GONE IN 60 SECONDS:
LAST-MINUTE BIDDING ON EBAY AUCTIONS

Abstract

This is a theoretical and empirical study of the delay of bid submission due to strategic last-minute bidding in eBay. We consider a two-period model in which two identical items are auctioned sequentially by different sellers. We show that there is a symmetric equilibrium in which the bidders wait until the last minute to submit their bids in the first auction. The last-minute bidding arises from the sequential structure of the sale and not, as in Ockenfels and Roth (2002, 2005), because last minute bids may not be recorded. Additionally, we prove that the last-minute bidding equilibrium yields the same expected revenue for both auctions. We test the predictions of the theory and confirm the existence of last-minute bidding using eBay bid data from 1,973 auctions of Texas Instruments TI-83 Graphing Calculators. Furthermore, we introduce a new and informative approach of studying bid submission times. We arrange the auctions consecutively according to their closing times and focus on the intervals in between.

1. Introduction

Historically, auctions have been one of the most prominent and popular selling mechanisms. In recent years, the pervasive spread of high-speed Internet has led to a significant spurt in online trading; internet auctions are a popular manifestation of this trend. Buyers and sellers can meet in this virtual market place to buy and sell a varied range of products without leaving the comfort of their home. The historic development of this low-cost trading mechanism thus created an entirely new market. In the United States eBay has become the dominant player in this market with expected sales as high as $4.4 billion in 2005. As of June 2005, its customer base has grown to 147 million and earnings climbed 53 percent relative to the corresponding figure the year before.[1]

EBay allocates the item through an auction with proxy bidding, which is a mechanism that bids on behalf of the bidder up to a reservation price specified by her. At the end of the auction, the bidder with the highest reservation price wins the item by a price equal to the second highest reservation price submitted. The auction thus resembles an ascending second-price auction. The auction lasts for a number of days that is chosen by the seller. Thus, by choosing the starting date and length of the auction, the seller also chooses the precise day and time it closes. There is no extension of auction length at the end of eBay auctions. This "hard-close" rule gives bidders an opportunity to delay their bid submissions and place their bids in the last minutes of the auctions. In a second-price setting with private values[2] Vickrey (1961) shows that bidders have a weakly dominant strategy to bid their valuations at any time during the auction. In practice, though, observation of bid submission times shows that bidders tend to wait until the very end of the auction to submit their bids.

[1] This information and more about eBay is available at www.forbes.com.
[2] Potential buyers know their own valuation of the item. All these individual valuations are independent of each other.

This delay in submission time until the end of the auction is called "Last-Minute Bidding" or "Sniping".

Researchers offer various explanations for the incidence of Last-Minute Bidding (LMB). Ockenfels and Roth (2002, 2005) argue that late bidding could be due to network congestion that might prevent some of the submitted bids from being recorded. This explanation seems plausible due to the firm ending rule of eBay auctions. When bids are submitted at the last minutes of the auction, an increase in the network activity decreases the probability of each bid getting recorded. As a result, bidders, under certain conditions, prefer to submit their bids at the last seconds of the auction hoping to win the auction for a lower price. Ockenfels and Roth show that there are two possible outcomes of their model: (1) all bidders submit their valuations at the last seconds of the auction; (2) everyone bids their valuations at any instance during the auction except for the last few seconds. However, we believe there is more to explain since it is commonly observed in eBay that both last-minute bids and earlier bids are present in same auctions. This co-existence cannot be explained with Roth and Ockenfels' model.

Bajari and Hortacsu (2003) offer another explanation. They show that LMB can arise in a common value environment.[3] In such an auction, bidders might be revealing some information about the true value of the item by bidding early in the auction. This would cause other bidders to bid more aggressively and the winner might end up paying more than she would otherwise. Although this explanation is reasonable, it is valid in a common value environment and does not apply to private value auctions.

An alternative explanation for LMB has been recently proposed by Barbaro and Bracht (2005). They present LMB as a best response to dishonest actions by the sellers or sellers' accomplices. Shill bidding and squeezing are examples for such

[3] Every potential buyer values the object the same. They differ in their own estimates of this common value.

actions.[4] These actions might cause some bidders to wait until the last minutes of the auction to submit their bids. However, we believe the numbers of bidders who actually snipe because of these reasons are insignificantly small.

The common feature of most of these papers is that they focus on stand-alone auctions. However, on eBay, many auctions overlap and they close one after another. For instance, while studying auctions on Texas Instruments Calculator (TI-83), we observe that on average 31 minutes elapse between closing times of two consecutive auctions. This number is much smaller during peak hours of the day when more users are actively bidding. Thus, it is critical to focus on multiple auctions.

We show that capturing the sequential nature of eBay auctions is crucial in understanding LMB. We consider a two-period model in which two identical items are auctioned sequentially by different sellers. We show that there is a symmetric equilibrium in which the bidders wait until the last minute to submit their bids in the first auction. Last-Minute Bidding arises endogenously from the sequential structure of our model. The existence of the second auction creates a common value component in the first auction. Observing other bidders' valuations affect a buyer's expected payoff from the second auction, and thus his willingness-to-pay in the first one. For a high-valuation buyer, revealing her valuation lowers the expected payoff for the other bidders from the second auction, hence they are willing to bid more aggressively in the current one. Thus, bidders are willing to reveal their valuations for the item only at the end of the first auction, when there is not enough time for other bidders to respond. Additionally, we show that the expected revenues for the sellers in both auctions are equivalent.

A similar argument has been proposed by Wang (2003). He also uses the sequential auctions setting to show that the unique symmetric perfect Bayesian

[4] *Shill bid* is the name given to a high bid the seller submits in her own auction using another account (or a friend does it for her). She learns the willingness to pay of the highest bidder. Then, the shill bid is canceled by the seller and replaced by the amount that is equal to the willingness to pay for the highest bidder. Thus, any potential gain for the bidder is *squeezed* from him by the seller.

equilibrium of the first auction that survives the deletion of weakly-dominated strategies features last-minute bidding.

In order to observe sequential bidding, we study the temporal distribution of bids on Texas Instruments Calculator (TI-83) auctions that we have collected. We organize our dataset as sequentially closing multiple auctions. In addition to showing the existence of LMB we also demonstrate that some of the late bidding is simply losers from previous auctions moving onto the next one. Nevertheless, we show that most LMB is still due to the strategic waiting of bidders until the last minutes of the auction as our theory predicts.

The rest of the paper is organized as follows. In section two we provide an overview of eBay. In section three, we introduce the model and our theoretical results. In section four we use the eBay data to statistically confirm our theoretical findings and study LMB in the closing intervals of the auctions. We present our conclusions in section five.

2. The EBay Auction

EBay attracts numerous bidders and sellers to its online market. As a result, there are countless items that are listed in many different categories and subcategories such as antiques, cars, coins and all kinds of electronics. Bidders can search for an item by using keywords and then narrow down the results using specific criteria related to the goods like price range or location.

EBay auctions are ascending bid auctions. The seller determines the length of the auction which can last for 1, 3, 5, 7 or 10 days.[5] The starting bid is also set by the seller before the auction commences. Any amount below the starting bid is not sufficient for the transaction to go through. The starting bid acts like a posted reserve price for an auction since it is observed by all the potential buyers. EBay also offers

[5] While the data were collected, 1-day auctions were not an option for the sellers in eBay.

sellers an option to set a secret reserve price that is not observable to the bidders. The reserve price is the lowest amount that the seller is willing to accept. Potential buyers are informed by eBay via email and a posting on the website whether the reserve price is met or not. Moreover, eBay charges the seller a certain fee for choosing this option. The fee depends on the amount of the reserve price she sets. However, the fee is fully refunded if the auction ends with a successful transaction.

In eBay, potential buyers bid by means of proxy bidding. By submitting a proxy bid, the bidder assigns an amount that she is willing to pay for a particular item and eBay's computer runs the bidding on her behalf. If the submitted proxy bid is the highest, then the highest-standing bid is the second highest proxy bid plus the bid increment.[6] As higher bids get submitted, eBay's computer keeps increasing the highest-standing bid on the bidder's behalf until she is outbid and is no longer the highest bidder. This bidding process continues until the auction ends. The winner is the buyer who submitted the highest proxy bid and pays the second-highest proxy bid plus the bid increment. Hence, we regard eBay as an ascending second-price auction. However, since a bidder can learn about others' proxy bids by increasing her own proxy bid, it is not precisely a sealed-bid auction. Therefore, it qualifies as a partially "sealed" bid ascending second-price auction.

At any point in the bidding, the bidders can observe the following information: the highest standing bid, time left for the auction to close, starting and ending time of the auction, ID of the high bidder including his/her feedback, ID of the seller including his/her feedback along with the percentage of positive feedback, location of the item, shipping and payment details, item specifics including a detailed description with one or more pictures of the item, starting bid, number of bids, quantity being sold and, in most cases, a counter showing how many prospective bidders have visited the auction site. Moreover, information on the bid history can be obtained with a single click on the "history" link. In this new page, the potential buyer observes all

[6] Bid increment is the lowest amount by which a bidder can raise the highest-standing bid. This number is predetermined by eBay and it depends on the highest-standing bid.

submitted bids, the IDs of the bidders along with the dates and times of the bid submissions.

EBay also offers a special Buy-It-Now (BIN) option to sellers. The BIN option gives sellers the opportunity to provide bidders with the possibility of buying the item forthwith by paying the BIN price. However, the BIN option disappears once a proxy bid that is equal to or higher than the reserve price is submitted. When this happens, the BIN option is removed from the auction web site by eBay. Since the focus of this study is last-minute bidding, we will focus on auctions without the BIN feature.

3. The Model

We study the following simplified version of the eBay auction setting. There are two potential sellers and three potential buyers of a good. All agents are risk-neutral, their preferences are presented by a utility function that is quasilinear in wealth, and they do not discount the future. There are two possible types of buyer. With probability $q \in (0,1)$, a buyer is a low type, and with the complementary probability she is a high type. The three buyers' types are independent. Each type of buyer wants at most one unit of the good, and values that one unit at V_L if she is a low type and at V_H if she is a high type, where $V_H > V_L > 0$. Each seller has one unit of the good to sell, and values the good at zero. This structure is common knowledge, although the buyers' realized types are private information.

The sellers run consecutive auctions. Each auction is a continuous-time, first-price auction with proxy bidding (described below). Seller 1's auction runs from time t_1 to time T_1, and the second auction runs from t_2 to T_2, where $t_1 < T_1 < t_2 < T_2$. The assumption that the auctions do not overlap is not important; what matters is that one auction ends before the other.

The auctions use a proxy-bidding program. A buyer who chooses to participate enters a proxy bid b, and the program bids on his behalf up to that level. If b exceeds h, the maximum of the previous high proxy bid and the reserve price, then the program automatically enters a bid of h plus a small, discrete increment ε. If two or more buyers enter the same proxy bid $b > h$ at the same time, then the tie is broken randomly with equal probabilities – the program records a bid of b for one of the buyers and no bid for the other(s). If a buyer with proxy bid b has the current high bid h, and another buyer enters a proxy bid b' between h and b, then the program automatically enters a new bid of b' for the first buyer and no bid for the second. Having entered a proxy bid, a buyer can neither retract it nor revise it downward, although he can increase it at any time before the end of the auction. When the auction ends, the item is awarded to the buyer with the highest proxy bid at a price equal to the second-highest proxy bid. That outcome is publicly observed.

Throughout, we distinguish between a buyer's proxy bid (the price that he gives the program) and his bid (which the program makes on his behalf as a function of the proxy bids of all bidders, the order in which they were entered). Bids are publicly observed, but proxy bids are private information (except to the extent that they are revealed by bids). Thus, the auction is first-price in bids, but it resembles a second-price auction in proxy bids.

We assume that the minimum bid increment ε is vanishingly small. In particular, we assume that a buyer with valuation V is indifferent between paying p for the good and paying $p + \varepsilon$, as long as $p < V$, but is unwilling to pay $V + \varepsilon$.

Buyers arrive at the first auction in random order. An arriving buyer knows whether he is the first, second, or third to arrive; early buyers observe the arrival of subsequent buyers. That assumption is consistent with the fact that most eBay auction sites have counters that display the number of visitors to the site. All buyers are present at the start of the second auction (although one may have already obtained an item, and thus will not participate).

A pure strategy for a buyer is whether to enter a proxy bid and if so, in what amount to enter the proxy bid, as a function of time, his own previous actions, the history of bids and the order of arrival at the first auction and (during the second auction) the outcome of the first auction. A buyer cannot enter a proxy bid in the first auction before it begins and he arrives, or in the second auction before it begins. Mixed strategies are defined straightforwardly.

We look for symmetric sequential equilibria. As an equilibrium selection device, we make the following assumption:

Assumption 1: *With vanishingly small probability, any proxy bid submitted by a buyer is rejected by the eBay mechanism at the end of the auction. That probability is independent of when the proxy bid was submitted and of whether or not the other buyers' proxy bids are rejected.*

In the second auction, therefore, each remaining buyer (that is, those who did not obtain the item in the first auction) will play his conditionally weakly dominant strategy to enter a proxy bid equal to his valuation at the end of the auction at time T_2, if he has not previously entered a proxy bid that high. Consequently, the outcome of the second auction is as follows:

Proposition 1: *The outcome of the second auction is the same as that of a second-price, sealed-bid auction.*

Assumption 1, together with the result of Proposition 1, implies that in equilibrium a low-type buyer will enter a proxy bid of V_L at some time during the first auction. Expecting no surplus in the second auction, he is willing to pay up to his valuation in the first auction, hoping that other buyers' proxy bids will be rejected.

The behavior of high-type buyers in the first auction is more complicated. A high type's continuation value (the surplus from participating in the second auction) depends on the type(s) of the other participants in that auction. If a high type is matched with only low types, then he will win the item at a price V_L (+ ε), yielding surplus $V_H - V_L$. If two or more high types participate, then the winning bid is V_H,

and no buyer gets any surplus. (Note that the surplus to a low-type buyer in the second auction is zero in any case.) There is thus a common-value component in the first auction – other buyers' valuations affect a high-type buyer's continuation payoff, and thus his willingness to pay.

That effect means that a high-valuation buyer is unwilling to reveal his type. Doing so reduces any other high type's expected continuation payoff: the probability that at least one opponent is a low type, so that there will be positive surplus from the second auction, falls from $1 - (1 - q)^2$ to q ($= q(2 - q) > q$). Consequently, Buyer i's high-type opponents are willing to bid more aggressively if he reveals himself to be a high type. In particular, another high type will outbid Buyer i unless he enters a proxy bid of at least $(1 - q)V_H + qV_L$ (the price that yields surplus $q(V_H - V_L)$). In addition, Assumption 1 implies that another high type's best response to Buyer i's proxy bid $b \geq (1 - q)V_H + qV_L$ is a proxy bid of $b - \varepsilon$: if Buyer i's proxy bid is rejected, then Buyer i will participate in the second auction, and there will be no surplus for the other high type if he loses the first auction. By revealing himself as a high type, then, Buyer i will get an expected surplus lower than what he would derive from the equilibrium calculated below, where types are not revealed until the end of the auction, when it is too late for buyers to respond to the information.

Thus, at most one high-valuation buyer will enter a proxy bid greater than V_L before the end of the first auction at time T_1. Otherwise, the publicly-observed high bid would exceed V_L and reveal the bidder as a high type. Whenever there is more than one high-type buyer, therefore, there must be proxy bids submitted at time T_1 – last-minute bidding. (There may also be proxy bids not exceeding V_L submitted earlier in the auction.)[7] We can therefore analyze the first auction as though it were a second-price, sealed-bid auction (with respect to the proxy bids). As described above, a low-type buyer will enter his weakly dominant proxy bid, V_L. A high type, in choosing a proxy bid, must balance two factors. On the one hand, entering a high

[7] In the context of a model that is close to ours but with a continuum of possible types of buyers, Wang (2003) uses a similar argument to show that in fact the unique symmetric perfect Bayesian equilibrium of the first auction that survives the deletion of weakly-dominated strategies features last-minute bidding.

proxy bid creates the risk of needlessly competing with a single other high type – whichever of them loses will win the second auction at price V_L. On the other hand, when both opponents have high valuations, then there will be no surplus in the second auction – the buyer's only chance for positive surplus is to win the first auction at a price up to V_H.

In fact, the sealed-bid auction does not have a symmetric equilibrium in pure strategies. The intuition is as follows: Suppose that each high type enters a proxy bid equal to $b^* > V_L$. Relative to sitting out the first auction and waiting for the second, proxy bidding b^* benefits the buyer when the other two bidders are also high types – but the buyer gets the benefit in that case only with probability one-third. In contrast, a proxy bid of b^* hurts the buyer with conditional probability one-half when only one of his opponents is a high type. (If both competitors have low valuations, then the buyer gets the same payoff, $V_H - V_L$, from entering b^* as from waiting.) If a high type deviates by bidding $b^* + \varepsilon$, he wins the first auction for sure. By doing so, he doubles his chances of incurring the cost, but triples the probability of gaining the benefit. Thus, if the buyer is willing to enter a proxy bid of b^* rather than wait, then he must strictly prefer to enter $b^* + \varepsilon$ instead, so there is no symmetric, pure-strategy equilibrium. A similar argument shows that in equilibrium, a high type's mixed proxy-bidding strategy cannot have any mass points.

Thus, the only symmetric sequential equilibria of the first auction have the following characteristics:

Proposition 2: *Given Assumption 1, the only symmetric sequential equilibria of the first auction call for low-type buyers to enter a proxy bid of V_L and for high-type buyers to randomize over the interval*

$$\left[V_L, (1-q)V_H + qV_L\right]$$

according to the cumulative distribution function

$$F^*(b) = \frac{q(b - V_L)}{(1-q)(V_H - b)}.$$

No more than one high type enters that proxy bid before the end of the auction at time T_1. Low types may enter their proxy bids at any time during the auction, and both types may enter other, lower proxy bids before time T_1.

At the end of the auction, the buyers with high valuations are indifferent between submitting a proxy bid anywhere in the support of the cdf F^*, entering a proxy bid above it, or not entering a proxy bid at all and waiting until the second auction, as long as the other buyers follow the equilibrium strategies. The expected payment for a high-type who submits proxy bid $(1 - q)V_H + qV_L$ and thus wins for sure is

$$(1-q)^2 V_H + \left[1-(1-q)^2\right]V_L,$$

yielding surplus

$$\left[1-(1-q)^2\right](V_H - V_L).$$

That value is the same expected surplus as waiting until the second auction, and by construction the same expected surplus from a proxy bid anywhere in the support of F^*.

The expected revenue for the first seller is calculated as the probability $(1 - q)^3$ that all three buyers are high types times the expectation under F^* of the second-highest of three proxy bids, plus the probability $3q(1 - q)^2$ that two buyers are high types times the expectation of the lower of two proxy bids, plus the complementary probability that at most one buyer is a high type times V_L. That expected revenue is equal to

$$(1-q)^3 V_H + \left[1-(1-q)^3\right] V_L ,$$

which is the same as the second seller's expected revenue. The winning bid in the second auction is V_L unless all three buyers are high types – one wins the first auction, and the other two compete up the price in the second to V_H.

As a result we have the following three qualitative results; 1) low types submit their valuations as proxy bid while the high types randomize over a certain interval according to a cumulative distribution function; 2) low types submit their proxy bids at any time during the auction but the high types (all except one) perform last-minute bidding (sniping); 3) both sellers of the two auctions have equal expected revenues.

III. The Data

Our dataset consists of information collected for Texas Instruments TI-83 Graphing Calculator auctions featured on eBay. The data includes every auction that took place between June 15th, 2003 and July 30th, 2003. We chose this item due to the availability of many data points as well as the homogeneous nature of the good itself. Dutch auctions, in which bidders compete for multiple quantities of an item, and private auctions, in which information about the bidders are not available, are excluded from our dataset. We have also excluded Buy-It-Now auctions from our dataset.[8] Unfortunately, the information about the final transaction of the item is not revealed by eBay. Therefore, we cannot distinguish between auctions ending with a successful sale and auctions ending without any sale. We assume that all auctions that attracted a bid ended with a sale and the winner was the buyer with the highest standing bid.

[8] Nevertheless, our dataset includes some wiped-out Buy-It-Now auctions which we are not able to exclude since we do not observe them. This is due to the impossibility of detecting those auctions after the BIN option disappears with a bid meeting the reserve price.

The sale prices for TI-83 calculators in retail stores are between 80 and 100 dollars for new items and between 40 and 60 dollars for used ones. The final prices of the auctions we have in our dataset differ quite substantially. The reasons for the variation are the used and new items as well as additions like the case or the manual of the calculator. However, our dataset also contains auctions offering either these additions without the calculator, or more quantities of the calculator bundled together. In order to be more consistent with the items being offered in auctions we decide to keep only the auctions which have a transaction price greater than or equal to 20 dollars and less than or equal to 110 dollars. Hence, in order not to lose crucial information, we are only dropping extremely low and high valued auctions.

There are 1,973 unique auctions that were completed in the 6-week period. We observe 4,652 bidders who participated in these auctions and submitted 26,488 bids in total.[9] Moreover, we observe 192 unique auctions that did not receive any bids or were cancelled. For this study we don't include no-bid auctions in our data set.

We collected the following information for each TI-83 auction in the 6-week period: auction number, seller and bidder ratings, starting bid, number of bids, auction length, winning bid, minimum bid and number of bidders. Table 1 presents a brief description of these variables. Additionally, we provide the descriptive statistics of the variables in Table 2. More information on the variables along with discussion of the summary statistics is provided below.

The winning bid is price that the winner pays at the end of the auction. Remember that this price is the willingness-to-pay of the second-highest bidder plus the bid increment. Our dataset shows that the TI-83 calculators on eBay sell for a bit less than 60 dollars on average. They also have a standard deviation around 17 dollars which is mainly due to the existence of used and new items.

Using the starting bid variable, the seller sets the amount her auction will start from. Any bid that is submitted has to be equal to or greater than this amount. A

[9] We detected 229 bids without any bidder ID. Fortunately, most of these bids do not affect the outcomes of the particular auctions.

correlated variable to the starting bid is the minimum bid, which is the lowest submitted bid that we observe in our dataset. Minimum bids are, on average, a little higher than the average starting bids set by the sellers. This is due to some inconsistency on eBay's method of recording the first submitted bid. In some auctions the minimum bid and the starting bid are identical however in some others the minimum bid is above the starting bid. This discrepancy does not have any effect on our results. Additionally, there is evidence of reasonable interest by potential buyers in eBay's TI-83 auctions. On average, these auctions last more than 5 days with about 7 bidders showing up and submitting more than 13 bids per auction.

Another important feature of eBay is the feedback score that both sellers and bidders receive. EBay users can leave feedback for each other anytime up to 90 days after the completion of a transaction and the feedback assigned is permanent. A positive comment earns the user a +1 point, a neutral one 0 points and a negative comment corresponds to -1 point. The feedback score is the sum of all the points that the user receives from other parties. In our sample, the mean seller and bidder ratings are 639 and 49 respectively. The sellers are more experienced than the bidders for TI-83 auctions on eBay. The median seller has a feedback point of 54 meaning she had around 60 transactions. The same number is only 5 for the bidders.[10]

IV. Analysis of eBay Data

We cannot distinguish among the bidder types in our dataset thus we cannot test our first prediction of low types bidding V_L and high types randomizing over a certain interval. However, we study the bid submission times to confirm the main result of our theoretical section; the existence of last-minute bidding. Additionally, we compare and statistically test sellers' revenues for auctions in which late-bidding

[10] In 22 auctions we do not observe the seller ratings. However, the data points we observe are enough to conclude that the sellers are fairly experienced. Therefore, it is a reasonable assumption to think that they follow revenue maximizing strategies in their auctions.

exists against the others. The results for this test are not completely consistent with our theoretical predictions. We show that the existence of more late-bidding for lower-valued items might be the reason for this inconsistency.

First we analyze the TI-83 calculator data in eBay and study the last-minute bidding (LMB) phenomenon. We introduce a new and informative approach of studying bid submission times. Many auctions on eBay run around the same time and, as a result, their ending times are fairly close to each other. Analogous to the strategy in our theory model, we believe that a potential buyer would always bid on the early closing auctions. In case she fails to win, then she moves on to one of the next closing auctions. Thus, we picture a trend of bidders preferring to bid on soon-to-end auctions. This trend is strengthened by the order eBay displays the auctions after a search for an item has been performed. The auctions are listed according to their closing times with the early closing ones at the top of the list. As a result, when the interval between closing auctions is short enough, it is hard to differentiate among two types of bidders. First type consists of bidders strategically waiting until the end of the auction to snipe. The second type of bidders are the ones just moving onto the next closing auction because they were not successful winning the previous one. Thus, we believe that the existence short intervals between consecutive closing auctions might strengthen the appearance of last-minute bidding on eBay.

For instance, consider an auction that closes two minutes after the previous one has ended. There will be bidders strategically waiting until the last minute of the auction to submit their bids. However, there will also be bidders who have lost in the previous auction and now moving on to this one since it is the one ending next. In some previous studies in the literature these two types of submissions would both be categorized as last-minute bids although they actually are not. However, we show that even after controlling for previous losers' bids, significant volume of strategic last-minute bidding exists in eBay.

In order to observe if these intervals are indeed small, we arrange the auctions sequentially by their closing times. In Table 3 we report the summary statistics of the

minutes between the closing times of consecutive auctions. We refer to the period that is between the closing times of auctions as the *Closing Interval*. The table reveals that, on average, the closing times for auctions are 31 minutes apart while having a very low minimum (0) and a relatively high maximum (714). The median for the closing intervals is 13 minutes. We infer that these intervals are highly skewed with most of them lasting only for very short periods of time. This means many of the auctions in our dataset close soon after the previous one ends. Table 4 presents us a closer look at the distribution of the *Closing Intervals*. We observe more than 6 percent of the auctions closing precisely at the same time and almost one third of them having 5 minutes or less between their closing times.

As mentioned earlier, the existence of very short *Closing Intervals* might lead to an uncertainty about the amount of LMB among bidders. Thus, an accurate way of measuring the extent of LMB is by studying the distribution of bid times relative to the length of the *Closing Interval*. Histograms in Figure 1 compose of bins representing 2 percent of the length of the respective *Closing Interval*. Our focus is on the submission times of the highest proxy bids submitted by bidders. In the figure, we gradually leave out some short *Closing Intervals* in order to distinctly observe their effect on the bid submission times. First histogram contains all *Closing Intervals* in our dataset. For the second graph we exclude very short *Closing Intervals*, the ones less than equal to a single minute. Similarly, for the third and fourth histograms we leave out less than equal to 3 and 5 minute *Closing Intervals* respectively.

The existence of some spikes prior to the end of the auctions is obvious for the first figure. For instance, in the first graph we observe that 5 percent of the bids are submitted before 2 percent of the *Closing Interval* has elapsed (first 15 seconds of a 13 minutes interval which is the median length in our dataset). However they fade away as we drop the *Closing Intervals* and the final histogram becomes almost smooth. This is due to the existence of quite short intervals and when we only consider longer ones, we observe a much smoother distribution.

In addition to that, the big spike in the first bin represents existence of last-minute bidding. In all four cases we see that between 20-25 percent of the proxy bids are submitted after the 98 percent of the *Closing Interval* has elapsed (last 12 seconds of a 10 minute interval). This number confirms the existence of strategic bidders waiting until the last minute to submit their bids. The magnitude of the first bin is closer to 25 percent in the first graph but falls down to 22 percent when we leave out some short *Closing Intervals*. As we have mentioned earlier, the reason for this drop is the strengthening effect of short intervals on LMB.

As a result, our data confirms the existence of last-minute bidding on eBay but also draws attention to the bidders from previous auctions submitting their bids without strategically waiting for the last minutes of an auction. We believe that researchers should pay attention to these bidders and should not consider them as a part of the last-minute bidding activity.

Next, we focus on sellers' revenue depending on the winning bid being submitted in the closing minutes of an auction or earlier. Our theory suggests that conditional on sale, the revenue for auctions with winning bids in the closing intervals should be higher than the revenue for auctions in which the winning bid is submitted prior to the last 5 minutes. In order to test if these average winning bids are significantly different, we run a two-sample t-test and report our results in table 5.[11] We also report the summary statistics for both types of winning bids in the same table. We observe that when the winning bid is submitted in the last 5 minutes, the average winning bid is $58.73. Alternatively, when it is submitted early on in the auction, the average bid is slightly lower, $58.12. From our test statistic, we conclude that the average last-minute winning bid is larger than the average early winning bid but this difference is not statistically significant.

We believe the main reason for the lack of a significant difference among the averages like our theory predicts is the heterogeneity among our dataset. As we have already mentioned, some items are brand new and some ones are used. The price

[11] Detailed information about this test is presented in the appendix.

difference among them is substantial and might be deriving the result for our t-test. We studied our dataset to see if the heterogeneity of items has an effect on last-minute bidding. We identify auctions in which the winning bid is greater than or equal to $80 as new. This is the lowest price any potential buyer can purchase a new TI-83 from another store. We identify the rest of the auctions in our dataset as used. In table 6 we study the submission time for the winning bid depending on the item being new or used. Similar to our t-test, we focus on winning bids coming in the last 5 minutes of an auction or prior to that. We see that the percentage of LMB is lower for new items than used ones. This is the reason for the inconsistent result of the t-test with our theory. Heterogeneity of the items in our dataset has an impact on our results for the seller revenues.

V. Conclusion

In this study, our main focus is on the incidences of the delay of proxy bid submission due to last-minute bidding. We specify a two-period, three-bidder model in which two identical items are auctioned sequentially by different sellers. Our results show that high-value types wait until the end to reveal their types by submitting proxy bids, while low-value types may submit a proxy bid at any time. The intuition is as follows: revealing one's type early decreases the opponent's expected utility from winning the subsequent auction, causing her to bid more aggressively in the current one. Additionally, we show that expected revenues for the sellers in both auctions are equivalent. The existence of LMB does not alter the expected revenue of the first seller.

Then, we analyze the data we have collected from eBay auctions to test our theoretical predictions. Our dataset comprises 1,973 auctions of Texas Instruments TI-83 Graphing Calculators and the bids submitted in those auctions. We arrange the auctions by their closing times. As a result, we confirm the existence of strategic last-minute bidding as our theoretical model predicts. Additionally, we draw attention to

the bids of losers' from previous auctions that might appear as strategic last-minute bidding when the interval between closing auctions is short enough.

Lastly, we compare and statistically test sellers' revenues for auctions in which late-bidding exists versus auctions with early submitted winning bids. The average revenue for auctions with late-bidding is indeed higher than the others but the difference is not statistically significant. We show that the existence of more late-bidding for lower-valued items might be the reason for the lack of significance.

Our theoretical specification of the sequential structure of sale describes the actual eBay setting more precisely compared to other studies in the auction literature. In our model last-minute bidding arises endogenously as a result of the sequential structure. An extension of this framework would be to incorporate simultaneous, overlapping and sequential auctions that can capture the complexity of trading in these markets.

Another way to extend our research is to analyze the Yahoo online auction site. Yahoo has many features in common with eBay but differs in some very interesting aspects. Yahoo is a thinner market relative to eBay, both in terms of variety of products traded and in terms of the volume of trading. Moreover, it offers different ending rules for auctions and has a distinctive buy-now feature. These variations can potentially affect the strategies of buyers and sellers quite differently, compared to eBay. It would be interesting to study how these distinctions affect players' strategies and auction outcomes in these two markets

APPENDIX

Table 1: Definitions of Variables

Variables	**Explanation**
Auction number	ID of the auction
Winning bid	Maximum recorded bid of each auction
Starting bid	Starting price of the auction that is set by the seller
Minimum bid	Minimum recorded bid of each auction
Auction length (hr)	Length of the auction measured in hours
Number of bids	Total number of bids at the end of the auction
Number of bidders	Total number of bidders at the end of the auction
Seller rating	Number representing the rating of the seller
Bidder rating	Number representing the rating of the bidder

Table 2: Summary Statistics for TI-83 Calculators

Variable	**Num. of Obs.**	**Mean**	**Median**	**Std. Dev.**	**Min**	**Max**
Winning bid	1973	58.35	54	17.76	20	107.52
Starting bid	1973	16.16	9.99	17.32	0.01	99.99
Minimum bid	1973	19.8	14	17.84	0.01	99.99
Auction length (hr)	1973	134.9	168	46.34	72	240
Number of bids	1973	13.43	13	7.07	1	54
Number of bidders	1973	7.14	7	3.05	1	18
Seller rating	1964	639.28	54	1782.42	-1	16832
Bidder rating	1957	48.96	5	182.18	-2	3788

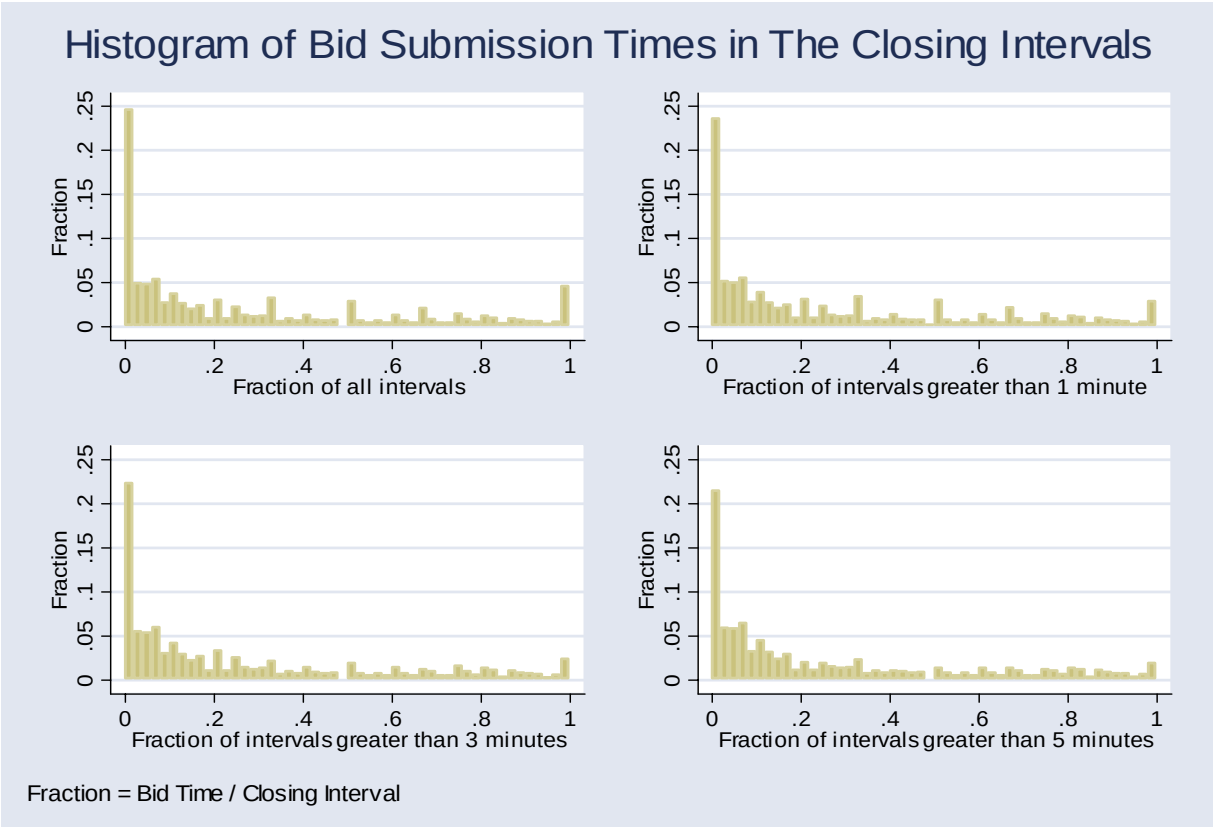
Table 3: Summary Statistics for the Closing Intervals

Variable	**Num. of Obs.**	**Mean**	**Median**	**Std. Dev.**	**Min**	**Max**
Closing Interval (minutes)	1972	30.95	13	59.98	0	714

Table 4: Distribution of Intervals with various lengths

Minutes in the Interval	Frequency	Percent (%)	Cumulative (%)
0	125	6.34	6.34
1	150	7.61	13.95
2	92	4.67	18.61
3	88	4.46	23.07
4	87	4.41	27.48
5	70	3.55	31.03
6-10	267	13.54	44.57
11-30	587	29.77	74.34
>30	506	25.65	100

Figure 1


Histogram of Bid Submission Times in The Closing Intervals
Fraction
0
.05
.1
.15
.2
.25
0
.2
.4
.6
.8
1
Fraction of all intervals
Fraction of intervals greater than 1 minute
Fraction of intervals greater than 3 minutes
Fraction of intervals greater than 5 minutes
Fraction = Bid Time / Closing Interval

T-TEST FOR THE WINNING BID

For this test, we group the winning bids into two. First group is the ones submitted in the last 5 minutes of an auction (*Last-minute*). The second group consists of auctions for which the winning bid is submitted earlier than the last five minutes (*Early*). We want to see if the difference between their average values is significantly different than zero. A two-sample t-test allows us to test the following hypothesis.

H_0 : Mean (*Last-minute winning bid*) - Mean (*Early winning bid*) = Difference = 0

H_1 : Difference $\neq$ 0

Table 5: T-test for the winning bid

Winning Bid	Obs.	Mean	Std. Err.	Std. Dev.
Last 5 minutes	745	58.73	0.64	17.35
Prior to last 5 minutes	1228	58.12	0.51	18.01
Combined	1973	58.35	0.40	17.76
Difference		0.62	0.83	
Degrees of freedom: 1971				

We get the following result and fail to reject the null hypothesis that the difference among the means is zero.

$$t = 0.7484$$

$$P > |t| = 0.4543$$

This suggests that the means are not significantly different. More precisely, we can conclude that the average *Last-minute winning bid* is larger than the average *Early winning bid* but this difference is not statistically significant.

Table 6: Late bidding in used and new items

Number of Winning Bids	Used item	New item
Last 5 minutes	635	110
Prior to last 5 minutes	1020	208
Total	1655	318
Percent of LMB	38.37	34.59

Proofs:

Proof of Proposition 1: Low types will not enter a proxy bid greater than V_L, because they might get negative surplus. If they enter a bid less than V_L, then a low type can deviate to V_L and increase his surplus in the case where the other buyer or buyers are low types, or where only the proxy bids of low types are not rejected.

By a similar argument, high-type buyers will enter a proxy bid equal to V_H. Thus, the outcome will be the same as that of a second-price, sealed-bid auction.

Q.E.D.

Proof of Proposition 2: First, an argument similar to the one used in the proof of Proposition 1 shows that low-valuation buyers will enter a proxy bid of V_L. Next, we demonstrate that high types will not reveal their type before the end of the auction. Suppose Buyer i is known to be a high type. Then the expected surplus from the second auction for any other high type is $q(V_H - V_L)$, as explained in the text. If another high type can win the first auction with positive probability by bidding $(1 - q)V_H + qV_L$, he will do so, because winning at or below that price yields surplus at least $q(V_H - V_L)$. If, on the other hand, the smallest proxy bid that Buyer i might enter, b, is greater than $(1 - q)V_H + qV_L$, then any other high type j will submit a proxy bid of $b - \varepsilon$, as long as $b \leq V_H$: that bid will win the item only if Buyer i's proxy bid is rejected. In that case, Buyer i will participate in the second auction, and there will be no surplus for Buyer j if he loses the first auction. In that event, Buyer j is willing to pay up to his valuation to win the first auction. He will not outbid Buyer i, though – he prefers to wait till the second auction and face the third buyer rather than to win the first at price greater than $(1 - q)V_H + qV_L$.

Thus, if there is at least one high type other than Buyer i,, then Buyer i can expect surplus no greater than $q(V_H - V_L)$ in the first auction. If the other two buyers have low valuations, then he get surplus $(V_H - V_L)$. Overall, his expected surplus if he wins the first auction is no greater than $(1 - q^2)q(V_H - V_L) + q^2(V_H - V_L)$, which is strictly less than $[1 - (1- q)^2](V_H - V_L)$, the expected surplus from the equilibrium described below, where types are not revealed until the end of the auction. If he loses the first auction, his expected surplus in the second auction is no greater than $[1 - (1- q)^2](V_H - V_L)$ – he gets no surplus if both opponents have high valuations. His surplus from both auctions, then, does not exceed his surplus from the equilibrium where his type is not revealed, and it is strictly less if he wins the first auction with positive probability. If he loses the first auction for sure after revealing his type, then he might be indifferent, but Assumption 1 breaks the tie in favor of having two chances to win. Thus, a high-valuation buyer will not reveal his type, which implies that no more than one high type will submit a proxy bid before time T_1, as mentioned in the text.

It remains only to show that the mixed strategy over proxy bids F^* is an equilibrium strategy for high types in the second-price, sealed bid auction at time T_1, given that low types enter a proxy bid of V_L, and that that equilibrium is unique.

Suppose that other players follow the equilibrium strategies, and that the equilibrium of the second auction is as described in Proposition 1. If a high type enters a proxy bid b in the support of F^*, then his expected surplus $S(b)$ from both auctions is

$$S(b) = (1-q)^2[F^*(b)]^2\left\{V_H - \frac{1}{[F^*(b)]^2}\int_{V_L}^{b} 2F^*(b')f^*(b')b'db'\right\}$$
$$+ 2q(1-q)\left[F^*(b)\left\{V_H - \frac{1}{F^*(b)}\int_{V_L}^{b} f^*(b')b'db'\right\} + [1-F^*(b)]\{V_H - V_L\}\right]$$
$$+ q^2\{V_H - V_L\}.$$

That is, with probability $(1 - q)^2$ he faces two other high types, which means that there will be no surplus in the second auction. Thus, his total expected surplus is the probability that he submits the highest proxy bid in the first auction, $[F^*(b)]^2$, times his surplus if he does win, which is the difference between V_H and the expected value of the higher of two proxy bids drawn from F^* conditional on being less than b. With probability $2q(1 - q)$, he faces one high type and one low type, so if he loses today he will win the second auction for price V_L. If he wins today, his surplus is V_H minus the expected value of a proxy bid drawn from F^* conditional on being less than b. With probability q^2 he faces two low types – he wins the first auction at price V_L.

On the other hand, suppose that he deviates by not entering a proxy bid in the first auction and waiting to the second, or, equivalently, by submitting a proxy bid less than V_L. (The other type of deviation, a proxy bid above the support of F, gives the same payoff as a proxy bid at the top of the support, so it cannot be strictly better.) His surplus from waiting $S(w)$ is

$$S(w) = (1-q)^2 0 + 2q(1-q)\{V_H - V_L\} + q^2\{V_H - V_L\}.$$

That is, if both opponents are high types, he is certain to face one in the second auction and get no surplus. If both are low types, he will win the second auction at price V_L. He also wins the second auction for V_L if one opponent has high valuation, because that opponent will wins the first auction if he follows the equilibrium strategy. Subtracting $S(w)$ from $S(b)$ yields the difference $D(b)$:

$$D(b) = (1-q)^2[F^*(b)]^2\left\{V_H - \frac{1}{[F^*(b)]^2}\int_{V_L}^{b} 2F^*(b')f^*(b')b'db'\right\}$$
$$+ 2q(1-q)\left[F^*(b)\left\{V_L - \frac{1}{F^*(b)}\int_{V_L}^{b} f^*(b')b'db'\right\}\right].$$

Note that $D(V_L) = 0$, implying that a high type is indifferent between waiting and bidding V_L. He must also be indifferent among all proxy bids b in the support of F^*, so we differentiate $D(b)$ and set the derivative equal to 0:

$$D'(b) = 2(1-q)\, f^*(b)\{F^*(b)(V_H - b)(1-q) + (V_L - b)q\} = 0..$$

Solving yields $F^*(b) = q(b - V_L) / (1 - q)(V_H - b)$, which was to be shown. So F^* is the unique mass-free equilibrium bidding distribution. We note that the equilibrium survives Assumption 1 – introducing a small probability that a proxy bid is rejected causes the equilibrium distribution to be adjusted in a continuous way to compensate.

To show that cannot be a mass point, first we show that there is no pure strategy equilibrium. Suppose that high types enter some proxy bid b with probability one. Then their expected surplus, calculated in the same way as $S(b)$, is

$$(1 - q)^2(1/3)(V_H - b) + 2q(1- q)[(1/2)(V_H - b) + (1/2)(V_H - V_L)] + q^2(V_H - V_L).$$

The surplus from waiting is the same as before. Subtracting yields difference $E(b)$:

$$E(b) = (1 - q)^2(1/3)(V_H - b) + 2q(1- q)(1/2)(V_L - b),$$

which must be weakly positive, or else high types would prefer to wait. Now suppose that a high type deviates by entering a proxy bid higher than b. He wins the first auction for sure, at price V_L if both opponents are low types and at price b otherwise. His expected gain relative to submitting proxy bid b is

$$(1 - q)^2(2/3)(V_H - b) + 2q(1- q)(1/2)(V_L - b),$$

which is strictly greater than $E(b)$, so he strictly prefers to deviate.

A similar argument shows that in equilibrium there is no mass point at any b: we perform the same exercise conditioning on the other buyers' proxy bids being b or less.

The final step to show uniqueness is to demonstrate that high types must enter a proxy bid with probability one in equilibrium. Suppose that with probability $p > 0$ a high type does not participate in the first auction, and with complementary probability he enters a proxy bid drawn according to some cumulative distribution function G. In that case, a high type has a profitable deviation in the first auction: when his strategy calls for him to wait, he instead submits a proxy bid V_L (+ ε). Then, in the event that his two opponents are high types and they both are waiting until the next auction, he gets positive surplus rather than none. Otherwise, the deviation does not affect his payoff. Thus, high types always enter a proxy bid, and the equilibrium calculated is unique. *Q.E.D.*

References

Ariely, D., Ockenfels, A. and Roth, A. "An Experimental Analysis of Ending Rules in Internet Auctions", Rand Journal of Economics.

Bajari, P. and Hortacsu, A. "The Winner's Curse, Reserve Prices and Endogenous Entry: Empirical Insights from eBay Auctions", Rand Journal of Economics, Summer 2003, vol. 3, No. 2, pp. 329-355.

Bajari, P. and Hortacsu, A. "Economic Insights from Internet Auctions", Journal of Economic Literature, vol. 42, June 2004, pp. 457-486.

Barbaro, S. and Bracht, B. "Shilling, Squeezing, Sniping: Explaining late bidding in on-line second-price auctions", University of Mainz working paper 2005.

Budish, E. and Takeyama, L. "Buy prices in online auctions: irrationality on the internet?" Economics Letters, vol. 72, 2001, pp. 325-333.

Hasker, K., Gonzalez, R. and Sickles, R. "An Analysis of Strategic Behavior in eBay Auctions," Rice University working paper 2003.

Krishna, V. "Auction Theory", Academic Press, 2002, San Diego, CA.

Ockenfels, A. and Roth, A., "Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon Auctions on the Internet," American Economic Review, 2002, vol. 92, issue 4, pp. 1093-1103.

Ockenfels, A. and Roth, A., "Late and multiple bidding in second price Internet auctions: Theory and evidence concerning different rules for ending an auction," Games and Economic Behavior, in press 2005.

Peters, M. and S. Severinov. "Internet Auctions with Many Traders," University of Wisconsin working paper 2004.

Reynolds, S. and Wooders, J. "Auctions with a Buy Price," University of Arizona working paper 2003.

Stryszowska, M. "Last-minute and Multiple Bidding in Simultaneous and Overlapping Second Price Internet Auctions" Tilburg University working paper 2005.

Vickrey, W. "Counterspeculation, Auctions and Competitive Sealed Tenders," Journal of Finance, 16, 1961, pp. 8-37.

Wang, J. "Is Last Minute Bidding Bad", UCLA Working Paper 2003.