Information Acquisition, Dissemination and Transparency of Monetary Policy

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Abstract

This paper examines the role of transparency in a benevolent monetary authority’s policies. The payoff for firms depend on unobservable underlying macroeconomic conditions and firms may incur a cost to acquire private information about macroeconomic conditions. The policy authority attempts to infer the underlying macroeconomic conditions from a noisy measure of aggregate actions and makes a public announcement to inform firms of this inference. High quality announcements provide incentive not to gather private information and to base actions solely on information contained in policy announcements. However, this makes the observed actions of firms less informative to the policy authority. JEL classification : E52, E58

. . . the problem of what is the best way of utilizing knowledge initially dispersed among all the people is at least one of the main problems of economic policy . . . - F. A. Hayek

Over the last decade as more central banks have adopted inflation targeting policies, there has been an increased emphasis on the role of transparency in the conduct of monetary policy. In an effort to increase transparency\(^1\) under inflation targeting policy regimes, countries such as Canada, New Zealand, the United Kingdom and Sweden announce explicit inflation targets, issue publications of inflation forecasts, and may even publish the ongoings of the policy committee’s meetings. The amount of attention paid to these publications certainly suggests that there could be an important information dissemination role played by policy announcements.\(^2\) Improper maintenance of this information channel could result in private sector agents making costly decisions based on inaccurate beliefs about the state of the economy.

A complication that arises in the conduct of policy is that this information channel works in both directions. The policy authority’s instrument setting may transmit information to members of the private sector that influences their actions. On the flip-side, actions taken by agents in the private sector provide data that may inform the policy authority about the state of the economy thereby affecting the setting of the policy instrument. Thus there is a feedback from the action of the policy authority to the data that it observes, from which it draws inference concerning the state of the economy. The information gathering and dissemination role of the policy authority begets the question of whether increased transparency is optimal when the informational content of actions is endogenous.

\(^1\) This paper examines the role of transparency in a benevolent monetary authority’s policies.

\(^2\) Improper maintenance of this information channel could result in private sector agents making costly decisions based on inaccurate beliefs about the state of the economy.
This paper highlights a potential danger of transparent policy when the policy authority must gather information from observing the actions of private sector agents.\(^3\) Agents populating the economy may incur a cost to acquire private information concerning underlying supply and demand conditions.\(^4\) As information acquisition costs may vary across private agents there may be asymmetric knowledge concerning underlying macroeconomic conditions. The policy authority is confronted with the task of gathering information about the unobservable fundamentals underlying the economy through its inference based on the observed actions of private sector agents. At the same time, the policy authority is charged with making a policy statement to better inform the private sector about the unobservable fundamentals. As the policy statement is publicly observed, when it is very informative, private sector agents have an incentive not to acquire private information. A trade-off emerges between increasing the accuracy of the beliefs amongst private sector agents and decreasing the quality of the information contained in private sector actions observed by the policy authority.

The main result of this paper is that the policy authority should be vague about its beliefs concerning macroeconomic conditions when doing so causes a sizable fraction of private sector agents to gather information about economic conditions thereby enabling the policy authority to make more informative policy statements. By increasing the noise in policy statements the authority increases the risk to agents who choose their actions based solely on inference of macroeconomic conditions from the policy statement. Thus a greater incentive for private sector agents to acquire private information about macroeconomic conditions is provided. This information is worked into the actions of informed private agents, hence increasing the information quality of data that the policy authority observes. With higher quality data the policy authority is better equipped to inform agents of the private sector through its statements. Therefore, in highlighting a new avenue by which policy transparency affects the economy, this paper exposes another margin on which policy authorities must exercise care when making policy decisions.

A model is constructed in which firms must set prices for their output and a policy authority attempts to influence the prices set by firms so that the prices best reflect underlying macroeconomic conditions. The profits of firms are determined in part by how their prices reflect underlying cost and demand conditions but there is a strategic complementarity component in that payoffs are also affected by the deviation of their price from the prices set by firms selling substitutable goods. All firms in the economy would like a clear measure of underlying macroeconomic conditions, however, it is costly for firms to acquire such information. Firms differ in their cost of information acquisition. By choosing their price optimally given their information, prices may reflect the private information of firms. Unfortunately, agents are hamstrung by their inability to directly communicate with one another and cannot observe prices set by other firms in the economy. It is assumed that a benevolent policy authority exists that has access to a noisy measure of aggregate activity,
that is, the policy authority is assumed to be able to observe a measure of the aggregate prices set by firms. This is the policy authority’s sole source of information about underlying macroeconomic conditions. It is this information that the policy authority would like to pass on to the agents in the economy through its policy statement, which in this paper is a “policy” signal.

A problem arises that if the policy authority is too clear about its information, then the agents in the economy pay more attention to the policy signal in choosing their action and thereby have little incentive to acquire private information. If this is the case, then the aggregate actions of the agents mostly reflect information the policy authority incorporates into its signal and not what they know through their private signals. Ironically, by trying to be more informative, the policy authority is able to gather less information thereby making the policy signal less informative. This is the major control problem faced by the authority in this paper.

When there is a sizable fraction of agents with relatively similar information acquisition costs and there is a sufficient degree of strategic complementarity in the payoff function of the private sector agents then the monetary authority will choose to make statements that do not perfectly reveal its beliefs to the private sector. In other words, non-transparent policy will be optimal. This result arises because all firms choose their prices partly to be as close to those of their competitors as possible. This element of the payoff function results in informed firms choosing prices that may not necessarily be those optimal given supply and demand conditions but rather to conform to prices chosen by firms that are not perfectly informed about macroeconomic conditions. Uninformed firms learn about economic conditions solely from policy announcements so if policy announcements are more informative then both the informed firms and the uninformed firms are better off. Cases exist where increasing the level of noise in policy statements will cause a large fraction of firms to pay their costs to become informed. In these cases, the information obtained by informed firms is worked into their prices allowing the data observed by the policy authority to be more informative about the underlying macroeconomic conditions. This allows for more informative policy statements to those for whom it is excessively costly to directly acquire information about the underlying economic conditions. It is precisely when this spillover effect dominates the harm of more noise in the policy statement that the policy authority should choose to be non-transparent.

The contribution of this paper to the monetary policy literature is that it considers the effect that policy announcements have on an economy where both the policy authority and the private sector are asymmetrically, imperfectly informed. In doing so it is recognized that the policy authority’s source of information are endogenous variables and the informational content of these variables depend on policy actions. Svensson and Woodford (2003, 2004), and Aoki (2003) address the problems that a monetary policy authority faces in controlling a forward-looking dynamic economy when either the policy authority and the private sector are equally ignorant about the state of the world or when the private sector is more knowledgeable about the state of the world.
than is the policy authority.

Recently, a paper by Morris and Shin (2002) studying the social value of public information has shown that increasing the quality of publicly observed policy announcements issued by a benevolent policy authority may be harmful to the economy. By emphasizing the interaction between imperfect information about unobservable payoff relevant fundamentals and strategic complementarities in agents’ actions, their paper shows that small errors in commonly observed exogenous policy signals can be amplified to the point where the economy would be better off having agents ignore the policy signal and act only on their imperfect private information. This paper differs in that the quality of the policy statement is endogenous and depends on the behaviour of the policy authority. As a result it is shown that the reason in favour of less transparency is quite different. It is optimal to reduce transparency in order for the policy authority to gather more information thereby making it possible for policy statements to be more informative. This makes clear the consequences of recognizing the fact that the policy authority’s source of information is an endogenous variable and that the informational quality of the authority’s data depends heavily on the actions of the policy authority.

The following section presents the environment to be studied which is an extension of the model presented in Morris and Shin (2002). In Section 2 the equilibrium is defined. Section 3 contrasts the mechanism highlighted in this paper from those commonly discussed in the monetary policy transparency literature. Section 4 presents an extension concerning discretionary policy under which the policy authority does not possess a commitment technology. Section 5 concludes.

1 The Model

The private sector agents in this model are firms who are setting the price of their good. Ex ante, firms are unaware of the underlying macroeconomic conditions that are relevant to their profit maximizing price. If these supply and demand conditions were known perfectly, then all firms would know exactly what price to charge. These market conditions can encompass anything from labour market conditions to the development of new cost reducing technologies. In the model, these conditions are captured by an unobserved random variable. It is assumed that these unobserved conditions affect all firms alike. Each firm faces a cost of acquiring information about the underlying macroeconomic conditions. These costs can be thought of as a research cost or effort expended to learn about market conditions. If a given firm incurs this cost then that firm learns perfectly about the macroeconomic conditions. It is assumed that firms can learn about the macroeconomic conditions because they have access to information about its workers, firm specific productivity, market competition, market demands, etc. There also exists a policy authority who would like prices to best reflect the unobservable economic fundamentals. The policy authority can send a policy signal concerning its beliefs about the realization of the unobservable, a signal
that is common knowledge. However, the policy authority gathers all its information from a noisy observation of private sector actions.

The timing is as follows: the policy authority decides on the quality of its policy signal, firms decide whether to become informed, nature chooses the shocks to the economy at which point informed firms learn the realization of macroeconomic conditions, and finally the prices chosen by firms and the policy signal are simultaneously determined. It is assumed that agents know all the parameters governing the economy when they make their decisions. An elaboration on details is given below.

### 1.1 The Agents in the Private Sector

There exists a continuum of firms indexed on the unit interval. Firm \( i \) chooses its price \( p_i \in \mathbb{R} \), and also decides whether to incur an idiosyncratic cost, \( c_i \), in order to obtain a signal about the realization of payoff relevant macroeconomic conditions. The underlying macroeconomic conditions will be denoted by the random variable \( \tilde{\theta} \). Following convention, random variables are denoted with a tilde, while the same variables without a tilde denote the realization of the random variable. Costs of information acquisition are drawn from a distribution \( G(c) \) that is continuously differentiable with density \( g(c) \). The information cost can be thought of as research costs incurred to learn about macroeconomic conditions. Let \( z_i \) denote firm \( i \)'s decision of becoming informed about \( \tilde{\theta} \).

\[
z_i = \begin{cases} 
1 & \text{if informed} \\
0 & \text{if uninformed} 
\end{cases}
\]

The payoff function for firm \( i \) is given by

\[
u_i(p, \theta) = -(1 - r)(p_i - \theta)^2 - r(p_i - P)^2 + k \int_0^1 (p_j - P)^2 dj - z_i c_i
\]

where \( r \) is a constant such that \( r \in (0, 1) \), \( k \) is a constant, \( p \) is the profile of all prices chosen by firms, \( P \) is the aggregate price level,

\[
P = \int_0^1 p_j dj
\]

and \( \theta \) is an unobserved random variable with \( \tilde{\theta} \sim N(0, \sigma_\theta^2) \). This payoff function captures the payoff function used in Morris and Shin (2002) as a special case when \( k = r \). When \( k = r \),

\[
u_i(p, \theta) = -(1 - r)(p_i - \theta)^2 - r(L_i - \bar{L}) - z_i c_i
\]

with

\[
L_i \equiv \int_0^1 (p_j - p_i)^2 dj
\]

and
Another special case that will be examined in the paper is the case then \( k = 0 \).

Clearly, aside from the information acquisition cost, the loss function has two components:

1. The first component is a standard quadratic loss in the distance between the underlying state variable \( \theta \) and firm \( i \)'s action \( p_i \).

2. The second component is the strategic complementarity term. When \( k = r \), the loss \( L_i \) is increasing in the average distance between \( i \)'s action and the action profile of the entire population. The effect of the quality of firm \( i \)'s forecast, as measured by \( L_i \), on total loss \( u_i \) depends on how good firm \( i \)'s forecast of \( \theta \) is relative to the measure of the average quality of forecasts across the population, \( \bar{L} \). When \( k = 0 \), we obtain a simpler strategic complementarity term in which the loss to the firm is increasing in the deviation of its price from the average price of all firms in the economy.

As \( r \neq 0 \), there is an externality in which an individual agent tries to second guess the decisions of other individuals in the economy. The parameter \( r \) puts weight on this externality. In the case \( k = r \), this externality is socially inefficient in that it is of a zero-sum nature. In the game of second guessing, the winners gain at the expense of the losers.

To put the variable \( \tilde{\theta} \) in context, it is useful to think of \( \theta \) as representing the price level that would prevail if everyone in the economy could observe the values of all the fundamental shocks buffeting the economy with common knowledge. These include all the unobserved market conditions referred to earlier. In this model, all the agents want to know what this price level should be but individual firms do not have direct observations of the fundamental shocks unless an information acquisition cost is incurred. In this setting the demand for their good will depend on their relative price. However, if all firms set prices too high (given fundamental shocks) then it may be better for a given firm to choose prices to closer reflect what it believes the fundamental shocks warrant.

Social welfare, defined as the average of individual payoffs is

\[
W = \int_0^1 u_i(p, \theta) di
= \int_0^1 \left[ -(1-r)(p_i - \theta)^2 - r(p_i - P)^2 + k \int_0^1 (p_j - P)^2 dj \right] di - \int_0^1 z_i c_idi
\]

When \( k = r \) the second and third terms drop out, and so a social planner who cares only about welfare seeks to keep \( p_i \) close to \( \theta \), for all \( i \) while minimizing the costs of doing so.

Before choosing its price, each firm must decide whether to incur the cost of becoming informed about \( \theta \). If a firm decides to become informed it will learn the realized macroeconomic conditions.
Otherwise, firms will only observe a signal issued by the policy authority concerning macroeconomic conditions. When firms decide whether or not to become informed about macroeconomic conditions, the quality of the policy signal, \( y \), which is described later, is common knowledge. Firm \( i \) will become informed if it expects a higher payoff when informed than if it remains uninformed. That is, \( z_i = 1 \) if

\[
E \left\{ E \left[ - (1 - r)(\tilde{p}_i - \tilde{\theta})^2 - r(\tilde{p}_i - \tilde{P})^2 + k \int_0^1 (\tilde{p}_j - \tilde{P})^2 \, dj - c_i |\tilde{\theta}, \tilde{y} \right] \right\} \geq E \left\{ E \left[ - (1 - r)(\tilde{p}_i - \tilde{\theta})^2 - r(\tilde{p}_i - \tilde{P})^2 + k \int_0^1 (\tilde{p}_j - \tilde{P})^2 \, dj |\tilde{y} \right] \right\}
\]

(2)

and \( z_i = 0 \) otherwise.

After making its information acquisition decision, each firm chooses the price of its good, \( p_i \), to solve the problem

\[
\max_{p_i} E_i \left[ u_i (p_i, \theta) \right] = E_i \left[ - (1 - r)(p_i - \theta)^2 - r(p_i - P)^2 + k \int_0^1 (p_j - P)^2 \, dj \right]
\]

(3)

for which the first order necessary condition is:

\[
p_i = (1 - r)E_i(\theta) + rE_i(P).
\]

(4)

Here \( E_i(\cdot) \) is the conditional expectations operator and accounts for whether firm \( i \) is informed about \( \theta \). When choosing their prices, all firms observe the realization of a policy signal, \( y \), to be discussed below. Therefore when choosing its price, firm \( i \)’s information set includes \( \theta \) and \( y \) if it is informed, and when firm \( i \) is uninformed, it only observes \( y \) when choosing its price. Hence the incentive for uninformed firms to extract all information about \( \theta \) contained in \( y \). Note that firm \( i \) treats \( \int_0^1 (p_j - P)^2 \, dj \) as a constant as its action is trivial given the continuum of firms. Thus agent \( i \) puts positive weight on \( \theta \) and the actions of everyone else. If \( \theta \) is common knowledge the equilibrium entails \( p_i = \theta \) for all \( i \) so social welfare is maximized in equilibrium. Therefore when perfect information obtains there is no conflict between individual rational actions and the socially optimal actions.

Denote by \( p_i(\mathcal{I}_i) \) the decision by firm \( i \) as a function of his information set \( \mathcal{I}_i \). \( \mathcal{I}_i \), which contains the pair \((z_i, \theta, y)\), captures all the information available to \( i \) at the time of the decision.\(^{10}\) As it is optimal for firms to make use of the information in its information set, \( p_i = p_i(\mathcal{I}_i) \). Notice that this set-up implies that the actions of the other firms, \( p_{-i} \), are not observable by firm \( i \).

### 1.2 The Monetary Policy Authority

Reiterating the timing of the model; the policy authority decides on the quality of its policy signal, firms decide whether to become informed, nature chooses a vector of shocks at which point informed firms learn the realization of macroeconomic conditions, and finally the prices chosen by firms and the policy signal are simultaneously determined. As the policy authority chooses the
quality of its signal before the state of the world is revealed to the informed firms it maximizes the unconditional expected social welfare function

$$E \left[ E_M(\tilde{W}) \right] = E \left\{ E \left[ \int_0^1 u(\tilde{p}_i, \tilde{z}_i) di | I_M \right] \right\}$$

$$= -E \left\{ \int_0^1 \left[ (1-r)(\tilde{p}_i + \tilde{\theta})^2 + r(\tilde{p}_i - \tilde{P})^2 - k \int_0^1 (\tilde{p}_j - \tilde{P})^2 dj \right] di + \int_0^1 \tilde{z}_i \tilde{c}_i di \right\}$$

The policy authority observes a noisy measure of the aggregate action of the agents and sends a policy signal that is seen by all agents and is common knowledge. In other words, the policy authority’s objective is to maximize the expected social welfare by informing the private sector of its expectations of $\theta$ conditional on observing its noisy measure of the aggregate price level, $P$. Formally, the policy authority observes $A$ where

$$A = P + \eta \quad \tilde{\eta} \sim N(0, \sigma^2_\eta). \quad (5)$$

In this model, the policy authority’s set of instruments is restricted to a linear report of its conditional expectations of $\theta$. This restriction is motivated by the policies of inflation targeting central banks. The idea is that central banks give statements reflecting what it believes are the inflationary pressures buffeting the economy. Denoting the policy instrument by $y$,

$$y = E[\tilde{\theta}|A] + u \quad \tilde{u} \sim N(0, \sigma^2_u). \quad (6)$$

The policy authority chooses $\sigma^2_u$ to maximize expected social welfare given that all the agents understand the policy authority’s problem. With common knowledge of the structure of the economy, all agents can solve the policy authority’s problem and are able to deduce the policy authority’s optimal choice for $\sigma^2_u$. Similarly, all firms understand the problem that each firm faces and can determine the fraction of firms, $x$, that will become informed for any given $\sigma^2_u$. This determines the optimal weights that the firms place on any announcement, $y$, by the policy authority. To be explicit the policy authority’s problem is

$$\max_{\sigma^2_u} E \left\{ \int_0^1 \left[ -(1-r)(\tilde{p}_i + \tilde{\theta})^2 - r(\tilde{p}_i - \tilde{P})^2 + k \int_0^1 (\tilde{p}_j - \tilde{P})^2 dj \right] di - \int_0^1 \tilde{z}_i \tilde{c}_i di \right\}$$

subject to equations (2)-(4) and (5)-(6).

2 A Rational Expectations Equilibrium

**Definition 1** A rational expectations equilibrium (REE) is a joint distribution of $\tilde{\theta}, \tilde{\eta}, \tilde{u}, \tilde{z}_i, \tilde{x}, \tilde{p}_i$ and $\tilde{y}$ such that

1. for informed firms,

$$p_i \in \arg \max_{\tilde{p}_i \in \mathbb{R}} \left\{ E \left[ -(1-r)(\tilde{p}_i + \tilde{\theta})^2 - r(\tilde{p}_i - \tilde{P})^2 + k \int_0^1 (\tilde{p}_j - \tilde{P})^2 dj | \tilde{\theta}, y(\tilde{\theta}, \tilde{\eta}, \tilde{u}) \right] \right\}$$
2. for uninformed firms \( p_i \) is optimal given an observed policy signal \( y \), that is

\[
p_i \in \arg\max_{\hat{p}_i \in \mathbb{R}} \left\{ E \left[ - (1 - r) (\bar{p}_i + \hat{\theta})^2 - r(\bar{p}_i - \bar{P})^2 + k \int_0^1 (\bar{p}_j - \bar{P})^2 d\bar{y}(\bar{\theta}, \bar{\eta}, \bar{u}) \right] \right\}
\]

3. \( z_i \) is optimal given \( c_i \) in that \( z_i = 1 \) if

\[
E \left\{ E \left[ - (1 - r)(\tilde{p}_i - \tilde{\theta})^2 - r(\tilde{p}_i - \bar{P})^2 + k \int_0^1 (\tilde{p}_j - \bar{P})^2 d\bar{y}(\tilde{\theta}, \tilde{\eta}, \tilde{u}) \right] \right\} \geq \left\{ E \left[ - (1 - r)(\bar{p}_i - \bar{\theta})^2 - r(\bar{p}_i - \bar{P})^2 + k \int_0^1 (\bar{p}_j - \bar{P})^2 d\bar{y}(\bar{\theta}, \bar{\eta}, \bar{u}) \right] \right\}
\]

4. \( \sigma^2_u \) is optimal in that

\[
\sigma^2_u \in \arg\max_{\sigma^2_u \in \mathbb{R}^+} E \left\{ \int_0^1 \left[ - (1 - r)(\tilde{p}_i + \tilde{\theta})^2 - r(\tilde{p}_i - \bar{P})^2 + k \int_0^1 (\tilde{p}_j - \bar{P})^2 d\bar{y}(\tilde{\theta}, \tilde{\eta}, \tilde{u}) \right] \right\} - \int_0^1 \tilde{z}_i \tilde{c}_i d\bar{t}
\]

In words, a rational expectations equilibrium is a joint distribution for the variables \( \tilde{\theta}, \tilde{\eta}, \tilde{u}, \tilde{z}, \tilde{x}, \tilde{p}_i \) and \( \tilde{y} \) such that, the policy authority chooses \( \sigma^2_u \) optimally and the firms make their information acquisition decisions optimally, knowing the structure of the economy which encompasses the values of \( \sigma^2_\theta \), and \( \sigma^2_\eta \) (the volatility in the fundamental \( \theta \) and the measurement error in the policy authority’s observable). Then conditional on the realizations of \( \tilde{\theta}, \tilde{\eta}, \tilde{u}, \tilde{y} \),

1. given expectations conditional on observing the pair \( (\theta, y) \), informed firms choose \( p_I \) optimally,\(^{11}\)

2. given expectations conditional on observing \( y \), uninformed firms choose \( p_U \) optimally,

3. given expectations conditional on observing \( A \), the policy authority chooses a policy signal, \( y \), that is generated by its choice of \( \sigma^2_u \) and its conditional expectations,

4. expectations for each firm \( i \) are formed such that given private signals and the equilibrium policy signal function, the actions chosen by agents result in the expectations generated by the observed private and policy signals,

5. the expectations of the policy authority are formed such that the policy signal function (ie. the policy rule), \( y(A, u) \), generates equilibrium beliefs for agents which result in actions that, when aggregated, result in the \( A \) which generates the policy signal as specified by the policy rule.

### 2.1 Equilibrium

In order to solve for the rational expectations equilibrium it is necessary to work backwards.\(^{12}\)

Suppose that a fraction of firms, \( x \), have become informed and learned the value of \( \theta \) in the first stage. Then these \( x \) firms will choose their price such that \( p_i = p_I \). The remaining \( 1 - x \) uninformed
firms choose their prices optimally given that they observe the policy signal $y$ so that $p_i = p_U$. From the firm's optimal policy rule it is known that

$$p_i = (1 - r)E_i(\theta) + rE_i(P).$$

Substituting for $P$ and writing $\bar{E}(\theta)$ for the average expectations of $\theta$ across agents we have

$$p_i = (1 - r)E_i(\theta) + (1 - r)rE_i[\bar{E}(\theta)] + (1 - r)r^2E_i[\bar{E}^2(\theta)] + ...$$

which upon continued iteration yields

$$p_i = (1 - r)\sum_{k=0}^{\infty} r^k E_i[\bar{E}^k(\theta)]. \qquad (7)$$

Given the linear-normal structure of the economy it is conjectured that $y(\theta, \eta, u)$ and $A(\theta, \eta, u)$ are normally distributed random variables. If so, then Bayesian beliefs for the uninformed firms are given as

$$E_U[\tilde{\theta}|y] = b_U y.$$ 

Noting that for the informed firm that $E_i(\tilde{\theta}) = \theta$ substitution of beliefs into equation (7) and forward iteration yield the price functions

$$p_I = \frac{(1 - r)}{(1 - rx)}\theta + \frac{r(1 - x)}{1 - rx} b_U y \qquad (8)$$

$$p_U = b_U y. \qquad (9)$$

Aggregating across firms

$$A = \frac{(1 - r)x}{1 - rx}\theta + \frac{(1 - x)}{1 - rx} b_U y + \eta$$

and so the monetary authority’s beliefs are given by

$$E_M(\tilde{\theta}) = f_A A_p.$$ 

where

$$A_p = A - f_U u.$$ 

The term $f_U u$ is the linear least squares projection of $A$ on $u$ and the term $b_U$ is the linear least squares projection coefficient of $y$ on $\theta$. The coefficients governing the belief functions can be expressed as

$$b_U = \frac{cov(y, \theta)}{var(y)}, \quad f_A = \frac{cov(A_p, \theta)}{var(A_p)}, \quad f_U = \frac{cov(A, u)}{var(A)} \quad (10)$$
Substitution of the authority’s expectations into its instrument rule and then solving the resulting expression for the policy instrument gives an expression for $y$ in terms of the exogenous variables,

$$
y = \frac{f_A(1 - r)x}{1 - rx - (1 - x)f_AB_U} \theta + \frac{f_A(1 - rx)}{1 - rx - (1 - x)f_AB_U} \eta - \frac{(f_Af_U - 1)(1 - rx)}{1 - rx - (1 - x)f_AB_U} u. \quad (11)
$$

Using this expression for the policy signal allows the policy observable to be expressed as

$$
A = \frac{(1 - r)x}{1 - rx - (1 - x)f_AB_U} \theta + \frac{(1 - rx)}{1 - rx - (1 - x)f_AB_U} \eta - \frac{(1 - x)(f_Af_U - 1)b_U}{1 - rx - (1 - x)f_AB_U} u \quad (12)
$$

From (11) and (12) it is clear that the policy signal and the policy authority’s observable are both functions of the realization of $\theta$, the measurement error in the policy authority’s observable, $\eta$, and the noise that the policy authority injects into its policy signal, $u$. The weights on these variables will depend on the coefficients governing the belief functions of the agents in the economy. These coefficients are complicated functions of the structural parameters of the economy, including the level of noise chosen by the policy authority, $\sigma_u^2$. Note that in equilibrium the expressions for $y$ and $A$ are linear functions of $\theta$, the measurement error, $\eta$ and the policy noise $u$. This allows for the use of linear least squares projection formulas to calculate the beliefs of the agents.

It is clear from (11) that the measurement error in $A$ is worked into the policy signal as the policy authority is unable to disentangle the measurement error from the realization of $\theta$. This prevents a fully revealing equilibrium from being obtained. From (12) it can be seen that the noise injected by the policy authority works its way into the policy authority’s observable. As the policy authority knows the realization of $u$, it can clean this noise out of $A$ when forming its expectations concerning $\theta$.

Now consider the stage during which the firms decide whether or not to become informed and the policy authority sets the quality of its signal, $\sigma_u^2$. Firm $i$ understands that if it pays its cost $c_i$ then it will choose a price $p_f$. Otherwise it will pick a price $p_U$. Thus firm $i$ will only become informed if

$$
E[u_i^P(\tilde{p}; \tilde{\theta}, \tilde{\eta}, \tilde{u})|\sigma_u^2] \geq E[u_i^P(\tilde{p}; \tilde{\theta}, \tilde{\eta}, \tilde{u})|\sigma_u^2]
$$

or

$$
c_i \leq \frac{(1 - r)^2}{(1 - rx)^2} E[(b_U\tilde{y} - \tilde{\theta})^2|\sigma_u^2]. \quad (13)
$$

Finally expected social welfare is given as

$$
E(W) = E \left\{ - (1 - r) \left[ x \left( \frac{r(1 - x)}{1 - rx} \right)^2 + 1 - x \right] + (k - r)x \left[ \frac{(1 - r)(1 - x)}{1 - rx} \right]^2 
+ (k - r)(1 - x) \left[ \frac{(1 - r)x}{1 - rx} \right] \right\} - \int_0^{c^*} c_i g(c_i) dc_i
$$

where $c^*$ denotes the highest cost of all informed firms.

11
It is instructive to notice that, everything else constant, uninformed firms pay more attention to the policy signal as the fraction of informed firms increases; that is \( \partial b_u / \partial x > 0 \). Thus, there is always a free-rider effect in that uninformed firms prefer not to incur the cost of becoming informed if the policy signal is more precise. The strategic complementarity component in payoffs adds an extra coordination effect. Furthermore it can be shown that the direct effects of increased policy noise is to worsen the welfare of the uninformed, \(-\partial E[(b_U \bar{y} - \bar{\theta})^2] / \partial \sigma_u^2 \leq 0\), and that the direct effect of increasing the fraction of informed agents on the welfare of the uninformed is positive, \(-\partial E[(b_U \bar{y} - \bar{\theta})^2] / \partial x \geq 0\).

**Lemma 1** Let the density \( g(c) = G'(c) \) be such that it is continuous and non-zero on \( \mathbb{R}_+ \). Let \( c^* \) be the highest cost type of all informed firms. For every level of policy noise, \( \sigma_u^2 \), there exists at least one cut-off cost, \( c^* \), such that the firm with this cost is indifferent between becoming informed or remaining uninformed.

**Proposition 1** There exists an equilibrium.

Differentiating the social welfare function with respect to \( \sigma_u^2 \) we obtain

\[
\frac{\partial E(W)}{\partial \sigma_u^2} = \left\{ -(1 - r) \left[ \left( \frac{r(1 - x)}{1 - rx} \right)^2 - 2x \left( \frac{r(1 - x)}{1 - rx} \right) \frac{r(1 - r)}{(1 - rx)^2} - 1 \right] + (k - r) \frac{(1 - r)^2}{(1 - rx)^2} (1 - 2x) + 2r(k - r)x(1 - x) \frac{(1 - r)^2}{(1 - rx)^3} \right\} \frac{\partial x}{\partial c^*} \frac{\partial c^*}{\partial \sigma_u^2} E(b_u \bar{y} - \bar{\theta})^2
\]

\[
+ \left\{ -(1 - r) \left[ x \left( \frac{r(1 - x)}{1 - rx} \right)^2 + 1 - x \right] + (k - r) \frac{(1 - r)^2x(1 - x)}{(1 - rx)^2} \right\} \frac{\partial E(b_u \bar{y} - \bar{\theta})^2}{\partial \sigma_u^2} + \frac{\partial E(b_u \bar{y} - \bar{\theta})^2}{\partial x} \frac{\partial c^*}{\partial \sigma_u^2} - c^* g(c^*) \frac{\partial c^*}{\partial \sigma_u^2}
\]

The first two lines on the right-hand side equals the benefits accrued to the fraction of firms that are the marginally informed. If there were no strategic complementarity in payoffs, so that \( r = k = 0 \), then these firms would choose \( p_i = \theta \) so that \( E(p_i - \theta)^2 = 0 \) and their gain from becoming informed would be \( E(b_U \bar{y} - \theta)^2 \). In other words, by becoming informed these firms would rid of any uncertainty. The change to total welfare from this gain would then be \( E(b_U \bar{y} - \bar{\theta})^2 \) times the change in the fraction of firms becoming informed, \( (\partial x / \partial c^*) \cdot (\partial c^* / \partial \sigma_u^2) \). However, with the presence of strategic complementarity in the payoff function, the informed do not choose their prices to be equal to \( \theta \). The informed firms place a weight \( [r(1 - x)]/(1 - rx) \) on the price that the uninformed firms will choose and with a fraction \( x \) of firms being informed, the change in expected welfare coming from the deviation of the informed firms’ price from \( \theta \) is given by the first two terms on the first line of equation (16).

Ignoring the last term in the expression, the last two lines on the right-hand side describe the gains from increasing policy noise that arise from the accuracy of setting prices conditional on
only the policy signal. There are direct costs of increasing policy noise which is given by the term \( \partial E(b_u \tilde{y} - \tilde{\theta})^2 / \partial \sigma_u^2 \). On the other hand, there are indirect benefits in that as more firms are pushed into becoming informed, the policy signal becomes more informative - a benefit captured by the term \( \partial E(b_u \tilde{y} - \tilde{\theta})^2 / \partial x \cdot (\partial x / \partial c^*) \cdot (\partial c^* / \partial \sigma_u^2) \).

**Lemma 2** In a stable equilibrium, as policy noise increases, the cut-off cost of the marginal informed firm cannot decrease, \( \partial c^* / \partial \sigma_u^2 \geq 0 \). In an unstable equilibrium, as policy noise increases, the cut-off cost of the marginal informed firm decreases, \( \partial c^* / \partial \sigma_u^2 < 0 \).

Thus, in the equilibria of interest, increases in policy noise push firms towards becoming informed.

As shown in the appendix, by making use of the indifference condition for the marginal informed firm, the change in expected social welfare can be rewritten as the sum of two components.

\[
\frac{\partial E(\tilde{W})}{\partial \sigma_u^2} = \left\{ \frac{r(1 - r)^2}{(1 - rx)^2} (1 - 2x + rx) + (k - r) \frac{(1 - r)^2}{(1 - rx)^2} (1 - 2x) + 2r(k - r)x(1 - x) \left( \frac{1 - r^2}{1 - rx} \right) \right\}
\cdot E(b_u \tilde{y} - \tilde{\theta})^2 \frac{\partial c^*}{\partial \sigma_u} - \left\{ (1 - r) \left[ x \left( \frac{r(1 - x)}{1 - rx} \right)^2 + 1 - x \right] - (k - r) \left( \frac{1 - r^2}{1 - rx} \right) \right\}
\cdot \left[ \frac{\partial E(b_u \tilde{y} - \tilde{\theta})^2}{\partial \sigma_u^2} + \frac{\partial E(b_u \tilde{y} - \tilde{\theta})^2}{\partial x} \frac{\partial x}{\partial c^*} \frac{\partial c^*}{\partial \sigma_u^2} \right]
\]

The first component describes the “strategic complementarity externality”. This term arises from two sources. First, informed firms choose their prices as a weighted sum of the realization of macroeconomic conditions, \( \theta \), and the average price chosen by all other firms. When firms make their information acquisition decision, they consider their own benefits arising from the strategic complementarity component to payoffs; if there is a large fraction of informed firms (more than one-half) then becoming informed allows price to be closer to the average price set by other firms. In increasing policy noise, the policy authority pushes some firms into acquiring information. Without the complementarity in payoffs, these firms would choose prices to perfectly reflect macroeconomic conditions and so the gain to welfare for each of these firms would be \( E[(b_u \tilde{y} - \tilde{\theta})^2] \). When \( r \neq 0 \), informed firms deviate from the price \( p = \theta \) by putting a weight \( [r(1 - x)]/(1 - rx) \) on the price set by uninformed firms when choosing their price. Thus when the fraction of informed firms, \( x \) increases, informed firms are also made better off because the weight on uninformed firms prices is reduced. These effects can be seen most clearly in the Morris and Shin set-up where \( k = r \). In the social welfare function for this case, when summed across all firms, the strategic complementarity component to payoffs nets to zero. Then the policy authority wants firms to choose their prices to best reflect underlying macroeconomic conditions. In the other case of interest, when \( k = 0 \), in addition to wanting the price of firms to best reflect underlying macroeconomic conditions, the policy authority wants the price of firms to reflect the average price of other firms. This means that when more firms become informed, the uninformed firms prices will be less accurate with respect to the average price while the informed firms will be more accurate with respect to the average price; there are winners and losers.
The second source of the strategic complementarity externality arises because the policy authority is also concerned about minimizing the costs of information acquisition. When policy noise is increased firms compare the benefits of becoming informed, which includes the changes in benefits arising from the strategic complementarity to payoffs, to the cost of becoming informed. This is clearly observed through the indifference condition for the marginally informed firm. Thus in considering the effects of policy noise on total costs of information acquisition, the policy authority is indirectly accounting for the effects of policy noise on the payoffs arising from the strategic complementarity. Therefore, the complementarity effect sneaks into the welfare evaluation via the costs of information acquisition.

Individual firms do not consider the effect that becoming informed has on the prices set by all the other firms. In making the information acquisition decision, each individual firm considers itself too small to have any impact on aggregate variables. However, when a positive measure of firms with identical costs become informed, the uninformed are made worse off on average because their guess concerning the prices set by all other firms is now less accurate as the uninformed do not know what price the informed agents are setting with certainty. In contrast with more informed firms, informed firms gain in the complementarity component to payoffs as their price is closer to the prices of more firms. The overall effect of this strategic complementarity effect on aggregate welfare is ambiguous and depends on the fraction of informed firms. Furthermore, the informed firms are better off because their prices are now closer to \( \theta \) which has a positive effect on welfare. In the special case, \( k = 0 \), it can be shown that the strategic complementarity effect is equal to zero, that is, the two sources to the strategic complementarity effect cancel each other. This allows for the isolation of the informational spillover effect which is discussed below.

The second component to social welfare is the “informational spillover” effect. This term captures the informativeness of the policy signal. When this spillover effect is positive it means that increasing policy noise results in a more informative policy signal. There are two forces at work here. First, as policy noise increases, the policy signal is less informative because the firms now have a harder time deciphering the component of the policy signal that is related to the policy authority’s beliefs and the component that is random noise. Second, this increases the value of being informed and so more firms are willing to incur the cost to become informed. With a larger fraction of informed firms, the policy authority’s observable is more informative, allowing the policy authority to better extract the truth about macroeconomic conditions, \( \theta \), from its data, \( A \). With better knowledge of \( \theta \) the policy authority’s beliefs are refined and are passed along to the private sector via its statement \( y \). It is when a large fraction of firms are pushed into acquiring information that this informational spillover dominates the direct costs of increased policy noise. As can be seen in the following proposition, it is the interaction between the complementarity parameter, \( r \), and the distribution of costs that generates this informational spillover.
Proposition 2  In stable equilibria there is a positive informational spillover effect when

\[
2r(1-r)^2 \frac{(1-r)x}{(1-rx)^3} E[(b_U \tilde{y} - \tilde{\theta})^2] g(c^*) > 1.
\]

From Proposition 2 it is apparent that a particular interplay between strategic complementarities in payoffs, \(r\), and the density of costs \(g(c)\) is necessary to generate a positive information spillover effect. To understand the role played by the complementarity, consider the case where \(k = r = 0\). In this case the indifference condition for the marginal informed firm is

\[
c^* = E[(b_u \tilde{y} - \tilde{\theta})^2|\sigma_n^2].
\]

If the informational spillover effect is positive then the uninformed firms are better off meaning that the expected deviation of an uninformed firm’s price from that best reflecting macroeconomic conditions is lower, \(dE(b_u \tilde{y} - \tilde{\theta})^2/d\sigma_n^2 < 0\). The net benefits of becoming informed are eroded and uninformed firms are willing to pay less for information about \(\theta\). Thus the cut-off \(c^*\) must fall. However, if the cut-off falls then there must be less informed firms and as \(\partial E(b_u \tilde{y} - \tilde{\theta})^2/\partial \sigma_n^2 > 0\) and \(\partial E(b_u \tilde{y} - \tilde{\theta})^2/\partial x < 0\) it is impossible for the spillover effect to be positive. The only way that the spillover effect can be positive is if more firms become informed to offset the direct harm caused by increased policy noise.

Now consider the indifference equation for the marginal informed firm when \(r \neq 0\).

\[
-(1-r) \left\{ \frac{r^2(1-x)^2}{(1-rx)^2} + \frac{r(1-r)}{(1-rx)^2} (1-2x) \right\} E[(b_U \tilde{y} - \tilde{\theta})^2|\sigma_n^2] - c^* = -E[(b_U \tilde{y} - \tilde{\theta})^2|\sigma_n^2]
\]

The first term on the left-hand side accounts for the penalty that informed firms incur for not setting their price equal to macroeconomic conditions, \(\theta\). The second term accounts for the difference in the complementarity payoff component between the pricing as an informed firm versus pricing as an uninformed firm. If increased policy noise is to result in sufficiently large informational spillovers then an uninformed firm’s price should reflect economic conditions better on average. This means that the right-hand side increases providing more incentive for the marginal firm to remain uninformed. For this spillover to occur a large number of firms must become informed requiring a higher threshold \(c^*\). As an equilibrium outcome, this can only occur if the first term on the left-hand side increase significantly to make-up for the increased threshold such that the marginal firm will choose to become informed. In other words there must be an increased incentive to become informed. The strategic complementarity in payoffs provides a mechanism for such incentives to arise. When a large fraction of firms are informed then a firm’s benefit from becoming informed increases (i) because the optimal price it sets will be closer to \(\theta\) and (ii) as more firms are informed, the payoff arising from the strategic complementarity is higher if price is set equal to the price that the large fraction of informed firms, \(x\), have set their prices to.

It turns out that the effects of informational spillovers on social welfare can be highlighted in the case where \(k = 0\). In this case it turns out that the strategic complementarity effect disappears from
the derivative $\frac{\partial E(W)}{\partial \sigma^2_u}$. To illustrate the effects, numerical examples are provided in which the costs follow a gamma distribution with parameters $\alpha$ and $\beta$. Figure 1 provides an example of a case where there is a positive informational spillover. In this example, the costs of information acquisition are tightly distributed across the firms and there is a positive complementarity in payoffs. As the density of costs becomes more dispersed, it becomes more difficult to obtain a positive informational spillover effect because small increases in policy noise do not generate a large flow of firms into the informed state. Figure 2 provides an example where the costs of information acquisition are more dispersed. In this case there is no benefit from deviating away from perfect transparency.

[Insert Figure 1 Here]

[Insert Figure 2 Here]

Figures 3 and 4 illustrate the effects of changing the distribution of costs when $k = r$. This is the payoff structure that mirrors that of Morris and Shin. In this case both the strategic complementarity effect and the informational spillover effect are present. In Figure 3, an example is provided in which a positive informational spillover effect overwhelms the decrease in benefits arising from the strategic complementarity effect for initial increases in policy noise allowing for social welfare to increase. In short, the informational spillover effect is large enough so that social welfare can be improved by giving some firms the incentive to acquire information. In contrast, Figure 4 presents an example where perfect transparency is optimal. When the positive information spillover effect is not obtained and the strategic complementarity effect is relatively small, it is optimal for the policy authority to be perfectly transparent ($\sigma^2_u = 0$).

[Insert Figure 3 Here]

[Insert Figure 4 Here]

Given the discussion of the main mechanism that allows for positive information spillover effects, it is clear that a model with cost complementarities and without payoff externalities can also
generate an informational spillover effect. Under a cost complementarity the cost of information acquisition for any given agent can be given by a function $c(x, c_i)$ where $x$ is the fraction of informed agents and $c_i$ is firm $i$’s idiosyncratic cost. Given a function $c(x, c_i)$ such that $c_x(x, c_i) < 0$, as more firms become informed, the cost of acquiring information falls for the firm at the margin of gathering information. One justification for such a cost function could be that as more firms become informed, rumours arise concerning methods with which to acquire knowledge about the underlying macroeconomic state-of-the-world.

To illustrate the positive implications of policy noise, Figures 5 and 6 display plots of price dispersion when there is only an informational spillover effect ($k = 0$). Clearly as more firms become informed, the better prices reflect the underlying macroeconomic conditions, $\theta$. This is shown by the curves that plot the expected squared deviations of prices from the underlying macroeconomic conditions. Notice that

$$
\frac{\partial E}{\partial \sigma^2_u} \left[ \int_0^1 (\hat{p}_i - \hat{\theta})^2 di \right] = \left[ \left( \frac{r(1-x)}{1-rx} \right)^2 - \frac{2rx^2(1-r)(1-x)}{(1-rx)^3} - 1 \right] g(c^*) \frac{\partial c^*}{\partial \sigma^2_u} E[(bU_{\hat{y}} - \hat{\theta})^2] + \left[ x \left( \frac{r(1-x)}{1-rx} \right)^2 + 1 - x \right] \left[ \frac{\partial E}{\partial \sigma^2_u} (bU_{\hat{y}} - \hat{\theta})^2 \frac{\partial c^*}{\partial \sigma^2_u} \frac{\partial x}{\partial c^*} \right]
$$

where the first term is negative and the second term is negative when there is a positive information spillover. The first term arises for two reasons. First when policy noise increases, some firms go from being uninformed to informed thus reducing the dispersion of prices around $\theta$. Furthermore, as informed firms set their prices partly to reflect the average price, which is a function of the measure of uninformed firms, the prices set by informed firms also become more reflective of macroeconomic conditions as the measure of uninformed firms decreases. Thus when there is a positive informational spillover effect, it should be expected that prices become more concentrated around the underlying macroeconomic fundamentals as policy noise increases.

Now consider the expected square deviation of prices from the aggregate price level. When all firms are either informed or when all firms are uninformed, all firms will set the same price which will equal the aggregate price level. Expected price dispersion around the aggregate price level will be minimized in these two cases.

[Insert Figure 5 Here]

[Insert Figure 6 Here]

It is also worth noting that as measurement error, $\sigma^2_\eta$, increases in the policy authority’s observ-
able, the benefits of reducing policy transparency are eroded. This is because the policy authority’s beliefs concerning the state of the underlying macroeconomic conditions are already imprecise and so the firms place little weight on the policy announcements already. Thus the required change in policy noise required to induce additional information acquisition is much larger than would be the case with less measurement error.

**Remark 1** As the noise in the policy signal is pushed to the limit, $\sigma_n^2 \to \infty$, the benefits from increased policy noise disappear.

### 3 Discussion

Recently, the paper of Morris and Shin (2002) discussed how public information is a “double-edged” instrument for a policy authority. When there is a strategic complementarity component to payoffs they argue that agents may overweight public information resulting in a magnification of mistakes in policy statements. Even worse, if private information were costly to obtain, agents may not obtain informative private information at all, instead relying on public information of inferior quality. They further argue that central banks must be careful to guard against the potential damage created by noise in their statements.

By endogenizing the source of noise in the public signal, this paper characterizes a different reason for the “double-edged” nature of the policy instrument. In the Morris and Shin model, the policy authority receives an exogenous noisy signal about the state of macroeconomic conditions. A policy authority can choose to add additional noise to its exogenous signal when making policy statements. Private sector agents are endowed with a noisy private signal in addition to observing the policy signal. Welfare can be lowered by decreasing the amount of policy noise in the public signal in situations where the noise in private signals is relatively small. Notice that in the Morris and Shin model, the informational quality of the public signal is *increased* when its noise is reduced. However, as the information quality of the public signal is considerably worse than that of the private signals, agents are better off weighing the two signals according to their signal-to-noise ratios. Unfortunately, agents overweight the public signal because it acts as a coordinating device in the “beauty contest” component of the economy.\(^{13}\)

In the present model, when there is a large mass of agents with low idiosyncratic costs of information acquisition, lowering the endogenous noise chosen by the policy authority results in lower welfare as agents place more weight on the policy signal. This results in a large fraction of firms foregoing the opportunity to acquire information at a low cost. Interestingly, by doing so, the informational quality of the policy signal is *decreased*, thus reducing its service to the private sector. This is the key to the results of this paper. It is the fact that lowering the amount of noise in the policy signal results in a worsening of the quality of the information in the policy signal that makes agents worse off.
It is also interesting to note that the information spillover effect adds a complementary channel to the strategic complementarity channel emphasized in Morris and Shin (2002). This additional channel allows the degree of complementarity in the payoff function, \( r \), to be reduced while still allowing for optimal policy noise to be strictly positive. By modelling the costly acquisition of information the model sidesteps the recent criticism of the original Morris and Shin model that without large degrees of complementarities in the payoff function, the Morris and Shin model is really an argument for perfect transparency.\(^{14}\)

Another closely related paper is that of Bernanke and Woodford (1997). In their paper they discuss the difficulties of inflation forecast targeting. The idea in their paper is that the policy authority wishes to minimize the variance of one-period ahead inflation. All the information concerning underlying inflationary pressures is known by the private sector and the monetary authority has to infer this information from the private sector’s inflation forecast. It is shown that it is impossible for the authority to eliminate all sources of variation in inflation from the underlying inflationary pressures because attempting to do so would cause the private sector forecasts to be void of any information about the underlying inflationary pressures. Hence the impossibility of the monetary authority to learn about these pressures in order to defend against them. In other words, the monetary authority will offset any useful information incorporated into the inflation forecasts so forecasters will not incorporate this information into their forecasts.

One remedy that they present is for central banks to infer information about unobservable inflationary pressures from variables other than the variables that they target in their objective function. So long as the private sector works information about inflationary pressures into other variables that are not targeted by the monetary authority, and that are observable by the monetary authority, then the authority has the ability to draw inference about the inflationary pressures without losing the ability to target inflation.

The implications of the results in this paper suggest that so long as private sector agents have imperfect information about the underlying inflationary pressures, any statement by the monetary authority revealing information about inflationary pressures can result in a reduction of information quality in the monetary authority’s observable. This means that even if the policy authority extracts information from non-targeted variables the quality of information in the non-targeted variables is diluted the more informative is the policy signal. Of course this is contingent on the actions of the private sector agents being functions of their beliefs about the unobserved underlying inflationary pressures. The key difference from the mechanism in Bernanke and Woodford’s paper is that agents have a choice of drawing inference from a policy signal of endogenous quality or a private signal of known high quality. As the amount of the policy noise is reduced there can be a shift in the fraction of agents drawing inference from private signals into drawing inference from the policy signal. The result is the endogenous quality of the authority’s observable data is
reduced, thereby reducing the endogenous quality of the policy signal.

4 Extension: Policy Under Discretion

In this section the policy authority is allowed to alter its choice of policy noise after the firms make their information acquisition decisions. A time-inconsistency problem is present in that after firms choose to become informed, the policy authority can then choose to re-optimize its level of policy noise. As the fraction of informed agents is now fixed, by reducing policy noise, the policy authority can now increase the expected payoff to uninformed agents at no cost.

Consider, as a simple example, the economy with $k = r$ and the case where the policy authority maximizes social welfare taking the fraction of informed agents as given. In this case, taking the derivative with respect to $\sigma_u^2$ yields

$$\frac{\partial E(\bar{W})}{\partial \sigma_u^2} = -\left\{x \left[ \frac{r(1-x)}{1-rx} \right] + 1 - x \right\} \frac{\partial E[(bU\bar{y} - \bar{\theta})^2]}{\partial \sigma_u^2} \leq 0$$

The key to notice here is that once the fraction of informed firms is fixed, increasing policy noise can only reduce social welfare as it makes the policy signal less informative to the uninformed, the only agents who make use of the policy signal. By backward induction, as firms understand the policy authority’s incentives, they will make their information acquisition decision under the knowledge that when prices are chosen there will be no policy noise.

Remark 2 Under discretionary policy, the zero policy noise equilibrium will be obtained.

An implication of this result is that the policy authority can never do better than the zero policy noise case, even though adding policy noise may increase social welfare. The benefits of large informational spillovers are impossible to obtain because the policy authority cannot commit to maintaining the level of noise in its policy signal in order to induce marginal firms to become informed.

5 Conclusion

This paper has presented an argument for the determination of the optimal level of transparency for a central bank. The model isolated a particular informational channel through which policy has effects. The appealing feature of the model is that the effect of policy only comes through announcements from the central bank that are common knowledge to the private sector. In understanding how agents form their beliefs about unobservable fundamentals and how these beliefs are mapped into the actions chosen by the agent, the policy authority chooses the optimal transparency in its announcements in order to affect the beliefs of private sector agents. In doing so,
the policy authority must trade-off increasing the quality of the information in its announcement with the amount of private information that agents build into their actions. This is important as the monetary authority has no private information and must rely on private sector agents incorporating their private information into their actions in order to form policy announcements that contain useful informational content. The results do not consider the effects of instruments such as the interest rate or the money supply. While isolating a particular channel of communication, this transmission mechanism seems germane given the recent literature on inflation targeting policies.

An interesting implication of this paper is that more transparency does not necessarily result in lower inflation. If a large fraction of private sector agents have easy access to high quality information then the policy authority may choose not to be transparent and low inflation may be achieved. If private sector agents have very different and dispersed costs to access quality information then the policy authority may choose to be very transparent leading to lower inflation than if it chooses to be vague. However, in this model, the private sector as a whole has all the information about the underlying inflationary forces. If the quality of this information is good then lack of transparency may lead to lower inflation outcomes. Thus while the mechanism emphasized in this paper is not the only channel impinging on the transparency decision of the central banks (for example, hedging against mistakes that may result in loss of credibility) it may provide something to ponder about when testing the transparency-inflation outcomes empirically.

It is important to note that the only type of uncertainty in this model faced by the agents is situational uncertainty; no agent knows the true situation of the economy. This is in contrast to model uncertainty where agents are not certain as to the true model of the economy. Model uncertainty is undoubtedly a major problem faced by monetary authorities and provides a reason for cautious statements by central banks. Caution and transparency are separate issues when discussing the informational content of policy announcements, though potentially they may provide identification problems for the private sector when interpreting the announcements. The model in this paper is designed to emphasize a theoretical argument and in doing so purposely abstracts from all other complications. Even the reduced form of the payoff functions are used in order to allow for the simplest exposition of the theoretical point.

Recent papers have pointed out that in particular structural models (see Hellwig (2005) as an example), the nature of an economy’s structure that gives rise to the complementarities can actually reverse the welfare implications of reduced policy transparency. For example, with the recent vintage of sticky price models, there are welfare losses to price dispersion while welfare losses from fluctuations in the aggregate price level may be of a lower order. Thus the policy authority would work to minimize on the dispersion of information across economic agents; a force in favour of increased policy transparency. However, one can imagine that a model in which agents have to invest to learn about the current frontier of technology could have a force in favour of a little
bit of policy opaqueness in order for agents to be provided some incentive to conduct explorative research that could stimulate the aggregate economy. The construction of such structural models of learning for policy analysis leaves much arduous work for the future.
A Solving for the Rational Expectations Equilibrium

In order to solve for the rational expectations equilibrium it is necessary to work backwards. Suppose that a fraction of firms, $x$, have become informed and learned the value of $\theta$ in the first stage. Then these $x$ firms will choose their price such that $p_i = p_I$. The remaining $1 - x$ uninformed firms choose their prices optimally given that they observe the policy signal $y$ so that $p_i = p_U$. From the firm’s optimal policy rule it is known that

$$p_i = (1 - r)E_i(\theta) + rE_i(P).$$

Substituting for $P$ and writing $E(\theta)$ for the average expectations of $\theta$ across agents we have

$$p_i = (1 - r)E_i(\theta) + (1 - r)E_i \left[ E(\theta) \right] + (1 - r)r^2E_i \left[ E^2(\theta) \right] + ...$$

which upon continued iteration yields

$$p_i = (1 - r)\sum_{k=0}^{\infty} r^k E_i \left[ E^k(\theta) \right]. \tag{A1}$$

Given the linear-normal structure of the economy it is conjectured that $y(\theta, \eta, u)$ and $A(\theta, \eta, u)$ are normally distributed random variables. If so, then Bayesian beliefs for the uninformed firms and the monetary authority are given as

$$E_U(\theta) = b_U y \tag{A2}$$
$$E_M(\theta) = f_A A_p \tag{A3}$$

respectively, where

$$A_p = A - f_U u.$$

The term $f_U u$ is the linear least squares projection of $A$ on $u$ and the term $b_U$ are the linear least squares projection coefficient of $y$ on $\theta$. The coefficients governing the belief functions can be expressed as

$$b_U = \frac{\text{cov}(y, \theta)}{\text{var}(y)}, \quad f_A = \frac{\text{cov}(A_p, \theta)}{\text{var}(A_p)}, \quad f_U = \frac{\text{cov}(A, u)}{\text{var}(A)} \tag{A4}$$

Noting that for the informed firm that $E_i(\theta) = \theta$ substitution of beliefs into equation (A1) and forward iteration yield the price functions

$$p_I = \frac{1 - r}{1 - rx} \theta + \frac{r(1 - x)}{1 - rx} b_U y \tag{A5}$$

$$p_U = b_U y \tag{A6}$$
for the informed and uninformed firms, respectively. Aggregating across firms

\[
A = \int_0^1 p(d i) + \eta
\]

\[
= \frac{(1 - r)x}{1 - rx} \theta + \frac{(1 - x)b_u y}{1 - rx} + \eta.
\]

Define the variable

\[A_p = A - f_u u\]

which is the data from which the policy authority learns about the realization of \(\theta\). The component \(f_u u\) accounts for the fact that the policy authority knows the realization of \(u\). Thus the policy authority can clean this noise out of its data, \(A\) when forming its expectations about \(\theta\). Then

\[E_M(\theta) = f_A A - f_A f_u u.\]

Given that \(y = E_M(\theta) + u\) it is easily verified that

\[
y = \frac{f_A (1 - r)x}{1 - rx - (1 - x)f_A b_u} \theta + \frac{f_A (1 - rx)}{1 - rx - (1 - x)f_A b_u} \eta - \frac{(f_A f_u - 1)(1 - rx)}{1 - rx - (1 - x)f_A b_u} u
\]

(A7)

\[
A = \frac{(1 - r)x}{1 - rx - (1 - x)f_A b_u} \theta + \frac{(1 - rx)}{1 - rx - (1 - x)f_A b_u} \eta - \frac{(1 - x)(f_A f_u - 1)b_u}{1 - rx - (1 - x)f_A b_u} u
\]

(A8)

which shows that in equilibrium the \(\tilde{y}\) and \(\tilde{A}\) are normally distributed allowing for the conjectured belief coefficients. The belief coefficients are found to be

\[b_u = \frac{(1 - r)^2 x^2 \sigma^4}{\sigma^2 + [(1 - r)^2 x^2 \sigma^2 + (1 - rx)^2 \sigma^2] \sigma^2_u}\]

(A9)

and

\[f_A = \frac{(1 - r)x(1 - rx) \sigma^2}{(1 - r)x[(1 - r)x + (1 - x)b_u] \sigma^2 + (1 - x)^2 \sigma^2_u}\]

(A10)

\[f_u = \frac{(1 - x)b_u}{(1 - rx)}.\]

(A11)

Now it is possible to specify the payoff functions of the informed and uninformed firms. The payoff to an informed firm is

\[u^I_i(p; \theta, \eta, u) = -(1 - r)\left(\frac{r(1 - x)}{1 - rx}\right)^2 (b_u y - \theta)^2 - (r - k)x \left[\frac{(1 - r)(1 - x)}{1 - rx}\right] (b_u y - \theta)^2 + k(1 - x) \left[\frac{(1 - r)x}{1 - rx}\right]^2 (b_u y - \theta)^2 - c_i\]

and the payoff to an uninformed firm is

\[u^U_i(p; \theta, \eta, u) = -(1 - r)(b_u y - \theta)^2 - (r - k(1 - x)) \left[\frac{(1 - r)x}{1 - rx}\right]^2 (b_u y - \theta)^2 + kx \left[\frac{(1 - r)(1 - x)}{1 - rx}\right]^2 (b_u y - \theta)^2.\]

It can easily be shown that

\[E(b_u \tilde{y} - \tilde{\theta})^2 = \frac{(1 - r)^2 x^2 (1 - rx)^2 \sigma^4 \sigma^2 + [(1 - r)^2 x^2 \sigma^2 + (1 - rx)^2 \sigma^2] \sigma^2_u}{(1 - r)^2 x^2 \sigma^4 + (1 - r)^2 x^2 \sigma^2 + (1 - rx)^2 \sigma^2_u} + [(1 - r)^2 x^2 \sigma^2 + (1 - rx)^2 \sigma^2_u].\]
Stepping back to the information acquisition decision, letting \( c^* \) denote the information cost for the marginal informed firm, \( c^* \) then must satisfy

\[
c^* = \frac{(1 - r)^2}{(1 - rx)^2} E[(b_U \tilde{y} - \tilde{\theta})^2 |\sigma^2_u].
\]  

(A12)

Notice that in equilibrium the fraction of informed agents, conditional on \( \sigma^2_u \) is common knowledge. Finally, expected social welfare is given by

\[
E(\tilde{W}) = E \left\{ (1 - r) \left[ x \left( \frac{r(1 - x)}{1 - rx} \right)^2 + 1 - x \right] + (k - r)x \left[ \frac{(1 - r)(1 - x)}{(1 - rx)} \right]^2 \right. \\
\left. + (k - r)(1 - x) \left( \frac{(1 - r)^2 x^2}{(1 - rx)^2} \right) E(b_U \tilde{y} - \tilde{\theta})^2 \right\} - \int_0^{c^*} c_i g(c_i) dc_i
\]  

(A13)

which the policy authority maximizes by choice of \( \sigma^2_u \).

B Proofs

B.1 Proof of Lemma 1

Consider the case of the marginal investing firm when \( x = 0 \). For the firm to be indifferent about acquiring information or not it must be that

\[
c_i = (1 - r)^2 \sigma^2_u \geq 0.
\]

Given the assumptions on \( g(c) \) this means that the marginal firm will choose to become informed when \( x = 0 \) and so \( c^* > 0 \). From the indifference conditions

\[
c^* = \frac{(1 - r)^2}{(1 - rx)^2} E[(b_U \tilde{y} - \tilde{\theta})^2 |\sigma^2_u]
\]

where the function on the right-hand side, the marginal benefit of being informed, is continuous and starts above zero. Consider the marginal benefit of becoming informed:

\[
\frac{\partial m b^I}{\partial c^*} = \left\{ \frac{2r(1 - r)^2}{(1 - rx)^3} E[(b_U \tilde{y} - \tilde{\theta})] + \frac{(1 - r)^2}{(1 - rx)^2} \frac{\partial E[(b_U \tilde{y} - \tilde{\theta})]}{\partial x} \right\} \frac{\partial x}{\partial c^*}
\]

As \( \partial x/\partial c^* = g(c^*) \), when we take the limit as \( c^* \to \infty \), \( g(c^*) = 0 \) and \( \partial m b^I/\partial c^* = 0 \). Given that \( g(c^*) \) exists and is continuous, there exists at least one crossing point at which the indifference equation holds. □

B.2 Proof of Proposition 1

This is direct given the result of lemma 1 and continuity of the expected social welfare in \( x \). □
B.3 Proof of Lemma 2

Consider the indifference equation for the marginally informed firm

\[ c^* = \frac{(1 - r)^2}{(1 - rx)^2} E[(b_u \tilde{y} - \tilde{\theta})^2|\sigma_u^2] \]

The left-hand side plots the explicit cost of becoming informed. The right-hand side plots the net benefits of being informed. Notice that the direct cost has a slope of one as the cut-off cost is varied. Taking the derivative of the benefits with respect to policy noise, while holding the cut-off cost constant, the change in net benefits is given by

\[ \frac{(1 - r)^2}{(1 - rx)^2} \frac{\partial E(b_u \tilde{y} - \tilde{\theta})^2}{\partial \sigma_u^2} \]

which is positive. Thus as policy noise increases, holding the fraction of informed firms constant, the guess of any firm concerning \( \theta \) is worse on average. This yields an incentive to become informed.

Now consider the change in the net benefits as the cut-off cost is varied, holding the level of policy noise constant. The derivative of the net benefits with respect to the cut-off cost yields

\[ \frac{\partial \text{Net Benefits}}{\partial c^*} = \frac{(1 - r)^2}{(1 - rx)^2} \left\{ 2r \frac{\partial E(b_u \tilde{y} - \tilde{\theta})^2}{\partial \sigma_u^2} + \frac{\partial E(b_u \tilde{y} - \tilde{\theta})^2}{\partial x} \right\} g(c^*). \]

The first term on the right-hand side is the change in benefits attributed to the strategic complementarity component to payoffs. As more firms are informed, the uninformed firms receive less expected benefits from forecasting the prices set by other firms because they do not know what price the informed firms will set as informed prices are dependent on the realization of \( \theta \). In contrast, informed firms are better off because they set prices as a function of the realization of \( \theta \) and the prices set by uninformed firms. The lower the fraction of uninformed firms, the closer is the price of informed firms to \( \theta \). In other words, on average, the strategic complementarity in payoffs pulls the optimal price of informed firms further away from \( \theta \) when a larger fraction of firms are uninformed. Thus the larger the fraction of uninformed firms, the more incentive the strategic complementarity gives firms to remain uninformed. It can then be easily shown that

\[ \frac{\partial c^*}{\partial \sigma_u^2} = \frac{(1 - r)^2 \frac{\partial E(b_u \tilde{y} - \tilde{\theta})^2}{\partial \sigma_u^2}}{1 - \frac{(1 - r)^2}{(1 - rx)^2} \left\{ 2r \frac{\partial E(b_u \tilde{y} - \tilde{\theta})^2}{\partial \sigma_u^2} + \frac{\partial E(b_u \tilde{y} - \tilde{\theta})^2}{\partial x} \right\} g(c^*)} \]  

\text{(B1)}

For the cut-off to decrease as policy noise increases it is necessary for the second term in the denominator to be overwhelmingly large. That is, the increase in the fraction of uninformed firms from a fall in the cut-off cost must be so large that the gains from guessing the same price as all uninformed firms makes up for the expected deviation of the price from \( \theta \). Notice for this case to
hold the net benefits from an increase in \( c^* \) must exceed unity. Thus the case in which \( \partial c^*/\partial \sigma_u^2 < 0 \) holds at the set of unstable equilibria. □

### B.4 Proof of Proposition 2

Totally differentiate the indifference equation for the marginal informed firm to obtain \( \partial c^*/\partial \sigma_u^2 \).

Substitute into equation (16) to obtain

\[
\frac{\partial E(\tilde{W})}{\partial \sigma_u^2} = \left\{ \frac{r(1-r)^2}{(1-rx)^2}(1-2x+rx) + (k-r)\frac{(1-r)^2}{(1-rx)^2}(1-2x) + 2r(k-r)x(1-x)\frac{(1-r)^2}{(1-rx)^3} \right\}
\]

\[
\cdot E(b_u\tilde{y} - \tilde{\theta})^2 \frac{\partial x}{\partial c^*} \frac{\partial c^*}{\partial \sigma_u^2} - \left\{ (1-r) \left[ x \left( \frac{r(1-x)}{1-rx} \right)^2 + 1 - x \right] - (k-r)\frac{(1-r)^2x(1-x)}{(1-rx)^4} \right\}
\]

\[
\cdot \left[ \frac{\partial E(b_u\tilde{y} - \tilde{\theta})^2}{\partial \sigma_u^2} + \frac{\partial E(b_u\tilde{y} - \tilde{\theta})^2}{\partial x} \frac{\partial x}{\partial c^*} \frac{\partial c^*}{\partial \sigma_u^2} \right].
\]

The second term is the informational spillover effect. Notice that for \( k = r \) and \( k = 0 \),

\[
(1-r) \left[ x \left( \frac{r(1-x)}{1-rx} \right)^2 + 1 - x \right] - (k-r)\frac{(1-r)^2x(1-x)}{(1-rx)^4} \geq 0.
\]

Thus the sign of the informational spillover effect depends on the sign of \( \Xi \), where

\[
\Xi = \left[ \frac{\partial E(b_u\tilde{y} - \tilde{\theta})^2}{\partial \sigma_u^2} + \frac{\partial E(b_u\tilde{y} - \tilde{\theta})^2}{\partial x} \frac{\partial x}{\partial c^*} \frac{\partial c^*}{\partial \sigma_u^2} \right].
\]

Using the derivative \( \partial c^*/\partial \sigma_u^2 \) from equation (B1) we can rewrite \( \Xi \) as

\[
\frac{\partial E[(b_u y - \theta)^2]}{\partial \sigma_u^2} \left\{ \frac{1}{1-\left( \frac{(1-r)^2}{(1-rx)^2} \right)^2} \left[ \frac{E[(b_u y - \theta)^2]}{1-\left( \frac{(1-r)^2}{(1-rx)^2} \right)^2} \left( \frac{2x}{1-rx} E[(b_u y - \theta)^2] + \frac{\partial E[(b_u y - \theta)^2]}{\partial x} g(c^*) \right) \right] \times \frac{\partial E[(b_u y - \theta)^2]}{\partial c^*} \right\}.
\]

In a stable equilibrium the denominator is positive. As \( \partial E[(b_u y - \theta)^2]/\partial \sigma_u^2 \) is positive, the sign of the informational spillover depends on the sign of the numerator. Specifically, the information spillover effect is positive when the numerator is negative. □
References


Notes

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1 The common (theoretical) measure of transparency is the distance between what the private sector believes to be the realization of the policy authority’s information set and the actual realization of the policy authority’s information set.

2 See Romer and Romer (2000) for an empirical argument that there may be informational content in policy instrument settings.


4 In the model that follows, for purposes of exposition, the environment is pushed to the extreme in that some agents may choose to be perfectly informed while others are imperfectly informed.

5 Much of the previous work on transparency stresses the incentive of the policy authority to be vague in order to prevent the private sector from learning about the objectives of the policy authority which is private information of the policy authority. The policy authority may choose not to be transparent about its objective function in order to be able to sacrifice inflation for output when desired. In other words, the policy authority does not want to be forthright in order to take advantage of its informational advantage. For some examples see Stein (1989), Cukierman and Meltzer (1986), and Faust and Svensson (2002). Geraats (2002) provides a comprehensive literature review.

6 One can also think of the problem of the firms in a Lucas Island world where firms want to know macroeconomic conditions because it helps them distinguish between the component of the demand for their good that is common across all firms versus idiosyncratic to their specific firm. The underlying mechanism of this paper should carry through.

7 Alternatively, a firm can receive a noisy signal about the underlying macroeconomic conditions if it incurs the cost of becoming informed. By allowing firms to receive a noiseless signal, the exposition is simplified immensely.

8 It may be useful to think of the firm setting the logarithm of its price as prices may be negative.

9 This set-up is adopted from the model of Morris and Shin (2002). The generalized Morris and Shin payoff function is adopted from Hellwig (2005). Woodford (2003) discusses possible sources of such complementarities in payoffs.

10 For notational convenience the parameters governing the economy and the general structure of the economy are omitted from $I_i$.

11 For notational convenience, variables subscripted with $I$ and $U$ denote variables corresponding to informed and uninformed firms respectively.
Proofs and details about the equilibrium solution are relegated to the Appendix.


See Svensson (2006) for such a critique.
Figure 1: Varying $\sigma_u^2$ ($\sigma_\theta^2 = 20, \sigma_\eta^2 = 2, r = 0.35, \alpha = 25, \beta = 0.2, k = 0$)
Figure 2: Varying $\sigma_u^2$ ($\sigma_\theta^2 = 20, \sigma_\eta^2 = 2, r = 0.35, \alpha = 3.5, \beta = 2, k = 0$)
Figure 3: Varying $\sigma_u^2 (\sigma_\theta^2 = 20, \sigma_\eta^2 = 2, r = 0.35, \alpha = 25, \beta = 0.2, k = r)$
Figure 4: Varying $\sigma_u^2$ ($\sigma_\theta^2 = 20, \sigma_\eta^2 = 2, r = 0.35, \alpha = 3.5, \beta = 2, k = r$)
Figure 5: Varying $\sigma_u^2$ ($\sigma_\theta^2 = 20$, $\sigma_\eta^2 = 2$, $r = 0.35$, $\alpha = 25$, $\beta = 0.2$, $k = 0$)
Figure 6: Varying $\sigma_n^2$ ($\sigma_\theta^2 = 20$, $\sigma_\eta^2 = 2$, $r = 0.35$, $\alpha = 3.5$, $\beta = 2$, $k = 0$)