Optimal Incentives and the Time Dimension
of Performance Measurement

Michael Raith
University of Rochester*
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Abstract: In many occupations, the consequences of an agent’s actions become known only over time. Should a principal then compensate an agent based on early but noisy information about performance, or later but more accurate information, or both? To answer this question, I study a two-period model in which a principal can pay a risk-averse agent based on both true output, which is realized with delay, and an early signal of output. The agent can borrow against future income, and can commit to a long-term contract. I show that under very general conditions the optimal wage contract depends on the early signal as well as on output even if the signal is merely a noisy version of output; that is, if the signal is uninformative of effort, given output. Specifically, for given output levels, better signals are on average associated with higher wages. Thus, an important characteristic of any performance measure is the time at which it is generated, which expands the range of signals useful for contracting well beyond that implied by the classic Informativeness Principle. The results also shed light on the use of forward-looking performance measures such as stock returns in managerial incentive contracts.

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*Correspondence: Michael Raith, Simon School of Business, University of Rochester, Rochester, NY 14627, USA; email: raith@simon.rochester.edu. I am grateful for many helpful suggestions from Alexei Alexandrov, Arpad Abraham, Rajiv Dewan, Guido Friebel, Romans Pancs, Allison Raith, Jan Zabojnik, Jerry Zimmerman and seminar participants at Queen’s University.
1 Introduction

In all but the simplest of occupations, the consequences of agents’ actions become known only over time. Managers, in particular, make investments, introduce new products, initiate mergers, etc. that may generate early information but whose success or failure is often known only much later. For instance, a manager’s investment of time and money into a research project may lead to a patent in the short run, whereas commercial success or failure occurs only years later.

This paper studies incentive contracting in a world in which performance measurement is not a one-time event. Suppose an agent’s action generates information over time that increases in accuracy as uncertainty about various influencing forces resolves itself, as in the examples above. Should one then compensate the agent based on early but less accurate information, or later but better information, or both? Understanding the tradeoff between the accuracy of a performance measure and the time at which it is generated is the central goal of this paper.

This tradeoff has not been studied previously but is one that firms routinely face when designing compensation plans. Accounting measures, for instance, are largely based on actual outcomes, whereas stock prices incorporate beliefs about the future. In a static context, one could look at the tradeoff between the two measures through the lens of Holmström’s (1979) Informativeness Principle. In a longer-term relationship, however, there is also a tradeoff between paying based on stock prices now and paying based on accounting measures later, as both may be measures of the same outcomes of actions, but which differ along a time vs. accuracy dimension.

Even in a dynamic setting, the Informativeness Principle provides a useful benchmark intuition that can withstand several counterarguments. It suggests that an early signal should not be used in a contract if it is merely a noisy version of a later outcome and if the outcome is feasible for contracting.

A first counterargument is that the agent may want to consume before the outcome of his action is known. But with access to credit, he could finance early consumption by borrowing against his future income, and would normally use the signal in deciding how much to consume early. This so far does not imply that the agent’s wage contract should depend on the signal. A second counterargument is that the agent may face borrowing constraints in an external credit market. But this could simply require the principal to act as the agent’s bank, with compensation based on early signals merely being a form of advance loan on later income. A third counterargument is that the agent may be unable to commit to a long-term contract. As I will explain in Section 2, however, this does not lead to a succession of short-term contracts as optimal outcome, and again does not violate the above informativeness argument.
In contrast to the above intuition, I show that under very general assumptions an optimal wage contract makes use of both the actual outcome and an early signal about it. This result holds even if the signal is uninformative of effort (given output), and does not rely on the assumption of credit constraints or horizon problems.

I derive this result within two models that have the same two-period structure. The first (studied in Section 2) adopts the popular framework of Holmstrom and Milgrom (1987, 1991) with linear contracts, exponential utility, and normally distributed noise terms. The second model (see Section 3) is a general model in the tradition of Mirrlees (1999) and Holmstrom (1979). In each model the agent faces a dynamic consumption problem as considered in the repeated moral hazard literature (e.g. Rogerson, 1985a, Fudenberg et al., 1990, or Chiappori et al., 1994), and the recent macroeconomic literature on consumption, saving and optimal social insurance; see Aiyagari (1994), Atkeson and Lucas (1995), Cole and Kocherlakota (2001) or Kocherlakota (2005). However, the issues studied in these literatures — such as the role of memory or the scope for spot-contract implementability — are quite different from the question I am concerned with here.

In each model, the agent exerts effort once, in the first period, but there are two different performance measures: output, which occurs in the second period, and a noisy signal of output, which becomes available in the first period. Both are observable and verifiable.\footnote{For an analysis of the principal’s decision whether to disclose a signal about performance to the agent, see Lizzeri, Meyer and Persico (2002).} The early signal is uninformative of effort in the sense of Holmstrom (1979), given output. A signal $z$ is “better” than a signal $z'$ if it corresponds to a first-order dominant shift in output conditional on the signal. The agent is risk-averse and maximizes a time-separable intertemporal utility function.

There are no frictions of the kinds mentioned above: both principal and agent can commit to a long-term contract that can be based on both output and the signal. That is, there are no horizon problems or problems resulting from possible renegotiation of the contract.\footnote{Much of the earlier microeconomic literature has focused on problems that arise when the parties cannot commit to long-term contracts, that is, cannot commit not to renegotiate the contract at an interim stage; see Fudenberg et al. (1990) or Chiappori et al. (1994). In contrast, the related macroeconomic literature typically assumes the feasibility of long-term contracts without renegotiation, see e.g. Atkeson and Lucas (1995) or Kocherlakota (2005). Likewise, to many of those who study managerial incentives it seems natural that at least firms (though perhaps not managers) can commit to long-term contracts if it is ex ante in their interest to do so, cf. Dutta and Zhang (2002).} Moreover, the agent has unrestricted access to a credit market, and can save or borrow in the form of
risk-free debt at the same interest rate as the principal. The agent’s use of the credit market is not publicly observable.\footnote{As in the related literatures on repeated moral hazard and on consumption and saving (see references in text), I rule out access to insurance contracts beyond the one provided by the principal. This incompleteness of the external market can be motivated by the agent’s private information about his assets and his effort choice; for an endogenization, see Cole and Kocherlakota (2001).}

Based on the above discussion, an intuitive benchmark solution is a wage contract that is contingent on output but not the signal. The agent still relies on the signal in choosing his first-period consumption because the signal is informative of output and hence the agent’s future consumption possibilities. Moreover, if the agent’s first-period consumption depends on the signal, then his second period-consumption does too due to the agent’s budget constraint. However, the usefulness of the signal for the agent’s consumption decisions does not directly imply that the wage contract should make use of the signal.

The main result for each model that it is in fact optimal for the wage contract itself to be contingent on the signal. In the linear model, a better signal is associated with a higher wage for any given output. In the general model, the same holds on average (using the density of output conditional on the signal as averaging weights).

The intuition is that a purely output-based contract exposes the agent to too much risk in the second period and too little in the first. With an output-based contract, first-period consumption necessarily has a lower variance than second-period consumption because the first-period decision is based on a noisy signal (the variances would be the same only with perfect signals).

The key is that the overall risk can be reduced while preserving incentives for effort, leading to a Pareto-improvement for the two parties. Suppose that incentives are shifted marginally from output onto the signal in a way that leaves effort unchanged. The advantage of doing so is that the agent can spread income shocks related to the signal over both periods, whereas income shocks in output fall on second-period consumption (to the extent they were not anticipated through the signal). This advantage may appear to be offset by the fact that the signal is noisier than output, thus increasing the agent’s risk. However, the noisiness of the signal is already accounted for in the agent’s optimal saving decision: the noisier the signal, the \textit{less} first-period consumption varies. At the margin, starting from a purely output-based contract, the second offsetting effect vanishes entirely while the first effect, the ability to spread shocks of two periods, remains. It follows that it is optimal to shift incentives onto the signal to some extent.
In the linear model, the main result does not rely on any assumptions other than those mentioned. The model is as tractable and intuitive as static versions of the Holmström-Milgrom framework, and leads to closed-form solutions for everything. Although uninformative in the standard sense, the early signal exhibits all the features of an informative signal. In particular, using the signal in addition to output in the contract allows the principal to offer stronger incentives overall, which is a version of the standard risk-incentives tradeoff.

In the general model, the results rely mainly on the monotone likelihood ratio property (MLRP), which is also a standard assumption in static models. I also assume that the agent’s felicity function exhibits nonincreasing absolute risk aversion, or NIARA. NIARA is a very weak assumption and is frequently encountered in the literature on consumption and saving. It implies convex marginal utility, which in turn generates a precautionary saving motive (see Cole and Kocherlakota, 2001). Here, however, the results are driven only by risk aversion and not precautionary saving; NIARA merely serves to rule out odd effects that might override the normal implications of the MLRP.

While the main result is fundamentally an optimal-insurance result, it is inseparably linked to the moral-hazard problem. The reason for the principal to use the signal in the contract is that the signal is informative of output, and hence the agent’s wage, and hence his consumption possibilities over both periods. A critical link in this chain is that wages do in fact depend on output, which they must if there is a moral hazard problem. Conversely, without moral hazard, the principal can pay the agent a constant wage. But then, the realization of output is irrelevant to the agent, and hence the signal too, which means it is also irrelevant to the principal.

Overall, this paper suggests that time is an important dimension of performance measurement, and that the range of signals potentially useful for contracting extends well beyond the classic Informativeness Principle. While it is trivial to observe that “earlier is better” for any given performance measure, the result shown here is much stronger: any early signal that contains any information about future outcomes is useful for contracting purposes even if it is strictly less informative than later performance measures, simply because it influences the agent’s early consumption decisions.\(^4\) In Section 4, I discuss further implications of the results.

\(^4\) The notion of “timeliness” has a long tradition in the accounting literature on desirable features of accounting information, but has previously lacked an agency-theoretic foundation; cf. Lambert (2001, Sections 4.7 and 6).
2 A Model With Linear Contracts

In this section, I study a model that employs the framework of Holmström and Milgrom (1987, 1991) with linear contracts, exponential utility, and normally distributed noise terms. This framework is widely used in economics and related fields because of its tractability and easy interpretability. For models that include intertemporal consumption decisions, see Dutta and Reichelstein (1999) or Dutta and Zhang (2002).

Consider a principal and an agent who interact over two periods $t = 0, 1$. In $t = 0$, after signing a compensation contract, the agent chooses an effort level $a$ that results in a one-time output

$$y = \theta a + \varepsilon_y$$

in $t = 1$, where $\theta$ scales the productivity of effort, and $\varepsilon$ is a normally distributed noise term with a mean zero and variance $\sigma^2_y$. Shortly after exerting effort, still in $t = 0$, both principal and agent receive a signal $z$ about the output $y$. It takes the form

$$z = \theta a + \varepsilon_z,$$

where $\varepsilon_z = \varepsilon_y + \zeta$, and $\zeta$ is normally distributed with a mean zero and variance $\sigma^2_\zeta$. Thus, we have $z = y + \zeta$, which means that the signal is a mean-preserving spread of output, and hence is uninformative in the sense of Holmström (1979).

Both the output $y$ and the signal $z$ are publicly observable and verifiable. The principal can pay the agent a wage in $t = 0$ that depends on $z$, and a wage in $t = 1$ that depends on $y$ and $z$. Restricting attention to linear contracts, we are looking for optimal contracts within the class

$$w_0 = \alpha_0 + \beta_0 z \quad \text{and} \quad w_1 = \alpha_1 + \beta_1 z + \gamma y.$$  

I assume that both principal and agent can commit to a two-period contract characterized by $(\alpha_0, \beta_0, \alpha_1, \beta_1, \gamma)$ in $t = 0$. At the end of this section, I illustrate that this section’s results also carry over to the case in which the principal but not the agent can commit to a long-term contract.

For simplicity, neither principal nor agent discount payoffs between periods (the model of Section 3 allows for discounting). Both principal and agent have unrestricted access to a credit market and can borrow or save any amount they wish at zero interest in the form of risk-free bonds with full repayment liability on part of the bond seller. The agent has no access to any outside insurance market. As is known from e.g. Chiappori et al. (1994), we can without loss of generality assume that only the agent saves or borrows. The agent decides how much to save or
borrow, and thus how much to consume in \( t = 0 \), after observing the signal \( z \). Let \( b(z) \) denote the agent’s (positive or negative) net borrowing in \( t = 0 \) as a function of \( z \); only the agent knows \( b(z) \). Given the wages \( w_0 \) and \( w_1 \) and borrowing \( b \), the agent’s consumption in each period is given by \( c_0 = w_0 + b \) and \( c_1 = w_1 - b \).

The principal’s and agent’s access to the credit market allows us to normalize the set of contracts, as is known from Fudenberg et al. (1990). Since principal and agent can save or borrow at the same interest rate (here, zero), they are indifferent between all wage contracts that lead to the same net present value of wages for each realization of \( z \) and \( y \). All that matters therefore is the agent’s total salary \( \alpha = \alpha_0 + \alpha_1 \) and the total bonus for \( z \), namely \( \beta = \beta_0 + \beta_1 \). It then suffices to derive an optimal contract of the form \( w = \alpha + \beta z + \gamma y \). In the analysis, I assume this wage is paid in its entirety in \( t = 1 \), after realization of \( y \), which means that the agent needs to borrow \( b > 0 \) in order to consume in \( t = 0 \). However, the contract can be implemented equivalently in any other way that preserves its NPV.

The principal is risk-neutral; her net profit is simply \( y - w_1 - w_2 \). The agent is risk averse; his utility is given by a time-separable function of net consumption in each period \( t = 0, 1 \):

\[
U(c_0, c_1, a) = u(c_0 - d(a)) + u(c_1),
\]

where \( u(x) = -\exp[-rx] \), \( r \) is the agent’s constant absolute risk aversion coefficient, and \( d(a) = k_a a^2 \) is the agent’s disutility of effort expressed in monetary terms, with scaling parameter \( k \). The agent has an outside option in each period that gives her a (certainty equivalent) reservation wage of \( w \). The agent’s expected utility from working for the principal must therefore be at least \( 2u(w) \) over both periods.

The principal chooses contract parameters \( \alpha, \beta, \) and \( \gamma \) to maximize her net profit subject to the agent’s individual rationality and incentive compatibility constraints and his optimal borrowing decision. Formally, we have:

\[
\max_{\alpha, \beta, \gamma, a, b} E_{z,y} \left[ y - \alpha - \beta z - \gamma y \right] \quad \text{s.t.}
\]

\[
E_{z,y} \left[ u(b - ka^2/2) + u(\alpha + \beta z + \gamma y - b) \right] \geq 2u(w),
\]

\[
a \in \arg \max_{a'} E_{z,y} \left[ u(b - ka'^2/2) + u(\alpha + \beta z(a') + \gamma y(a') - b) \right],
\]

\[
b \in \arg \max_{b'} u(b' - ka'^2/2) + E_y \left[ u(\alpha + \beta z + \gamma y - b') \right].
\]

In (5)-(7), all expectations are taken over \( z \) and \( y \), which in effect means over the noise terms \( \varepsilon_y \) and \( \varepsilon_z \). In (8), the first-period utility is deterministic; the second-period utility is stochastic and expectations are taken over \( y \), conditional on the realized \( z \).
To state the optimal solution of (5)-(8), note that it follows from (1) and the definition of \( \zeta \) that the variance of \( \varepsilon \) is \( \sigma^2_y + \sigma^2_\zeta \), and the covariance between \( y \) and \( z \) is \( \sigma^2_y \). Define \( \varphi \) as the theoretical coefficient of a regression of \( y \) on \( z \):

\[
\varphi = \frac{\text{Cov}(y, z)}{\text{Var}(z)} = \frac{\sigma^2_y}{\sigma^2_y + \sigma^2_\zeta}.
\]

We can then show:

**Proposition 1** An optimal solution to the problem (5)-(8) is given by

\[
\alpha = w - \frac{(\beta + \gamma)^2 \theta^2}{2k} + \frac{r}{2} \left[ (1 - \varphi) \gamma^2 + \frac{(\beta + \varphi \gamma)^2}{2\varphi} \right] \sigma^2_y,
\]

\[
\beta = \frac{\varphi \theta^2}{(1 + \varphi) \theta^2 + k r \sigma^2_y} = \varphi \gamma,
\]

\[
\gamma = \frac{\theta^2}{(1 + \varphi) \theta^2 + k r \sigma^2_y}.
\]

The agent’s equilibrium choices are given by

\[
a = (\beta + \gamma) \frac{\theta}{k} \quad \text{and} \quad \quad b(z) = \frac{1}{2} \left[ \alpha + \gamma (1 - \varphi) \theta a + \frac{k}{2} a^2 - \frac{r}{2} \gamma^2 (1 - \varphi) \sigma^2_y + (\beta + \varphi \gamma) z \right].
\]

Proof: see the Appendix.

Deriving Proposition 1 involves a few steps (including computing the agent’s optimal borrowing as function of \( z \)) but is conceptually standard. The most significant aspect of Proposition 1 is that \( \beta \) is positive and not zero, even though \( z \) is uninformative of effort, given \( y \). This means that a better signal leads to a higher wage for any given level of output.

To understand this result, it is helpful to consider the principal’s tradeoff between output- and signal-based incentives while holding incentives for effort fixed.\(^5\) As the proof shows, the total surplus between principal and agent is given by

\[
\theta a - \frac{k}{2} a^2 - \frac{r}{2} \left[ \frac{(\beta + \varphi \gamma)^2}{2\varphi} \right] \sigma^2_y,
\]

where the first term is expected output, the second is the agent’s disutility, and the third is a risk premium which depends on the variance of the agent’s first-and second-period consumption. Since the agent’s effort \( a \) is proportional to \( \beta + \gamma \) in equilibrium, suppose we fix the sum \( \delta \equiv \beta + \gamma \)

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\(^5\) I would like to thank Romans Pancs for suggesting this analysis.
and then consider the choice between $\beta$ and $\gamma = \delta - \beta$, with $\beta = 0$ as the benchmark solution. The variance in the third term of (10) then becomes (omitting the scaling factor $\sigma_y^2$)

$$
\frac{(\beta + \varphi \gamma)^2}{2\varphi} + (1 - \varphi)\gamma^2 = \frac{\varphi \delta + (1 - \varphi)\beta^2}{2\varphi} + (1 - \varphi)(\delta - \beta)^2.
$$

(11)

The second term in (11) is the residual variance of the agent’s second term consumption, conditional on knowing the realization of $z$. The first term is the variance of the agent’s first-period consumption. It is scaled by $1/\phi > 1$ which accounts for the noisiness of the signal. It is also scaled by $1/2$, which accounts for the fact that shocks in $z$ are optimally spread over both periods. It is this latter effect that drives the result, which can be seen by evaluating the derivative of (11) with respect to $\beta$ at $\beta = 0$: starting from an output-based contract ($\beta = 0$), increasing $\beta$ marginally increases the variance of first-period consumption by only half as much as it decreases the variance of second-period consumption, leading to an unambiguous decrease in risk. This decrease does not push $\beta$ all the way towards $\delta$ because $z$ is strictly noisier than $y$. At $\beta = 0$, however, the noisiness of $z$ ($\phi$ in the denominator in (11)) is already fully absorbed in the agent’s optimal borrowing decision ($\phi \delta$ term in the numerator (11)).

When we optimize over the strength of incentives $\delta$ as well, a standard risk-incentives trade-off kicks in: by shifting incentives partly onto $z$ and reducing the agent’s income risk, the principal can also provide stronger incentives overall while satisfying the agent’s participation constraint. To see this, suppose $\beta$ is constrained to be zero. The optimal coefficient $\gamma$ (obtained by maximizing (42) in the Appendix with respect to $\gamma$ while $\beta = 0$) then is

$$
\gamma_0 = \frac{2\theta^2}{2\theta^2 + (2 - \varphi)k\sigma_y^2}.
$$

(12)

It is straightforward to show that the sum $\beta + \gamma$ in Proposition 1 exceeds $\gamma_0$ in (12). Since the agent’s equilibrium effort in the two scenarios is proportional to $\beta + \gamma$ and $\gamma_0$, respectively, it follows that the agent’s effort, and hence expected output too, are higher if the wage contract is based on $z$ in addition to $y$.

The parameters $k$, $r$ and $\sigma_y^2$ affect both $\beta$ and $\gamma$ in the usual way. The signal $z$ thus has the same features as an informative signal in the standard static model. Although $z$ is not informative of the agent’s effort, it is informative of the agent’s future consumption possibilities, which, as we have seen, renders it useful as performance measure.

Changes in $\sigma_z^2$, the variance of the signal noise $\varepsilon_z | \varepsilon_y$ (see (2)), also have implications familiar from standard models: the smaller $\sigma_z^2$, the larger is $\phi$ according to (9), and therefore the larger $\beta$ and the smaller $\gamma$ according to Proposition 1. The agent’s equilibrium effort is decreasing in
\(\sigma^2_\xi (\text{increasing in } \phi)\) because the sum \(\beta + \gamma\) is increasing in \(\phi\).\(^6\)

As mentioned, one can implement the contract according to Proposition 1 in many ways. One option is to choose \(w_0\) and \(w_1\) such that the agent neither saves nor borrows, which means that in effect that the principal acts as the agent’s bank. Simply set \(w_0 = b(z)\) according to Proposition 1, and then set \(w_1 = \alpha + \beta z + \gamma y - w_0(z)\). These are still linear wage functions because \(b(z)\) is linear too. As it turns out, this specification eliminates \(z\) from the second-period wage function. To see this, note that according to Proposition 1, \(\partial b(z)/\partial z = (\beta + \phi \gamma)/2\), which because of \(\beta = \phi \gamma\) equals \(\beta\) itself. This in turn means that the second-period wage depends on \(y\) but not on \(z\): the agent still faces income risk because of \(y\), but after the agent has determined his optimal first-period consumption based on \(z\), he is no longer exposed to any risk associated with \(z\).

Under the normalization \(b(z) = 0\), the contract of Proposition 1 can potentially be empirically distinguished from a pure output-based contract where \(w_0(z)\) is simply an advance loan. Under the optimal contract, as argued above, \(w_1\) depends on \(y\) but not on \(z\). In contrast, under an output-based contract with \(\beta = 0\) and \(\gamma_0\) as given by (12), the second-period wage \(w_1 = \alpha + \beta z + \gamma y - w_0(z)\) is negatively related to \(z\), which is straightforward to verify by plugging in the expressions for \(\gamma\) and for \(w_0(z) = b(z)\) from the Proposition. It follows that a regression of the second-period wage on output and the early signal should yield a zero coefficient for the early signal under the optimal contract, but a negative coefficient under the output-based contract.\(^7\)

A different implementation of the optimal contract serves to show that if the agent could not commit to a two-period contract, the outcome would still be the same. Suppose the contract is designed such that the agent’s expected wage in \(t = 1\), evaluated in \(t = 0\) conditional on \(z\), does not depend on the realization of \(z\). To accomplish this, note that at \(t = 0\) the expected output is given by \(E[y|z] = \varphi z + (1 - \varphi)\theta a\) as a result of standard Bayesian updating. By setting \(\beta_1 = -\phi \gamma\) in (3), one then obtains a wage function \(w_1 = \alpha_1 + \gamma (y - \varphi z)\) with the feature that \(E[w_1|z]\) does not depend on \(z\) because the \(\varphi z\) terms in it cancel out. This contract is clearly optimal under the assumption of commitment as it is just an implementation of the contract of Proposition 1.

\(^6\)In the limit \(\sigma^2_\xi = 0\) or \(\phi = 1\), one might intuitively expect to arrive at a contract with full weight on \(\beta\) and with \(\gamma = 0\). But since \(y\) and \(z\) are identical in this limit, and since and the timing of payments never matters, any contract with the same \(\beta + \gamma\) leads to the same outcome. The limit implied by Proposition 1, namely \(\beta = \gamma = \theta^2/[2\theta^2 + kr\sigma^2_\eta]\), is therefore a correct solution.

\(^7\) Because of the correlation between \(z\) and \(y\), a regression of \(w_1\) on \(z\) alone should yield a positive coefficient in both cases.
The same solution also works even if the agent cannot commit to a two-period contract. As by construction the agent’s continuation utility does not depend on \( z \), all one needs to do is to choose \( \alpha_1 \) large enough to satisfy the agent’s second-period participation constraint, which imposes no further cost because it can be balanced by setting \( \alpha_0 = \alpha - \alpha_1 \) with \( \alpha \) according to Proposition 1. Thus, the agent’s inability to commit to a long-term contract does not lead to a short-term contract but instead a long-term contract designed to get the agent to stay voluntarily! In Raith (2008), I show that this solution is in fact the unique optimal optimal when the agent cannot commit to a two-period contract.

3 General Analysis

I now study a general model that contains the model of Section 2 as special case. It combines a static moral-hazard model in the tradition of Mirrlees (1999) and Holmström (1979) with a dynamic consumption problem as it is considered in the macroeconomic literature, see e.g. Aiyagari (1994) or Cole and Khocherlakota (2001). For a similar setup without early signal, see Abraham and Pavoni (2008).

3.1 Model

The general setup of the model and the timing is the same as in the previous section. A principal and an agent interact over two periods \( t = 0, 1 \). In \( t = 0 \), after signing a compensation contract the agent chooses an effort level \( a \geq a \). The agent’s effort results in a stochastic output \( y \) in \( t = 1 \) that belongs to the principal. The agent consumes in both periods; I specify his utility function further below.

The output \( y \) is drawn from the set \( Y = [\underline{y}, \overline{y}] \subset \mathbb{R} \equiv \mathbb{R} \cup \{-\infty, \infty\} \). The distribution of \( y \) depends on the agent’s effort \( a \); specifically, the distribution of \( y \) conditional on \( a \) has a cumulative distribution function (c.d.f.) \( P(y|a) \) and a density function \( p(y|a) \). I assume that \( p \) is twice differentiable in both \( y \) and \( a \), and satisfies the Monotone Likelihood Ratio Property (MLRP), that is,

\[
\frac{d}{dy} \left( \frac{p_a(y|a)}{p(y|a)} \right) \geq 0.
\]

Shortly after exerting effort, still in \( t = 0 \), both principal and agent receive a signal \( z \in Z \) about the output \( y \), where \( Z = [\underline{z}, \overline{z}] \subset \mathbb{R} \). It is intuitively clear that conditioning the agent’s wage on \( z \) in addition to \( y \) is optimal if \( z \) provides incremental information about the agent’s effort. To rule out this uninteresting case, I assume that \( z \) is uninformative in the sense of
Holmström (1979). That is, $y$ is a sufficient statistic for $(y, z)$ with respect to $a$. Formally, this means that there exists a function $q : Y \times Z \rightarrow \mathbb{R}_+$ such that for all $(y, a)$, the density $p(y|a, z)$ can be factorized according to
\[
p(y|a, z) = q(y, z)p(y|a).
\]
(13)

I assume that $q$ is continuous and differentiable in both $y$ and $z$, and that $\int_{\mathbb{R}} q(y, z)dz = 1$.

I assume that for any two signals $z > z'$, the signal $z$ is a “better” signal in the sense that $z$ corresponds to a first-order stochastic dominant shift of $y|a$ relative to $z'$.

More formally, denote by $r(y, z|a)$ the density of $y$ conditional on $a$, that is,
\[
r(y, z|a) = \frac{p(y|a)q(y, z)}{\int_{\mathbb{R}} p(y'|a)q(y', z)dy'},
\]
and let $R(y, z|a) = \int_{\mathbb{R}} r(y', z)dy'$ denote its c.d.f. I then assume that for any $z, z' \in Z$,
\[
z > z' \Rightarrow R(y, z|a) \leq R(y, z'|a) \quad \forall \ y \in Y,
\]
and
\[
R(y, z|a) < R(y, z'|a) \quad \text{for a positive measure of } y \in Y.
\]
(14)

The following is a well-known result from utility theory, adapted to this model:

**Lemma 1** Consider any $z, z' \in Z$ with $z > z'$ and some mapping $x : Y \rightarrow \mathbb{R}$ that is continuous and differentiable in $y$. Then if $x$ is weakly increasing in $y$, then $E[x(y)|z] \geq E[x(y)|z']$, and
\[
\int_{\mathbb{R}} r_z(y, z)x(y)dy \geq 0.
\]
(15)

These inequalities are strict if $x$ is strictly increasing, and the opposite results hold if $x$ is weakly or strictly decreasing.

Proof: see the Appendix.

Both $y$ and $z$ are publicly observable and verifiable. The principal can pay the agent a wage in $t = 0$ that depends on $z$, and a wage in $t = 1$ that depends on $y$ and $z$. A general wage contract is thus characterized by $w = (w_0, w_1)$, where $w_0 : Z \rightarrow \mathbb{R}$ and $w_1 : Y \times Z \rightarrow \mathbb{R}$. Both players can commit to such a two-period contract.

Both principal and agent have unrestricted access to a credit market and can borrow or save any amount they wish in $t = 0$ at an interest rate $r$ in the form of risk-free bonds. Both parties

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8 This imposes much more structure on the signal set $Z$ than is required for the results. More generally, it would suffice to assume that $Z$ is infinite and that the marginal distribution of $(y, z)$ over $Z$ is atomless. The main result (Proposition 4) could then be stated for any two signals that happen to be ordered by first-order stochastic dominance.
therefore discount monetary amounts that accrue in \( t = 1 \) by the same factor \( \delta = 1/(1 + r) \). Again, we can assume without loss of generality that only the agent saves or borrows, which he does in \( t = 0 \) after observing the signal \( z \). Let \( \delta b(z) \) denote the agent’s net borrowing in \( t = 0 \) as a function of the signal \( z \), which means he has to repay \( b(z) \) in \( t = 1 \).

The principal is risk-neutral; her net profit is the expected net present value of output minus the agent’s wage. In computing the net present value, we can make use of (13), that is, the fact that the distribution of signals \( z \) depends only on \( y \) but not directly on \( a \):

\[
V(a, w) = \int_y^{\bar{y}} \int_z^{\bar{z}} \left[ -w_0(z) + \delta(y - w_1(y, z)) \right] q(y, z) p(y|a) \, dz \, dy. \tag{16}
\]

The agent maximizes a time-separable utility function

\[
U(c_0, c_1, a) = u(c_0, a) + \beta u_1(c_1),
\]

where \( c_t \) is the agent’s consumption in \( t = 0, 1 \), \( u : C \times [a, \infty) \to \mathbb{R} \) is a function of consumption and effort, \( C \subset \mathbb{R} \) is a closed real interval with \( \min C =: \underline{c} \), \( u_1(c) \) is defined as \( u(c, a) \) and \( \beta \geq 0 \) is a subjective discount factor. The function \( u(c, a) \) is thrice continuously differentiable. It is strictly increasing and strictly concave in \( c \), and strictly decreasing and weakly concave in \( a \). The cross derivative \( u_{ca}(c, a) \) may be positive, negative, or zero. This specification encompasses a wide range of more particular cases, such as an additively separable utility of the form \( u(c, a) = u(c) - d(a) \), or the model in Section 2 with “monetary” disutility.

I assume that the agent exhibits non-increasing absolute risk aversion (NIARA), which means that \(-u_{cc}(c, a)/u_c(c, a)\) is non-increasing in \( c \). Likewise, I assume that \(-u_{ca}(c, a)/u_c(a, c)\) is non-increasing in \( c \). The latter assumption already follows from NIARA if \( u(c, a) \) takes the form \( \phi(a) + \psi(a)v(c) \) (as in Grossman and Hart, 1983); that is, if \( u \) is either multiplicatively or additively separable in consumption and effort, as is the case in most models. NIARA is a very weak assumption that covers many standard utility functions used in theoretical models (CARA, DARA, CRRA) and is consistent with most empirical evidence on risk preferences.

NIARA is also closely related to the notion of prudence \((u_{ccc} > 0\), or convex marginal utility\), which plays a central role in the study of precautionary saving. More precisely, given differentiability, NIARA translates into \( u_{ccc}u_c \geq (u_c)^2 \) or

\[
\frac{u_{ccc}}{u_c} \geq \frac{u_{cc}}{u_c}. \tag{17}
\]

This means that NIARA is equivalent to the condition that the agent’s absolute degree of prudence in the sense of Kimball (1990) (the term on the left-hand side of (17)) is greater than or equal to the agent’s absolute degree of risk aversion.
Since utility is monotonic in consumption, we have $c_0 = \max\{w_0(z) + \delta b(z), \underline{c}\}$ and $c_1 = \max\{w_1(y, z) - b(z), \underline{c}\}$. For the agent to be solvent in every state of the world therefore requires the "natural" borrowing constraint $b(z) \in B(z, w) := [c/\delta, \min_y\{w_1(y, z)\} - \underline{c}]$, cf. Aiyagari (1994). We can then express the agent’s utility as a function of his choices $a$ and $b(z)$ as follows:

$$U(a, b(z), w) = \int_y \int_z [u(w_0(z) + \delta b(z), a) + \beta u_1(w_1(y, z) - b(z))] q(y, z) p(y|a)dzdy. \quad (18)$$

Denote the agent’s reservation utility for the two-period time horizon by $U > -\infty$.

As in Section 2, we can normalize the set of contracts. As the principal and the agent can save or borrow at the same interest rate $r$, they are indifferent between all wage contracts that lead to the same net present value of wages for each realization of $z$ and $y$. In this model, it is convenient to normalize wage contracts by setting $b(z) = 0$ for all $z \in Z$, and imposing a zero-borrowing incentive constraint for the agent. This means that the principal in effect acts as the agent’s bank.

Naturally, the agent’s desire to smooth consumption over both periods will lead to an optimal wage payment $w_0(z)$ that depends on the realized signal. But this payment might simply be an advance loan to the agent that is subsequently subtracted from the second-period wage. Define the firm’s total (NPV) payment to the agent as

$$\bar{w}(y, z) = w_0(z) + \delta w_1(y, z).$$

Below I examine whether and how $\bar{w}(y, z)$ depends on $z$ if $z$ is uninformative.

The principal chooses the contract $w$ to solve

$$\max_{w, b(z), a} V(a, w) \quad \text{s.t.} \quad U(a, b(.), w) \geq U, \quad \text{and} \quad (a, b(z)) = (a, 0) \in \arg \max_{\hat{a}, \hat{b}(z)} U(\hat{a}, \hat{b}(z), w) \quad \text{s.t.} \quad a \geq 0 \quad \text{and} \quad \hat{b}(z) \in B(z, w) \forall z \in Z, \quad (19)$$

where (20) is the agent’s participation constraint, and (21) is the agent’s incentive constraint with respect to his effort choice and his subsequent borrowing decision, including the normalization $b(z) = 0$ for all $z$. While effort and optimal saving are chosen sequentially by the agent, they can be represented as simultaneous choices in (21) by expressing $b$ as a function of $z$.

I assume that the problem (19)-(21) has a unique interior solution (with $a > a$ and $c_0, c_1 > \underline{c}$ in all states of the world) that can be obtained by replacing (21) with the relevant first-order conditions (that is, the first-order approach is valid). Examples satisfying these assumptions are easy to find. As is well known from the conventional static moral-hazard model, however,
simple sufficient conditions for the validity of the first-order approach are notoriously difficult to identify, see Rogerson (1985b) and Jewitt (1988). As my goal in this paper is to characterize the role of an early signal for an optimal contract, I take existence and uniqueness as given.

I also assume that at the optimal solution, all relevant second-order conditions are strictly satisfied. This is a purely technical assumption that enables me to use the implicit function theorem to derive the main characterization result. I rely on the implicit function theorem instead of lattice-theoretic methods because the relevant objective function is not supermodular, as we shall see.

### 3.2 Characterization of the Optimal Contract

The incentive constraint (21) leads to the following first-order condition for the choice of $a$:

$$\int_{\bar{y}}^{\bar{y}} \int_{\bar{z}}^{\bar{z}} \{ [u(w_0(z), a) + \beta u_1(w_1(y, z))] q(y, z)p_a(y|a) + u_a(w_0(z), a)q(y, z)p(y|a) \} \, dz \, dy = 0.$$  \hfill (22)

Moreover, for $b = 0$ to be the optimal solution to (21) for any signal $z$ requires

$$\int_{\bar{y}}^{\bar{y}} \left[ \delta u_c(w_0(z), a) + \beta u_1'(w_1(y, z)) \right] q(y, z)p(y|a) \, dy = 0 \quad \forall \, z,$$  \hfill (23)

which is the agent’s Euler equation (cf. Hall 1978). The relaxed version of the problem (19)-(21), using the first-order approach, is

$$\max_{w, b(z), a} V(a, w) \quad \text{s.t.} \quad (20), (22), (23).$$  \hfill (24)

The corresponding Lagrangian is

$$L(w_0, w_1, a) = \int_{\bar{y}}^{\bar{y}} \int_{\bar{z}}^{\bar{z}} \left[ -w_0(z) + \delta(y - w_1(y, z)) \right] q(y, z)p(y|a) \, dz \, dy$$

$$+ \lambda \left\{ \int_{\bar{y}}^{\bar{y}} \int_{\bar{z}}^{\bar{z}} \left[ u(w_0(z), a) + \beta u_1(w_1(y, z)) \right] q(y, z)p(y|a) \, dz \, dy - U \right\}$$

$$+ \mu \int_{\bar{y}}^{\bar{y}} \int_{\bar{z}}^{\bar{z}} \left\{ [u(w_0(z), a) + \beta u_1(w_1(y, z))] q(y, z)p_a(y|a) + u_a(w_0(z), a)q(y, z)p(y|a) \right\} \, dz \, dy$$

$$+ \int_{\bar{z}}^{\bar{z}} \xi(z) \int_{\bar{y}}^{\bar{y}} \left\{ [\delta u_c(w_0(z), a) + \beta u_1'(w_1(y, z))] q(y, z)p(y|a) \right\} \, dy \, dz,$$  \hfill (25)

where $\lambda$ and $\mu$ are the multipliers associated with the participation and effort incentive constraints, respectively, and $\xi(z)$ is a continuum of multipliers for the agent’s Euler equation.

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9 In the present context with intertemporal consumption, Abraham and Pavoni (2008) show that in addition to conditions familiar from the earlier literature, NIARA and a Frisch elasticity of leisure of less than one together ensure that the agent’s problem (21) is globally concave.
corresponding to each signal \( z \). It is convenient to express the Lagrangian (25) in nested form as follows. Denote by \( \bar{q}(z|a) = \int_y^0 q(y,z)p(y|a) \) the density of any signal \( z \). Then we have

\[
L(w_0, w_1, a) = \int_z z \bar{q}(z|a)L_z(w_0, w_1, a)dz,
\]

where

\[
L_z(w_0, w_1, a) = -w_0(z) + \lambda u(w_0(z), a) + \mu \left\{ \int_y^0 \frac{p_a(y|a)}{p(y|a)} r(y,z)dy \right\} u(w_0(z), a) + u_a(w_0(z), a)
\]

\[+ \xi(z)\delta u'(w_0(z), a) + \beta \int_y^0 r(y,z)L_{yz}(w_1, a)
\]

and

\[
L_{yz}(w_1, a) = -\delta \beta w_1(y,z) + \lambda u_1(w_1(y,z)) + \mu \frac{p_a(y|a)}{p(y|a)} u_1(w_1(y,z)) - \xi(z)u_1'(w_1(y,z)).
\]

As is apparent from this nested Lagrangian, to find the optimal second-period wage function \( w_1(y,z) \) one needs to maximize each \( L_{yz} \) pointwise. The first-period wage \( w_0(z) \) is chosen to maximize \( L_z \) subject to the Euler equation (23); the first-order conditions also yield the multiplier \( \xi(z) \). As each signal \( z \) and its associated wages \( w_0(z) \) and \( w_1(y,z) \) have zero mass in the overall Lagrangian \( L \), they are found by taking \( a, \lambda \) and \( \mu \) as given. The latter three variables are chosen to satisfy the first-order conditions for maximization of \( L \).

The nested structure means that for given \( a, \lambda \) and \( \mu \), we can determine \( w_0(z), w_1(y,z) \) and \( \xi(z) \) for each \( z \) separately. They are determined by the following system (I will drop \( z \) as argument or subscript where no confusion is possible). From (23), we have

\[
f(z) = \delta u_c(w_0, a) - \beta \int_y^0 u_1'(w_1(y,z))r(y,z)dy = 0.
\]

Next, the condition \( \partial L_z/\partial w_0(z) = 0 \) can be written as

\[
g(z) - \lambda u_c(w_0, a) + \mu \left[ \int_y^0 \frac{p_a(y|a)}{p(y|a)} r(y,z)dy u_c(w_0, a) + u_a(w_0, a) \right] + \xi \delta u_{cc}(w_0, a) - 1 = 0.
\]

Finally, \( \partial L_{yz}/\partial w_1(y,z) = 0 \) leads to

\[
h(y,z) = -\lambda u_1'(w_1) + \mu \frac{p_a(y|a)}{p(y|a)} u_1'(w_1) - \xi u_1''(w_1) - \frac{\delta}{\beta} = 0
\]

for all \( y \), or equivalently,

\[
\frac{\delta}{\beta u_1'(w_1)} = \lambda + \mu \frac{p_a(y|a)}{p(y|a)} - \xi \frac{u_1''(w_1)}{u_1'(w_1)}.
\]

Conditions (28)-(30) along with (22) imply
Proposition 2 For the problem (24) with the Lagrangian (25), we have \( \mu > 0 \) and \( \xi(z) \geq 0 \) for all \( z \).

Proof: see the Appendix.

The proof first establishes \( \xi(z) \geq 0 \), which is similar to Rogerson’s (1985a) result for a repeated-moral hazard problem: if the agent did not have access to credit, the principal would optimally “front-load” the agent’s wage, meaning that the agent would want to save (not borrow) if he could. In the model here, the same holds, to the effect that lack of access to credit would lead to a wage contract for which the left-hand side of (23) is negative; hence \( \xi \geq 0 \) if saving is possible. The arguments that lead to \( \mu > 0 \) are largely standard but also rely on \( \xi(z) \geq 0 \).

Equation (31) in conjunction with NIARA utility and MLRP immediately leads to the following extension of a result due to Abraham and Pavoni (2008):

Proposition 3 The optimal second-period wage \( w_1(y, z) \) is non-decreasing in \( y \) for any \( z \in Z \).

Proof: As is shown in Proposition 2, the multipliers \( \mu \) and \( \xi(z) \) are nonnegative. Rewrite (31) as

\[
\frac{\delta}{\partial w'_1(w_1)} + \xi \frac{u''_1(w_1)}{u'_1(w_1)} = \lambda + \mu \frac{p_a(y|a)}{p(y|a)}.
\]  

(32)

The right-hand side is non-decreasing in \( y \) due to MLRP. The left-hand side is increasing in \( w_1 \) due to concavity of \( u_1 \) (first term) and NIARA (second term). It follows that in order to satisfy (32), \( w_1(y, z) \) must be non-decreasing in \( y \).

My main interest, however, is in examining whether and how the solution to the system (28)-(30) varies with \( z \). To be able to do so using conventional methods of comparative statics, I assume that the relevant second-order conditions are strictly satisfied. I will show in the next result that for \( w_1(y, z) \), the second-order condition

\[ h_1(y, z) \equiv \partial h(y, z)/\partial w_1(y, z) = \partial^2 L_{yz}/\partial w^2_1 < 0 \]

already follows from other assumptions. In contrast, the sign of \( g_0(z) \equiv \partial g(z)/\partial w_0(z) = \partial^2 L_z/\partial w^2_0 \) is indeterminate because \( \xi \geq 0 \) and \( u_{ccc}(w_0, a) > 0 \). However, the second-order
condition for \( w_0(z) \) does not require \( g_0(z) < 0 \); it only requires that \( L_z \) be concave with respect to deviations in \( w_0 \) that satisfy \( f = 0 \) (cf. (28)) as well. In other words, what we need is that any second variation in \( w_0 \) and simultaneously \( w_1(y) \) satisfying \( f = 0 \) is negative in a neighborhood of the optimal contract. The relevant Legendre condition leads to the following result.

**Lemma 2** For \( L_z \) to be strictly concave in \( w_0 \) subject to the constraint (28) requires that

\[
 g_\xi^2 - g_0\eta > 0, \tag{33}
\]

where

\[
g_\xi \equiv \frac{\partial g}{\partial \xi} = u_{cc}(w_0, a), \quad \eta \equiv -\beta \int_y^\bar{y} \left( \frac{h_\xi}{h_1} \right)^2 r(y, z) dy, \quad \text{and} \quad h_{xi}(y) \equiv \frac{\partial h(y)}{\partial \xi} = -u''(w_1).
\]

Proof: see the Appendix.

As mentioned, the main question of interest is how the total wage \( \bar{w}(y, z) = w_0(z) + \delta w_1(y, z) \) varies with \( z \). In the linear model, we obtained the very strong result that if \( z_1 > z_0 \), then \( \bar{w}(y, z_1) > \bar{w}(y, z_0) \) for every \( y \). This strong result no longer holds in the general case. In fact, not even the ex-ante expected wage,

\[
\int_y^\bar{y} \bar{w}(y, z)p(y|a)dy,
\]

is necessarily increasing in \( z \) (a numerical example is available upon request). However, it is still true that \( z_1 > z_0 \) implies \( \bar{w}(y, z_1) > \bar{w}(y, z_0) \) on average for the distribution \( y|z_1 \), which can be interpreted as follows.

Since the signals are ordered by stochastic dominance, one would intuitively expect that if \( z_1 > z_0 \), then the expected total wage conditional on \( z_1 \) exceeds that for \( z_0 \); that is, \( E[\bar{w}(y, z_1)|z_1] > E[\bar{w}(y, z_0)|z_0] \). This difference in expected wages can be decomposed as follows:

\[
E[\bar{w}(y, z_1)|z_1] - E[\bar{w}(y, z_0)|z_0] = \int_y^\bar{y} \bar{w}(y, z_1)r(y, z_1)dy - \int_y^\bar{y} \bar{w}(y, z_0)r(y, z_0)dy
\]

\[
= \int_y^\bar{y} [\bar{w}(y, z_1) - \bar{w}(y, z_0)]r(y, z_1)dy + \int_y^\bar{y} \bar{w}(y, z_0)[r(y, z_1) - r(y, z_0)]dy. \tag{34}
\]

The second term in (34) is clearly non-negative because of Lemma 1 and Proposition 3 (and strictly positive if the MLRP holds as a strict inequality). In words, holding the wage function fixed, the expected total wage conditional on \( z_1 \) is higher than for \( z_0 \) simply because \( z_1 \) corresponds to a “better” distribution over \( y \).

The main result of this section is that the first term in (34), too, is non-negative (and in general strictly positive):
Proposition 4 If \( z_1, z_0 \in Z \) with \( z_1 > z_0 \), then (a) \( w_0(z_1) \geq w_0(z_0) \) and (b)

\[
\int_y [\bar{w}(y, z_1) - \bar{w}(y, z_0)] r(y, z_1) dy \geq 0.
\]

The inequalities in (a) and (b) are strict if the MLRP holds as a strict inequality.

Proof: see the Appendix.

Part (a) seems intuitive: a higher signal corresponds to a higher expected total wage, which in turn means that it is optimal for the agent (and thus for the principal acting as the agent’s bank) to consume more in the first period as well. This would be clear if the total wage depended only on output and not on the signal. Given that it does, though, proving part (a) requires examining the whole system (28)-(30).

The main result is part (b): the total wage payment \( \bar{w}(y, z) \) itself is on average higher for better signals. The intuition is the one given in the Introduction and in Section 2: holding effort incentives constant, the principal can shift incentives to some extent from \( y \) to \( z \) in a way that lowers the agent’s overall consumption risk by decreasing second-period risk by more than first-period risk increases.

That this result takes a less stark form than in Section 2 should not come as a surprise. There, we obtained a very strong result because contracts were restricted to be linear (and thus by definition monotonic in \( z \)) and because effort depends simply on the sum \( \beta + \gamma \). Here, shifting risk between the two periods while preserving effort incentives takes a more complicated form because the agent’s effort incentives depend on the likelihood ratios in (29) and (30). In particular, monotonicity of \( \bar{w}(y, z) \) in \( z \) no longer holds for each individual \( y \). Nevertheless, the logic of the result is the same as in the linear model.

The above results, in conjunction with Rogerson’s (1985a), suggest that the agent’s access to credit and the availability of a contractible early signal have opposite consequences for the principal. We know from Rogerson (1985a) that without access to credit, optimal wages in a repeated moral hazard context satisfy an “inverse” Euler equation (akin to condition (45), but with \( \xi \) set to zero), whereas with access, the agent’s Euler equation as constraint imposes a cost on the principal; see also Abraham and Pavoni (2008). The same holds in the presence of a contractible early signal, as Proposition 2 establishes.

In contrast, the contractibility of an early, noisy signal always makes the principal better off. For the case in which the agent has access to credit, this follows directly from Proposition 4, according to which the optimal contract depends on the signal. The same, however, also holds if the agent does not have access to credit: the no-access case emerges as special case of the
analysis if all $\xi(z)$ are set to zero, in which case Proposition 4 still holds because its proof only relies on nonnegative multipliers $\xi(z)$. In this sense, my results are orthogonal to Rogerson’s.\(^{10}\)

4 Conclusion and Discussion

Holmström’s (1979) sufficient-statistic result is the cornerstone of the theory of incentive contracting. It states that an optimal incentive contract should make use of a signal if and only if it provides incremental information about the agent’s effort.

The standard theory envisions performance measurement as a one-time event. In reality, however, information about the consequences of an agent’s actions often arrives over time, implying that later performance measures are often more accurate than earlier ones. This paper examines incentive contracting in a setting where both early but noisy and later but more accurate measures of performance are available.

I show that introducing time, and more specifically, an intertemporal consumption decision for the agent, enlarges the set of useful performance measures relative to Holmström’s result under very weak assumptions. Specifically, I show that even if an early signal is strictly noisier than a later performance measure, it is still optimal to condition the wage contract on the signal as long as the agent relies on it in making early consumption decisions.

The results suggest that the time at which a performance measure is generated is a quality that is distinct from those known from standard theory, such as variance and correlation with the principal’s payoff. The virtue of evaluating performance early, that is in “timely” fashion, has long been emphasized in the accounting literature, but has previously not been examined formally.

Recognizing the time dimension of performance measurement helps to understand the use of complex measures such as stock returns in managerial incentive contracts, as opposed to simple measures such as current profits. Stock prices, for instance, incorporate information about future

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\(^{10}\) I have assumed that an early signal exists and explored the benefits of its use in a contract. A different question is whether the existence of an early signal makes the principal better or worse off even if the signal is not contractible but the agent uses it for his saving decision. The answer is not obvious a priori: a standard risk-incentives argument would suggest an improvement, but from Rogerson (1985a) we also know that it can make the principal worse off if the agent is better able to smooth consumption over time. I have not pursued this question in the general model. For the linear model, it can be shown that the observability of an early signal leads to stronger incentives and a higher payoff for the principal even if the signal is not contractible. Thus, in that model at least the principal is better off if an early signal is observable, and still better off if it is contractible.
firm profits, but in very noisy fashion. Waiting for some of those profits to materialize and a new price to form surely provides more accurate information about a manager’s present actions. As I have shown, however, even if the manager’s time horizon is long enough, it is optimal to base the manager’s compensation on today’s stock price simply because the information contained in it guides the manager’s current consumption decisions.

Understanding the tradeoff between time and accuracy in performance measurement also helps to explain the economic origins of accounting rules and procedures by which cash flows are transformed into accounting numbers such as earnings. Research over the past ten years has shown how practices concerning the allocation of sunk investment costs over time can be interpreted as optimal solutions to agency problems, see Rogerson (2008). Less well understood economically are the rules and practices by which future gains and losses are incorporated into today’s earnings as “accruals”. By definition, such accruals are based on early information about yet-to-be-realized outcomes. In a companion paper (Raith 2008), I argue that accounting practices related to the recognition of future revenue as earnings can be interpreted as mimicking an optimal incentive contract in a setting similar to the one studied here.

Appendix: Proofs

Proof of Proposition 1: The distribution of $y$ conditional on $z$ is normal with mean

$$E[y|z] = \theta a + \varphi \varepsilon z = \varphi z + (1 - \varphi) \theta a$$

and variance

$$\text{Var}(y|z) = \sigma^2_y - \text{Cov}(y, z) \frac{1}{\text{Var}(z)} \text{Cov}(y, z) = \sigma^2_y - \frac{\sigma^4_y}{\sigma^2_y + \sigma^2_\zeta} = (1 - \varphi) \sigma^2_y.$$ 

It follows that conditional on $z$, the agent’s consumption in $t = 1$, namely $w - b = \alpha + \beta z + \gamma y - b$, is normally distributed conditional on $z$ and has a certainty equivalent of

$$CE_2(b) = \alpha + \beta z + \gamma E(y|z) - b - \frac{r}{2} \gamma^2 \text{Var}(y|z)$$

$$= \alpha + \beta z + \gamma [\varphi z + (1 - \varphi) \theta a] - b - \frac{r}{2} \gamma^2 (1 - \varphi) \sigma^2_y. \quad (35)$$

At the end of the first period, the agent decides how much to borrow by solving (8). The first-order condition for utility maximization with respect to $b$ is given by

$$u' \left( b - \frac{k}{2} a^2 \right) = E_y[u'(w - b)]. \quad (36)$$
Since \( u'(x) = -ru(x) \), equating the expected marginal utilities in (36) is equivalent to equating the expected utilities themselves, or equating their certainty equivalents. Thus (36) is equivalent to 
\[
b(\varepsilon) = \frac{1}{2} \left\{ \frac{k}{2} a^2 + \alpha + \beta \varepsilon + \gamma \varphi \varepsilon + (1 - \varphi) \theta a - r \gamma^2 (1 - \varphi) \sigma_y^2 \right\}.
\]
Given the agent’s optimal saving decision, his total expected utility over both periods, conditional on \( z \), is given by
\[
U(z) = 2u \left( \frac{1}{2} \left\{ \alpha + \beta z + \gamma \varphi z + (1 - \varphi) \theta a - \frac{k}{2} a^2 - r \gamma^2 (1 - \varphi) \sigma_y^2 \right\} \right),
\]
or expressed as function of \( \varepsilon_z \) instead of \( z \),
\[
U(\varepsilon_z) = 2u \left( \frac{1}{2} \left\{ \alpha + \beta (\theta a + \varepsilon_z) + \gamma (\theta a + \varphi \varepsilon_z) - \frac{k}{2} a^2 - r \gamma^2 (1 - \varphi) \sigma_y^2 \right\} \right). \tag{37}
\]
The argument of \( u \) on the right-hand side of (37) is normally distributed with mean
\[
\frac{1}{2} \left\{ \alpha + (\beta + \gamma) \theta a - \frac{k}{2} a^2 - \frac{r}{2} \gamma^2 (1 - \varphi) \sigma_y^2 \right\}, \tag{38}
\]
and variance \((\beta + \varphi \gamma)^2 \text{Var}(\varepsilon_z)/4 = (\beta + \varphi \gamma)^2 \sigma_y^2/(4 \varphi)\). It follows that the agent’s ex-ante expected utility \( \int_{\varepsilon_z} U(\varepsilon_z) \) has a certainty equivalent of
\[
CE_1 = \alpha + (\beta + \gamma) \theta a - \frac{k}{2} a^2 - \frac{r}{2} \gamma^2 (1 - \varphi) \sigma_y^2 \tag{39}
\]
(note that the factor 1/2 in (38) cancels the factor 2 in (37)). At the beginning of the first period, the agent chooses \( a \) to maximize (39), which leads to
\[
a = \frac{(\beta + \gamma) \theta}{k}. \tag{40}
\]
Ensuring the agent’s participation at \( t = 0 \) requires that \( CE_1 = 2w \). The principal’s expected profit is given by
\[
(1 - \beta - \gamma) \theta a - \alpha \tag{41}
\]
As usual in the Holmström-Milgrom framework, maximizing the principal’s profit subject to the agent’s participation constraint is equivalent to maximizing the sum of the principal’s profit (41) and the agent’s certainty equivalent (39), which simplifies to
\[
\theta a - \frac{k}{2} a^2 - \frac{r}{2} \left( \frac{(\beta + \varphi \gamma)^2}{2 \varphi} + (1 - \varphi) \gamma^2 \right) \sigma_y^2. \tag{42}
\]
After substituting the optimal effort level (40) into (42), maximizing with respect to \( \beta \) and \( \gamma \) leads to the expressions for \( \beta \) and \( \gamma \) stated in the proposition. Finally, solving (39) for \( \alpha \) and plugging in the optimal effort (40) leads to \( \alpha \) as stated in the proposition.
Proof of Lemma 1: Using partial integration, we have

\[ E[x(y)|z] - E[x(y)|z'] = \int_y^\bar{y} [r(y, z) - r(y, z')]x(y)dy \]

\[ = \int_y^\bar{y} [(R(y, z) - R(y, z'))x(y)]dy - \int_y^\bar{y} [R(y, z) - R(y, z')]x'(y)dy, \]

which is positive because the first term vanishes while the integral is negative because of first-order dominance and \( x' \geq 0 \). Equation (15) follows by continuity. If \( x \) is strictly increasing, the inequalities are strict because by assumption, the conditional distributions \( y|z \) and \( y|z' \) are strictly different, cf. (14).

Proof of Proposition 2: I first show that \( \xi(z) \geq 0 \) for all \( z \). For any \( z \), (29) is equivalent to (suppressing \( z \) as argument in the wage functions)

\[ \lambda = \frac{1}{u_c(w_0, a)} - \mu \int_y^\bar{y} \frac{p_a(y|a)}{p(y|a)}r(y, z)dy - \xi \frac{u_{cc}(w_0, a)}{u_c(w_0, a)}. \]  \hfill (43)

Next, write (31) as

\[ \lambda = \frac{\delta}{\beta u_1'(w_1)} - \mu \frac{p_a(y|a)}{p(y|a)} + \xi \frac{u''_1(w_1)}{u_1'(w_1)}, \]

and integrate over all \( y \) to obtain

\[ \lambda = \int_y^\bar{y} \frac{\delta}{\beta u_1'(w_1)}r(y, z)dy - \mu \int_y^\bar{y} \frac{p_a(y|a)}{p(y|a)}r(y, z)dy + \xi \int_y^\bar{y} \frac{u''_1(w_1)}{u_1'(w_1)}r(y, z)dy. \]  \hfill (44)

Combining (43) and (44), we have

\[ \int_y^\bar{y} \frac{\delta}{\beta u_1'(w_1)}r(y, z)dy - \frac{1}{u_c(w_0, a)} = -\xi \left[ \frac{\delta}{\beta} \frac{u_{cc}(w_0, a)}{u_c(w_0, a)} + \int_y^\bar{y} \frac{u''_1(w_1)}{u_1'(w_1)}r(y, z)dy \right] \]  \hfill (45)

The left-hand side of (45) is nonnegative because

\[ \int_y^\bar{y} \frac{\delta}{\beta u_1'(w_1)}r(y, z)dy - \frac{1}{u_c(w_0, a)} \geq \frac{\delta}{\beta} \int_y^\bar{y} u_1'(w_1)r(y, z)dy - \frac{1}{u_c(w_0, a)} = 0, \]  \hfill (46)

where the first inequality in (46) follows from Jensen’s inequality and the second from (28). This means that the right-hand side of (45) must be nonnegative too, and since the term in square brackets is strictly negative, it follows that \( \xi(z) \) must be nonnegative.

Next, suppose \( \mu \leq 0 \) contrary to the claim, and rewrite (22) as follows:

\[ \int_y^\bar{z} \int_z^y \left\{ [u(w_0(z), a) + \beta u_1(w_1(y, z))]q(y, z)dz \frac{p_a(y|a)}{p(y|a)}p(y|a)dy + \int_y^\bar{y} \int_z^y u_a(w_0(z), a)q(y, z)p(y|a) \right\} dzdy = 0. \]  \hfill (47)

Since \( \xi(z) \geq 0 \), it follows from (32) and the arguments of the proof of Proposition 3 that \( u_1(w_1(y, z)) \) is (weakly) decreasing in \( y \) (whereas \( w_0(z) \) may be increasing or decreasing in \( z \)). This means
that for any \( z \), the term \( u(w_0(z), a) + \beta u_1(w_1(y, z)) \) in (47) is decreasing in \( y \), which means the same must hold if the term is integrated over all \( z \). Overall, in the first term of (47) the factor \( p_a(y|a)/p(y|a) \) is increasing in \( y \) by MLRP, whereas the other factor
\[
\int_\bar{z}^y [u(w_0(z), a) + \beta u_1(w_1(y, z))] q(y, z) dz
\]
is decreasing in \( y \). The factors are thus negatively correlated, and since \( \int_y p_a(y|a)p(y|a)dy = \int_y p_a(y|a)dy = 0 \), it follows that the first term in (47) is nonpositive. Since the second term is negative, the effort incentive constraint is violated; contradiction. It follows that \( \mu > 0 \).

**Proof of Lemma 2:** I first show that \( h_1(y) < 0 \) and hence \( \eta > 0 \). From (30),
\[
h_1(y) = \frac{\partial h(y,z)}{\partial w_1(y,z)} = \lambda u''_1(w_1) + \mu \frac{p_a(y|a)}{p(y|a)} u''_1(w_1) - \xi u'''_1(w_1)
\]
\[
= u''_1(w_1) \left[ \lambda + \mu \frac{p_a(y|a)}{p(y|a)} - \xi \frac{u'''_1(w_1)}{u''_1(w_1)} \right].
\]
(48)
Because of (17) and \( \xi \geq 0 \), the term in square brackets is no less than the right-hand side of (32), which means that it is no less than \( \delta/|\beta u'_1(w_1)| > 0 \). It follows that \( h_1(y) \) is negative, and that \( \eta \equiv -\beta \int_y^\bar{y} \frac{(h_w)^2}{h_1} r(y,z) dy \) is positive.

As explained in the text, we require concavity of \( L_z \) not in \( w_0 \) alone but in \( (w_0, w_1(y)) \) that satisfy \( f = 0 \) or
\[
\delta u_c(w_0, a) = \beta \int_y^\bar{y} u'_1(w_1(y, z)) r(y, z) dy.
\]
Given that the cross derivatives \( \partial^2 L_{yz}/(\partial w_0 \partial w_1) \) are zero, the relevant Legendre condition in its strict form therefore is that the second variation of \( L_z \),
\[
\frac{\partial^2 L_z}{\partial w_0^2} dw_0^2 + \beta \int_y^\bar{y} \frac{\partial^2 L_{yz}}{\partial w_1^2} [dw_1(y)]^2 r(y, z) dy
\]
\[
= g_0 dw_0^2 + \beta \int_y^\bar{y} h_1(y)[dw_1(y)]^2 r(y, z) dy
\]
(49)
must be negative for any \( (dw_0, dw_1(y)) \) that satisfy
\[
\delta u_{cc}(w_0, a) dw_0 = \beta \int_y^\bar{y} u''_1(w_1(y, z)) dw_1(y) r(y, z) dy.
\]
(50)
Suppose now that contrary to the claim of the Lemma, \( g_0^2 - g_0 \eta \leq 0 \) or \( g_0 \geq g_0^2/\eta \) (which is where the result \( \eta > 0 \) enters). For infinitesimal \( d\varepsilon \), consider the deviation \( dw_1(y) = h_\varepsilon(y)/h_1(y) d\varepsilon \). To satisfy (50), we need
\[
\delta u_{cc}(w_0, a) dw_0 = \beta \int_y^\bar{y} u''_1(w_1(y, z)) \frac{h_\varepsilon(y)}{h_1(y)} d\varepsilon r(y, z) dy,
\]
23
which simplifies to \( g_\xi dw_0 = \eta \, d\xi \) or \( dw_0 = (\eta/g_\xi) d\xi \). Thus, with \( g_0 \geq g_\xi^2/\eta \) and \( dw_0 = \eta/g_\xi d\xi \), the first term in (49) is
\[
g_0(dw_0)^2 \geq \frac{g_\xi^2}{\eta} \frac{\eta^2}{g_\xi^2} (d\xi)^2 = \eta(d\xi)^2,
\]
while the second term in (49) is
\[
\beta \int_y^\theta h_1(y)[dw_1(y)]^2 r(y, z) dy = \beta \int_y^\theta h_1(y) \left[ \frac{h_\xi(y)}{h_1(y)} \right]^2 (d\xi)^2 r(y, z) dy = -\eta(d\xi)^2.
\]
Overall, therefore, (49) is non-negative, in violation of strict concavity. It follows that \( g_\xi^2 - g_0 \eta > 0 \) is a necessary condition.

**Proof of Proposition 4:** Given differentiability of \( r(y, z) \) with respect to \( z \), we can differentiate (28)-(30) with respect to \( z \), which leads to, respectively:
\[
\delta u_{cc}(w_0, a) \frac{\partial w_0}{\partial z} - \beta \int_y^\theta u_1'(w_1(y)) \frac{\partial w_1(y)}{\partial z} r(y, z) dy + f_z = 0, \tag{51}
\]
\[
g_0 \frac{\partial w_0}{\partial z} + g_\xi \frac{\partial \xi}{\partial z} + g_z = 0, \tag{52}
\]
\[
h_1 \frac{\partial w_1(y)}{\partial z} + h_\xi \frac{\partial \xi}{\partial z} = 0. \tag{53}
\]
Let us look at some of the coefficients of this system of equations in more detail: first,
\[
f_z = -\beta \int_y^\theta u_1'(w_1(y)) r_z(y, z)
\]
is non-negative by Lemma 1 because \( w_1 \) is non-decreasing by Proposition 3 and hence \( u_1' \) is non-increasing because of concavity. It is strictly positive if the MLRP holds strictly. Likewise,
\[
g_z = \mu \int_y^\theta \frac{p_\alpha(y|a)}{p(y|a)} r_z(y, z) \, dy \, u_{cc}(w_0)
\]
is nonnegative by Lemma 1 because of MLRP (and positive if the MLRP holds strictly). Next, we have
\[
g_0 = u_{cc}(w_0, a) \left\{ \lambda + \mu \left[ \int_y^\theta \frac{p_\alpha(y|a)}{p(y|a)} r_z(y, z) \, dy + \frac{u_{acc}(w_0, a)}{u_{cc}(w_0, a)} \right] + \xi \delta \frac{u_{ccc}(w_0, a)}{u_{cc}(w_0, a)} \right\},
\]
\[
g_\xi = \delta u_{cc}(w_0, a) \text{ and hence}
\]
\[
\frac{\delta g_0}{g_\xi} = \lambda + \mu \left[ \int_y^\theta \frac{p_\alpha(y|a)}{p(y|a)} r(y, z) \, dy + \frac{u_{acc}(w_0, a)}{u_{cc}(w_0, a)} \right] + \xi \delta \frac{u_{ccc}(w_0, a)}{u_{cc}(w_0, a)}. \tag{54}
\]
As comparison, the first-order condition for \( w_0 \) (29) can be written as
\[
\frac{1}{u_c(w_0, a)} = \lambda + \mu \left[ \int_y^\theta \frac{p_\alpha(y|a)}{p(y|a)} r(y, z) \, dy + \frac{u_{ac}(w_0, a)}{u_c(w_0, a)} \right] + \xi \delta \frac{u_{cc}(w_0, a)}{u_c(w_0, a)}. \tag{55}
\]
Combining (54) and (55), we have
\[ \frac{1}{u_c(w_0, a)} - \frac{\delta g_0}{g_\xi} = \mu \left[ \frac{u_{ac}(w_0, a)}{u_c(w_0, a)} - \frac{u_{acc}(w_0, a)}{u_{cc}(w_0, a)} \right] + \delta \xi \left[ \frac{u_{cc}(w_0, a)}{u_c(w_0, a)} - \frac{u_{ccc}(w_0, a)}{u_{ccc}(w_0, a)} \right]. \] (56)

Since by assumption both \(-u_{cc}(c, a)/u_c(c, a)\) and \(-u_{cca}(c, a)/u_{ca}(c, a)\) are non-increasing in \(c\), both terms on the right-hand side of (56) are nonnegative (cf. (17)), and hence \(\delta g_0/g_\xi \leq 1/u_c(w_0, a)\). Finally, \(h_\xi(y) = -u_1''(w_1)\), and recall from the proof of Lemma 2 that
\[ h_1(y) \leq u_1''(w_1) \frac{\delta}{\beta u_1'(w_1)}. \]

It follows that \(h_\xi(y)/h_1(y) \geq -\beta u_1'(w_1)/\delta\).

We are now ready to prove the claims of the proposition. Part (a): From (53),
\[ \frac{\partial w_1(y)}{\partial z} = -\frac{h_\xi(y)}{h_1(y)} \frac{\partial \xi}{\partial z}. \] (57)

Substitute into (51) to obtain
\[ \delta u_{cc}(w_0, a) \frac{\partial w_0}{\partial z} + \beta \int_y^g u_1''(w_1(y)) \frac{h_\xi(y)}{h_1(y)} \frac{\partial \xi}{\partial z} r(y, z) \, dy = -f_z, \] (58)

where the first term on the left-hand side equals \(g_\xi \frac{\partial w_0}{\partial z}\), while the second term reduces to
\[ -\beta \frac{\partial \xi}{\partial z} \int_y^g [h_\xi(y)]^2 \frac{\partial \xi}{\partial z} \, r(y, z) \, dy = \eta \frac{\partial \xi}{\partial z}. \]

Equations (52) and (58) therefore lead to the system of equations
\[ g_\xi \frac{\partial w_0}{\partial z} + \eta \frac{\partial \xi}{\partial z} = -f_z \quad \text{and} \quad g_0 \frac{\partial w_0}{\partial z} + g_\xi \frac{\partial \xi}{\partial z} = -g_z. \]

By Lemma 2, we have \(m \equiv g_\xi^2 - g_0 \eta \) is strictly positive, and using Cramer’s rule, we have
\[ \frac{\partial w_0}{\partial z} = -\frac{1}{m} (g_\xi f_z - \eta g_z) \quad \text{and} \]
\[ \frac{\partial \xi}{\partial z} = -\frac{1}{m} (g_\xi g_z - g_0 f_z). \] (59, 60)

Since \(m\) and \(\eta\) are positive, \(f_z\), and \(g_z\) are non-negative, and \(g_{zi}\) is negative, \(\frac{\partial w_0}{\partial z}\) must be non-negative, which proves part (a). Moreover, if the MLRP is strict, then \(f_z, g_z > 0\) and hence \(\frac{\partial w_0}{\partial z} > 0\).

Part (b): given differentiability in \(z\), we can write the integral in the proposition as
\[ \int_y^g [\bar{w}(y, z_1) - \bar{w}(y, z_0)] r(y, z) \, dy = \int_{y_0}^y \left[ \int_{z_0}^{z_1} \frac{\partial \bar{w}(y, z)}{\partial z} \, dz \right] r(y, z) \, dy \]
\[ = \int_{z_0}^{z_1} \left[ \int_{y_0}^y \frac{\partial \bar{w}(y, z)}{\partial z} \, dy \right] r(y, z) \, dz. \] (61)
A sufficient condition for (61) to be nonnegative is that the inner integral in square brackets is nonnegative for every \( z \in [z_0, z_1] \). Using the definition of \( \tilde{w}(y, z) \), the latter integral can be written as

\[
\int_y^{\tilde{y}} \left[ \frac{\partial w_0(z)}{\partial z} + \delta \frac{\partial w_1(y, z)}{\partial z} \right] r(y, z_1) dy = \frac{\partial w_0(z)}{\partial z} + \delta \int_y^{\tilde{y}} \frac{\partial w_1(y, z)}{\partial z} r(y, z_1) dy. \tag{62}
\]

Because of (59), (60) and (57), the expression in (62) has the same sign as

\[
\eta g_z - g_0 f_z - \delta \int_y^{\tilde{y}} \left( g_0 f_z - g_0 g_z \right) \frac{h_{\xi(y)}}{h_1(y)} r(y, z_1) dy
\]

\[
= \eta g_z - g_0 f_z - \delta g_0 \int_y^{\tilde{y}} \frac{h_{\xi(y)}}{h_1(y)} r(y, z_1) dy + \delta g_0 \int_y^{\tilde{y}} \frac{h_{\xi(y)}}{h_1(y)} r(y, z_1) dy g_z. \tag{63}
\]

The coefficients of the \( g_z \)-terms in (63) are strictly positive. It therefore suffices to show that the \( f_z \)-terms are non-negative, or equivalently, since \( f_z \leq 0 \),

\[
- \left[ g_\xi + \delta g_0 \int_y^{\tilde{y}} \frac{h_{\xi(y)}}{h_1(y)} r(y, z_1) dy \right] \geq 0 \iff 1 + \delta g_0 \int_y^{\tilde{y}} \frac{h_{\xi(y)}}{h_1(y)} r(y, z_1) dy \geq 0 \tag{64}
\]

Recall that

\[
\frac{\delta g_0}{g_\xi} \leq u_c(w_0, a) \quad \text{and} \quad \frac{h_{\xi(y)}}{h_1(y)} \geq -\frac{\beta u'_1(w_1)}{\delta}.
\]

It follows that the left-hand side of (64) is greater than or equal to

\[
1 - \frac{\beta}{\delta u_c(w_0, a)} \int_y^{\tilde{y}} u'_1(w_1) r(y, z_1) dy. \tag{65}
\]

Condition (28), in turn, can be written as

\[
1 = \frac{\beta}{\delta u_c(w_0, a)} \int_y^{\tilde{y}} u'_1(w_1) r(y, z) dy.
\]

Therefore, (65) equals

\[
\frac{\beta}{\delta u_c(w_0, a)} \int_y^{\tilde{y}} u'_1(w_1) [r(y, z) - r(y, z_1)] dy,
\]

which must be nonnegative by Lemma 1 because \( u'_1 \) is decreasing in \( y \) (given Proposition 3) and \( z_1 \) first-order dominates \( z \) for all \( z \in [z_0, z_1] \). This completes the proof of part (b). \( \blacksquare \)

References


