Net Foreign Assets, Productivity and Real Exchange Rates in Constrained Economies

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Preliminary work

Abstract: Empirical evidence suggests that real exchange rates (RER) behave differently in developed and developing countries. We develop an exogenous 2-sector growth model in which RER determination depends on the country’s capacity to borrow from international capital markets. The country faces a constraint on capital inflows. With high domestic savings, the country converges to the world per capita income and RER only depends on productivity spread between sectors (Balassa-Samuelson effect). If the constraint is too tight and/or domestic savings too low, RER depends on both net foreign assets (transfer effect) and productivity. We then analyze the empirical implications of the model and find that, in accordance with the theory, RER is mainly driven by productivity and NFA in constrained countries and exclusively by productivity in unconstrained countries.

Key-words: Real exchange rate; capital inflows constraint; overlapping generations; exogenous growth.

Classification J.E.L.: E39; F00; F20.

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1 Introduction

A recurrent question in International Macroeconomics is what are the main long-run determinants of real exchange rates (RER). There is, however, no consensus yet on this question. Among the most often quoted determinants we can find productivity, terms of trade and net foreign assets (NFA) [see Chinn (2006)]. Empirical evidence suggests that these determinants change significantly as we vary periods and countries considered.

The literature on the Balassa-Samuelson (BS) theory show that RER appreciation may be related to productivity growth but not systematically. It seems to have special relevance for countries like Japan, some OECD countries [Canzoneri et al. (1999)] and transition economies [Égert et al 2003, 2006]. Ito et al. (1999) show that RER and growth are positively correlated in Japan, Korea, Taiwan, Hong Kong whereas the correlation remains negative for Indonesia, Thailand, Malaysia, Philippines and China. Hong Kong, Taiwan and Singapore combine a high growth rate and a small appreciation. For other Asian countries except China, Singapore, Taiwan and Thailand, Chinn (2000a) finds that productivity explains RER only when public spending and oil prices are taken into account. Chinn (2000b), using panel data, finds that the RER requires around 5 years to converge to the level predicted by BS. In a recent paper, Bergin et al (2008) also report that the BS effect is not stable through time, but it appears to have become more important in recent years. The fit of the standard BS theory to explain RER changes seems to be very poor and largely country- and period-specific.

Recently, in line with the theory that emphasises the role of foreign assets for equilibrium RERs, Lane and Milesi-Ferretti (2004) develop a model to highlight the transfer effect - which relates the RER to NFA. Using a database that covers 64 industrial and less developed countries between 1970 and 1998, they show that a rise in NFA appreciates the RER, especially for countries that have low income, low openness or foreign exchange restrictions. However, the theoretical model they present falls short of explaining why developing countries that face a constraint on capital inflows experiment a higher transfer effect than others.

Linking together this diverse set of results, our study contributes to this literature in two ways. First, it presents a model that reconciles these empirical findings in which RER determination depends on the country’s capacity to borrow from international capital markets. Second, it tests whether the behavior of the data is consistent with the main results of the model focus-
ing on whether the long-run and short-run causal relationships between the RER and its main determinants depend on the financial constraints faced by countries.

Although not our primary focus, the paper is also related to the growth and convergence literature. Our model stresses that a RER appreciation may help a developing country catching-up with the world per capita income. Indeed, because of the way the constraint is specified, a RER appreciation attracts capital flows\(^1\) and may fill in a lack of domestic savings that accelerates growth temporarily or increases permanently long-run income per capita. In the exogenous growth literature, some models with constraints on capital inflows have been developed [Barro, Mankiw and Sala-i-Martin (1995), Lane (2001)]. They exhibit a common property: the constraint on capital inflows slows down economic convergence but does not stop it\(^2\). The convergence speed they obtained is empirically plausible [Mankiw, Romer and Weil (1992)] while it was not the case in closed economy models. In the long-run, however, per capita income of the developing country systematically converges to the world per capita income: the developing country is no longer constrained in steady state. In contrast, by making use of overlapping generations, our model allows the steady state to be constrained or unconstrained. The credit constraint we impose can not only slow down absolute convergence but also prevent it from occurring even in the long-run. This model then predicts convergence clubs rather than absolute convergence: a developing country with lack of domestic savings or unstable institutions may not converge even in the long-run - since it reaches a constrained steady state with lower income per capita. Bosworth and Collins (2003), amongst others, have provided evidence in favor of such type of convergence and against absolute convergence.

We use an overlapping generations setting of a constrained economy initially developed by Obstfeld and Rogoff [1996] in which we introduce two production sectors [Gente, 2006] a non-traded and a traded sector. We assume that the amount the country can borrow on the international capital market is an exogenous fraction of per-capita income. This fraction represents the trust of foreign investors about local institutions and creditworthiness. If the constraint is not too tight - or if there are high domestic savings - the constrained economy will

\(^1\)In Rodrick (2007), capital inflows are related to traded inputs and a real appreciation also increases capital inflows as in our model.

\(^2\)This is the case for small open economy models only. Quah (1996) develops a model that exhibits convergence clubs.
become unconstrained in the long-run. Otherwise, if investors are not confident - or there are low domestic savings - the developing country will converge to the constrained steady state. The RER behaviour differs widely between those two kinds of steady states.

In the unconstrained steady state, the RER will exclusively be determined by the Balassa-Samuelson effect. Conversely, in the constrained steady state, the RER will depend on supply and demand of non-traded goods. A productivity shock operate through a demand effect and not only through the Balassa-Samuelson effect. In the same way, an international transfer from abroad will appreciate the RER whereas there is no transfer effect at all in the unconstrained steady state. This transfer effect is higher in less open economies. This is consistent with Lane and Milesi-Ferretti (2004) empirical results. Assuming perfect mobility of factors between sectors, global income depends on both capital intensity and RER. A RER appreciation as well as an increase in savings relaxes the constraint and may promote convergence. Then a positive transfer from abroad may increase permanently income per capita. Indeed, through a RER appreciation, the transfer effect loosens the constraint. It then increases capital inflows and help the country converging to a higher long-run per capita income, at the unconstrained steady state.

These implications of the model for the RER behavior are then tested using a simple econometric model based on Lane and Milesi-Ferretti (2004). We estimate separate RER models for financially constrained and unconstrained economies, selected using the Chinn and Ito (2007) measure of external financial openness. The findings are supportive of the implications of the model in the long-run.

The paper is organised as follows. Section 2 presents some stylized facts, Section 3 the theory model, Section 4 presents the econometric evidence and Section 5 provides some conclusions.

2 Stylized facts

According to the literature, RER determinants change as we vary periods and countries considered. Our theoretical motivation is based on the idea that that these different RER behaviours reflect the fact that all countries do not have the same access to the world capital market. This section puts forward some empirical facts about this access to international capital flows using

Figure 1

Figure 1 presents some cross-country evidence on the relationship between per capita GDP and NFA/GDP. It plots the average ratio of NFA over GDP against average GDP per capita (US$ PPP converted) over the period 1970-1998 for 55 countries. It shows a positive but non-linear relationship suggesting that richer countries accumulate more NFA. The more indebted countries are the ones with the lowest GDP per capita levels. However, the relationship appears to become flatter after income levels of about 6,000US$. The ratio of NFA over GDP lies between a limited range of -130% to 90% in our sample. Richer countries do not have lower liabilities/GDP ratios than poorer countries but do have higher Assets/GDP ratios [See Appendix A]. This fact may be reflecting globalisation and diversification in investment strategies: domestic savings and domestic investment are less related today than they were thirty years ago. The crucial assumption of the model we develop is that the access to international capital flows differs between countries. Hereafter, we review evidence that justifies this assumption.

A rough way to measure the access to international capital flows is to characterize the relationship between the ratio of NFA to GDP and GDP per capita in each country by estimating

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3These are the same countries used in our empirical analysis in Section 5 (listed in Appendix A).
a relationship as follows:

\[
\frac{NFA_t}{GDP_t} = \alpha + \beta Y_{pc,t-1} + u_t
\]

We estimate the \( \beta \) coefficient for each of the 55 countries. A positive relationship suggests, according to the standard neoclassical open-economy model, that a rise in income per capita leads to a current account surplus and increases NFA. This is true as long as the country has a perfect access to the international capital market. However, a negative relationship reflects that a rise in per capita income leads to current account deficits and NFA decreases. This may occur in the standard neoclassical model only when the country faces a constraint on international capital flows. In this case, a rise in per capita income may suggest to international lenders that the country has a higher ability to refund loans in the future. Then a country that becomes richer can also be more indebted.

Figures 2 depict this beta coefficient per country and its T-ratio against the logarithm of GDP per capita. The results show that the coefficient appears to be close to zero for the richer countries in the sample. For lower income countries, however, we find both positive and negative (significant) \( \beta \) coefficients. This seems to indicate that access to the international capital market does not have a strong correlation with GDP per capita. Poorer countries may either be constrained (negative beta) or unconstrained (positive beta), which is also true of rich countries although their coefficients appear to be very small. This implies that separating countries in terms of their income levels in RER regressions may not entirely capture the effect of external financial constraints. For this reason, we prefer to use alternative measures of the access to international capital flows. One such measure is the capital openness index (KAOPEN) developed by Chinn and Ito (2007) (see Section 5). The higher KAOPEN index, the more open the country is to international capital flows.

\footnote{With this regression we only aim at unveiling correlations rather than inferring causal relations. The equation was also estimated using contemporaneous values of \( Y_{pc,t} \) and \( Y_{pc,t-2} \) and the results remained almost identical.}
Figure 3

Figure 3 plots the ratio NFA/GDP versus the KAOPEN index (averages for the sample period). It shows that the less financially open economies have lower NFA in terms of GDP and always negative NFA. This evidence supports the intuition according to which economies whose institutional arrangements impose controls and interventions on the international exchange of assets face constraints on capital flows.

Figure 4

Figure 4 plots the beta coefficient estimated earlier against the average KAOPEN index. For more financially open economies the coefficient is small, although positive in the majority of
cases. For less open economies there seems to be a positive relationship: the higher the KAOPEN index, the higher the $\beta$ coefficient is reflecting, the importance of impediments to financial flows as a determinant of external constraints. Negative $\beta$ coefficients are also more often observed in less open economies than in poor countries. Hence, the KAOPEN index appears to be a relevant variable to empirically separate constrained and unconstrained economies to test the predictions of our theoretical model. The model assumes that there is a constraint on NFA accumulation like

$$B_{t+1} \geq -\eta N_t y_t$$

where $B_{t+1}$ denotes the NFA of the domestic country in terms of traded goods and $\eta > 0$ is the proportion of global income ($N_t y_t$) the domestic country can borrow. Our $\beta$ coefficient is a proxy of $(-)\eta$ for indebted countries. The $\eta$ parameter reflects the access the country has to international capital flows and may be related to institutional features. The smaller $\eta$ the more constrained the country. In the next section, we investigate the RER behaviour in such constrained economies (negative beta). Then we test empirically the predictions of the model.

3 The model

The model is a variant of the small open economy overlapping generations model of Obstfeld and Rogoff (1996) in which we introduce two production sectors: a tradable sector and a non-tradable sector. In this setting, the real exchange rate $R$ denotes the relative price of non-tradable to tradable good. The country faces a constraint on capital inflows. We develop a model in which agents only work in their first period of life.

3.1 Individuals

The economy consists of a sequence of two-period lived individuals. In the second period of his life, each individual gives birth to $1+n$ others so that the per period rate of population growth is

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5Our purpose in this paper is not to explain the $\eta$ parameter or estimate it, but to emphasize the role that this constraint can play in an open economy model.

6According to ..., the relative price of non-traded to traded good is a relevant proxy for the real exchange rate in a long-run horizon.
At time $t$, each generation consists of $N_t$ identical individuals who make decisions concerning consumption and savings.

Intertemporal preferences of an individual belonging to generation $t$ are represented by

$$U(c_t, d_{t+1}) = \beta \ln c_t + (1 - \beta) \ln d_{t+1}$$

where $c_t$ and $d_{t+1}$ are respectively composite consumption when adult and composite consumption when old; $\beta \in (0, 1)$ denotes individuals’ thrift.

Let $x = c, d$ denote individual consumption at each period of life, $x_N$ and $x_T$ be respectively the spending allocated to non-traded and traded goods. Instantaneous preferences are defined according to:

$$u(x_T, x_N) = x_T^\alpha x_N^{1-\alpha}, \quad 0 < \alpha < 1$$

Following Obstfeld and Rogoff, the small economy faces a capital inflows constraint (Equation 1). The consequence of this assumption is that the domestic return on capital may be higher than the world one. During the first period of life, individuals offer labor inelastically and distribute their net earnings $w_t$ among own consumption spending $\pi_t c_t$ and savings

$$\pi_t c_t + (1 + n) k_{t+1} + (1 + n) b_{t+1} = w_t$$

where $k_{t+1}$ is the whole capital stock per young agent in terms of traded goods $k_{t+1} = K_{t+1}/N_{t+1}$ and net foreign assets per young are $b_{t+1} = B_{t+1}/N_{t+1}$. The price of the tradable good is normalized at unity. We denote the composite consumption good by $x \equiv x_T^\alpha x_N^{1-\alpha}$ with $x = c, d$ to specify the same preferences among the two goods at each life period and $\pi_t$ the consumer price index. National savings can be held into two forms, capital stock and net foreign assets. Since the returns on these stocks are different, the agents choose both the amount of their savings $s_t$ and their allocation between the two assets $k_{t+1}$ and $b_{t+1}$. To take into account this arbitrage, we assume following Obstfeld and Rogoff that the constraint on capital inflows holds at the microeconomic level. This assumption means that banks cannot lend more than $\eta y_t$ to each individual at the world market interest rate $\bar{r}$. Agents know both the world and domestic returns on capital. A spread between these two returns is a new potential source of income for them: they can borrow from the world market to lend on the domestic market and realize a capital gain.
When old, individuals are retired and consume the proceeds of their savings according to
\[ \pi_{t+1} d_{t+1} = \left(1 + r^d_{t+1}\right) \left(1 + n\right) k_{t+1} + (1 + \bar{\pi}) (1 + n) b_{t+1} \]  
(5)

The domestic return on capital is the market interest rate \( r^d_{t+1} \) whereas the world return \( \bar{\pi} \) is fixed according to the small open economy assumption.

The maximization program of an individual born in period \( t \) solves in two steps. First, the individual chooses \( \pi_t c_t \) and \( b_{t+1} \) to maximize life-cycle utility (2) under the budget constraints (4), (5) and the capital inflows constraint
\[ b_{t+1} \geq - \frac{\eta y_t}{1 + n} \]  
(6)

Second, he shares his consumption spending (\( \pi x \)) between the two goods \( x_N \) and \( x_T \) to maximize instantaneous utility (3) under the spending constraint \( \pi x = R x_N + x_T \). Hence, the allocation of total consumption spending between the two goods at each period is
\[ x_T = \alpha \pi x \]
\[ R x_N = (1 - \alpha) \pi x \]

where the price index is \( \pi = \phi (\alpha) R^{1-\alpha} \), with \( \phi (\alpha) \equiv \alpha^{-\alpha} (1 - \alpha)^{\alpha-1} \).

### 3.2 Production Sectors

Investment transforms instantaneously a unit of tradable good in a unit of installed capital: \( K_{t+1} = I_t \) and capital fully depreciates after one period (\( \delta = 1 \)). The representative firm produces over the two sectors, the (non) traded sector is denoted by \( T \) (\( N \))

\[ \max_{I_t,K_{Tt},K_{Nt}} F(K_{Tt},L_{Tt}) + RH (K_{Nt},L_{Nt}) - wL_t - I_t \]

s.c. \( K_{t+1} = I_t \)
\[ K_{Tt} + K_{Nt} = K_t \]  
(7)
\[ L_{Tt} + L_{Nt} = L_t \]  
(8)

with \( L_t \) total labor supply, \( K_i \) and \( L_i \) respectively the amount of capital stock and labor supply dedicated to the \( i \) sector, \( i = T, N \). Dropping time indices, optimal allocation of factors is given by
\[ a_T f^\prime (k_T) = a_N Rh^\prime (k_N) \]  
(9)
\[ a_T \left[ f(k_T) - k_T f'(k_T) \right] = a_N R \left[ h(k_N) - k_N h'(k_N) \right] \]  \tag{10}

where capital intensity is \( k_i \equiv K_i / l_i \), the share of labor used in sector \( i \) is \( l_i = L_i / L \), \( i = T, N \), and intensive production functions are \( F(k_T, 1) \equiv f(k_T) \), \( H(k_N, 1) \equiv h(k_N) \). According to (9) and (10), \( k_N \) and \( k_T \) depend only on RER whereas the allocation of labor depends both on capital intensity and RER. Hence, \( k_N \equiv k_N(a_T, a_N, R) \) and \( k_T \equiv k_T(a_T, a_N, R) \), while \( l_N \equiv l_N(a_T, a_N, k, R) \) and \( l_T \equiv l_T(a_T, a_N, k, R) \). From (7), (8), (9) and (10), the optimal factor allocation satisfies

\[
\frac{dk_N}{dR} = \frac{a_T f}{R^2 a_N h''(k_N - k_T)}
\]

\[
\frac{dk_T}{dR} = \frac{Ra_N h}{f' a_T (k_N - k_T)}
\]  \tag{11}

Similarly, \( \partial l_N / \partial k \lesssim 0 \) if \( k_N \lesssim k_T \) and \( \partial l_N / \partial R > 0 \). When the tradable sector is capital intensive, a real appreciation leads to an increase in both capital intensities \( k_N \) and \( k_T \) whereas labor moves from the traded to the non-traded sector. These factor movements reflect that a real appreciation makes the non-tradable sector more attractive. Assuming perfect intersectoral mobility, the returns on capital \( r^d = a_T f'(k_T(a_T, a_N, R)) - 1 \equiv r^d(a_T, a_N, R) \) and labor \( w = a_T [ f(k_T(a_T, a_N, R)) - f'(k_T(a_T, a_N, R)) k_T(a_T, a_N, R) ] \equiv w(a_T, a_N, R) \) only depends on the RER \( R \) and productivity. A RER appreciation, profitable to the non-traded sector which is labor intensive, will increase wage and reduces domestic interest rate. An exogenous rise in traded (non-traded) productivity increases (decreases) domestic interest rate and (increases) reduces wage when the traded sector is capital intensive. Hereafter - unless otherwise slated - we omit productivity when there is no productivity change: \( k_T \equiv k_T(R) \), \( k_N \equiv k_N(R) \), \( l_N \equiv l_N(k, R) \), \( r^d \equiv r^d(k, R) \), \( w \equiv w(k, R) \).

Per capita global income depends on both RER and per capita capital stock: \( y = (1 + r^d(R)) k + w \equiv y(R, k) \) with

\[
\frac{\partial y}{\partial k} = 1 + r^d
\]  \tag{12}

\[
\frac{\partial y}{\partial R} = Ra_N h(k_N(R)) l_N(k, R)
\]  \tag{13}

A RER appreciation and a rise in per capita capital stock both exert a positive effect on global income.
3.3 The temporary equilibrium in the constrained case

Hereafter we focus on the case where the capital inflows constraint binds, at least initially, with a traded sector capital intensive. This creates a gap between domestic and world returns on capital. This gap - in accordance with risk premia phenomenon - reflects that developing countries do not have access to perfect international capital markets: the return on domestic capital \( r_{t+1}^d \) must be higher than the world market interest rate \( \bar{r} \) to offset bad economic conditions in these countries.

Young and old agents’ consumption functions are

\[
\pi_t c_t = \beta \left[ w_t - \frac{r_{t+1}^d - \bar{r}}{1 + r_{t+1}^d} (1 + n) b_{t+1} \right]
\]

(14)

\[
\pi_{t+1} d_{t+1} = (1 - \beta) \left[ (1 + r_{t+1}^d) w_t - \left( r_{t+1}^d - \bar{r} \right) (1 + n) b_{t+1} \right]
\]

(15)

Individuals consume a proportion \( \beta \) of their life-cycle income during the first period of life and the remaining when old. Life-cycle income consists of:

- the wage \( w \)
- the capital gain agents may realize borrowing at world rate \( \bar{r} \) to invest in domestic capital whose return \( r^d \) is higher than \( \bar{r} \)
- the redistribution premium: the return on the transfer between generations is \( 1 + n \) whereas the domestic market of capital offers \( 1 + r^d \).

The period-\( t \) temporary equilibrium conditions are

(i) **Capital market equilibrium.** Given the optimal intersectoral factor allocation \( k_T (R) \) and \( k_N (R) \), net foreign assets per capita are given by

\[
b_{t+1} = -\frac{\eta y (R_t, k_t)}{1 + n}
\]

(16)

Let \( \Gamma (R_{t+1}) \equiv \eta \left[ r^d (R_{t+1}) - \bar{r} \right] \left[ 1 + r^d (R_{t+1}) \right]^{-1} \) be the *arbitrage premium* which depends on the interest rate gap between domestic and world capital markets and on proportion \( \eta \) of the wage bill agents can borrow. The higher \( \Gamma \) the higher the capital gain agents realize. Therefore, capital per worker is

\[
k_{t+1} = \left[ 1 - \beta + \eta - \beta \Gamma (R_{t+1}) \right] \frac{w (R_t)}{1 + n} - \eta \frac{1 + r^d (R_t)}{1 + n} k_t
\]

(17)
(ii) **Labor market equilibrium.** The inelastic labor supply $N_t$ is equal to the labor demand $L_t$. Given the capital market equilibrium, the wage $w$ equalizing labor supply and demand is defined by

$$w(R_t) \equiv a_T \left[ f(k_T(R_t)) - k_T(R_t) f'(k_T(R_t)) \right]$$  \hspace{1cm} (18)

(iii) **Non-tradable goods market equilibrium.** There are $N_t$ young agents and $N_{t-1}$ old agents. Hence, the equilibrium on the non tradable goods market is

$$(1 - \alpha) \left( N_t \pi_t c_t + N_{t-1} \pi_d d_t \right) = R_t Y_N(R_t, k_t)$$  \hspace{1cm} (19)

with $Y_N(R_t, k_t) \equiv l_N(R_t, k_t) N_t h(k_N(R_t))$. Consumption spending is given by equations (14) and (15).

Equation (17) describes the allocation of saving between both assets. It offers a first dynamic relationship between RER and capital intensity. Using (16), (18) and (19), with consumption spending given by (14) and (15), we get a second dynamic relationship between $R$ and $k$.

The intuition behind the dynamics is the following. In such a constrained economy, the amount the country can borrow on world market is limited to a $\eta/(1 + n)$ fraction of the global income. In this 2-sector 2-factor model, global income does not only depend on capital intensity but also depends on RER. A RER appreciation - or an increase in capital intensity - increases global income in terms of traded good and then loosens the constraint. The country can borrow more, increases its capital stock and global output, loosening the constraint again. This mechanism will help the country converging to an unconstrained steady state if non-traded consumption is sufficiently high and if the constraint is not too tight (if $\eta$ not too small). Otherwise the country will remain constrained in the long-run\(^7\).

In the literature, there are constrained economy models like Barro, Mankiw and Sala-i-Martin (1995) or Lane (2001) that focused on the convergence speed issue. Indeed, in those constrained economy models the country systematically converges to an unconstrained steady state and the point is to know at what speed. At the opposite, this paper uses a model in which the country may converge in the long-run to a steady state that could either be constrained or

\(^7\)When the tradable sector is labor intensive, this is the RER depreciation that helps the country to converge to an unconstrained steady state. We do not focus on this case in what follows because it is less frequently observed and corresponds to a preliminary stage of development [Ito, Isard and Symanski (1999)].
unconstrained. The question here is to study the relationship between net foreign assets and RER in both types of equilibrium.

4 Steady state

There are two kinds of steady states: a constrained one and an unconstrained one. However, these two steady states do not exist simultaneously. The country may converge to an unconstrained steady state if the constraint is not too severe (high $\eta$), domestic saving is high (low $\beta$) or if agents consume enough services (low $\alpha$). The relationship between RER, NFA and productivity depends on the kind of steady state the economy converges to.

4.1 Constrained or unconstrained?

This subsection aims at determining the threshold level of the constraint, $\tilde{\eta}$ such that if $\eta < \tilde{\eta}$, the country will remain constrained in the long-run (see Appendix B). We will then proceed into three stages. First, we describe the two kinds of steady state that could either be constrained or unconstrained. Then we determine the threshold level of the constraint $\tilde{\eta}$.

4.1.1 A constrained steady state

The constrained steady state is denoted by a *. If the country remains constrained even in the long-run, the steady state $(k^*, R^*)$ is defined by the following system

\begin{equation}
    k^* = \frac{w(R^*) (1 - \beta) + \eta - \beta \Gamma (R^*)}{1 + \eta \frac{1 + r^d(R^*)}{1 + n}}
\end{equation}

\begin{equation}
    \left[ \beta + \frac{1 + r^d(R^*)}{1 + n} (1 - \beta) \right] \left[ 1 + \frac{r^d(R^*) - \bar{r}}{1 + r^d(R^*)} \eta \right] w(R^*) = \frac{y_N (R^*, k^*) R^*}{1 - \alpha}
\end{equation}

Equation (20) gives the long-run allocation of saving. In this constrained economy, capital per capita $k$ is financed by domestic saving plus capital inflows. Equation (21) is the long-run non-traded good market clearing condition. Both long-run capital intensity and RER...
are determined by those two conditions. Then, the constraint gives net foreign assets: \( b^* = -\eta y (R^*, k^*) / (1 + n) \).

### 4.1.2 An unconstrained steady state

An overbar – denotes the unconstrained steady state. This unconstrained steady state \((\bar{k}, \bar{R})\) is determined by equations (22) and (23). It is the standard steady state that occurs in a two-sector two-factor small open economy model. The developing country has a perfect access to the international capital market:

\[
r^d(\bar{R}) = \bar{r} \tag{22}
\]

That is domestic return on capital converges to the world one. Equation (22) determines the long-run RER that depends only on the world interest rate\(^\text{10}\). The long-run RER determines the wage and hence the demand for services (left hand side of equation (23)). Domestic capital \(\bar{k}\) clears the non-traded good market:

\[
\left[ \beta + \frac{(1 + \bar{r})(1 - \beta)}{1 + n} \right] w(\bar{R}) = \frac{y_N(\bar{R}, \bar{k}) \bar{R}}{1 - \alpha} \tag{23}
\]

Finally, the net foreign assets fill the gap between domestic capital \(\bar{k}\) and domestic saving:

\[
\bar{b} = w(\bar{R}) \frac{1}{1 + n} (1 - \beta) - \bar{k} \tag{24}
\]

In this unconstrained steady state, the Balassa-Samuelson analysis holds since RER only depends on the supply side of the model.

### 4.1.3 The threshold level

The level of the constraint, \(\eta\), is exogenous and could be interpreted as the penalty imposed by international investors to a developing country because of political instability, corruption, inflation. In this paper, we take this penalty as given and determines whether this \(\eta\)–penalty is too severe or not to allow the developing country to converge to the unconstrained steady state. Let \(\tilde{\eta}\) be the threshold level of the constraint such that

- when \(\eta \geq \tilde{\eta}\), the country converges to the unconstrained steady state and we recover the standard small open economy setting.

\(^{10}\)The RER is also determined eventually here by the productivity spread between sectors as well:

\[
r^d(a_N, a_T, R) = \bar{r} \quad \text{with} \quad \partial r^d / \partial a_N < 0, \partial r^d / \partial a_T > 0 \quad \text{when the traded sector is capital intensive.}\]
- when $\eta < \tilde{\eta}$, the country converges to the constrained steady state and remains constrained in the long-run.

A special case of this model would be $\eta = 0$ where the country would be so constrained that net foreign assets would be zero. This case would correspond to a closed economy setting.

A rise (drop) in $\tilde{\eta}$ makes the convergence to the (unconstrained) constrained steady state more likely to occur. We can characterize the threshold level $\tilde{\eta}$ in a simple Cobb-Douglas case.

**Example: The Cobb-Douglas case** We assume Cobb-Douglas technologies in both sectors.

Let the long-run propensity to consume the non-traded good be

$$\Psi = (1 - \alpha) \left[ \beta + (1 - \beta) \left( 1 + \bar{r} \right) / (1 + n) \right]$$

After a bit of algebra (see Appendix B) we can show that

$$\tilde{\eta} = \frac{(1 - \beta) (1 - \nu) - \frac{1 + \bar{r}}{1 + n} [\nu + \Psi (\rho - \nu)]}{\nu + \Psi (\rho - \nu) - (1 - \nu)}$$

We have $\nu - \Psi (\nu - \rho) - (1 - \nu) < 0$ when traded sector is capital intensive\(^{11}\). This implies that a rise in the propensity to consume the non-traded good promote convergence to the unconstrained steady state. The intuition behind this result is simply that a rise in non-traded goods consumption tends to appreciate the RER. This RER appreciation relaxes the constraint and help the country reaching the unconstrained steady state. In the same way, the threshold level $\tilde{\eta}$ depends on $n$ and $\beta$ since population growth and time preference influence both propensity to consume $\Psi$ and savings: $\partial \tilde{\eta} / \partial n > 0$ when $\Psi < 1$, $\partial \tilde{\eta} / \partial \beta < 0$ when $\bar{r} < n$.\(^{17}\)

**Calibration** We assume that half of consumption is spent on non-traded good. Assuming that each generation lives for 25 years, the world interest factor is $1 + \bar{r} = 0.37$ which means that the world real interest rate is about 1.25% per year, and $n = 0.6$ corresponds to a rate of population growth of 1.9%. In accordance with Beine et al. (2001), let $\beta = 0.6$ to have a domestic rate of time preference of around 3.56%. Using those figures, the threshold level is $\tilde{\eta} = 0.39$. Figure 5 represents the long-run equilibrium. The constrained steady state is represented for $\eta = 0.2$ and the unconstrained steady state for $\eta = 0.7$. In the constrained steady state, the domestic real interest rate exceeds the world interest rate

\(^{11}\)This is the case when $1 - \nu > \rho$ and $\Psi < 1$.\(^{17}\)
and more resources are allocated to the production of the traded good. Since domestic interest rate and RER are negatively related, the RER is lower in the constrained steady state than in the unconstrained one.

Figures 5: A constrained or an unconstrained steady state

The loci (CM) and (NTM) depict, respectively, the capital market and non-traded good market equilibria.

If (CM) intersects (NTM) below (WIR), the steady state is the unconstrained one.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Constrained steady state</th>
<th>Unconstrained steady state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{k}/k$</td>
<td>0.7394</td>
<td>0.3770</td>
</tr>
<tr>
<td>$\bar{r}/r$</td>
<td>0.4168</td>
<td>0.3770</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>0.4653</td>
<td>0.4168</td>
</tr>
</tbody>
</table>

4.2 NFA and RER

The steady state can either be constrained or unconstrained. The relationship between NFA and RER will depend on the nature of the steady state the economy converges to. Let $T_t$ denote a transfer received from abroad. Then capital market equilibrium becomes

$$s_t + T_t = (1 + n) [b_{t+1} + k_{t+1}]$$

We can consider $T$ as a gift from foreigners. Like savings, this gift will be dedicated to the assets accumulation.

4.2.1 Unconstrained steady state

Long-run equilibrium is given by equations (23) and (24)

$$\left[ \beta + \frac{(1 + \bar{r}) (1 - \beta)}{1 + n} \right] w(\bar{R}) = \frac{yN(\bar{R}, \bar{k})}{1 - \alpha} \bar{R}$$

$$\bar{b} = T + \frac{w(\bar{R})}{1 + n} (1 - \beta) - \bar{k}$$

(25)

and unchanged equation (22). A transfer will increase NFA. Since RER is exclusively determined by productivity and world interest factor: there is no transfer effect and NFA do not cause the RER.
4.2.2 Constrained steady state

In the constrained steady state, long-run equilibrium is given by equations (20) and (21) and the constraint still binds

\[ b^* = \frac{-\eta y (R^*, k^*)}{1 + n} \]

The introduction of the transfer \( T \) changes equation (20) that becomes

\[ k^* = \frac{1}{1 + \eta} \frac{1 + \nu (R^*)}{1 + n} \left[ \frac{w (R^*)}{1 + n} \left[ (1 - \beta) + \eta - \beta \Gamma (R^*) \right] + \frac{T}{1 + n} \right] \]

A transfer will have two kinds of effects

(i) a direct effect: a rise in \( T \) increases capital intensity. Since the non-traded sector is labor intensive, this rise in capital reduces non-traded output and leads to a RER appreciation.

(ii) an indirect effect: a transfer increases global production and loosens the constraint. As a result capital stock increases more and this reinforces the RER appreciation. The higher \( \eta \) - the more the country is allowed to borrow on international market - the higher the RER appreciation.

As in Lane and Milesi-Ferretti (2004), the transfer effect increases with the size of the non-traded sector: the less open (low \( \alpha \)) the country, the higher the direct effect. However, our model shows analytically that the transfer effect depends also on the country’s access to external borrowing. It holds only in the constrained economy case that is when\(^{12} \eta < \tilde{\eta} \). Conversely, when \( \eta \geq \tilde{\eta} \) there is no transfer effect at all.

We can notice that another consequence of the transfer would be to help the constrained economy reach an unconstrained steady state. Indeed, a positive transfer tends to reduce \( \tilde{\eta} \) (See Appendix B for details)

\[ \tilde{\eta} = \frac{(1 - \beta)(1 - \nu) - \frac{1 + \nu}{1 + \rho} \left[ \nu + (1 - \alpha) \Psi (\rho - \nu) \right] + \frac{\nu T^*}{1 + \rho} \left( \frac{1 + \rho}{\eta T^*} \right) }{\nu - \Psi (\nu - \rho) - (1 - \nu)} \]

A positive transfer from abroad may reduce \( \tilde{\eta} \) and promote convergence - help the country reach an unconstrained steady state.

In the unconstrained steady state, the relationship between NFA and RER is univariate: it goes from the RER to NFA. Conversely, in the constrained steady state, there is a bilateral

\(^{12}\)In this constrained economy case, the more constrained the country, the lower the transfer effect.
relationship between NFA and RER

(i) From RER to NFA: a RER appreciation attracts capital inflows. Since the constraint
binds, a RER appreciation increases global output and relaxes the constraint: a rise in RER
causes a decrease in NFA.

(ii) From NFA to RER: a positive transfer effect.

4.3 RER and Productivity

In this 2x2 model (2 sectors, 2 mobiles factors) global output increases not only with capital in-
tensity but also with RER appreciation [See equations (12) and (13)]. However, the relationship
between RER and output still depend on the nature of the steady state the economy converges
to.

4.3.1 Unconstrained steady state

Long-run equilibrium is given by equations (23) and (24). The RER is exclusively determined
by the world interest rate and productivity spread between sectors according to

$$r^d (a_T, a_N, R) = \bar{r}$$  \hspace{1cm} (26)

An increase in traded productivity will directly generate a RER appreciation (Balassa-Samuelson
effect). Then, this RER appreciation leads to a rise in output: the higher the saving rate and
population growth, the higher the output rise.

4.3.2 Constrained steady state

RER and capital intensity clears non-traded goods market and the long-run equilibrium is given
by

$$b^* = -\frac{\eta y (a_T, R^*, k^*)}{1 + n}$$

$$k^* = \frac{1}{1 + \eta \frac{1 + r^d (a_T, R^*)}{1 + n}} \left[ \frac{w (a_T, R^*)}{1 + n} (1 - \beta) + \eta - \beta \Gamma (a_T, R^*) \right]$$

where $\partial r^d / \partial a_T > 0$ and $\partial w / \partial a_T < 0$. The constraint always binds so that NFA are determined
by output. The Balassa-Samuelson theory does not hold here in the sense that equation (22)
no longer applies. The RER does not only depend on productivity and world interest rate but
instead results from demand and supply of non-traded output. A rise in traded productivity \( a_T \) leads to changes in both demand and supply of non-traded goods and will generate more precisely

(i) an ambiguous effect on non-traded goods demand\(^{13} \) due to a rise in domestic return on capital combined with a wage decrease.

(ii) a decrease in non-traded output

(iii) an ambiguous effect on global output

The third effect will affect the country’s capacity to borrow on international market. A rise in global output will relax the constraint, increases capital stock and decreases non-traded output. Conversely, a decrease in global output will retighten the constraint, reducing capital stock and increasing non-traded output. Since this third effect is ambiguous, the relationship between traded productivity and RER is difficult to characterize in this constrained steady state. For economies with high rate of time preference and/or not allowed to borrow enough on international markets, the Balassa-Samuelson effect may be reversed: a rise in traded productivity may lead to a RER depreciation. Otherwise (high \( \beta \) and/or high \( \eta \)), the RER still appreciates as in the unconstrained case but operating here through a demand effect and not only through a productivity channel as in the unconstrained case.

5 Econometric tests

In this section we present empirical evidence on the determinants of the RER in order to analyze the main conclusions from the theory model presented in the previous section. Our strategy follows two steps. We first test for the steady state solution of the model by analyzing the long-run cointegration vector for the RER and its main determinants and split the sample according to the degree of restrictiveness to foreign capital. We then estimate a system Error Correction Model (ECM) where we analyze causal relationships between the variables in both the short and the long-run. It is worth noting that, far from testing directly the model, we aim at analyzing whether its main conclusions are reflected in the behavior of the data. Throughout our analysis, we make use of the Lane and Milesi-Ferretti (2007) dataset, which we match with the Chinn-Ito

\(^{13}\)With a simulation exercise, we could show in the high majority of cases, that demand on non-traded goods decrease.
We estimate an equilibrium (log) real effective exchange rate \( LREER_t \) equation where we consider relative productivity (proxied by log-relative per-capita income \( LYD_t \)) and net foreign assets as a percentage of GDP (\( NFA_t \)) as the main steady state determinants of the RER. This specification is derived from equations (26) (20) (21) that determined long-run RER. Following Lane and Milesi-Ferretti (2004), we also consider the (log) terms of trade \( LTT_t \) as a control variable. The estimated equation takes the form:

\[
LREER_t = \beta_1 LYD_t + \beta_2 NFA_t + \beta_3 LTT_t + \varepsilon_t
\]  

(27)

We estimated the equation using a panel modified version of the Dynamic-OLS estimator. Following Phillips and Loretan (1991), we augmented the regression not only with leads and lags of the variables, but also with lags of the residuals. This estimator, which is equivalent to a Maximum Likelihood estimator, is not only efficient but ensures that the parameters are normally distributed. In our application, given the yearly nature of the data, we used one lag (and lead) augmentation\(^{14}\).

Given our interest on the impact of external financial access on the determination of the RER, we proceeded to split the sample into constrained and unconstrained economies. For this, as argued in Section 2, we used the KAOPEN index recently developed by Chinn and Ito (2007).\(^{15}\) This is an index that measures the extent of openness in capital account transactions of an economy. It is based on the first standardized principal component of a series of binary variables accounting for the presence of multiple exchange rates, capital account transaction restrictions, current account transactions restrictions and the appropriation of export proceeds. Low values indicate high capital account restrictions and hence constraints on the access to international finance.

In order to obtain an optimal sample split, we used the methodology developed by Hansen (2000) based on threshold estimation. It would be desirable to obtain a sample split for the whole

\(^{14}\)The results from using a DOLS(1,-1) estimator are very similar.

\(^{15}\)One could think of using derived measures like the \( \beta \) coefficient estimated in Section 2 to capture capital constraints. However, this is an estimated parameter subject to estimation uncertainty and cannot be used for sample splitting exercises. The properties of these tests using estimated coefficients are simply not known.
panel data. However, threshold sample splitting techniques for nonstationary variables are not available in the literature. For this reason we applied the sample splitting method on a cross-sectional estimate where we regress the first difference of $LREER$ on the first difference of the independent variables. In order to avoid problems of initial and final anomalous observations, the first difference is defined as the difference between the average value of the variable for 1985-1998 and the average for 1975-1985. It is worth noting, though, that splitting the sample between countries with positive and negative average KAOPEN values gave remarkably similar results\textsuperscript{16}. The results from the sample splitting tests are reported in Table 1\textsuperscript{17}.

<table>
<thead>
<tr>
<th>Table 1: Sample splitting results using KOPEN index and sample averages of variables</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Threshold estimate</strong></td>
</tr>
<tr>
<td><strong>95% Confidence Interval</strong></td>
</tr>
<tr>
<td><strong>LM test</strong></td>
</tr>
<tr>
<td><strong>Bootstrapped p-value</strong></td>
</tr>
<tr>
<td><strong>No. countries Regime 1 (KOPEN&gt;threshold)</strong></td>
</tr>
<tr>
<td><strong>No. countries Regime 2 (KOPEN&lt;threshold)</strong></td>
</tr>
</tbody>
</table>

Notes: The table shows the results from the Hansen (2000) threshold method for sample splitting. We used a simple cross-sectional estimate using sample averages of the variables to determine the sample split. The threshold variable is KOPEN from Ito and Chinn (2007). The bootstrapped CVs were obtained using 1,500 bootstrap draws.

We report the value of the threshold of KAOPEN for sample splitting, the LM test and it’s bootstrapped p-value and the number of countries in each regime\textsuperscript{18}. These show that KAOPEN is a significant variable for sample splitting and the split point is at a value of -0.53. Figure 6 shows the recursive Likelihood Ratio test depending on the threshold, variable, which is

\textsuperscript{16}This is also the case if we split the sample by observation rather than by country using positive and negative KAOPEN values as indicator function. This way, however, is problematic as we need data that is continuous in time and we lose observations when the same country switches from positive to negative KAOPEN.

\textsuperscript{17}Other sample splitting variables were considered such as NFA and YD, but they did not yield significant results. We also used an arbitrary split considering countries with positive and negative KOPEN and the results remained very similar.

\textsuperscript{18}We also used $\Delta LYD$ and $\Delta NFA$ as potential thresholds, but the LM test rejected the hypothesis that the sample could be split using these variables.
minimized at -0.53.

All the variables were checked for stationarity and the I(1) specification could not be rejected. We then proceeded to test for cointegration using $LREER_t$, $NFA_t$, and $LYD_t$ as dependent variables in the cointegration vector. Table 2 presents the results from the group-ADF test of Pedroni (1999)$^{19}$ and shows that only the equations using $LREER_t$ as dependent variable constitute long-run equilibrium relations.

Table 2: Panel cointegration tests, Group-ADF statistics

<table>
<thead>
<tr>
<th>Dep. variable</th>
<th>Whole sample</th>
<th>Panel A (KAOPEN&gt;-0.53)</th>
<th>Panel B (KAOPEN&lt;-0.53)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LREER_t$</td>
<td>-2.63***</td>
<td>-1.84*</td>
<td>-1.97**</td>
</tr>
<tr>
<td>$LYD_t$</td>
<td>-0.51</td>
<td></td>
<td>-0.74</td>
</tr>
<tr>
<td>$NFA_t$</td>
<td>0.65</td>
<td></td>
<td>-1.58</td>
</tr>
</tbody>
</table>

Note: (***), (**) and (*) show rejection of the null hypothesis of no cointegration at 1%, 5% and 10% statistical level respectively.

$^{19}$We also checked for cointegration using the panel ADF test and the group and panel PP tests proposed in Pedroni (1999). The results are invariant to extracting group means to account for cross-sectional correlation.
The results for the cointegration vectors are reported in Tables 3 to 5.

**Table 3: Long-run cointegration vector. Whole sample.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LYD</td>
<td>0.044</td>
<td>0.039</td>
<td>[.265]</td>
</tr>
<tr>
<td>NFA</td>
<td>0.320</td>
<td>0.028</td>
<td>[.000]</td>
</tr>
<tr>
<td>LTT</td>
<td>0.070</td>
<td>0.025</td>
<td>[.005]</td>
</tr>
<tr>
<td>R²</td>
<td></td>
<td>0.883</td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td></td>
<td>1,320</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4: Long-run cointegration vector. Unconstrained economies.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LYD</td>
<td>0.477</td>
<td>0.048</td>
<td>[.000]</td>
</tr>
<tr>
<td>NFA</td>
<td>0.021</td>
<td>0.035</td>
<td>[.549]</td>
</tr>
<tr>
<td>LTT</td>
<td>0.218</td>
<td>0.038</td>
<td>[.000]</td>
</tr>
<tr>
<td>R²</td>
<td></td>
<td>0.887</td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td></td>
<td>528</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5: Long-run cointegration vector. Constrained economies.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LYD</td>
<td>-0.177</td>
<td>0.043</td>
<td>[.000]</td>
</tr>
<tr>
<td>NFA</td>
<td>0.482</td>
<td>0.033</td>
<td>[.000]</td>
</tr>
<tr>
<td>LTT</td>
<td>-0.001</td>
<td>0.028</td>
<td>[.819]</td>
</tr>
<tr>
<td>R²</td>
<td></td>
<td>0.865</td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td></td>
<td>792</td>
<td></td>
</tr>
</tbody>
</table>

The results for the whole sample show a positive relationship of the three variables considered with the RER. This is similar to the results in Lane and Milesi-Ferreti (2004), but in our case relative income appears to have an insignificant effect. The results, though, are not directly comparable due to differences in the sample of countries. The results from the sample splitting show that for unconstrained economies only \(LYD\) and \(LTT\) appear as significant long-run determinants of the RER. Conversely, for constrained economies, \(NFA\) becomes strongly significant, and \(LYD\) appears to have a negative impact on the RER. This lends support to the theoretical hypothesis that NFA is only important for constrained economies and YD is the main driver of RER for unconstrained economies (see Subsections 4.4 and 4.5). In the model, the transfer effect
(positive effect from NFA to RER) holds only in constrained economies whereas the Balassa-
Samuelson effect (positive effect from YD to RER) always holds in unconstrained economies.
For the more constrained economies, the Balassa-Samuelson effect may be reversed: a rise in
YD may cause a drop in RER. This is supported by econometric results.

Using the estimated long-run vectors, we then proceed to estimate ECMs for each of the
variables involved, treating \( LTT \) as strictly exogenous. Given the well known bias of the OLS
estimator for dynamic stationary panels, we estimated the ECMs by system GMM. We used as
instruments the second and third lags of the variables and the lagged level\(^{20}\). The results are
reported in Tables 6 to 8, where we show the coefficients and their p-values and a J-test for over-
identifying restrictions. In all cases the J-test indicated that the system is overidentified. The
coefficient on the ECM indicates whether or not the cointegration vector is a long-run attractor
and hence can be used to test for long-run causality, whereas the rest of the coefficients indicate
short-run causality.

<table>
<thead>
<tr>
<th>( \Delta LRER_{t-1} )</th>
<th>( \Delta LYD_{t-1} )</th>
<th>( \Delta NFA_{t-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.284 [.004]</td>
<td>-0.107 [.015]</td>
<td>-0.015 [.330]</td>
</tr>
<tr>
<td>0.027 [.265]</td>
<td>0.011 [.685]</td>
<td>0.026 [.001]</td>
</tr>
<tr>
<td>0.122 [.164]</td>
<td>-0.108 [.162]</td>
<td>0.565 [.000]</td>
</tr>
<tr>
<td>-0.044 [.321]</td>
<td>0.559 [.001]</td>
<td>0.048 [.007]</td>
</tr>
<tr>
<td>-0.151 [.000]</td>
<td>0.018 [.191]</td>
<td>0.003 [.617]</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.071</td>
<td>0.153</td>
</tr>
<tr>
<td>J-test [p-val]</td>
<td>18.87 [0.22]</td>
<td></td>
</tr>
</tbody>
</table>

Notes: numbers in brackets [] are p-values of the coefficients. The J-test is a test for
over-identification restrictions for the system.

\(^{20}\) The results are invariant to the inclusion of the lag level.
Table 7: GMM estimation of the system ECM model. Unconstrained economies.

<table>
<thead>
<tr>
<th></th>
<th>Eq for ΔLRER, t-1</th>
<th>Eq for ΔLYD, t-1</th>
<th>Eq for ΔNFA, t-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔLRER, t-1</td>
<td>-0.060 [.629]</td>
<td>-0.041 [.350]</td>
<td>0.001 [.954]</td>
</tr>
<tr>
<td>ΔLTT, t-1</td>
<td>0.074 [.009]</td>
<td>-0.007 [.844]</td>
<td>0.011 [.328]</td>
</tr>
<tr>
<td>ΔLYD, t-1</td>
<td>0.412 [.001]</td>
<td>0.028 [.781]</td>
<td>0.344 [.004]</td>
</tr>
<tr>
<td>ΔNFA, t-1</td>
<td>0.072 [.095]</td>
<td>0.512 [.005]</td>
<td>0.029 [.292]</td>
</tr>
<tr>
<td>ΔECM, t-1</td>
<td>-0.159 [.000]</td>
<td>-0.009 [.754]</td>
<td>0.002 [.872]</td>
</tr>
<tr>
<td>R²</td>
<td>0.099</td>
<td>0.179</td>
<td>0.092</td>
</tr>
<tr>
<td>J-test [p-val]</td>
<td>18.99 [0.21]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: see Table 4.

Table 8: GMM estimation of the system ECM model. Constrained economies.

<table>
<thead>
<tr>
<th></th>
<th>Eq for ΔLRER, t-1</th>
<th>Eq for ΔLYD, t-1</th>
<th>Eq for ΔNFA, t-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔLRER, t-1</td>
<td>0.357 [.007]</td>
<td>-0.215 [.000]</td>
<td>-0.012 [.531]</td>
</tr>
<tr>
<td>ΔLTT, t-1</td>
<td>0.004 [.912]</td>
<td>-0.048 [.118]</td>
<td>0.029 [.008]</td>
</tr>
<tr>
<td>ΔLYD, t-1</td>
<td>0.024 [.829]</td>
<td>-0.292 [.005]</td>
<td>0.663 [.000]</td>
</tr>
<tr>
<td>ΔNFA, t-1</td>
<td>-0.039 [.540]</td>
<td>0.922 [.000]</td>
<td>0.043 [.084]</td>
</tr>
<tr>
<td>ΔECM, t-1</td>
<td>-0.159 [.000]</td>
<td>0.027 [.128]</td>
<td>0.004 [.616]</td>
</tr>
<tr>
<td>R²</td>
<td>0.049</td>
<td>0.126</td>
<td>0.045</td>
</tr>
<tr>
<td>J-test [p-val]</td>
<td>15.95 [0.38]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: see Table 5.

The results for unconstrained economies indicate that, in the long-run, only the RER is temporally caused by its determinants as only the ECM for RER is significant. In the short run, relative income causes the RER and NFA causes relative income. Finally, NFA also appears to be positively affected by LYD indicating a two-way causality between these variables. From the point of view of the theory model, this is partially supportive since in unconstrained economies

(i) RER is caused by productivity

(ii) NFA are caused by income and savings

(iii) Income is caused by capital intensity, and capital intensity results from domestic savings and NFA.

For constrained economies long-run causality is similar to that for unconstrained economies. This indicates that the determinants of the RER are weakly exogenous. In the short run, we find that NFA has a positive impact on relative income and the RER a negative one. Income also appears to cause NFA in the short run with a positive sign.
We analyzed further the results to check for the possibility that they are sensitive to the inclusion of some countries in the sample. The robustness of our findings was tested using a cross-validation approach. We assume a function:

\[ \Phi_{it}(x_{it}, z_{it}, \delta) = u_{it} \quad i \in [1, N], t \in [1, T] \] (28)

where \( x_{it} \) is the vector of endogenous variables; \( z_{it} \) is the vector of exogenous variables; \( \Phi_{it} \) is a vector function representing the estimated cointegration functional form or the EC model; \( \delta \) are the estimated parameters, and \( u_{it} \) is the error term. Denote by \( \hat{\delta}_{(i-1)} \) the estimate of \( \delta \) obtained when we omit country \( i \in [1, N] \) from the sample. In this case, \( \hat{\delta}_{(i-1)} \) is the cross-validated estimate of \( \delta \) when we omit country \( i = 1, ..., N \). This procedure allows us to check for correct statistical inference, especially when the number of cross-sections is small.

We analyzed the robustness of all the estimated coefficients in both panels for the long-run and the ECM term in the EC model. The estimates remained remarkably stable throughout, which further confirms the advantages of our sample split as no further parameter instability appears to be present in the model. Figures 7-1 and 7-2 present the estimated coefficient (denoted
by a diamond) for the panel when we drop each one of the countries ± one standard error.

**Figure 7-1: Beta coefficients sensitivity analysis, high KAOPEN**

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**Beta coefficient on YD and standard errors**

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**Beta coefficient on NFA and standard errors**
For reasons of space we only present the coefficients of the long-run vector for $\text{LREER}_{it}$ and $\text{YD}_{it}$, but the other coefficients, including the one for the ECM, also show the same stable pattern. It can be easily seen that dropping any of the countries does not produce a substantial change in the estimated coefficients and their significance. Any changes are of a small order of magnitude, both statistically and economically.

6 Conclusion

Empirical evidence suggests that significant real exchange rate (RER) determinants change as we vary periods and countries considered. We develop an overlapping generations two-factor
two-sector model of a small open economy in which the way RER is determined varies with the country's capacity to borrow on international markets. We assume the country is constrained on capital inflows. A special feature of that model, conversely to the existing literature, is that the steady state can either be constrained or unconstrained. The way capital, net foreign assets (NFA) and real exchange rate are determined depends on the nature of the steady state. In the unconstrained steady state, the RER only depends on productivity spread between sectors - a Balassa-Samuelson effect. In the constrained steady state, the RER does not only depend on productivity but also on savings determinants and capital inflows (transfer effect).

We then study the implications of the model using econometric evidence on the determinants of the RER for financially constrained and unconstrained economies. We split the sample endogenously using the Chinn and Ito (2007) capital account openness variable as an index of external financial constraints. Our results validate partly the theoretical implications of the model: RER is mainly driven by productivity and NFA in countries that face foreign exchange restrictions and exclusively by productivity in countries that have perfect access to international capital market.
APPENDIX

A  Empirical evidence

The 55-country database details are given by Table 9

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<td>CAN JAP</td>
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<tr>
<td>SAUDI ARABIA</td>
<td>SAUDI ARABIA</td>
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</tbody>
</table>

Figure 8 plots the beta coefficient of the per-country regressions of Assets/GDP on GDP per capita against GDP per capita.

Figure 9 plots the beta coefficient of the per-country regressions of Liabilities/GDP on GDP
per capita against GDP per capita.

**Figure 9**

Liabilities vs GDPpc

B Calculation of $\tilde{\eta}$

Under what conditions will the domestic interest rate $r^d$ converge to the world one? Will domestic saving be enough to drive $k^*$ to $\bar{k}$ given the borrowing constraint?

Let $\tilde{k}^*$ denote the steady state capital per capita obtained in the constrained steady state (see equation (20)) in which $R = \bar{R}$

$$
\tilde{k}^* = \frac{w(\bar{R})}{1 + n} \frac{1 - \beta + \eta}{1 + \eta} $n
$$

since $\Gamma(R) = 0$. Then the critical level of the constrained $\tilde{\eta}$ is such that $\tilde{k}^* = \bar{k}$. Let the production functions be

$$
f(k_T) = \nu \eta, \quad 0 < \nu < 1 \quad (30)
$$

$$
h(k_N) = \rho \eta, \quad 0 < \rho < 1 \quad (31)
$$

Let $\Psi = (1 - \alpha)[\beta + (1 - \beta)(1 + \bar{r})/(1 + n)]$ denotes the aggregate propensity to consume non-traded good. Then using $k = (1 - l_N) k_T + l_N k_N$, the convenient Cobb-Douglas specification and equations (23) and (29) the critical level of the constraint $\tilde{\eta}$ is given by

$$
\tilde{\eta} = \frac{(1 - \beta)(1 - \nu) - \frac{1 + \eta^2}{1 + \eta}[\nu - \Psi(\nu - \rho)]}{\nu - \Psi(\nu - \rho) - (1 - \nu)} \quad (32)
$$
We can also calculate $\tilde{\eta}$ when there is a transfer in the long-run. The temporary equilibrium in the constrained case becomes with the transfer

$$k_{t+1} = [1 - \beta + \eta - \beta T(R_{t+1})] \frac{w(R_t) - \eta}{1+n} - \eta \frac{1 + r^d (R_t) k_t + T_t}{1+n}$$

with $T$ per-capita transfer at time $t$.

The constrained steady state is

$$k^* = \frac{w(R^*) (1 - \beta) + \eta - \beta T(R^*)}{1+n + \eta \frac{1 + r^d (R^*)}{1+n}} + \frac{T^*}{1 + n + \eta (1 + r^d (R^*))}$$

which gives

$$\tilde{k}^* = \frac{w(\bar{R}) (1 - \beta) + \eta}{1+n + \eta \frac{1 + \bar{r}}{1+n}} + \frac{T^*}{1 + n + \eta (1 + \bar{r})}$$

The unconstrained steady state capital stock $\bar{k}$ is unchanged and then we can calculate $\tilde{\eta}$ such that $\tilde{k}^* = \bar{k}$

$$\tilde{\eta} = \frac{(1 - \beta)(1 - \nu) - \frac{1+n}{1+\nu} [\nu + \Psi (\rho - \nu)] + \frac{\nu T^*}{1+\nu} \left( \frac{1+\rho}{1+\nu} \right)^{1+\rho}}{\nu - \Psi (\nu - \rho) - (1 - \nu)}$$

C Convergence to the Steady State

This unique steady state could either be constrained or unconstrained depending on $\eta - \tilde{\eta}$. In the long-run, three cases have to be distinguished:

- Case 1: $k_0 \geq \bar{k}$ and the country converges at the first period to the unconstrained steady state. The country will be a debtor (creditor) vis-à-vis the rest of the world if $\beta$ is low (high) - we assume that $n > \bar{r}$ to be compatible with the small open economy assumption.

- Case 2: $k_0 < \bar{k}$ and $\eta < \tilde{\eta}$. The country will never converge to the unconstrained steady state except if the government encourages savings or households to consume more non-tradable goods.

- Case 3: $k_0 < \bar{k}$ and $\eta > \tilde{\eta}$. The country converges to the unconstrained steady state.

Hereafter we focus on Case 2. The objective of the following subsection is to determine what the government can do to influence $\tilde{\eta}$, promote capital inflows and make the convergence more likely to occur.
References


