C remained unsold even though its price fell back to the minimum opening bid of 3. The reason is that the two regional bidders (1 and 2) with an interest in C had low values for it (2 and 4), and had no additional activity above what they were provisionally winning or actively competing for. More generally, the price following a withdrawal falls back to the second-highest bid for the license, which tends to be high since most withdrawals occur late in the auction. Bidders who are constrained in their activity, as is typically the case late in the auction, are often unwilling or unable to pick up a high-priced unsold unit.

To summarize, unsold licenses in the SMR auctions are a consequence of the interaction between exposure risk and the limited bid-withdrawal option intended to alleviate this problem. For example, Brunner et al. (2007) report *no* unsold licenses in their low-complementarities treatment without exposure risk, and there are no unsold licenses in the no-synergy experiment discussed in section 6.2. Banks et al. (2003) report SMR experiments without withdrawal options, in which case there are no unsold licenses but bidders incur large losses. These observations also suggest that all licenses are more likely to be sold when bidders' value draws are such that the exposure problem is less severe. In the experiment, there were no unsold licenses in 7 of the SMR auctions. But even for these auctions, the average revenue for SMR was 10% lower compared to HPB (69.6% versus 76.5%) while their efficiencies were virtually the same (93.6% versus 93.4%). Finally, the option to withdraw provisionally winning bids does not protect the national bidder from losses, see Figure 4.

The reduced performance of SMR in our design with value complementarities corroborates previous results regarding the adverse effects of exposure risk on efficiency and revenue (e.g. Bykowsky, Cull, and Ledyard, 2000). In addition, our findings suggest that the SMR procedure is disadvantageous for efficient providers for whom it is essential to acquire a national license (e.g. a major new entrant).

5.2. The Threshold Problem in MPB

The awarding of national licenses also provides a perspective on why HPB yields higher revenues and efficiencies than the more flexible package bidding format, MPB. There were several MPB auctions in which the national bidder won many licenses on the large circle when it was not optimal to do so, whereas there are relatively few such cases under HPB. To see how the national bidder was sometimes able to obtain all licenses under fully flexible package bidding (MPB) even when it was not optimal to do so, consider the round-by-round results of auction 2 in wave 1, shown in Table 3. The optimal allocation involved only a single license for the national bidder, but there were only three rounds in which the national bidder was not the provisional winner on all 12 licenses. In each of these three rounds, the regional bidders were not able to coordinate a very strong response in the sense that their provisionally winning bids left numerous provisionally unsold licenses (5 out of 12 licenses in rounds 3 and 4, and 2 out of 12 licenses in round 21). With fully flexible bidding, the regional bidders were bidding on "home-made" overlapping packages that did not "fit" in the sense that the revenue maximizing allocation left unsold licenses, which made it easier for the national bidder to regain provisional winner status in the subsequent rounds. In contrast, when HPB was used with the same draws, the regional bidders were able to effectively block the national bidder, and the resulting efficiency was 10 percentage points higher: 83% in MPB versus 93% in HPB (efficiency was 94% in HPB_2, 95% in HPB_3-odd, and 89% in HPB_3-even).

Motivated by this example, we focused on rounds in which the national bidder wins nothing and counted the number of licenses provisionally won by the regional bidders from the large circle (licenses A-L). The results are shown in Figure 5. Under MPB, the regional bidders are able to coordinate their bids such that they provisionally win all 12 licenses only 10% of the time when the national bidder is not winning any licenses. More than 65% of the time they provisionally win 10 licenses or less (out of 12), resulting in prices for the 12 licenses that can easily be topped by the national bidder. In contrast, under HPB, the regional bidders are able to coordinate and provisionally win 11 or 12 licenses more than 95% of the time. Our experiments nicely demonstrate that the threshold problem we observe with flexible package bidding is not merely a "free-rider" problem but also a coordination problem – indeed, the latter is more pronounced in our data. To our knowledge, our paper is the first to clearly demonstrate the existence of a threshold problem for smaller bidders in auctions with flexible package bidding.

We are not claiming that HPB will yield better performance in terms of efficiency and

	National Circle			Regional Circle	
Round	National	Regional	Unsold	Regional	Unsold
1	12	0	0	2	4
2	12	0	0	2	4
3	0	7	5	5	1
4	0	7	5	5	1
5	12	0	0	2	4
6	12	0	0	3	3
7	12	0	0	3	3
8	12	0	0	3	3
9	12	0	0	4	2
10	12	0	0	4	2
11	12	0	0	4	2
12	12	0	0	4	2
13	12	0	0	4	2
14	12	0	0	6	0
15	12	0	0	6	0
16	12	0	0	6	0
17	12	0	0	6	0
18	12	0	0	6	0
19	12	0	0	6	0
20	12	0	0	6	0
21	0	10	2	5	1
22	12	0	0	6	0
23	12	0	0	6	0
Optimal	0	12	0	6	0

Table 3. *Round-By-Round Outcomes for MPB Auction 2, Wave 1*

revenue in all environments. For instance, if the hierarchical pre-packaging completely mismatches bidders' preferences, the resulting exposure problem that all bidders face would likely reduce bids and revenues. Alternatively, if there is no bidder with an interest in the national package, mis-coordination would be more easily resolved by regional bidders who would be provisional winners in all rounds. The design of our experiment is based on the belief that the FCC will be able to craft economically relevant packages for at least some of the bidders and that there would be one or more bidders (e.g. *de novo* entrants) interested in a national package. Furthermore, as the results from treatment HPB_3-even demonstrate, the performance of HPB is robust to some degree of "mis-packaging." Even an extremely simple hierarchy structure such as HPB_2 leads to improved performance relative to the other formats. The experiments discussed in the next section provide a further robustness check in environments with richer sets of value synergies.

6. Two Follow-Up Experiments

6.1. Complements and Substitutes

The SMR was ideally designed to deal with substitutes since bidders can switch back and forth between equally desired licenses in response to changes in prices. Substitutes are

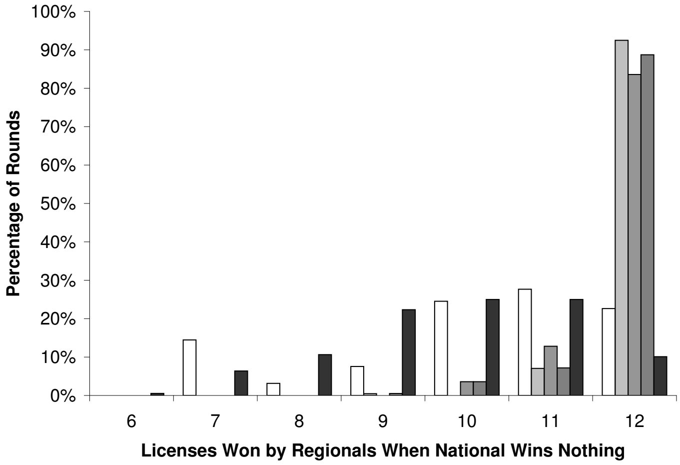

Figure 5. *Number of Licenses Regional Bidders Provisionally Win from the Large National Circle (Licenses A-L) in Rounds Where the National Bidder Wins Nothing*

SMR (white), HPB_2 (light gray), HPB_3-odd (medium gray), HPB_3-even (dark gray), MPB (black)

likely to play some role when multiple blocks of identically sized bandwidth are sold in the same regions, as is often the case in spectrum auctions.[16] We also tested an alternative design with 2 separate bands of 6 licenses, with positive synergies for licenses within a band and negative synergies between bands. The "upper band" consists of licenses A through F and the "lower band" consists of licenses G through L. Table 4 lists the synergy factors that can be used to determine the value of any combination of licenses. The synergy factor in the table is multiplied by the sum of values for the licenses in a combination. For example, if a bidder wins three licenses within the upper band and no licenses in the lower band then the column labeled 3 and the row labeled 0 apply. In this case, the total value for the package equals the sum of the individual values plus an additional 80%. Likewise, if the bidder wins three licenses in the upper band and two in the lower band, then the value of the combination is the sum of the values in the upper band times 1.6 plus the sum of the values in the lower band times 1.2.

Note that the numbers in Table 4 are increasing within each row to reflect complements, while substitution effects are captured by the decreasing numbers within each column. To see how bidders' valuations are super-additive for some combinations, consider a national bidder who acquires licenses A and B from the upper band. For simplicity,

[16]As a matter of theory, it is possible to design simultaneous ascending auctions that converge monotonically to Walrasian package prices in the pure complements case discussed above. This result assumes "straightforward bidding," i.e. allocating bidding activity to licenses and packages with the highest profit margins based on current prices. In practice, bidders typically do not bid straightforwardly (Brunner et al., 2007). More importantly, the auction would require non-linear pricing, i.e. the price for a package is not necessarily equal to the sum of the prices for the licenses it contains. As a result, the auctioneer and bidders would have to keep track of many, many prices (e.g. 262,143 in the experiment reported above).

		Licenses Won within the Band					
		1	2	3	4	5	6
Licenses Won in the other Band	0	1.00	1.43	1.80	2.13	2.40	2.63
	1	0.95	1.35	1.70	2.00	2.25	2.45
	2	0.90	1.28	1.60	1.88	2.10	2.28
	3	0.85	1.20	1.50	1.75	1.95	2.10
	4	0.80	1.13	1.40	1.63	1.80	1.93
	5	0.75	1.05	1.30	1.50	1.65	1.75
	6	0.70	0.98	1.20	1.38	1.50	1.58

Table 4. *Synergy Factors for Licenses Won within a Band*

assume the stand-alone values for the licenses are $v(\text{A}) = 15$ and $v(\text{B}) = 15$, which is the average over the range [10, 20] used in the experiment. It follows from the the top row in Table 4 that the combination is worth $v(\text{AB}) = 43$ (calculated as $= 1.43 \times 30$), which exceeds the sum of the individual values: $v(\text{AB}) > v(\text{A}) + v(\text{B})$. Similarly, the value of acquiring licenses G and H from the bottom band is 43 on average. Next consider the value for acquiring the combination ABGH, which is readily calculated as $v(\text{ABGH}) = 77$ $(= 1.28 * 60)$ and is less than the sum of the values of the two combinations from each line: $v(\text{ABGH}) < v(\text{AB}) + v(\text{GH})$. The sub-additive nature of bidders' valuations across bands is more dramatic for larger combinations of licenses. For example, the value of winning a combination of 6 licenses from a single band is 236 when each of the 6 licenses is worth 15 individually. Winning an additional 6 licenses from the other band raises the total value only to 284, i.e. $v(\text{ABCDEFGHIJKL}) = v(\text{ABCDEF}) + v(\text{GHIJKL}) - 188$.

Subjects in the experiment did not have to multiply synergy factors times sums of values. Rather, the *j*Auctions bidder interface showed the total value (including positive and/or negative synergies) for all the pre-defined packages and for any "custom" package of interest. In our HPB treatment, bids could be submitted for individual licenses and for pre-defined packages (AB, CD, EF and ABCDEF for the upper band and GH, IJ, KL and GHIJKL for the lower band). Under SMR, no bids could be placed for the pre-defined packages although bidders could see their values. Under all three auction formats, bidders could view the values of any desired combination of licenses but only under MPB were bidders allowed to submit bids for such custom packages.

As before, bidders were assigned roles randomly at the start of each auction, with IDs 1 through 3 for regional bidders and IDs 4 and 5 for national bidders. Regional bidders are interested in four licenses in each band with values drawn independently from a uniform distribution on [10, 30].[17] National bidders have values for all 6 licenses in each band;

[17]Bidder 1 values A, B, C, and F in the "upper" A–F band and licenses G, H, I, and L in the "lower" G–L band. Bidder 2 has values for B, C, D, E and H, I, J, K and bidder 3 has values for A, D, E, F and G, J, K, L.

	Efficiency	Revenue	Regional's Profit	National's Profit	Rounds	Unsold
Complementarities Design						
SMR	85.1%	65.6%	3.0%	-2.6%	17.8	2.1
HPB	93.0%	76.5%	2.4%	1.6%	12.2	0.1
MPB	89.7%	70.8%	2.8%	2.3%	15.1	1.0
Mixed-Synergy Design						
SMR	85.4%	78.0%	1.7%	1.1%	15.4	1.5
HPB	93.2%	85.7%	0.7%	2.6%	12.8	0.0
MPB	94.5%	84.6%	1.1%	3.3%	15.3	0.1
No-Synergy Design						
SMR	99.4%	77.1%	7.1%	0.4%	5.8	0.0
HPB	99.2%	77.3%	7.0%	0.5%	6.0	0.0
MPB	98.8%	78.3%	6.5%	0.4%	5.8	0.0

Table 5. *Summary Statistics for the Three Synergy Designs*

these draws are from a uniform distribution on [10, 20]. These ranges of values were selected to ensure that the optimal allocations involved awards of a national license in some but not all auctions. We conducted 6 waves of SMR, MPB and HPB sessions with 6 auctions in each session, using new random draws for each wave. In all auctions, regional bidders started with an activity limit of 8 and national bidders with an activity limit of 12. Other procedural elements (cash conversion rate, bid increment, common knowledge of value distributions, subject pool, number of practice periods) were unchanged from the experiment reported above. One procedural change that we implemented, however, was that bidders were unable to see the IDs and associated bids made by others ("anonymous bidding").

Efficiencies and revenues for this mixed-synergy design are shown in the middle part of Table 5 (the results for the complementarities treatment are shown in the top part for comparison, with pooled results for the different HPB treatments, and the bottom part shows the results of an experiment discussed in the next subsection). In this mixed-synergy design, revenue and efficiency increases of roughly 10% were observed when switching from SMR to HPB, which are about the same magnitude as those observed in the complementarities design. Again, the improved performance of HPB is accomplished in fewer rounds on average, as can be seen from the "Rounds" column in Table 5.

Result 5. *With mixed synergies, efficiencies are ranked* HPB $\sim$ MPB $\succ^{***}$ SMR *and revenues are ranked* HPB $\succ^{**}$ SMR *and* MPB $\sim$ SMR*. The national bidder's profit is ranked* HPB $\sim$ MPB $\succ^{***}$ SMR *while a regional's profit is ranked* HPB $\sim$ MPB $\sim$ SMR*. The numbers of unsold licenses are ranked* SMR $\succ^{***}$ HPB $\sim$ MPB*.*

Support. See Table A1 in the Appendix for an overview of session averages across auction formats. There are six averages for each auction format, corresponding to the six value waves, which are evaluated using a Wilcoxon matched-pairs sign-rank test. Comparing efficiency levels, the difference in ranks between HPB and SMR and between MPB and SMR is 21 ($p = 0.01$). Comparing revenue levels, the difference in ranks between HPB and SMR is 19 ($p = 0.04$) and between MPB and SMR is 15 ($p = 0.17$). For the national bidder's profit, the difference in ranks between HPB and SMR and between MPB and SMR is 21 ($p = 0.01$). For a regional bidder's profit, the difference in ranks is 13 ($p = 0.30$) between SMR and HPB and 11 ($p = 0.46$) between SMR and MPB. Finally, comparing numbers of unsold licenses the difference in ranks between SMR and HPB and between SMR and MPB is 21 ($p = 0.01$). □

6.2. No-Synergy Design

A final experiment we ran is based on a *no-synergy* design with payoffs that are strictly linear or additive, i.e. $v(P_1 \cup P_2) = v(P_1) + v(P_2)$ for any two packages P_1 and P_2. This design can be implemented by replacing all the synergy factors in Table 4 by 1. We conducted 5 waves of SMR, MPB and HPB sessions using the exact same setup as in the mixed-synergies design (e.g. two bands, two national bidders and 3 regional bidders, no information about others' IDs and bids, etc.). Giving bidders the opportunity to bid on packages has no adverse effects for efficiency and revenue in the no-synergies design, as the bottom part of Table 5 shows.

Result 6. *Without synergies, efficiencies are near perfect for all three auction formats and are not significantly different across formats, nor are the revenues, bidders' profits, the number of unsold licenses, and the auction's duration.*

Support. See Table A1 for an overview of session averages across formats. There are five averages for each format, corresponding to the five value waves. Evaluating differences with a Wilcoxon test reveals that none are significant at the 10% level. □

7. Conclusion

In this paper we investigate how the constrained structure of hierarchical package bidding (HPB) compares with alternatives that involve flexible package bidding (MPB) or no package bidding (SMR). The design entails an environment with value complementarities, which are likely significant for *de novo* entrants wishing to establish a national footprint. In addition, the design features a "spike" of adjacent high-value licenses, which introduces an asymmetry that exaggerates the threshold problem. Since different blocks of spectrum within the same region may be considered substitutes, we conducted two additional experiments: one with a stylized model that incorporates both complements and substitutes and the other one without any synergy effects. Within this general modeling framework, we used sets of randomly generated values that induce a wide range of possible market structures.

With complementarities, the results of the laboratory auctions reveal a clear advantage for HPB even though the pre-made packages allowed under HPB did not match the preferred packages for half of the regional bidders (while those bidders were part of the optimal assignment). HPB yields significantly higher auction revenues and efficiencies than SMR and MPB for all pairwise comparisons. Comparable improvements in revenues and efficiencies of HPB over SMR were observed in the mixed-synergy design, while the relative performance of HPB was not significantly reduced in the no-synergy design. These performance differences were not due to increased fine-tuning over a large number of bidding rounds, since the HPB auctions actually tended to have fewer rounds.

The lower efficiencies and revenues observed for SMR in the presence of value complementarities could have been anticipated from prior work. The option of withdrawing bids in SMR, which was introduced as a partial remedy of the exposure problem, generated a higher incidence of unsold licenses compared to the other auction formats. Previous experiments that did not allow bid withdrawals resulted in fewer or no unsold licenses, but there the effects of exposure risk are indicated by the low efficiencies and the negative earnings observed. And in our experiments, even in those cases where no license went unsold, SMR yields less revenue than HPB.

What came as a surprise was the relative ranking of HPB and the more flexible MPB.[18] One factor that contributed to this difference is that the home-made packages constructed under the flexible MPB format tended to overlap, causing a "fitting problem" that made it difficult for strong regional bidders to unseat a national package bid. Indeed, the number of licenses awarded to the national bidder was much higher than the optimal number under MPB, but not under HPB and SMR. More importantly, in rounds when the national bidder won nothing, regional bidders were unable to coordinate their bids under MPB while their coordination problems were virtually non-existent with pre-defined hierarchically-structured packages. Pre-packaging has the obvious disadvantage that the chosen packages may not be optimal, but in a non-overlapping hierarchical structure they are chosen to "fit," which enables bidders to coordinate their bids and avoid threshold problems with positive effects for efficiencies and revenues.

Combining results from the three synergy designs provides a clear picture. The SMR

[18]In an interesting recent contribution, Milgrom (2007) shows how limiting bidders' strategy spaces in combinatorial auctions, as in our HPB design, can eliminate certain undesirable (low-efficiency, low-revenue) outcomes.

auction, which has been used by the FCC for more than a decade (and copied by similar regulatory agencies around the world), performs well in the linear environment it was designed for. But so does the HPB auction, whose simple assignment and pricing rules reduce to those of the SMR auction when the absence of synergies results in license-by-license competition. Moreover, when bidders are interested in aggregating combinations of licenses to build a regional or national network, the HPB auction significantly outperforms SMR. It enables efficient *de novo* entrants to establish a national footprint, a level of aggregation that is virtually impossible under the commonly used SMR format due to exposure risk.

The adverse effects of exposure risk are not just of academic concern – they played an important role in the design of the upcoming 700MHz auction. The current wireless industry is highly concentrated and the 700MHz auction provides the last opportunity for a new firm to enter the market. For an entrant to be successful in the wireless market, however, it has to acquire a nationwide footprint. The solution to this important market design problem is to allow for package bidding. But package auctions can be complex and can result in coordination or "threshold" problems for smaller bidders (as our MPB data indicate). While package auctions have been discussed for more than a decade, the FCC never adopted any of the existing formats due to concerns about complexity.

The hierarchical package bidding format proposed here is a 'paper & pencil' package auction: trivial to implement with assignment and pricing rules that are transparent and easily verifiable by bidders as the auction proceeds. The predefined packages are chosen to fit, thereby eliminating coordination problems. Of course, one has to be careful in generalizing the relative performance of HPB to other environments. But the experiments show that its improved performance is robust to some degree of package misspecification and occurs even for a simple two-layer hierarchy with a single nationwide license. HPB is a prime example of "economic engineering," i.e. the combination of applied mechanism design with wind-tunnel laboratory testing. It offers a simple and transparent solution to a complex market design problem, and puts economic research right at the heart of the FCC's most important auction to date.

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Appendix A: Summary Data Table

	Complementarities Design					Mixed-Synergy Design			No-Synergy Design		
	SMR	HPB_2	HPB_3-odd	HPB_3-even	MPB	SMR	HPB	MPB	SMR	HPB	MPB
Efficiency											
Wave 1	88.5%	90.0%	92.4%	91.5%	88.7%	89.1%	92.8%	96.8%	99.6%	99.3%	99.9%
Wave 2	91.3%	92.0%	94.6%	92.4%	86.6%	89.0%	92.2%	94.2%	99.4%	99.4%	97.2%
Wave 3	80.2%	88.3%	94.6%	92.0%	89.0%	73.9%	91.8%	96.0%	99.4%	99.9%	99.6%
Wave 4	81.9%	94.5%	92.4%	92.3%	86.1%	87.5%	97.9%	90.6%	98.9%	99.5%	99.7%
Wave 5	83.8%	95.8%	96.0%	94.8%	98.2%	85.2%	89.9%	95.1%	99.5%	97.7%	97.4%
Wave 6						87.7%	94.4%	94.0%			
Average	**85.1%**	**92.1%**	**94.0%**	**92.6%**	**89.7%**	**85.4%**	**93.2%**	**94.5%**	**99.4%**	**99.2%**	**98.8%**
Revenue											
Wave 1	67.3%	79.0%	71.9%	81.5%	70.3%	81.2%	83.2%	88.7%	77.5%	76.9%	79.7%
Wave 2	64.2%	73.2%	76.8%	77.2%	65.3%	77.1%	84.9%	80.1%	74.4%	76.5%	74.1%
Wave 3	65.5%	74.0%	77.0%	73.5%	71.9%	65.1%	86.1%	86.6%	76.9%	75.1%	76.2%
Wave 4	62.1%	72.6%	78.7%	78.7%	71.0%	79.6%	90.1%	88.1%	76.8%	75.8%	77.5%
Wave 5	68.6%	77.5%	77.9%	77.3%	75.5%	81.9%	86.9%	84.4%	80.1%	82.2%	84.0%
Wave 6						83.3%	82.7%	79.7%			
Average	**65.6%**	**75.3%**	**76.5%**	**77.7%**	**70.8%**	**78.0%**	**85.7%**	**84.6%**	**77.1%**	**77.3%**	**78.3%**
Penalties	**4.3%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**3.2%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**
Profit Nationals											
Wave 1	-1.3%	-0.2%	0.8%	-2.6%	1.7%	2.3%	4.2%	2.5%	0.8%	0.7%	0.6%
Wave 2	-0.8%	0.0%	0.0%	0.3%	0.1%	2.9%	4.1%	4.0%	0.3%	0.3%	0.7%
Wave 3	-4.5%	0.1%	-0.7%	-0.4%	1.2%	0.9%	2.8%	2.1%	0.2%	0.4%	0.2%
Wave 4	-3.9%	0.8%	-0.3%	2.0%	0.3%	1.2%	1.8%	2.1%	0.3%	0.5%	0.3%
Wave 5	-2.3%	9.9%	8.4%	6.5%	8.3%	-1.9%	-0.5%	3.4%	0.3%	0.4%	0.4%
Wave 6						1.4%	3.4%	5.4%			
Average	**-2.6%**	**2.1%**	**1.6%**	**1.1%**	**2.3%**	**1.1%**	**2.6%**	**3.3%**	**0.4%**	**0.5%**	**0.4%**
Profit Regionals											
Wave 1	3.0%	1.8%	3.3%	2.1%	2.8%	1.1%	0.4%	1.1%	6.8%	7.0%	6.3%
Wave 2	4.4%	3.0%	3.0%	2.4%	3.6%	2.1%	-0.3%	2.1%	8.1%	7.4%	7.2%
Wave 3	2.0%	2.3%	3.0%	3.1%	2.6%	2.4%	0.0%	1.8%	7.3%	8.0%	7.7%
Wave 4	2.8%	3.6%	2.3%	1.9%	2.5%	1.8%	1.4%	-0.6%	7.2%	7.5%	7.2%
Wave 5	2.6%	1.5%	1.6%	1.8%	2.4%	2.4%	1.3%	1.3%	6.3%	4.9%	4.2%
Wave 6						0.5%	1.6%	1.1%			
Average	**3.0%**	**2.4%**	**2.6%**	**2.2%**	**2.8%**	**1.7%**	**0.7%**	**1.1%**	**7.1%**	**7.0%**	**6.5%**
Unsold Licenses											
Wave 1	2.2	0.3	0.2	0.3	1.2	1.2	0.0	0.0	0.0	0.0	0.0
Wave 2	1.5	0.0	0.2	0.0	1.8	1.2	0.0	0.0	0.0	0.0	0.0
Wave 3	2.7	0.3	0.3	0.0	1.2	2.8	0.0	0.0	0.0	0.0	0.0
Wave 4	2.7	0.3	0.0	0.0	0.8	1.5	0.0	0.3	0.0	0.0	0.0
Wave 5	1.5	0.0	0.0	0.0	0.2	1.5	0.0	0.0	0.0	0.0	0.0
Wave 6						1.0	0.0	0.0			
Average	**2.1**	**0.2**	**0.1**	**0.1**	**1.0**	**1.5**	**0.0**	**0.1**	**0.0**	**0.0**	**0.0**
# Rounds											
Wave 1	20.0	11.5	8.8	12.0	20.2	11.8	13.5	15.8	4.8	6.0	4.5
Wave 2	18.0	13.2	16.7	13.8	14.0	17.8	13.8	14.3	6.5	5.8	6.2
Wave 3	17.0	13.0	12.7	12.0	15.3	20.0	11.3	15.2	7.5	6.2	6.7
Wave 4	19.5	8.5	14.5	14.8	16.3	11.7	12.3	11.0	4.0	6.3	4.7
Wave 5	14.5	8.0	13.3	10.0	9.7	14.3	15.3	19.8	6.3	5.8	6.7
Wave 6						16.8	10.7	15.5			
Average	**17.8**	**10.8**	**13.2**	**12.5**	**15.1**	**15.4**	**12.8**	**15.3**	**5.8**	**6.0**	**5.8**

Table A1. *Average Performance Measures by Session and Treatment*