Chapter 4

Saving and Investment

The models I have discussed so far are missing a central component of the General Theory; the idea that investment is the driving force of business cycles. Chapter 4 introduces this idea by developing a model with saving and investment. In the General Theory, Keynes argued that the distinction between these concepts was central to his theory of effective demand. I will explain this distinction with a two-period model populated by three generations of households. One of these generations, the young in the first period, save for the future and invest in capital to produce commodities in the second period. In Walrasian general equilibrium models saving and investment are brought into equality by changes in intertemporal prices. In the Keynesian model they are equated by changes in employment. Explaining the difference between these two mechanisms is the main purpose of this chapter.

In earlier chapters I showed that there may be many equilibria, indexed by the stance of fiscal policy. That is not an entirely satisfactory account of Keynes' message since Keynes saw fiscal policy as the remedy to mass unemployment, not its cause. To explain this idea, Section 4.10 introduces fiscal policy into the two period model and shows how government may design a tax-transfer system to restore full-employment.

4.1 The Model Structure

The chapter builds a model economy that has all the same features as the one commodity environment that I introduced in Chapter 2. In addition it has an extra period and a produced factor of production, capital. This richer structure allows me to discuss the idea that unemployment is produced by a lack of investment spending. Whereas effective demand in Chapter 2 was a function of fiscal policy; in this chapter it will also depend on the beliefs of investors.

Two competing dynamic general equilibrium models are widely used in macroeconomics. The first assumes the existence of a representative family that makes decisions for the infinite future. The second is the overlapping generations model of Allais (1947) and Samuelson (1958). This latter model is the natural general equilibrium environment in which to discuss Keynesian economics since the representative agent environment places strong restrictions on the equilibrium interest rate that limit the possibility to discuss meaningful fiscal policies.

I will assume that there are two periods, labeled 1 and 2 and three generations labeled $0, 1$, and 2. In period 2 there are two generations alive. Generation 0 is old and owns the capital stock. Generation 1 is young and owns an endowment of time. At the end of period 1, generation 0 dies. At the beginning of period 2 generation 2 is born and is endowed with a production technology. Throughout the chapter a superscript will index the period in which a generation was born and a subscript will index calendar time, thus x_t^s is the date t value of the variable x associated with the generation born in period s.

4.2 Households

This section describes, in turn, the economic choices made by agents of each generation. I will begin by describing the choices made by the old and the young in the first period; I refer to them as generations 0 and 1. I will then move on to the second period of the model and introduce the choices of a third generation that I refer to as generation 2. The decisions of generations 0 and 2 are limited and most of the action in this model takes place with the choices made by generation 1. In later chapters I will adapt the same structure by adding more periods and more generations each of which behaves like generation 1.

4.2.1 The Initial Old

There are two coexistent generations in period 1. Generation 0, solves the problem

$$\max_{\{C_1^0\}} j^0\left(C_1^0\right), \tag{4.1}$$

subject to the constraint

$$p_1 C_1^0 \leq \left[(1-\delta)\, p_1 + r_1\right] K_1. \tag{4.2}$$

There is unique commodity in each period that may be consumed or accumulated to be used as capital in production in the subsequent period. This commodity has money price p_1 in period 1. K_1 is an initial stock of capital owned by generation 0, r_1 is the money rental rate for capital in period 1, δ is the depreciation rate and C_1^0 is consumption of generation 0 in period 1. Since I assume that the utility function $j^0\left(C_1^0\right)$ is increasing in C_1^0, the household's decision problem has the trivial solution

$$p_1 C_1^0 = \left[(1-\delta)\, p_1 + r_1\right] K_1, \tag{4.3}$$

which directs the household to consume all of its wealth.[1]

4.2.2 The Initial Young

As in previous chapters I assume a unit measure of households with preferences over current consumption of household members. The representative generation 1 household receives utility from consumption in periods 1 and 2 and solves the problem

$$\max_{\{C_1^1, C_2^1, K_2, H_1\}} j^1\left(C_1^1, C_2^1\right) = g_1 \log\left(C_1^1\right) + g_2 \log\left(C_2^1\right), \tag{4.4}$$

where the preference weights g_1 and g_2 sum to 1,

$$g_1 + g_2 = 1. \tag{4.5}$$

[1] Throughout the book I will abstract from the bequest motive. Adding bequests will not change the main message of the book provided bequests are given because the giver obtains direct utility from the size of the gift.

Each household member is endowed with a single unit of time in period 1 and a fraction H_1 of all members search for a job where

$$H_1 \leq 1. \tag{4.6}$$

Since leisure does not yield utility, H_1 will be chosen to equal 1. Of the workers that search, a fraction L_1 find a job and the remaining U_1 are unemployed, hence,

$$L_1 + U_1 = H_1 = 1. \tag{4.7}$$

The relationship between H_1 and L_1 is given by the expression

$$L_1 = \tilde{q} H_1, \tag{4.8}$$

where $\tilde{q}$ is taken parametrically by the household.

Generation 1's allocation problem is subject to the sequence of budget constraints

$$p_1 C_1^1 + p_1 K_2 \leq w_1 L_1, \tag{4.9}$$

$$p_2 C_2^1 \leq \left(r_2 + p_2 \left(1 - \delta\right)\right) K_2, \tag{4.10}$$

where w_1 is the money wage in period 1, K_2 is capital carried into period 2, r_2 is the money rental rate in period 2, and p_2 is the money price in period 2. Households may borrow and lend with each other at money interest rate i and hence the intertemporal budget constraint is,

$$p_1 C_1^1 + \frac{p_2}{1+i} C_2^1 \leq w_1 L_1. \tag{4.11}$$

The solution to this problem is characterized by the consumption allocation decisions

$$p_1 C_1^1 = g_1 w_1 L_1, \tag{4.12}$$

$$\frac{p_2 C_2^1}{1+i} = g_2 w_1 L_1, \tag{4.13}$$

and the no-arbitrage condition,

$$1 + i = \left(\frac{r_2}{p_2} + 1 - \delta\right) \frac{p_2}{p_1}, \tag{4.14}$$

that defines the money interest rate i at which households have no desire to borrow or lend with each other.

4.2.3 The Third Generation

In period 2 there is a third generation that solves the problem

$$\max_{\{K_2^\alpha, C_2^2\}} j^2\left(C_2^2\right), \tag{4.15}$$

$$p_2 C_2^2 \leq p_2 Y_2 - r_2 K_2, \tag{4.16}$$

where output Y_2 is produced with the technology

$$Y_2 \leq K_2^\alpha. \tag{4.17}$$

The solution to this problem is given by the expression,

$$C_2^2 = Y_2 - \frac{r_2}{p_2} K_2. \tag{4.18}$$

In later chapters, when I introduce an infinite horizon model, each generation will be modeled like that of generation 1. To keep this two-period example as simple as possible I assume, in this chapter, that generation 2 owns the technology described by Equation (4.17) and that members of this generation rent capital from generation 1 and produce output Y_2. There is no labor market in period 2.

4.3 Firms

I have described production in period 2. This section describes the choices made by firms in period 1. Since the structure of this problem is a special case of the problem described in Chapter 3, I will be relatively brief in my description.

There is a large number of competitive firms each of which solves the problem

$$\max_{\{Y_1, K_1, V_1, L_1, X_1\}} p_1 Y_1 - w_1 L_1 - r_1 K_1, \tag{4.19}$$

subject to the constraints,

$$Y_1 \leq A K_1^\alpha X_1^{1-\alpha}, \tag{4.20}$$

$$L_1 = X_1 + V_1, \tag{4.21}$$

$$L_1 = q V_1. \tag{4.22}$$

As in Chapters 2 and 3, the firm must choose a feasible plan $\{Y_1, K_1, V_1, L_1, X_1\}$ to maximize profit taking the wage w_1, the rental rate r_1, the price p_1 and the recruiting efficiency q as given. A firm that allocates V_1 workers to recruiting will hire $qV_1 = L_1$ workers of which X_1 will be allocated to productive activity.

The solution to this problem is characterized by the first-order conditions

$$(1-\alpha)\frac{Y_1}{L_1} = \frac{w_1}{p_1}, \tag{4.23}$$

$$\alpha\frac{Y_1}{K_1} = \frac{r_1}{p_1}, \tag{4.24}$$

and the factor price frontier

$$p_1 = \left(\frac{w_1}{[1-\alpha]\,Q}\right)^{1-\alpha}\left(\frac{r_1}{\alpha}\right)^{\alpha}, \tag{4.25}$$

where the aggregate productivity variable

$$Q = \left(1 - \frac{1}{q}\right), \tag{4.26}$$

is taken as given by the individual firm.

4.4 Search

The search technology is identical to that described in Chapter 2. There is a match technology of the form,

$$\bar{L}_1 = \bar{H}_1^{1/2}\bar{V}_1^{1/2}, \tag{4.27}$$

where $\bar{L}$ is employment, equal to the measure of workers that find jobs when $\bar{H}_1$ unemployed workers search and $\bar{V}_1$ workers are allocated to recruiting by firms. Households choose $\bar{H} = 1$ and hence

$$\bar{L}_1 = \bar{V}_1^{1/2}. \tag{4.28}$$

In a symmetric equilibrium, (4.21), (4.28) and (4.26) imply

$$Q = \left(1 - \bar{L}_1\right). \tag{4.29}$$

4.5 The Social Planner

How would a benevolent social planner arrange production and consumption in this economy? This section addresses that question by studying the solution to the following constrained optimization problem.

$$\max_{\left\{K_2,L_1,C_1^0,C_1^1,C_2^1\right\}} \lambda_0 j^0\left(C_1^0\right)+\lambda_1 j^1\left(C_1^1,C_2^1\right)+\lambda_2 j^2\left(C_2^2\right), \tag{4.30}$$

$$C_1^0+C_1^1+K_2 \leq K_1^\alpha L_1^{1-\alpha}\left(1-L_1\right)^{1-\alpha}+K_1\left(1-\delta\right), \tag{4.31}$$

$$C_2^1+C_2^2 \leq K_2^\alpha+\left(1-\delta\right)K_2, \tag{4.32}$$

where the numbers λ_i are welfare weights that sum to 1,

$$\lambda_1+\lambda_2+\lambda_3=1. \tag{4.33}$$

The social planner chooses K_2, the amount of capital to carry into period 2, L_1, employment in period 1, and a way of allocating commodities to individuals, $\{C_1^0, C_1^1, C_2^1, C_2^2\}$. His problem is characterized as the maximization of (4.30) subject to the two feasibility constraints (4.31) and (4.32).

The solution to this problem requires that the two inequalities (4.31) and (4.32) should hold with equality and, in addition, the following six first order conditions should be satisfied,

$$-\mu_1+\mu_2\alpha K_2^{\alpha-1}=0, \tag{4.34}$$

$$\left(1-\alpha\right)K_1^\alpha L_1^{1-\alpha}\left(1-L_1\right)^{1-\alpha}\left(\frac{1}{L_1}-\frac{1}{1-L_1}\right)=0, \tag{4.35}$$

$$\lambda_0 j_1^0\left(C_1^0\right)=\mu_1, \tag{4.36}$$

$$\lambda_1 j_1^1\left(C_1^1,C_2^1\right)=\mu_1, \tag{4.37}$$

$$\lambda_1 j_2^1\left(C_1^1,C_2^1\right)=\mu_2, \tag{4.38}$$

$$\lambda_2 j_1^2\left(C_2^2\right)=\mu_2. \tag{4.39}$$

The six equations (4.34) – (4.39), and the two constraints (4.31) and (4.32) determine the six variables, K_2,L_1, C_1^0, C_1^1, C_2^1 and C_2^2 and the two Lagrange multipliers μ_1 and μ_2 associated with the inequality constraints (4.31) and (4.32).

The most important of these conditions is Equation (4.35) which implies that optimal employment in period 1, call this L_1^*, occurs when

$$L_1^* = \frac{1}{2}. \tag{4.40}$$

This problem differs from a conventional social planning problem since there is an externality in the technology that is internalized by the social planner - this is the occurrence of the term $1 - L_1$ in the production function in period 1. In all other ways, the problem is conventional. Given L_1, the planner chooses how to allocate commodities across individuals and across time. I will show below that the existence of this externality may make it difficult or impossible for a market economy to make the right employment decision and I will formalize this idea in the concept of a demand constrained equilibrium. But if this problem can be corrected, the existence of commodity and asset markets implies that a decentralized economy can produce a Pareto efficient allocation.

4.6 Investment and the Keynesian Equilibrium

We are used to thinking of general equilibrium in Walrasian terms. Agents take prices and endowments as given and form demands – an equilibrium is a set of prices and an allocation of commodities such all markets clear and no individual has an incentive to alter his allocation through trade at equilibrium prices.

For the Keynesian model we will require a different equilibrium concept since, by construction, there are not enough markets to determine equilibrium allocations. This section extends the DCE equilibrium concept of Chapter 2 to the two-period model. I will use this extended concept to introduce the idea that investment determines economic activity and I will show that there is an interval such that, for any value of investment expenditure in that interval, there exists a demand constrained equilibrium.

Since the Keynesian model is missing an equation, there are many equivalent candidates for an equation with which to close it. Following the General Theory, this chapter closes the model with the assumption that investors form a set of beliefs about the future. Keynes called this 'animal spirits'. But this assumption has many representations all of which will be consistent with a self-fulfilling equilibrium. In this chapter I have chosen to represent the assumption by assuming that the value of capital is determined by the beliefs of investors.

Specifically, let

$$\tilde{I}_1 \equiv p_1 K_2. \tag{4.41}$$

I will refer to $\tilde{I}_1$ as investment although this is a misnomer since it is in fact the money value of the next period's capital stock. I have chosen this definition because it simplifies the equilibrium concept. By assuming that entrepreneurs have fixed beliefs about the appropriate value of $\tilde{I}_1$, I will be able to separate the equation that determines aggregate demand and employment from the equations that determine relative prices. The resulting dichotomy allows me to provide an interpretation of textbook Keynesian models that has a firm microfoundation.

If $\tilde{I}_1$ is large I will say that investors are optimistic and if $\tilde{I}_1$ is small they are pessimistic. I will show that there is value $\bar{I}_1$ such that for any value of

$$\tilde{I}_1 \in \left[0, \bar{I}_1\right], \tag{4.42}$$

there is an equilibrium, characterized by values for prices $\{p_1, p_2, w_1, i\}$, consumption allocations $\{C_1^0, C_1^1, C_2^1, C_2^2\}$ employment L_1, unemployment U_1, productions Y_1 and Y_2 and capital K_2 such that no individual has an incentive to change his behavior given the prices and the quantities demanded and supplied for commodities in each period and for borrowing, lending and capital in the asset markets. In the labor market, employment is determined by matching the equilibrium numbers of searchers on each side of the market.

4.7 The Definition of Equilibrium

This section extends the definition of a demand constrained equilibrium from Chapter 2 to the two-period model with capital. Since this concept is based on ideas from the General Theory I will also refer to it as a Keynesian equilibrium and I will refer to equilibrium values of variables in the model with the superscript K, for Keynes. These values are to be contrasted with the superscript $*$ that denotes the social planning optimum when the planner uses the welfare weights λ_i.

Definition 4.1 *(Demand Constrained Equilibrium) Let $\bar{I}_1$ be given by the equation,*

$$\bar{I}_1 = (1-\alpha)(1-b), \tag{4.43}$$

where

$$b = \alpha + g_1 (1 - \alpha). \tag{4.44}$$

For any given $\tilde{I}_1 \in \left[0, \bar{I}_1\right]$ *a symmetric demand constrained equilibrium (DCE) is*
(i) a six-tuple of prices $\{p_1, p_2, w_1, r_1, r_2, i\}$,
(ii) a production plan $\{Y_1, Y_2, K_2, V_1, L_1, X_1\}$,
(iii) a consumption allocation $\{C_1^0, C_1^1, C_2^1, C_2^2\}$ *and*
(iv) a pair of numbers $\tilde{q}$ *and* q*: with the following properties.*
1) Feasibility:

$$Y_1 \leq A K_1^{\alpha} X_1^{1-\alpha}, \tag{4.45}$$

$$Y_2 \leq K_2^{\alpha}, \tag{4.46}$$

$$C_1^0 + C_1^1 + K_2 - K_1 (1 - \delta) \leq Y_1, \tag{4.47}$$

$$C_2^1 + C_2^2 \leq Y_2 + K_2 (1 - \delta) \tag{4.48}$$

$$L_1 \leq V_1^{1/2}, \tag{4.49}$$

$$X_1 + V_1 = L_1, \tag{4.50}$$

$$K_2 = \frac{\tilde{I}_1}{p_1}. \tag{4.51}$$

2) Consistency with optimal choices by firms:

$$\frac{r_1}{p_1} = \alpha \frac{Y_1}{K_1}, \tag{4.52}$$

$$\frac{w_1}{p_1} = (1 - \alpha) \frac{Y_1}{L_1}, \tag{4.53}$$

$$\frac{r_2}{p_2} = \alpha \frac{Y_2}{K_2}, \tag{4.54}$$

$$p_1 = \left(\frac{w_1}{[1 - \alpha] Q}\right)^{1-\alpha} \left(\frac{r_1}{\alpha}\right)^{\alpha}. \tag{4.55}$$

3) Consistency with optimal choices by households:

$$p_1 C_1^0 = [(1 - \delta) p_1 + r_1] K_1, \tag{4.56}$$

$$p_1 C_1^1 = g_1 w_1 L_1, \tag{4.57}$$

$$\frac{p_2 C_2^1}{1+i} = g_2 w_1 L_1, \tag{4.58}$$

$$p_2 C_2^2 = p_2 Y_2 - r_2 K_2. \tag{4.59}$$

4) Search market equilibrium:

$$\tilde{q} = L_1, \tag{4.60}$$

$$q = \frac{L_1}{V_1}, \tag{4.61}$$

$$L_1 = V_1^{1/2}. \tag{4.62}$$

In Section 4.8 I will show that a DCE exists and in Section 4.9 I show how to compute the prices and allocations associated with this equilibrium.

4.8 Aggregate Demand and Supply

To show existence of a Keynesian equilibrium, this section develops aggregate demand and supply equations and shows that the equality of aggregate demand and supply results in an equilibrium employment level L_1^K that is feasible and that satisfies the optimality conditions of households. In Section 4.9 I show that there exist prices that support this allocation as a demand constrained equilibrium. An important feature of a Keynesian equilibrium is that that there is a different demand constrained equilibrium for every value of $\tilde{I}_1$ in the interval $\left[0, \bar{I}_1\right]$: All of these equilibria have the property that no investor has an incentive to deviate from his plan.

As in previous chapters I will choose the money wage w_1 as the numeraire and I define aggregate supply to be the money value of gdp at which employers are indifferent to hiring L_1 workers. The function $\phi(L_1)$ that has this property is found from the first order condition for labor (4.23) and is given by the expression

$$Z_1 = \frac{1}{1-\alpha} L_1 \equiv \phi(L_1). \tag{4.63}$$

As in the General Theory I refer to $Z_1 \equiv p_1 Y_1$ as the aggregate supply price of employment, L_1.

Period 1 aggregate demand, D_1 is equal to

$$D_1 = \left[p_1\left(C_1^0 + C_1^1\right)\right] + \left[\tilde{I}_1 - (1-\delta)\, p_1 K_1\right], \tag{4.64}$$

where the first term in square brackets is the money value of aggregate consumption and the second is the money value of investment. Using Equations (4.3) and (4.12), leads to the expression

$$D_1 = [(1-\delta)\, p_1 + r_1]\, K_1 + g_1 w_1 L_1 + \tilde{I}_1 - (1-\delta)\, p_1 K_1,$$

which, by using the first-order conditions, (4.23) and (4.24) can be simplified as follows

$$D_1 = bZ_1 + \tilde{I}_1, \tag{4.65}$$

where,

$$b = \alpha + g_1 (1-\alpha)\,. \tag{4.66}$$

This is the point where the definition of investment as $\tilde{I}_1$, the money value of period 2 capital, rather than $p_1 I_1$, the money value of additions to capital, leads to a considerable simplification of the equations that determine equilibrium. If I had chosen $p_1 I_1$ as the object of investors' beliefs, the equation that determines equality of aggregate demand and aggregate supply would have contained the additional term $-(1-\delta)\, K_1 p_1$. There is no conceptual difficulty in following this alternative definition but it would break the separation of the equations that determine equilibrium prices from those that determine aggregate demand and supply.

The Keynesian equilibrium occurs when $D_1^K = Z_1^K$. Imposing this condition and solving Equations (4.63) and (4.65) leads to the following expression for the equilibrium value of the aggregate supply price,

$$Z_1^K = \frac{1}{1-b}\tilde{I}_1. \tag{4.67}$$

Equilibrium employment, L_1^K is equal to

$$L_1^K = (1-\alpha)\, Z_1^K. \tag{4.68}$$

The Keynesian equilibrium is illustrated in Figure 4.1. Since employment must lie in the interval $[0,1]$ and aggregate supply is defined by Equation (4.63) it follows that the maximum value of aggregate supply is equal to $1/(1-\alpha)$. It follows from the linearity of the aggregate demand and supply equations that there exists a Keynesian equilibrium for any value of $\tilde{I}_1 \in \left[0, \bar{I}\right]$ where

$$\bar{I} = (1-\alpha)(1-b)\,. \tag{4.69}$$

The following section establishes this claim formally by showing how the other variables of the model are determined.

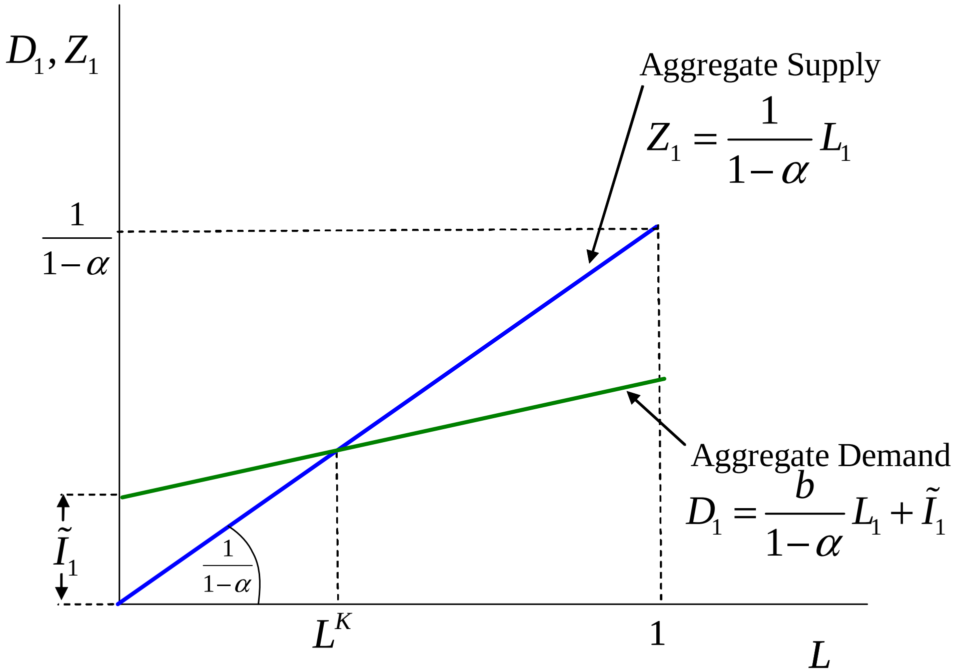

Figure 4.1: Aggregate Demand and Supply

4.9 Finding Values for the other Variables

I have shown how aggregate supply and employment are determined in period 1 in a Keynesian equilibrium. It remains to be shown how the prices $p_1^K, p_2^K, \left(1+i^K\right)$, the consumption allocations $C_1^{0K}, C_1^{1K}, C_2^{1K}$ and C_2^{2K}, the capital stock K_2^K and the outputs Y_1^K and Y_2^K are determined in equilibrium. This section applies some simple algebra by rearranging first-order conditions and budget identities and may be skipped without loss of content if the reader is inclined.

4.9.1 First Period Price and Output

I turn first to the determination of prices and of the physical value of output, Y_1^K in the Keynesian equilibrium. The equilibrium price in period 1, p_1^K can be found by solving the equation

$$p_1^K = \frac{Z_1^K}{Y_1^K}, \tag{4.70}$$

where

$$Y_1^K = AK_1^{\alpha} \left(L_1^K\right)^{1-\alpha} \left(1 - L_1^K\right)^{1-\alpha}, \tag{4.71}$$

is the physical value of output. Using (4.68) this gives the following expression for the money price p_1^K

$$p_1^K = \frac{1}{A\left(1-\alpha\right)} \left(\frac{L_1^K}{K_1^{\alpha}}\right)^{\alpha} \frac{1}{\left(1 - L_1^K\right)^{1-\alpha}}, \tag{4.72}$$

as a function of the endowment of capital, K_1 and the value of employment at the Keynesian equilibrium, L_1^K. p_1^K is an increasing monotonic function of L_1^K, reflecting the fact that the real wage falls as investors become more optimistic and the economy moves up the aggregate supply curve. This is the same mechanism that was discussed in Chapters 2 and 3.

4.9.2 Second Period Capital and Output

I now turn to the variables K_2^K and Y_2^K. Given $\tilde{I}_1$ and p_1^K it follows from the definition of $\tilde{I}_1$ that

$$K_2^K = \frac{\tilde{I}_1}{p_1^K}, \tag{4.73}$$

and hence

$$Y_2 = \left(K_2^K\right)^{\alpha}. \tag{4.74}$$

4.9.3 Rental Rates and Consumption Allocations

Next consider the determination of real rental rates and consumption allocation to each generation. Generation 0 consumes the amount C_1^{0K} which is found from the budget equation,

$$C_1^{0K} = \left(1-\delta\right) K_1 + \frac{r_1^K}{p_1^K} K_1. \tag{4.75}$$

The real rental rate r_1/p_1 is found from the first order condition for rental capital in period 1

$$\frac{r_1^K}{p_1^K} = \alpha \frac{Y_1^K}{K_1}. \tag{4.76}$$

Equations (4.75) and (4.76) imply,

$$C_1^{0K} = (1-\delta) K_1 + \alpha Y_1^K.$$

The second period real rental rate r_2^K/p_2^K is found from the first-order conditions,

$$\frac{r_2^K}{p_2^K} = \alpha \frac{Y_2^K}{K_2^K}, \tag{4.77}$$

and generation 2's consumption is

$$C_2^{2K} = Y_2^K - \frac{r_2^K}{p_2^K} K_2^K = (1-\alpha) Y_2^K. \tag{4.78}$$

Generation 1's consumption in period 2 is found from market clearing

$$C_2^{1K} + C_2^{2K} = (1-\delta) K_2^K + Y_2^K, \tag{4.79}$$

as

$$C_2^{1K} = (1-\delta) K_2^K + \alpha Y_2^K. \tag{4.80}$$

4.9.4 Second Period Prices

Finally we can solve for $p_2^K / \left(1+i^K\right)$, the present value of p_2^K, from Equation (4.13) as

$$\frac{p_2^K C_2^{1K}}{1+i^K} = g_2 w_1 L_1^K. \tag{4.81}$$

which can be rearranged to give

$$\frac{p_2^K}{1+i^K} = \frac{g_2 (1-\alpha) Z_1^K}{C_2^{1K}}. \tag{4.82}$$

It is worth pointing out that p_2^K and i^K are not separately defined in this model since there is no role for a separate unit of account in period 2.

4.10 Fiscal Policy in a Keynesian Model

A Keynesian equilibrium can result in any value of employment in the interval $[0,1]$, but the social planner will choose $L^* = 1/2$. It follows that unless investors happen fortuitously to choose the correct value of $\tilde{I}_1$, the Keynesian equilibrium may be one of over or under employment. This section describes the Keynesian remedy for this problem by putting fiscal policy into the model.

Since there are three generations, a fiscal policy could conceptually consist of a level of government expenditure and a set of taxes and transfers indexed by the age of the household. I will exclude government expenditure since that raises the issue of public goods and instead I will consider policies that consist of an income tax rate τ, levied in period 1, and a transfer payment T to generation 1. I will show that for any value of $\tilde{I}_1$, there exists a tax-transfer policy $\{\tau, T\}$ that implements the full employment level of employment, L_1^*.[2]

Conceptually, it is possible to tax wage income and capital income at different rates and the generational burden of these taxes will differ. I will be concerned with the question; can fiscal policy maintain full employment? The answer to this question is yes and further, there are many policies that can implement full employment. In light of the multiplicity of solutions I will show here not only that a policy of this kind exists but also that there is a policy that is distributionally neutral. This solution is equivalent to the demonstration in textbook Keynesian models of the existence of a balanced budget multiplier; a value of taxes and lump-sum transfers that leaves government debt unchanged.

To implement a policy that is distributionally neutral let τ_1 be the tax rate on labor income and let T_1 be a lump-sum transfer to generation 1 in period 1. There are no taxes or transfers in period 2 and there is no tax on rental income. These assumptions imply that aggregate demand is given by the expression

$$D_1 = \alpha Z_1 + g_1 \left[(1 - \tau_1) (1 - \alpha) Z_1 + T_1 \right] + \tilde{I}_1. \tag{4.83}$$

The first term on the right side is the consumption from rental income of the old generation in period 1. The term in square brackets is after-tax income of the young generation. Their labor income $(1 - \alpha) Z_1$ is taxed at rate τ and they receive a transfer T. The parameter g_1 is the marginal propensity to consume for these individuals. It is clear from this expression that there will be many choices of T and τ that force the equilibrium value, at which D_1 equals Z_1, to occur at the planning optimum Z_1^*; but most of these solutions will cause the government to accumulate debt that will need to be repaid in

[2] I have called this full employment in line with the language of the General Theory although the model introduced here admits of over employment as well as under employment.

the second period. Instead, let us confine ourselves to tax transfer policies for which

$$T = \tau_1 Z_1^*, \tag{4.84}$$

where

$$Z_1^* = \frac{1}{2(1-\alpha)}, \tag{4.85}$$

is the value of aggregate supply in the social planning optimum.

Substituting (4.84) into (4.83) and solving for the Keynesian equilibrium leads to the expression,

$$Z_1^K = \frac{\tilde{I}_1 + \alpha g_1 T_1}{1 - \alpha - (1-\alpha) g_1}. \tag{4.86}$$

Combining Equations (4.84) and (4.86), it follows that the lump-sum transfer or tax required to maintain full employment is given by the expression,

$$T_1^* = \frac{Z^* (1 - \alpha - (1-\alpha) g_1) - \tilde{I}_1}{\alpha g_1}, \tag{4.87}$$

and from (4.84), the tax rate (or subsidy) that implements this equilibrium is

$$\tau_1^* = \frac{T^*}{Z_1^*}. \tag{4.88}$$

If $\tilde{I}_1$ is too low then the equilibrium without intervention displays Keynesian unemployment and the optimal balanced budget policy is supported by an income tax and a lump-sum transfer. If $\tilde{I}_1$ is too high then the equilibrium displays over-employment and the optimal balanced budget policy is a wage subsidy and a lump-sum tax.

4.11 Concluding Comments

The main early criticisms of Keynes' work were theoretical, not empirical. It was pointed out that the General Theory does not have a satisfactory theory of the labor market. In Chapters 2 and 3 I constructed one-period demand-driven models to address these criticisms. Both of these chapters were based on a search-theoretic model of the labor market. Their purpose was to provide a microfoundation to the Keynesian theory of aggregate supply. Recall that the aggregate supply function $\phi(L_1)$ is a relationship between Z_1, the supply price measured in dollars, and L_1, employment. Z_1 is "that expectations of proceeds [...i.e. nominal gdp...] that will just make it worth the while of the entrepreneurs to give that employment".

Chapter 4 has developed the first, and simplest, of several models that embody Keynes' theory of effective demand. Aggregate demand D_1 is "the proceeds that entrepreneurs expect to receive from the employment of L_1 men" and it can be broken into two components, consumption and investment, each measured in dollars. I have provided a microfounded model in which consumption expenditure is a linear function of income and investment expenditure is determined by beliefs of investors about future productivity, so called 'animal spirits'. A Keynesian equilibrium, formally defined in this chapter as a demand constrained equilibrium, is a value of employment at which aggregate demand and aggregate supply are equal.

Keynes emphasized that saving and investment are equated not by the interest rate, but by the level of economic activity. This chapter has provided an interpretation of that idea. We are used to teaching macroeconomics in the language of Walrasian general equilibrium theory. In Walrasian theory it is prices that clear markets and it is a change in the rate of interest that equates saving and investment. By adding a search externality and removing the spot market for labor I have provided a framework where there are not enough Walrasian prices to equate demands and supplies for all of the quantities. This framework goes beyond the General Theory by providing an explicit microfoundation to the Keynesian idea of aggregate supply and in this sense it is not exactly what Keynes said. Arguably, it is what he ought to have said.