Abstract

Long-term predictions with a system of dynamic panel models can have tricky properties since the time dimension in regional (cross) sectional models is usually short. This paper describes the possible approaches to make long-term-ahead forecast based on a dynamic panel set, where the dependent variable is a cross-sectional vector of growth rates. Since the variance of the forecasts will depend on number of updating steps, we compare the forecasts behavior of a aggregated and a disaggregated updating procedure. The cross section of the panel data can be modeled by a spatial AR (SAR) or Durbin model, including heteroscedasticity. Since the forecasts are non-linear functions of the model parameters we show what MCMC based approach will produce the best results. We demonstrate the approach by an example where we have to predict 20 years ahead of regional growth in 99 Austrian regions in a space-time dependent system of equations. We show that such system estimates of cross-sectional growth rates produce reasonable long-term forecasts that extend usual approaches of "convergent" growth models.

1 Introduction

This paper discusses long-term forecasts from the spatial econometric models. Consider a cross-sectional model with a \((n \times 1)\) vector \(g_t\) of growth rates. We denote the vector \(y_t = \ln(\tilde{y}_t)\) as the log of the associated vector of levels \(\tilde{y}\), where the growth rates were calculated as \(g_t = \ln(\tilde{\gamma}_t/\tilde{\gamma}_{t-1}) = y_t - y_{t-1}\). \(x_t\) is a matrix/vector of exogenous variables in differenced log-form. In the following we use the concept of Spatial Autoregressive Models (SAR), as defined in [?]. The regional data set is the one from [?], which is used for long-term traffic model evaluations.

2 A system of SAR growth equations

Forecasting a dynamic difference equation system shifts the focus from the coefficients to the possible future paths of the (vector of) observations. Such stability condition have been worked out in the theory of systems of difference equations. In the following we propose a system of 2 spatial growth rate equations: Now we extend this approach to a \(2n\) equation system with \(x\) and \(y\) being \(2 \times n\)-dimensional vectors and \(W_x\) and \(W_y\) being spatial lags (growth or level) variables, respectively. Then the system can be written...
with \( g_1 = y_t - y_{t-1} \) and \( g_2 = x_t - x_{t-1} \) as
\[
\begin{align*}
g_{t,1} &= \alpha_1 W g_{t,1} + \alpha_2 W g_{t,2} + \alpha_3 x_t + \epsilon_{1t}, \\
g_{t,2} &= \beta_1 W g_{t,1} + \beta_2 W g_{t,2} + \beta_3 x_t + \epsilon_{2t}, \\
\epsilon_t &\sim N[0, \text{Diag}(\sigma_1^2, \sigma_2^2) \otimes I_n]
\end{align*}
\] (1)

Now we write the equation system in the level variables as
\[
\begin{align*}
y_t &= (I_n + \alpha_1 W)y_{t-1} + \alpha_2 W x_{t-1} + \epsilon_{1t}, \\
x_t &= \beta_1 W y_{t-1} + (I_n + \beta_2 W)x_{t-1} + \epsilon_{2t}.
\end{align*}
\] (4)

Writing this equation system as
\[
z_t = Az_{t-1} + \epsilon_t
\]
with \( z_t = (y_t, x_t)' \) \( \epsilon_t \sim N[0, \text{Diag}(\sigma_1^2, \sigma_2^2) \otimes I_n] \). In the same way as before we can compute the Jacobian derivative matrix using matrix differential calculus as e.g. in [?]. Then the \( 2n \times 2n \) matrix is given by
\[
A = \begin{pmatrix}
I_n + \alpha_1 W & \alpha_2 W \\
\beta_1 W & I_n + \beta_2 W
\end{pmatrix}
\] (6)

As before, we can guarantee the long-term steady-state stability by making sure that all eigenvalues are less than \(|1|\). This property can be easily built into the MCMC procedure of a SAR system model.

Let us denote the 3 conditional distributions by \( p(\rho \mid \theta^c), p(\beta \mid \theta^c), \) and \( p(\sigma^2 \mid \theta^c) \) where \( \theta = (\rho, \beta, \sigma^2) \) denotes all the parameter of the model and \( \theta^c \) the complementary parameters in the f.c.d.’s. The only the conditional distribution for the \( \beta \)-vector has to be adjusted by accepting only those \( \beta \)'s that obey the forecasting stability condition, i.e. the eigenvalues of the Jacobian matrix (??).

3 GDP and firm growth in 94 Austrian regions

The aim is to forecast GDP and firm growth for 94 Austrian regions using an interdependent system of equations:
\[
\begin{align*}
g_{1,t} &= c_1 + \tau_1 W_2 g_{1,t-1} + \rho_1 W_1 g_{2,t} + \sum_{k=1}^{8} \gamma_{1,k} d u_k + \epsilon_{1,t} \\
g_{2,t} &= c_2 + \tau_2 W_2 g_{2,t-1} + \rho_2 W_1 g_{1,t} + \sum_{k=1}^{8} \gamma_{2,k} d u_k + \epsilon_{2,t}
\end{align*}
\] (7a)
where $g_{1,t}$ is GDP growth, $g_{2,t}$ is firm growth, $du(k)$, $k = 1, \ldots, 8$ are dummies for the 8 federal regions of Austria, $W_1$ and $W_2$ are different spatial weighting matrices.

### 3.1 Estimation results

The model presented in table ?? were estimated using the different neighborhood structures. We chose the model which best fitted the data in terms of $R^2$. The two neighborhood structures are represented by the matrices $W_1$ and $W_2$, where $W_1$ is the six nearest neighbors weighting matrix and $W_2$ is an inverted travel time matrix using road travel times. For GDP growth it can be observed that neighbouring firm growth enters negatively and significantly and that GDP growth is positively correlated through time. Firm growth appears to be positively correlated with spatially lagged GDP growth, which offers the interpretation that high regional growth is stimulating the setting up process of firms. If a region experienced high firm growth one period ago it also do in the subsequent period.
### Table 1: System Estimation

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>GDP</th>
<th>firm</th>
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<tr>
<td>c</td>
<td>0.02</td>
<td>-0.002</td>
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<tr>
<td></td>
<td>0.06</td>
<td>0.006</td>
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<td>$W_{1g_{firms,t}}$</td>
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<td></td>
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<td>$W_{2g_{gdp,t−1}}$</td>
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<td></td>
<td>1.06</td>
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<tr>
<td>$W_{1g_{gdp,t}}$</td>
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<td>0.898***</td>
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<td>0.241</td>
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<td>$W_{2g_{firms,t−1}}$</td>
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<td>0.399*</td>
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<td>0.267</td>
</tr>
</tbody>
</table>

| Nobs | 94 | 94 |
| Nvars | 11 | 11 |
| $R^2$ | 0.39 | 0.39 |
| $\bar{R}^2$ | 0.32 | 0.32 |
| ndraws | 15000 | 15000 |
| Nomit | 200 | 200 |

***, **, * denotes significance at the 1, 5, 10 percent level, standard errors in parentheses
3.2 Forecasting

The results of the forecasts for the estimations in table ?? are shown in figures ?? - ???. Figure ?? shows the growth rates of 2003 on the x-axis and the forecasted rates on the y-axis for GDP (upper panel) and firm (lower panel). Only two regions account for less growth in 2023 than in 2003 (below the 45° line). Most of the regions substantially gain in growth, due to the high autoregressive influence and travel time improvements. The lower panel shows a quite different picture for firm growth. Some of the regions experience decreasing growth patterns and some zero or even negative growth.

The dispersion in the growth rates for GDP and firm over the regions after 20 years is shown in Figure ???. Predicted GDP growth rates range from 0.02 to 0.075 with an median of slightly under 0.04. After 20 years the whole box-plot shifts upwards and the upper quantile slightly increases. The dispersion in the firm growth rate (lower panel) are much higher. Three upper and one lower outlier can be observed and the growth rates range from negative to 15%.

The convergence pattern over time is depicted in Figure ?? for an example region. Due to neighborhood effects, negative and huge positive interactions within the system, we see a stable oscillating growth path. Stability was taken into account as only coefficient satisfying the conditions outlined in chapter ?? have been used.
4 Conclusions

We have shown that a spatial cross-sectional system of growth equations can be used successfully for long-term forecasting. Using stability conditions we can overcome the problem of few observations in the time dimension. We have also shown that the forecasts follow the pattern of economic convergence. Further studies will show that more refined space-time systems can improve the forecasts.

References


Figure 2: Dispersion of growth rates across regions, based on the system method

Figure 3: Forecasted growth path of example region 16, 20 periods ahead
