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Intermediated Quantities and Returns

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ABSTRACT

There is a large amount of intermediated borrowing and lending between households. The average difference in borrowing and lending rates is over 2 percent. In this paper, we develop a model economy that displays these facts and matches not only the returns on assets but also their *quantities*. The heterogeneity giving rise to borrowing and lending and differences in equity holdings is the result of differences in preferences for making bequests. In equilibrium, the lenders are annuity holders and the borrowers are the equity holders. The borrowing rate and return on equity are the same in our model which has no aggregate uncertainty. As there are intermediation costs, the lending rate is less than the borrowing rate and there is an equity premium. Within age cohorts, human capital endowments and inheritances are identical. A consequence of this is that there is almost no dispersion in consumption, yet there is a sizable dispersion in net worth and a huge dispersion in equity holdings.

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1. Introduction:

The *homogenous household* construct is of little use in modelling borrowing and lending between households. In equilibrium, with most models using this construct the shadow price of consumption at date *t+1* in terms of consumption at date *t* is such that the amount of borrowing and lending is zero. Homogenous household models are thus of little use in matching the *quantities* of assets held and *intermediated*.

To address this issue, we construct an alternative construct that incorporates household heterogeneity in the form of differences in the strength of preferences for bequests. Incorporating this household heterogeneity allows us to capture a key empirical fact: there is a very large amount of borrowing and lending between households[1]. This borrowing is done directly by households to finance owner occupied housing, by proprietorships and partnerships to finance their business, and indirectly by shared ownership corporations to partially finance the corporations. We abstract from the small amount of borrowing and lending between households and assume that all borrowing and lending between households is intermediated through financial institutions such as banks and pension funds. For the United States, in 2005 the amount intermediated was approximately 1.3 times the GDP.[2]

In light of the finding that the premium for bearing non-diversifiable aggregate risk is small in worlds consistent with growth and business cycle fact, our analysis abstracts from *aggregate risk*[3]. The only uncertainty that people face is idiosyncratic risk

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[1] Age heterogeneity alone gives rise to little borrowing and lending between households as found in Diaz-Giménez et al. (1991).

[2] See section 7 (calibration) for details.

[3] Prescott and McGrattan (2000) find that the equity premium is small in the growth model if it is restricted to be consistent with growth and business cycle facts. Lattau and Uhlig (2000) introduce habit formation into the standard growth model and find that the equity premium is small if the model parameters are

about the duration of their lifetime after retirement. All households in an age cohort have identical preferences for consumption. They differ only with respect to their preference for making bequests. In equilibrium, those with a strong preference for bequests accumulate equity assets and borrow during their working lives, and upon retirement, use equity income for consumption and interest payment on their debt. Upon death they bequeath all their assets. Households with no bequest motive buy annuities during their working years and use annuity benefits to finance their consumption over their retirement years.

The intermediation technology is constant returns to scale with intermediation costs being proportional to the amount intermediated. To calibrate the constant of proportionality, we use Flow of Funds Account statistics and data from National Income and Product Accounts. The calibrated value of this parameter equals the net interest income of financial intermediaries, divided by the quantity of intermediated debt and is approximately 2 percent.[4]

In the absence of aggregate uncertainty, the return on equity and the borrowing rate are identical, since the households who borrow are also marginal in equity markets. In our framework, government debt is intermediated at zero cost and thus its return is equal to the household lending rate. The equity premium relative to government debt is the intermediation spread for household borrowing and lending. The divergence between borrowing and lending rates gives rise to an equity premium even in a world *without aggregate uncertainty.*

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restricted to be consistent with the business cycle facts. Many others using the growth model restricted to be consistent with the macro economic growth and business cycle facts have found the same thing.

[4] See Section 7 (calibration) for details.

The paper is organized as follows: the economy is specified in Section 2. In Section 3, we discuss the decision problem of the households. Sections 4 deals with the relevant balance sheets, Section 5 with the aggregation of individual behavior, and Section 6 characterizes the balanced growth equilibrium. We calibrate the economy in Section 7. In Section 8, we present and discuss our results. Section 9 concludes the paper.

2. The Economy

In order to build a model that captures the large amount of observed borrowing, lending, and the large amount of resources used up in this process, we introduce three key features of reality. The first feature is differences in bequest preferences, the second is an uncertain length of retirement, and the third is costly intermediation of borrowing and lending between households. This leads some households to buy *costly* annuities which make payments throughout retirement years. As the people choosing this option have large expected promised annuity payments, they are big lenders. Others people with high bequest utility save more when young and do not buy costly annuities.

The model is an overlapping generation model and we consider the balanced growth path competitive equilibrium. All households born at a given date are identical in all respects except for bequest preference parameter α. Households have identical preferences with respect to consumptions over their lifetime, so the only dimension over which they differ is α. Those with a large α (type-B) borrow and hold the equity; others with no preferences for bequest (type-A) lend by acquiring annuities.

What motivates bequests? While a casual consideration of bequests naturally assumes that they exist because of parents' altruistic concern for the economic well being of their offspring, results in Hurd (1989) and Kopczuk and Lupton (2004), among others (see also Wilhelm (1996), Laitner and Juster (1996), Altonji et al. (1997), and Laitner and Ohlsson (2001)), suggest otherwise: households with children do not, in general, exhibit behavior in greater accord with a bequest motive than do childless households. This, we think leads us to conclude the existing literature supports our assumption that people some people have preferences for making bequests. These empirical results lead us to eschew the perspective of Barro (1973) and Becker and Barro (1988), who postulate that each generation receives utility from the consumption of the generations to follow, and simply model bequests as being motivated by a well defined "joy of giving"[5] as in Abel and Warshawsky (1998)) and Constantinides et al. (2007) . We emphasize that our result are not sensitive to the reason why people leave bequests.

Housholds

Any systematic consideration of bequests mandates that the analysis be undertaken in an overlapping generations model context. Consequently, we analyze an overlapping generations economy and determine its balanced growth behavior. Each period, a set of individuals of measure one, enter the economy. There are two types entering at each date, type-A with no utility from making a bequest and type-B with a whose utilty is an increasing function of the amount they bequeat. The measure of type $i \in \{A, B\}$ is μ_i. The total measure of people born at each date is 1, so $\mu_A + \mu_B = 1$.

Individuals have finite expected lives. They enter the labor force at age 22, work for T years and then retire.[6] Model age j is 0 when a person begins their working life. The first year of their retirement is model age $j = T$.

[5] See also De Nardi, Imrohoroglu and Sargent (1999), De Nardi (2004), and Hansen and Imrohoroglu (2007).

[6] We implicitly assume that parents finance the consumption of their children under the age of 22 – in other words, children's consumption is a part of their parents' consumption.

All workers receive an identical wage income. Wage income grows at the economy's balanced growth rate γ. At retirement, individuals face idiosyncratic uncertainty about the length of their remaining lifetime. Their retirement lifetimes are exponentially distributed. Once retired, the probability of surviving to the next period is $\sigma = (1-\delta)$, where δ is the probability of death. Expected life is $T + 1/\delta$. We emphasize there is no aggregate uncertainty.

Individuals of type α, born at time *t,* order their preferences over age contingent consumption and bequests by

$$\text{(2.1)} \qquad \sum_{j=0}^{T} \beta^j \log c_{t+j,j} + \sum_{j=T+1}^{\infty} \beta^j \sigma^{j-T} \log c_{t+j,j} + \sum_{j=T+1}^{\infty} \alpha\delta\beta^j \sigma^{j-T-1} \log b_{t+j,j}$$

Here $\beta < 1$ is the discount factor and α is the strength of bequest parameter. Variable $c_{t+j,j}$ is the period consumption of a *j* year old born at time *t,*[7] conditional on being alive at time *t* + *j*. An individual who is born at time *t* and dies at age $j-1$ consumes nothing at time *t* + *j* and bequeaths $b_{t,t+j}$ units of the period *t* + *j* consumption good and consumes nothing subsequently. Each generation supplies one unit of labor inelastically for $j = 0,1,\ldots,T-1$. Thus, aggregate labor supply is $L = T$ given that the measure of each generation is 1.

We only need to analyze the decision problems of an individual of a type α individual born at time *t* = 0. The solution to the problem for a type α born at any other *t* can be found using the fact that along a balanced growth path

[7] In this paper, the first subscript represents calendar time and the second subscript represents the age at that time.

(2.2) $$c_{t,j} = (1+\gamma)^t c_{0,j}$$

Further, to simplify the notation, we use c_j to denote the consumption of a *j* year old at time *j* rather than $c_{j,j}$. An analogous change of notation applies to the other *variables.*

Production Technology

The aggregate production function is

(2.3) $$Y_t = F(K_t, z_t L_t) = K_t^{\theta}(z_t L_t)^{1-\theta}$$

(2.4) $$z_{t+1} = (1+\gamma)z_t .$$

K_t is the capital, L_t is labor, and z_t is the labor augmenting technological change parameter, which grows at a rate γ. The parameter z_0 is chosen so that $Y_0 = 1$.

Output is produced competitively so

(2.5) $$\delta_k + r_e = F_K(K_t, z_t L_t)$$

(2.6) $$e_t = F_L(K_t, z_t L_t)$$

where δ_k is the depreciation rate, r_e is both the household borrowing rate and the return on equity, and e_t is the wage rate.

Income is received either as wage income E_t or gross capital income R_t. Thus

(2.7) $$Y_t = E_t + R_t ,$$

where $E_t = L_t e_t = (1-\theta)Y_t$ and $R_t = (\delta_k + r_e)K_t = \theta Y_t$. Components of output are consumption C_t, investment X_t and intermediation services I_t; thus

(2.8) $$Y_t = C_t + X_t + I_t$$

Along a balanced growth path, investment $X_t = (\delta_k + \gamma)K_t$ and $K_{t+1} = (1+\gamma)K_t$.

Financial Intermediary Technology

The intermediation technology displays constant returns to scale, with the intermediation cost in units of the composite output good being proportional to the amount borrowing and lending intermediated. The cost is ϕ times the amount of borrowing and lending between households. The intermediary also intermediates between households lending to the government. There are no costs associated with this intermediation. The intermediary effectively pays interest rate r on its lending to households and receives interest rate r_e on its lending to households. Given the technology, equilibrium interest rates must satisfy

$$r_e - r = \phi$$

The lending contract between households and intermediaries is not the standard one, but rather an annuity contract. When a household buys an annuity, the household is lending to the intermediary. Upon retirement a household begins receiving idiosyncratic-event-contingent payments. The present value of annuity contracts using interest rate *r* is 0. This leads us to refer to r as the household lending rate. In equilibrium, competitive intermediaries will offer any annuity contract with the property that the expected present value of benefits is equal to the present value of premiums using r in the present values calculations.

During their working years, individuals can accumulate equity and borrow. If a household enters into an annuity contract at age j = 0, the pension fund reserves for that contract is an asset of that individual.[8] It starts out at zero and grows over the households working life. Upon retirement for our calibrated economy it continues to grow, but at a slower rate. At death it falls to zero. Thus, a household's asset holdings at a given point in time are pension fund reserves and equities. The household's liabilities are the household's private debt.

Government Policy

The government finances interest payments on its debt by issuing new debt and by a taxing labor income at rate τ. The government's period *t* budget constraint is

(2.12) $$(1+r)D_t^G = \tau E_t + D_{t+1}^G.$$

$D_{t+1}^G = (1+\gamma)D_t^G$ in balanced growth . Therefore,

(2.13) $$(r-\gamma)D_t^G = \tau(1-\theta)Y_t$$

In addition, the government pursues a tax-rate policy that pegs[9] r, which is the interest rate on government debt. This being a balanced growth analysis, government debt grows at rate $\gamma > 0$, which means the government deficits are positive and grows at rate γ as well.

Finally the intermediary holds government debt and there are no intermediation costs associated with holding this asset on the part of the intermediary.

[8] The Flow-of-Funds household sector net worth sector lists pension fund reserves as part of household net worth.

[9] In this paper, we fix this to be 3 percent. This is discussed further in the section on calibration.

3. Optimal Individual Decisions

We consider the optimal individual decision problem, taking as given (i) the size of the inheritance the individual will received at model age 30 (chronological age 52), (ii) wages at each date of their working life, (iii) the labor income tax rate τ, and (iv) the borrowing and lending rates r_e and *r*. Aggregate bequests at $t = 0$ are $\bar{b}$ and given balanced growth at date *j* are $\bar{b}(1+\gamma)^j$. The inheritance of a person born at $t = 0$ at age 30 if that person adopts the no annuity strategy is $\bar{b}(1+\gamma)^j$ given the measure of every age cohort is 1 and all in the 30 year old cohort receive the same bequest.

The inheritance of those choosing the annuity strategy is $\bar{b}(1+\gamma)^j(1+r)/(1+r_e)$ as there are financial intermediation costs associated with the bequests they receive. This result warrants some explanation. The inheritance of a thirty year old at time *t*= 30s is transmitted to the intermediary, who uses it to buy an additional annuity for that person, to increase its reserves for pensions, and to cover the intermediation costs bequests. The estate delivers $\bar{b}(1+\gamma)^j/(1+r_e)$ units of capital. The return on this capital net of intermediation costs is $(1+r_e-\phi)=(1+r)$. Multiplying the number of units of capital times the return on capital net of intermediation costs is the amount a household choosing the annuity strategy inherits.

The first problem facing an individual is whether to choose the annuity strategy A or the no annuity strategy B. It will turn out that a type-A will choose the annuity strategy while a type-B will choose the no annuity strategy. The second problem is to determine the lifetime consumption and savings decisions conditional on the strategy chosen. We determine, given α, the optimal consumption-saving behavior for each strategy and the resulting lifetime utility, and then determine which of the two strategies is best for that individual type.

A convention followed is that a bar over a variable denotes a constant. In the case where the constant depends upon a person type, that is on α, this functional dependence is indicated. This is necessary, as the best strategy will differ across household types.

3.1 The Best No Annuity Strategy

This problem can be spit into two sub-problems. The first problem is the one after retirement, which is stationary and is solved using dynamic programming techniques. The state variable is net worth, which is in units of the *current period consumption good*. The value of a unit of *k* is $(1+r_e)k$ to a household choosing the no annuity strategy. The second problem is to determine of consumptions and savings over the working life.

The problem becomes stationary and recursive at retirement age T with net worth *w* being the state variable. The value function $f(w)$ is the maximal obtainable expected current and future utility flows if a retiree is alive and has net worth *w*. The optimality equation is

(3.1)
$$f(w) = \max_{c,w'} \{\log c + \sigma\beta f(w') + \delta\,\beta\,\alpha \log w'\}$$
$$s.t. \quad c + \frac{w'}{(1+r_e)} \le w$$

The solution to this optimality equation has the form:

(3.2)
$$f(w) = \bar{f}_1(\alpha) + \bar{f}_2(\alpha)\log w\,,$$

where

(3.3)
$$\bar{f}_2(\alpha) = \frac{1+\alpha\ \beta\delta}{1-\sigma\beta}\,.$$

The optimal consumption/saving policy for retirees is

$$
\begin{aligned}
&c = w \,/\, \overline{f}_2(\alpha) \\
&w' = (1 + r_e)(w - c)
\end{aligned}
\tag{3.4}
$$

The bequests, conditional on $j - 1$ being the person's last year of life, is

$$
b_j = w_j \qquad\qquad j > T \tag{3.5}
$$

The problem facing an individual at birth who follows the no annuity strategy, (which we call strategy B because it is the one that those with a sufficiently strong preference for making a bequest choose) is,

$$
\begin{aligned}
&U^B(\alpha) = \max_{\{c_j\}_{j=0}^{T-1}, w_T} \{\sum_{j=0}^{T-1} \beta^j \log c_j + \beta^T [\overline{f}_1(\alpha) + \overline{f}_2(\alpha) \log w_T]\} \\
&\text{s.t} \\
&\sum_{j=0}^{T-1} \frac{c_j}{(1+r_e)^j} + \frac{w_T}{(1+r_e)^T} \le v_0^B = \sum_{j=0}^{T-1} \frac{(1-\tau)e_0(1+\gamma)^j}{(1+r_e)^j} + \frac{\overline{b}}{(1+r_e)^{30}}
\end{aligned}
\tag{3.6}
$$

Here v_0^B is the present value of wages and bequest at birth of an individual born at $t = 0$. The solution (see Appendix for more details) is

$$
\begin{aligned}
&c_j^B = \overline{c}(\alpha)\beta^j (1+r_e)^j v_0^B \qquad\qquad j < T \\
&w_T^B = (1 - \sum_{j=0}^{T-1} \overline{c}(\alpha)\beta^j)(1+r_e)^T \; v_0^B
\end{aligned}
\quad ,
\tag{3.7}
$$

where $\qquad \overline{c}(\alpha) = \dfrac{(1-\beta)}{1-\beta^T + (1-\beta)\beta^T f_2(\alpha)}$.

The preretirement age j net worth of an individual following this strategy satisfies

$$
\begin{aligned}
&w_0^B = 0 \\
&w_j^B = (1+r_e)(w_{j-1}^B - c_{j-1}^B + (1-\tau)e_0(1+\gamma)^{j-1}) \qquad \text{for } 1 \le j < T, j \ne 30 \\
&w_{30}^B = (1+r_e)(w_{29}^B - c_{29}^B + (1-\tau)e_0(1+\gamma)^{29}) + \overline{b}(1+r_e)^{30}
\end{aligned}
\tag{3.8}
$$

3.2. The Best Annuity Strategy

The best annuity strategy for a type-α is the solution to the following:

$$U^A(\alpha) = \max_{\{b_j, c_j\}}\{ \sum_{j=0}^{T} \beta^j \log c_j + \sum_{j=T+1}^{\infty} \beta^j \sigma^{j-T} \log c_j + \sum_{T+1}^{\infty} \beta^t \sigma^{j-T-1} \delta\, \alpha \log b_j]\}$$

(3.9) s.t

$$\sum_{j=0}^{T} \frac{c_j}{(1+r)^j} + \sum_{j=T+1}^{\infty} \frac{\sigma^{j-T} c_j}{(1+r)^j} + \sum_{j=T+1}^{\infty} \frac{\sigma^{j-T-1} \delta\, b_j}{(1+r)^j} \leq v_0^A$$

where r is the lending rate and

(3.10) $$v_0^A = \sum_{t=0}^{T-1} \frac{(1-\tau) e_0 (1+\gamma)^t}{(1+r)^t} + \frac{\overline{b}^A}{(1+r)^{30}}$$

The constant v_0^A is the present value of future wage income and inheritances ($\overline{b}^A = \overline{b}(1+\gamma)^j (1+r)/(1+r_e)$) using the lending rate r of a person born at $t = 0$. The superscript *A* denotes the annuity strategy and not an individual type. It will be the case that in equilibrium type-*A* will choose strategy *A*.

There are other constraints, specifically, that the worker choosing this strategy does not borrow, that is $e_j - c_j \geq 0$ for $j < T$. For the economies considered in this study, these constraints are not binding and can therefore be ignored. If, however, the economy were specified such that the no-borrowing constraint were binding for some *j*, then the solution below would not be the solution to the problem formulated above.

The nature of the annuity contract is that the payment to a retiree who is alive at age $j \geq T$ is c_j. If the individual dies at age *j*, payment b_j is made to that person's estate. The solution to this program is

(3.11) $$c_j^A = \overline{c}(\alpha)(1+r)^j \beta^j \; v_0^A \qquad j \geq 0$$

(3.12) $$b_j^A = \alpha\bar{c}(\alpha)(1+r)^j\beta^j v_0^A \qquad j \geq T+1$$

The net worth of an individual choosing this strategy is the pension fund reserves associated with that individual's annuity contract. Pension fund reserves (from the point of view of the intermediary) for a given annuity contract for an individual born at $t = 0$ at age *j* in equilibrium equals the expected present value at time $t = j$ of payments that will be made less the value (at time $t = j$ as well) of premiums that will be received.

For workers, they can be determined as the present value of past premiums. Thus, pension fund reserves for individuals' annuity holders born at $t = 0$ at age *j* satisfy

(3.13)
$$\begin{aligned} w_0^A &= 0 \\ w_j^A &= \left(w_{j-1}^A - c_{j-1}^A + (1-\tau)e_0(1+\gamma)^{j-1}\right)(1+r) && \text{for } 1 \leq j < T, j \neq 30 \\ w_j^A &= \left(w_{j-1}^A - c_{j-1}^A + (1-\tau)e_0(1+\gamma)^{j-1}\right)(1+r) + \bar{\mathrm{b}}^{\mathrm{A}} && \text{for } j = 30 \end{aligned}$$

For retirees, conditional on being alive, pension fund reserves for individuals born at $t = 0$ at age *j* are equal to the expected present value of the future payments

(3.14) $$w_j^A = \sum_{t=0}^{\infty}(1-\delta)^t \frac{c_{j+t}^A}{(1+r)^t} + \sum_{t=0}^{\infty}\delta(1-\delta)^{t-1}\frac{b_{j+t}^A}{(1+r)^t} \qquad j > T$$

3.3 Best Strategy

The best strategy is the no annuity strategy if $U^B(\alpha) > U^A(\alpha)$. The best strategy is the annuity strategy if $U^A(\alpha) > U^B(\alpha)$.

Proposition 1: The function $U^B(\alpha) - U^A(\alpha)$ has positive slope.

Proof: See the Appendix.

There exists an $\alpha^* > 0$ that partitions individuals into two groups: individuals with $0 \le \alpha < \alpha^*$ choose to annuitize while those with $\alpha > \alpha^*$ hold equity and borrow. Given that the borrowing rate and the return on equity are equal, there is a portfolio indeterminacy for those following the no annuity strategy. There is no aggregate indeterminacy as type-B hold all the equity[10] and the intermediary none.

Plotted in Figure 1 is the difference in utilities for the two strategies, as a function of α, for our calibrated economy. We see for the calibrated economy that people with bequest preference parameter $\alpha < 0.182$ choose to annuitize[11].

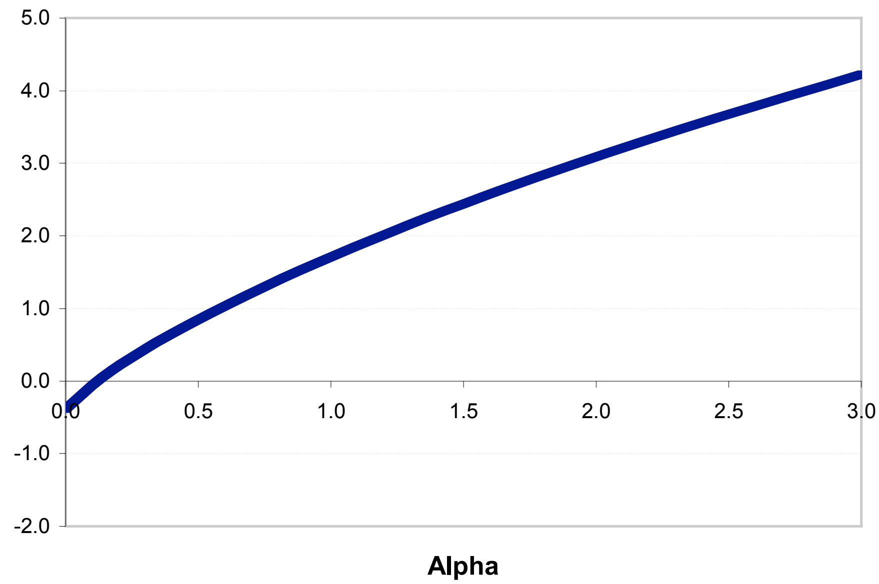
[10] All corporate equity and debt is treated as equity and debt of the owners of the corporation.

[11] Our finding that households with a low bequest preference will annuitize is consistent with the result in Yarri(1965), who finds households with no preference for bequests annuitize at retirement and leave no bequests.

Figure 1

Utility Difference between the Best No Annuity and Best Annuity Strategy:

$$U^B(\alpha) - U^A(\alpha)$$



Section 4: Aggregate Behavior of the Household Sector

Aggregate Consumption

Aggregate consumption depends upon the labor tax rate τ and inheritance factor $\overline{b}$ as well as the prices $\{e, r, r_e\}$. Equilibrium prices do not depend upon the household side, and can be determined from the policy choice of *r* and profit maximizing conditions. Having formulated the optimal consumption strategies for the two types of individuals, we characterize the aggregate consumption, asset holdings and bequest at time $t = 0$ by individual type given $\overline{b}$ and τ for the equilibrium prices. Two aggregate equilibrium relations must be solved for the two endogenous variables $\overline{b}$ and τ.

There are two types of households $i \in \{A, B\}$. The type-*A* has $\alpha_A = 0$ and will in equilibrium choose the annuity strategy A given the model economy. The type-*B* has $\alpha_B > 0$. The measure of type-*i* of age *j* at $t = 0$ is

(4.1) $$\mu_j^i = \begin{cases} \mu_0^i & j \leq T \\ (1-\delta)^{j-T} \mu_0^i & j > T \end{cases}$$

The aggregate consumption of the type-i households at time 0 is C^i

(4.2) $$C^i(\overline{b}, \tau) = \mu^i \sum_{j=0}^{T-1} c_j^i (1+\gamma)^{-j} + \mu^i \sum_{j=T}^{\infty} (1-\delta)^{j-T} c_j^i (1+\gamma)^{-j} .$$

Here we have used the fact that each subsequent generation has a consumption-age profile that is higher by a factor of $(1+\gamma)^j$ in balanced growth.

Aggregate consumption is

(4.3) $$C(\overline{b}, \tau) = C^A(\overline{b}, \tau) + C^B(\overline{b}, \tau).$$

Aggregate Asset Holdings

The aggregate net worth at time 0 of types $i \in \{A, B\}$ are,

(4.4) $$W(\bar{b}, \tau) = \mu_0^i \sum_{j=0}^{T} w_j^i (1+\gamma)^{-j} + \mu_0^i \sum_{j=T+1}^{\infty} (1-\delta)^{j-T} w_j^i (1+\gamma)^{-j}$$

Net worths are prior to consumption and receipt of wages income and includes net interest income and dividend income. In the case of the intermediary it includes intermediation cost liabilities. Net worth is prior to consumption and is denominated in units of the consumption good.

Aggregate Inheritance

At time zero the measure of the people aged $j > T$ who *die* and leave a bequest is $\mu_0^B \delta \sigma^{j-T-1}$, thus the total bequests given by these households is:

$$B_j = \mu_0^B \delta \sigma^{j-T-1} w_j^B \qquad j > T$$

Hence the *aggregate bequests* at time 0 are:

(4.5) $$B_0 = \sum_{j=T+1}^{\infty} B_j (1+\gamma)^{-j}$$

Since we assume that bequests are equally distributed and received at age 30, the *inheritance* of someone who is 30 years old at time 0 depends on which strategy they are following. If a person follows strategy B, that person's inheritance is

$$\bar{b}^B = \frac{B_0}{\mu_0^A + \mu_0^B} = B_0 .$$

If a person follows strategy A , that person's inheritance is

$$\bar{b}^A = B_0 \frac{1+r}{1+r_e} .$$

Aggregate Private Debt

The aggregate indebtedness of a type-B satisfies

(4.7) $$D^B(\bar{b},\tau) = K - W^B(\bar{b},\tau)\,/\,(1+r_e)\,,$$

as the price of existing capital in terms of the consumption good is $(1+r_e)$ and the household is obligated to make payment $(1+r_e)D^B(\bar{b},\tau)$.

Section 5: Balance Sheets

Assets and liabilities are beginning of period numbers and are in units of the consumption good. We consider only economies for which there is intermediated borrowing and lending in equilibrium. Given there is a large amount of intermediated borrowing and lending, these economies are the ones of empirical interest.

Type-A Sector: The assets of the type-*A* consist of assets in their pension accounts. They have no liabilities. The value of these pension assets (in terms of the consumption good) is: Pension Assets = $(1+r)D^B(\bar{b},\tau)+(1+r)D^G(\bar{b},\tau)$

Balance Sheet of Type-A Households

Assets	**Liabilities**
Pension Assets	0
	Net worth

Hence their net worth satisfies

$$W^A(\bar{b},\tau) = (1+r)D^B(\bar{b},\tau)+(1+r)D^G(\bar{b},\tau)$$

Type-*B* Sector: Those following the no annuity strategy have debt $D^B(\overline{b},\tau)$ and hold all the equity in the economy K. Their balance sheet is

Balance Sheet of Type-*B* Households

Assets	**Liabilities**
$(1+r_e)K$	$(1+r_e)D^B(\overline{b},\tau)$
	Net worth

Here we have adjusted the assets and liabilities by a factor $(1+r_e)$ to get the net worth in units of the consumption good. Their net worth is

$$W^B(\overline{b},\tau) = (1+r_e)K - (1+r_e)D^B(\overline{b},\tau)$$

Financial Intermediary Sector: The assets of the financial intermediary are the liabilities of the government and the type-B households, while its liabilities are the pension assets of type-A households and the amount payable for intermediation services. The net worth of the financial intermediaries is zero.

Assets	**Liabilities**
Government Debt = $(1+r)D^G(\overline{b},\tau)$	Pension assets = $(1+r)[D^B(\overline{b},\tau)+D^G(\overline{b},\tau)]$
Private debt = $(1+r_e)D^B(\overline{b},\tau)$	Amounts payable for intermediation services = $D^B(\overline{b},\tau)(r_e - r)$
	Net worth = 0

Government: The assets of the government are the present value of the tax receipts on labor income while its liabilities are the debt it has outstanding.

Balance Sheet of the Government

Assets	**Liabilities**
$\frac{\tau(1-\theta)Y}{r-\gamma}$	$D^G(\overline{b},\tau)$
	Net worth = 0

Since labor is supplied inelastically and taxed at a rate τ, the government effectively owns a fraction τ of an individual's time endowment (now and in all future periods). In our model economy, the net worth of the government is zero and government debt is an asset for debt holders in our model.

Section 6: Equilibrium Relations

From the Production Side

We determine the value of a set of balanced growth variables at $t = 0$. All variables grow at rate γ except aggregate labor supply, which is constant and equal to 40. Given *Y* has been normalized to 1 at time zero, the cost share relationships determine time *0* capital stock *K* and wage *e*:

(6.1) $$(r_e + \delta_k)K = \theta Y$$

(6.2) $$eL = (1-\theta)Y$$

From the intermediary's problem, the lending rate satisfies

(6.3) $$r_e = r + \phi$$

Three Equilibrium Conditions

Prices $\{e, r, r_e\}$ are determined from policy and technology. Therefore only $\overline{b}$ and τ are needed to completely specify the household budget constraints. Conditional on these variables, aggregate consumption, $C(\overline{b}, \tau)$, and aggregate intermediation, $I(\overline{b}, \tau)$, will be determined by aggregating individual household variables.

One aggregate equilibrium condition is the aggregate resource constraint,

(6.4) $$C(\overline{b}, \tau) + X + \phi I(\overline{b}, \tau) = K^{\alpha} L^{1-\alpha}.$$

where $X = (\delta_k + \gamma)K$ is investment. Intermediation services satisfy

(6.5) $$I(\overline{b}, \tau) = K - \frac{W^B(\overline{b}, \tau)}{(1 + r_e)}.$$

We assume that type-*B* hold all the capital and the intermediary none. This is done to resolve the unimportant indeterminacy. Increasing the amount of capital held by a type-*B* and type-*B* indebtedness by the same value amount does not affect a type-B net worth, which is what matters. This portfolio shift of the type-B is offset by a portfolio shift of the intermediary. The aggregate indebtedness of a type-B is denoted by $D^B(\overline{b}, \tau)$ and it is equal to $I(\overline{b}, \tau)$.

The second equilibrium condition is that the inheritance of people at a point in time equals aggregate bequests at that point in time. We consider $t = 0$ and let $B(\overline{b}, \tau)$ be the aggregate bequest at that time. The second equilibrium condition is

(6.6) $$\overline{b} = B(\overline{b}, \tau)$$

There is a third equilibrium condition, namely the government's budget constraint. Equating payments to receipts, $(1 + r)D_t = \tau E_t + D_{t+1}$.

Given $D_{t+1} = (1+\gamma)D_t$, $E_0 = (1-\theta)Y_0$, and Y_0 has been normalized to 1.0, the time zero government budget constraint is

(6.7) $$(r-\gamma)D(\bar{b},\tau) = \tau(1-\theta)$$

Equation (6.7) determines government debt.

Equilibrium

The first two equilibrium conditions are linear in $(\bar{b},\tau)$, so solving for a candidate solution is straightforward. This solution is the equilibrium only if in addition (i) the best strategy for type-*B* is the no annuity strategy; (ii) the best strategy for type-*A* is the annuity strategy; (iii) $D^B > 0$; and (iv) $c^A_{0,j} < (1-\tau)e_o$. The reason for the last constraint is that these equilibrium conditions hold provided that the no-borrowing constraint on annuity holders is not binding and it will not be binding if (iv) holds.

Section 7: Calibration

The parameters that needs to be "calibrated" are the parameters related to the households $\{\alpha^A, \alpha^B, \beta, \mu^A, \mu^B, T, \delta\}$; the intermediation technology parameter $\{\phi\}$; the goods technology parameters $\{\theta, \delta_k, \gamma\}$; and the policy parameter $\{r\}$. The other policy parameters $\{\tau, D^G\}$ are endogenous. Many of these parameters are well documented in the literature; others are not.

We proceed by listing them with selected values and a brief motivation

Parameters associated with individuals

$\beta = 0.99$ (Annuity holders *c* grow at almost over their lifetimes)

$\delta = 0.05$ (Implies a post retirement life expectancy of 20 years)

$\alpha^{A} = 0$ (Assumption: Type-A individuals have low bequest intensity)

$\alpha^{B} = ???$ (Assumption: Type-B individuals have high bequest intensity)

$T = 40$ (Retire at chronological age 63)

$\mu^{A} = 0.154$ (Specified so that the amount intermediated matched U.S. data)

$\mu^{B} = 1 - \mu^{A}$

Intermediation parameters

$\phi = .02$ (Consistent with the average difference in borrowing and lending rates)

Policy parameters

$r = 0.03$ (Assumption about government fiscal policy)

The motivation for this policy is that this has been the approximate return of on debt.

Goods production parameters

$\theta = 0.3$ (Capital cost share)

$\gamma = 0.02$ (Average growth rate of U.S. per capita output)

$\delta_{k} = 0.05$ (Consistent with capital output ratio = 3, given $r_{e} = .05$)

In calibrating ϕ we proceed by estimating the value added by the financial intermediation sector. The major source of revenue for this sector is the difference in interest payments received from borrowers and interest payments paid to lenders. Using data from NIPA[12] for year 2000 the former amounted to $1,480 billion (0.148 times GNI) and the latter to $940 billion (0.094 times GNI. To estimate the services associated with intermediating borrowing and lending, we first subtracted services furnished without

[12] The data used is from NIPA (2000) tables 7.11 and 2.4.5.

payment by the financial intermediaries, because we view these services as corresponding mostly to transaction services. These amounted to $187 billion. Thus, the value added by the financial intermediation sector is $ 353 billion or about 3.5 percent of GNI. A significant amount of intermediation services is purchased by non financial business. We do not have a good measure of this number. We estimate that it is about 0.8 times GNI which leads to a number of 0.026 times GNI being household borrowing/lending intermediation services.

Using data from the *Flow of Funds*,[13] we estimate the total amount of intermediated borrowing and lending between households to be 1.3 times GNI (See Table 1 below). The implied intermediation spread is thus 2.0 percent. Some intermediate borrowing is by young type-A people in the form of consumer debt. This led us to estimate the difference in average household borrowing and lending rates to be 2 percent and in turn the calibrated $\phi = 0.02$.

We estimated borrowing and lending between households by determining total household holdings of debt assets in year 2000. Not all of this corresponds to the household debt in our model. Some is intermediated borrowing and lending between young people of the same type. Some is lending for precautionary reasons and for transaction purposes (including currency held). Considerations such as these lead us to calibrate the measure of type-B so the amount of intermediated borrowing and lending was 1.3 times GNI.

[13] The data is from the Flow of Funds (2000) table B.100.e.

Table 1

Financial Intermediary Sector Accounts Relative to GNI Year 2000

Interest received by financial intermediaries	0.1484 GNI	Table 7.11 NIPA Line 28
Interest paid by financial intermediaries	0.0941 GNI	Table 7.11 NIPA Line 4
Net Interest	0.0543GNI	
Less Services furnished by financial intermediaries without payment	0.0167 GNI	Table 2.4.5 NIPA Line 89
Less Bad debt expense[14]	0.0107 GNI	
Intermediation services associated with household borrowing and lending[15]	0.0269 GNI	
Total amount intermediated[16]	1.3076 GNI	

Section 8: Results.

We considered three values for α^B, a parameter for which we have little information. For each value of α^B we search for the μ^B for which the intermediate borrowing and lending between households is approximately 1.3 times gross national income (GNI). The results are summarized in Table 2, which shows the aggregate results are not sensitive to the strength of the bequest parameter α^B.

[14] NIPA table 7.16 line 12

[15] Net interest less transaction services, which are assumed equal to Services furnished without payment by FI.

[16] From FoF year 2000 Table B.100b.e. This number is Assets (line 1) minus Tangible Assets (line 2) minus Equity Shares at Market Value (line 6) minus equity of unincorporated business. The last number was obtained from Table B.100 (line 28).

Table 2

Summary of Aggregate Results

Economy	$\alpha^B = 1$	$\alpha^B = 3$	$\alpha^B = 6$
μ^A	0.833	0.846	0.863
μ^B	0.167	0.154	0.137
Nation Accounts			
C_A	0.633	0.644	0.657
C_B	0.131	0.120	0.107
X	0.210	0.210	0.210
I	0.026	0.026	0.026
Y	1.000	1.000	1.000
Depreciation	0.15	0.15	0.15
Compensation	0.70	0.70	0.70
Profits	0.15	0.15	0.15
Net Worth			
Type-A	6.45	6.54	6.66
Type-B	1.78	1.78	1.78
Government Debt/Y	4.96	5.05	5.16
Bequest/Y	0.0355	0.0374	0.0399
Tax rate	0.0708	0.0721	0.0738

Table 3

Inheritance as Fraction of Wealth at Entry into Workforce

	$\alpha^B = 1$	$\alpha^B = 3$	$\alpha^B = 6$
Type-*A*	0.046	0.048	0.051
Type-*B*	0.037	0.039	0.041

Given the aggregate results are insensitive to α^B, subsequently we deal only with the case $\alpha^B = 3$.

Total bequests in our model are large, larger than for the U.S. economy. Total bequests reported on U.S. estate tax forms plus total charitable contribution reported on individual tax forms were only 0.4 percent of GNI in year 2000. This number is far smaller than the 3.7 percent number for our model economy. We do not view this as that problematic for two reasons. First, our model has no population growth when in fact population in the U.S. has been growing a little over 1 percent a year the last 50 years. With population growth the fraction of people leaving bequests in a given year is smaller and the fraction of the population that are workers is higher. The 1 percent population growth rate decreases bequests as a share of GNI by a factor of 2 or 3.

The second reason why the model's high bequest GNI ratio is not problematic is that much of bequests are not reported on tax records. Bequests given prior to death for estate tax reasons and for the joy of seeing others benefiting from them are not reported to tax authorities. There are unreported bequests associated with the transfer of family businesses to a younger generation. Further, most estates in year 2000 were worth less that $600,000 and therefore not reported on estate tax forms. Converting the inheritance to the annual wage, type-A individuals receive 2.14 times their chronological age annual wage when they are 52 years old. With 1 percent population growth this would be reduced by over a factor of 2. These considerations suggest that bequests are not excessive in our model world.

One variable of interest is the fraction of wealth that is inherited. A significant component of wealth is human capital, which is the present value of wages. It is about 95.5 percent and would be higher if there were population growth. These results are for a type-*A*, who discounts using a 3 percent rate. The share is a little lower for type-**B** who use a 5 percent discount rate. Anything that reduces the ratio of bequests to GNI reduces this number, so for the United States this number is probably al least 0.98 .

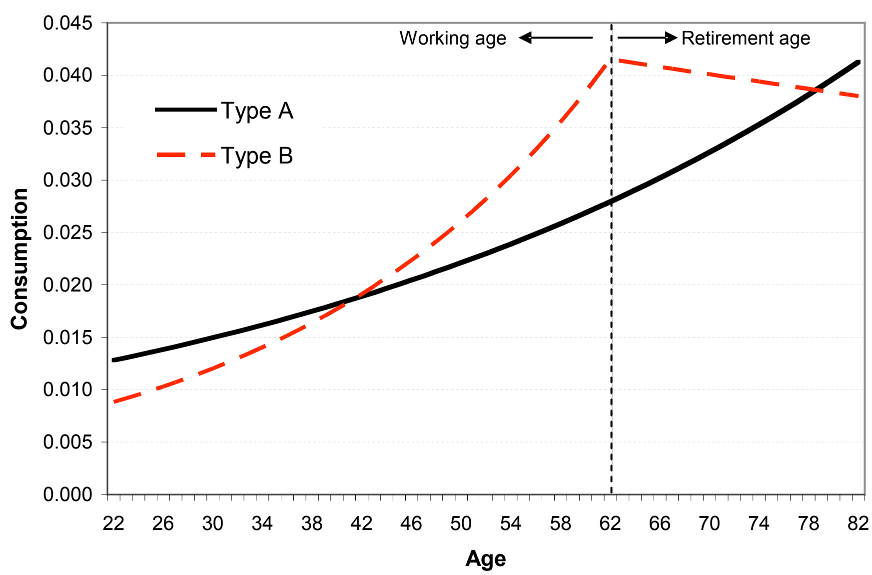
Government debt may appear large relative to explicit U.S. government debt, which is only 0.3 times GNI in recent times. In fact, the estimates of implicit Social Security Retirement and Medicare promises are over 3 times GNI by most estimates. Further, with population growth this number would be significantly smaller. Thus model government debt is not large. One point is that if the government prohibited bequests, the steady state capital stock would be the same, namely 3 times GNI with the given government policy.

Some Micro Findings

Our abstraction, we think, show intermediation costs that give rise to differences in household borrowing and lending rates accounts for about 2 percent of the difference in the household lending rate and the return on equity. Unlike this macro finding, the model's micro findings are not a positive theory of the distributions of consumption, net worth, and equity holdings and consequently must be interpreted with care. With this caveat, the micro distributional relations for our model economy are as follows.

Figure 2 plots the lifetime consumption patterns of the two types. Type-*A*'s consumption grows at a constant annual rate of 1.97 percent throughout their lifetime. Type-*B*'s starts out lower and grows more rapidly during their working life with this growth rate being 3.95 percent. Upon retirement the consumption growth rate turns negative, falling to -0.44 percent, but except for the older type-*B* retirees is higher than an equal age type-*A*.

Figure 2
Life Time Consumption Patterns



Cross sectional consumption

Figure 3 plots cross sectional consumption by age for the two types. All type-*A* that are alive have virtually the same consumption. Young type-*B* workers have lower consumption and older workers have higher consumption. For the type-*B*, the older the retiree the smaller is the consumption level.

Figure 3

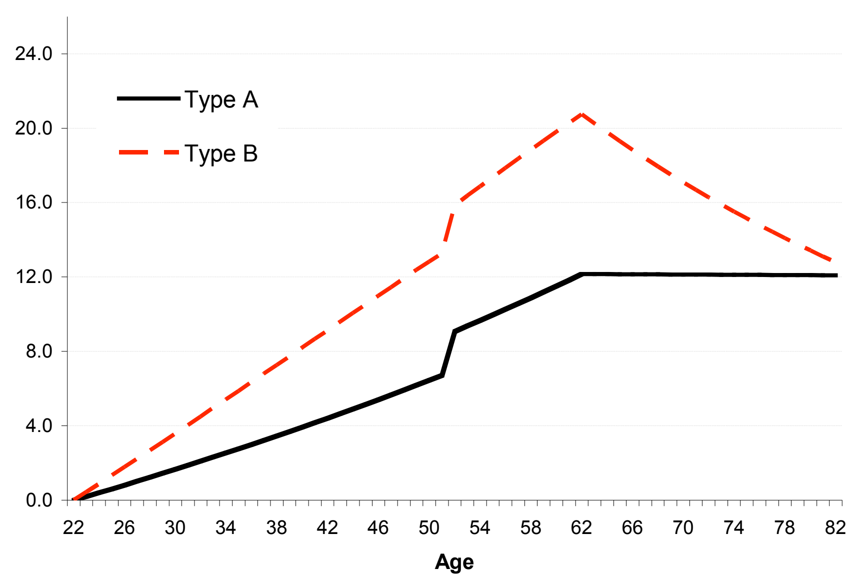
Consumption by Age

Net worth by age

In Figure 4 we plot net worth relative to current annual wage income, which has a stationary distribution. At retirement the net worth of a type-A is 12 times annual wage incomes and that of a type-B is 21 times annual wage income at time of retirement. The disparity in age corrected net worth is modest being a maximum about 1.8 prior to retirement. After retirement it falls until age 84 and then the type-*A* start having the larger new worth, but the number of survivors drops by 5 percent a year. The jump at chronological age 52 is due to inheritance.

Figure 4

Net Worth as a Function of Age in Units of Annual Wage Income



Lorenz Curves

Figure 5 plots the Lorenz curves for consumption, net worth, and capital or equity holdings. In the case of capital we assume all type-B have the same ratio of debt to capital in their portfolios in order to resolve the portfolio indeterminacy at the individual level. We truncated the distribution at age 112, so the curves are only very good approximations, and not exact.

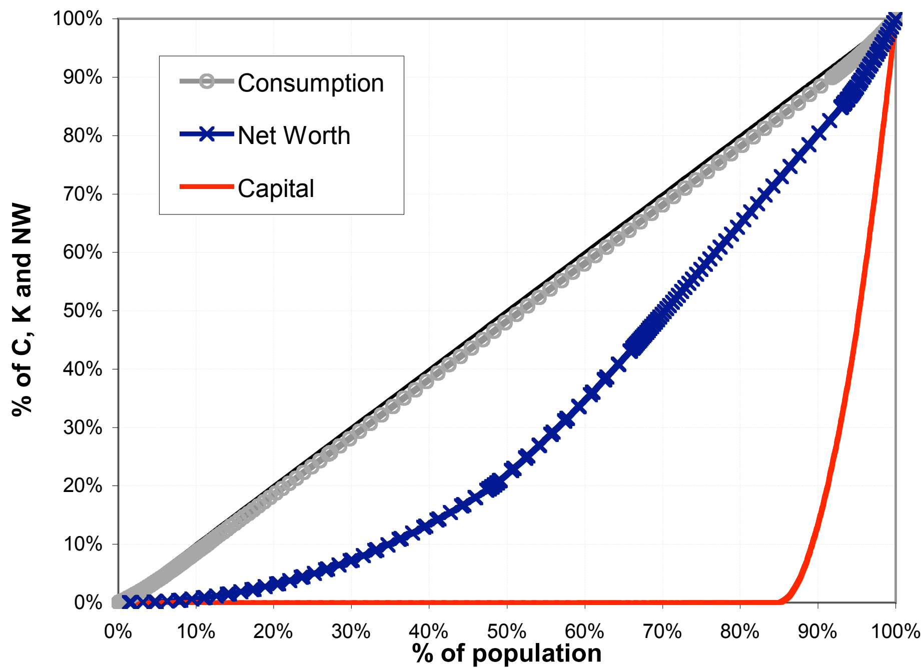
The principal findings are that there is almost no disparity in consumption levels, modest disparities in net worth levels, and huge disparity in capital holdings. Type-*B* are the only ones holding the capital stock, so 12 percent of the population owns 100 percent of the capital. There is some dispersion in capital holdings within the type-*B* sub population and 4.6 percent of the population own half the capital stock. This shows that the dispersion in capital holding is a bad proxy for dispersion in consumption.

In our model world all have the same human capital endowment. If the model were modifies to having people earn proportionally different wages, to a first approximation all that need be done is to scale the variables in this model.[17] This would add to disparity in consumption, net worth, and capital stock holdings.

[17] If bequests were distributed proportional to the human capital factor, the scaling result would hold exactly.

Figure 5

Lorenz curve for consumption, net worth and capital



This picture shows the usual Lorenz curve for consumption, total wealth and capital.

Cost of financial market constraints

What are the gains to a household of having access to the equity market at no intermediation cost? Table 4 reports the cost of not having this access, (which was the case for most Americans prior to the development of low cost indexed mutual funds) as being about 10 percent of wealth at entry into workforce. This wealth is the present value of labor income and inheritance.

Table 4

Cost to an A of not Having Access to the

Annuity Market in Units of Wealth at Entry into Workforce

α^B	Change in v_0^A
1	0.82%
3	0.86%
6	0.98%

Table 5

Cost to a B of not Being Permitted to hold Equity Directly

in Units of Wealth at Entry into Workforce

α^B	Change in v_0^B
1	3.97%
3	9.71%
6	15.74%

These tables show the percentage increase in either e_0 or v_0^k necessary to compensate an $i \in \{A, B\}$ in wealth equivalents if forced to switch to a system other than their most preferred choice. Since both, consumption and bequest are linear functions of initial wealth; the percentage changes in both consumption and bequest are the same as the percentage change in initial wealth.

What are the costs to a type-A if for some reason such as adverse selection problems or legal constraints, they do not have access to annuity markets, and must use the equity option for saving? The cost is small being approximately 0.9 percent of lifetime consumption.

Section 9: Concluding Comments

In this paper, we develop a heterogeneous household economy where households differ as to their preferences for bequest. In equilibrium, households with a low desire to bequeath, lend and hold annuities, while those with a high desire to bequeath borrow and hold equity. This is important, for the amount borrowed by households must equal the amount lent by households. Our simple framework mimics reality with respect to *both* the amount of intermediated borrowing and lending between households *and* the average spread in borrowing and lending rates resulting from intermediation costs.

We find that incorporating the divergence between borrowing and lending rates can account for a half of the historically observed equity premium, which we define to the difference in the average return on equity and the average after-tax return on productive assets from national account data.

Our analysis in this paper is admittedly stylized. However, we believe the abstraction is well suited to address the impact of the costs associated with financial intermediation on the equity premium and for enhancing our quantitative economic intuition as to the reason for the high disparity in equity holdings. We view this as a first step in what we think may prove to be a productive research program.

Possible extensions include building in differential survival rates and addressing the issues of adverse selection and moral hazard when pricing annuities. This extension might justify our requirement that people choose between the annuity and the no annuity strategy early in their careers. We expect these extensions to yield theories that, in addition to matching the quantity intermediated and the intermediation spreads, also match the stocks of assets held. We will, of course need detailed statistics on individual asset holdings to investigate these issues.

This research program, if successful, will interface with the literature on household lifetime consumption behavior. Such an interface will require an extension as the bequest motive is not the only factor that differentiates people. There surely are differences in preferences with respect to consumption today versus consumption in the future and differences in preferences that give rise to differences in lifetime labor supply. Our analysis suggests that asset holdings and consumption over the lifetime must be jointly considered.

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