Human Capital Risk in Life Cycle Economies

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Abstract

I study the effect of market incompleteness on the aggregate economy in a model where agents face idiosyncratic, uninsurable human capital investment risk. The environment is a general equilibrium life-cycle model with a version of a Ben-Porath (1967) human capital accumulation technology, modified to incorporate risk. A CARA-normal specification keeps household decisions independent of individual shock realizations. I study stationary equilibria of calibrated cases in which idiosyncratic uninsurable risk arises from specialization risk and career risk. Specialization risk is such that both mean and variance of the return from training are increasing in the endogenous decision to invest in human capital. In the case of career risk, however, only the mean return is increasing in the decision to invest in human capital. With career risk only, stationary equilibria resemble those studied by Aiyagari (1994), and one concludes that the impact of uninsurable idiosyncratic risk is relatively small. With a significant amount of specialization risk however, stationary equilibria are severely distorted relative to a complete markets benchmark. One aspect of this distortion is that human capital is only about 57 percent as large as its complete markets counterpart. This suggests that the two types of risk have very different and quantitatively significant general equilibrium implications. Keywords: Human capital risk, life-cycle, incomplete markets. JEL codes: E20, E21, E24.

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1 Introduction

1.1 Human capital risk

Dispersion of labor earnings increases over the life-cycle, a well documented feature of the US data. According to Deaton and Paxson (1994), within-cohort labor income inequality increases with age. In related work, Huggett Ventura and Yaron (2007) investigate the reasons behind this rise in earnings dispersion over the life-cycle which is primarily due to age effects in a partial equilibrium framework. Their study finds that about one-third of the variation in lifetime earnings is due to idiosyncratic human capital shocks. Other cross sectional studies also indicate that agents face a great deal of uncertainty when making their schooling decisions.\footnote{Carneiro, Hansen and Heckman (2003) find that the substantial heterogeneity in the returns to schooling is unpredictable at the time when schooling decisions are made. In related work, Cunha, Heckman, and Navaro (2005) conclude that 40% of the variability in the returns to schooling is unforecastable at the time students decide to go to college, implying that this uncertainty is not due to observable factors like ability differences or differences in initial conditions but purely due to idiosyncratic shocks.}

Taken together, it appears that investment in human capital is risky and part of the labor income uncertainty that agents face over their life-cycle is a manifestation of this idiosyncratic human capital risk. In addition, it is widely understood that human capital investment is uninsurable—there is a clear lack of complete markets with respect to this investment.

One main consequence of this type of labor income uncertainty is that it could deter investment in human capital, possibly leading to underaccumulation of human capital and overaccumulation of relatively less risky physical capital, in comparison to a case where agents can insure against this risk via complete markets. If a mechanism like this is at work in actual economies, the impact of market incompleteness on the aggregate economy could be large,\footnote{Even more so if one takes the view that human capital is an engine of growth.} possibly calling for policy intervention to mitigate the effect of this risk on household decisions to invest in training.\footnote{Krebs (2003) comes to this conclusion in an endogenous growth model where investment in risky human capital is modeled as a portfolio decision and physical capital is the risk-free asset.}
I study the macroeconomic implications of labor income uncertainty arising from the risky nature of human capital investment. The specification here allows us to directly see the impact of risk on the process of human capital accumulation, isolate and quantify the effect of risk on individual decisions, and comment on divergent views in this literature on the role of market incompleteness on the aggregate economy.

1.2 Main ideas

In this paper, investment in human capital is studied as a time allocation problem in a life-cycle framework. Here agents allocate time away from the labor market when they are young to acquire education. Using a version of a Ben-Porath (1967) production function for human capital that allows for risky human capital, I study how uninsurable risk impacts an individual’s decision to train in a general equilibrium life-cycle model. If the returns to training are uncertain and uninsurable, agents may try to self-insure by holding larger precautionary savings and, more importantly, they may also endogenously alter their training decisions to mitigate the effect of human capital risk.

I consider two types of uninsurable idiosyncratic risks, namely, specialization risk and career risk. Higher risk is compensated by higher return, and higher return is often associated with higher levels of education. In the formulation of the returns from training, this aspect of education is captured by the specialization risk. The specialization risk is such that the endogenous decision to train increases both the expected returns from training and its variance, while the career risk only affects the mean return from training. The career risk is additive in the human capital formation technology and is the most common formulation in the literature which studies the impact of uninsurable idiosyncratic risk on the aggregate economy. Risks that look like the multiplicative specialization risk of this paper were first studied by Angeletos and Calvet (2006), but not in a human capital setting.

I abstract from the risks associated with physical capital investment.
Market incompleteness arising because of these idiosyncratic uninsurable risks make the wealth distribution a relevant state for individual decisions, often making problems in this class intractable. Several papers, including Calvet (2001) and Angeletos and Calvet (2006), use constant absolute risk aversion (CARA) utility function and normally distributed shocks in order to ensure that an individual’s risk-taking decision is independent of wealth. I employ the same technique along with certain other assumptions to ensure that an individual’s decisions are independent of wealth. Due to the lack of wealth effects, within a generation all agents make identical decisions but they still differ in their labor quality, labor income, and consumption because of the different realizations of the two shocks. In the life-cycle model that I consider, therefore, there are both types of heterogeneity, within generation and across generation. Across generation heterogeneity is inherent in the life-cycle models.

I study calibrated versions of the model to assess the quantitative importance of incomplete markets. Risk related parameters—the variances of the two shock processes in the human capital accumulation technology—are chosen to match the portion of the variance in labor earnings over the life-cycle that is due to age effects. I study a baseline case which has a mixture of the two types of shocks, and also a more extreme case where there is only career risk. Such an analysis allows us to clearly see how each of these risks, the specialization and the career risk, influence the aggregate economy by altering an individual’s decisions. Along with exploring the macroeconomic implications of market incompleteness due to risky human capital, I also explore the life-cycle features implied by this model.

1.3 Main findings

I first establish that the stationary equilibrium of this model with only career risk has properties similar to Aiyagari (1994). Aiyagari studied the macroeconomic impact of uninsurable idiosyncratic labor income risk arising due to shocks to labor endowments in a model where households live forever and where there is no human capital. The career-risk-only case of
the present model has implications similar to Aiyagari (1994). In particular, the precautionary savings induced by this risk has only a small quantitative impact on the macroeconomy.

I then study the baseline calibration where both shocks play a role. In the baseline calibration, all the variance in labor income early in life when agents are investing in training is due to specialization risk. Later in life, both risks play a role. I find that the effects of the specialization risk dominate and there is a very large impact on macroeconomic variables in the stationary equilibrium. In particular, there is a 43 percent underaccumulation of human capital relative to the complete markets case. Accordingly, since labor quality is dramatically lower, output, physical capital, consumption and other variables are also drastically affected by the idiosyncratic uncertainty.

I conclude that uninsurable idiosyncratic specialization risk has a large impact on macroeconomic equilibrium, but that uninsurable idiosyncratic career risk does not.

Does human capital risk have a significant impact on actual economies? It may if the shocks resemble the baseline calibration. But if most of the risk in human capital investment is due to career risk, then the influence could diminish markedly. In a way, the quantitative analysis nests both the views that are commonly seen in the literature, one following the tradition of Aiyagari (1994) that argues that the quantitative effects of incomplete markets are small, and a relatively recent view associated with Angeletos and Calvet (2006) that suggests that these effects could be large. One conclusion is that empirical studies based on micro data would need to identify the relative importance of these shocks in order to assess the implications of market incompleteness on the aggregate economy.

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5This is despite the fact that there are no borrowing constraints or wealth effects in the present model.

6Neither of these authors talked explicitly about human capital. Aiyagari (1994) studied labor income risk whereas Angeletos and Calvet (2006) analyzed capital income risk. This paper is closer to Aiyagari but provides a different mechanism to explain the variance in labor income.
Apart from aiding our analysis in thinking about human capital investment and matching some of the salient features of the aggregate economy, the life-cycle model stays consistent with some of the features of the life-cycle that are often studied in partial equilibrium settings, for example the shape of mean earnings and the variance of labor earnings over the life-cycle.

1.4 Recent related literature

Using an incomplete markets framework, several papers since Bewley (1977) have investigated the core implications of uninsurable idiosyncratic labor income risk on the aggregate economy. These papers typically abstract from aggregate uncertainty. In Aiyagari (1994) individuals face uninsurable labor endowment risk which makes their labor income risky. In addition, households are borrowing constrained in the credit market. In such a setup, households self-insure by increasing their precautionary savings when they face greater income risk. The steady state is characterized by higher capital stock and lower interest rates due to market incompleteness and imperfect credit markets. However the quantitative implications of this risk are not very striking.

Angeletos and Calvet (2006) study how capital income uncertainty arising due to idiosyncratic entrepreneurial risk, production as well as endowment risk, impacts the steady state and transitional dynamics of a neoclassical growth model. They find that the macroeconomic implications of risk are not as straightforward as in Aiyagari (1994). On the one hand higher entrepreneurial risk reduces the demand for investment. But at the same time, increase in uninsurable income risk raises savings which lowers the

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7 Krusell and Smith (1998) among others incorporate both idiosyncratic and aggregate risk.

8 In a recent paper, Marcet, Obiols-Homs and Weil (2007) endogenize labor supply decisions using Huggett’s (1997) framework, another standard incomplete markets model with labor endowment shocks. They find that market incompleteness could potentially have quantitatively significant implications when leisure is a normal good and if labor elasticity is larger than consumption elasticity.
interest rate and therefore raises investment demand. Thus the overall effect of risk on investment is ambiguous.

While Angeletos and Calvet (2006) study the aggregate implications of market incompleteness in a model with risky entrepreneurial income, a related paper, Krebs (2003), studies the impact of labor income risk in a model with risky human capital and risk-free physical capital. In Krebs (2003) this risky human capital is also the engine of growth. Human capital investment in his model is a portfolio decision where the agents decide what fraction of their savings to invest in risky human capital and how much to invest in the risk-free physical capital. He shows that risk lowers investment in human capital which in turn lowers growth and welfare. Policy recommendations based on his model would suggest that programs that mitigate idiosyncratic risk are superior so long as total assets held by the households, both physical and human capital, increases.

Based on Krebs it is not clear whether the quantitatively significant macroeconomic implications of risky human capital are due to market incompleteness or because human capital is the engine of growth. To isolate the role of uninsurable risky human capital, I abstract from growth in the steady state. Even though in this paper I ask a question similar to Krebs (2003), the framework used here is quite different and is more consistent with the traditional analyses of human capital investment. I consider a general equilibrium life-cycle model with a Ben-Porath production process for risky human capital investments.

A feature of this paper is that it can also produce some of the life-cycle features seen in the data in a general equilibrium framework. There is growing empirical-theoretic, primarily partial equilibrium literature studying the life-cycle features in the data, for example the rise in the variance of earnings and consumption and the hump-shaped mean earnings and consumption over the life-cycle. Deaton and Paxson (1994), Huggett, Ventura and Yaron (2007), Storesletten, Telmer and Yaron (2004), Heathcote, Storesletten and Violante (2005) among others study these issues. Deaton and Paxson (1994) use a complete markets framework to study the life-
cycle features of their model and compare them to the cohort data in the U.S. They find that the age-profile of the dispersion of earnings and consumption increases over the life-cycle in the data and that these features can be explained by non-separable preferences over consumption and leisure and skill heterogeneity. Storesletten, Telmer and Yaron (2004) argue that Deaton and Paxson’s analysis has inconsistent implications for hours. As a result these authors attempt to explain the same features of the data using an incomplete markets framework where agents face idiosyncratic, uninsurable earnings risk, similar to Aiyagari (1994) and others but in a life-cycle context. In a related paper, Huggett, Ventura and Yaron (2007) use a partial equilibrium model with risky human capital to explain these same features in the data.

The paper is organized as follows. Section 2 presents the model while section 3 discusses some intuition using a two-period overlapping generation model. In the final section I present quantitative results.

2 Model

The economy has an infinite sequence overlapping generations of agents who live for $T$ periods. Time is discrete and is indexed by $t = 0, 1, 2, \ldots$ In each period, a continuum of ex ante identical young agents with unit mass is born. Each agent is endowed with two units of time in each period until they retire. There is no population growth. Physical capital is risk-free and there are no credit market imperfections. In terms of notation, subscripts indicate when the agent was born and the stage in an agent’s life-cycle or the time in the model is in the parenthesis. In general, the aggregate variables do not have a subscript.

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9 Deaton and Paxson (1994) also studied data from Great Britain and Taiwan which had features similar to the United States.

10 Recent literature on the incomplete markets, Magill and Quinzii (1996), Krebs (2003), and Angeletos and Calvet (2006) abstracts from credit market imperfections.
2.1 Incomplete markets

Each agent consumes a final good in every period such that the preferences of agent $i$ born at date $t$ are given by

$$ U = E \sum_{j=0}^{T-1} \beta^j u(c_i(t + j)) $$

where $c_i(t + j)$ is the consumption of the final good of agent $i$ who is born at date $t$ at stage $t + j$ in the life-cycle, the discount factor is $\beta$, and $E$ is the expectation operator. For tractability I assume that the utility function has the CARA form such that $u(c) = -\frac{1}{a} \exp[-ac]$ where $a$ is the coefficient of absolute risk aversion.

The budget constraint of the agent for $0 \leq j < J_R$, where $J_R$ is the exogenous date at which agents retire, is given by

$$ c_i(t + j) = w(t + j) \{ (1 - \tau_i(t + j))x + x_i(t + j) \}
+ R(t + j - 1)s_i(t + j - 1) - s_i(t + j) $$

and

$$ c_i(t + j) = R(t + j - 1)s_i(t + j - 1) - s_i(t + j) $$

for $J_R \leq j \leq T - 1$. Time allocated to training by agent $i$ is $0 \leq \tau_i(t + j) \leq 1$, $s_i(t + j)$ is the holdings of real risk-free asset (capital) and $x_i(t + j)$ is the labor quality, measured in efficiency units, at date $t + j$. The wage rate per efficiency unit in period $t + j$ is $w(t + j)$ and $R(t + j)$ is the gross return on the real risk-free asset holdings from period $t + j$ to $t + j + 1$. Since there are no bequests, agents enter the first period without any assets and do not save in the last period of their life. Agents born at any date $t$ however inherit the average labor quality $x(t)$ prevalent at that date and as a result $x_i(t) = x(t)$.

To ensure that all agents in the same stage of their life-cycle, agents within a generation, make the same training and real asset holding decisions that are independent of the actual realization of the two shocks we assume that the labor quality of an agent $x_i(t + j)$, measured in efficiency
units, enters additively such that it does not interact with the training decision in period $t + j$. The parameter $x > 0$ converts units of time into efficiency units.\footnote{One aspect of this specification is that in a stationary equilibrium, the marginal cost of acquiring training is the same across all agents and is independent of their own individual labor quality.} If $\tau^i(t + j) = 1$, agents receive $w(t + j)x^i_t(t + j)$ in labor income and when agents stop training, $\tau^i(t + j) = 0$, they allocate both units of their time to the labor market and their labor income becomes $w(t + j)\{x + x^i_t(t + j)\}$. Capital income is the only source of income after retirement.

The uncertainty in the returns from training is what makes investment in human capital risky in this paper. After allocating $\tau^i(t + j)$ units of time to training at date $t + j$, an agent is uncertain about the number of efficiency units received in the subsequent period. Therefore, the returns from training are uncertain at the time when the training decisions are made. We assume the following functional form for the training technology

$$h^i_t(t + j + 1) = \gamma(t + j)\tau^i_t(t + j)\phi^i_t(t + j + 1) + \eta^i_t(t + j + 1)$$  \hspace{1cm} (4)

for $0 \leq j \leq J_R$. Here $0 < \phi < 1$ is the elasticity parameter and $\gamma(t + j)$ is the productivity of an agent who invested $\tau^i_t(t + j)$ time in training. The productivity could vary over different stages of the life-cycle of an agent but it is the same for all the agents within a generation. The two idiosyncratic shocks, the specialization shock $\epsilon^i_t(t + j + 1)$, and the career shock $\eta^i_t(t + j + 1)$, are normally distributed, independent and identical across agents within a generation and independent across generations. Across generations, therefore, the risk associated with the returns from training could vary implying that $\epsilon^i_t(t + j) \sim N(1, \sigma_\epsilon(t + j))$ and $\eta^i_t(t + j) \sim N(0, \sigma_\eta(t + j))$. The expected return from training in period $t + j + 1$ is

$$Eh^i_t(t + j + 1) = \gamma(t + j)\tau^i_t(t + j)^\phi$$

and the variance is

$$Vh^i_t(t + j + 1) = \left(\gamma(t + j)\tau^i_t(t + j)^\phi\right)^2 \sigma^2_\epsilon(t + j + 1) + \sigma^2_\eta(t + j + 1).$$
Under complete markets, \( \epsilon_i(t+j) = 0 \) and \( \eta_i(t+j) = 0 \) for all the individuals and at all dates.

The specialization risk \( \epsilon_i(t+j+1) \) is such that for the same level of risk, if individuals devote more time to training, they face a positive risk-return trade-off.\(^{12}\) Allocating more time to training increases the expected future returns but at the same time it also increases the variance of the returns from training. For instance, a lawyer who invests many years in training faces a higher return from training but he also faces a greater risk relative to a nurse who devotes fewer years in training and hence faces lower risk-return trade-off. The career risk \( \eta_i(t+j+1) \) can be interpreted as the general uncertainty associated with working in a job, whether the match is good, whether the agent is compatible with other workers, and other aspects of the labor market that are not incorporated in the model. If there is only career risk, \( \sigma_c(t) = 0 \), then for the same level of risk, acquiring more training only increases the mean return from training but leaves the variance unaltered.

The uncertainty in the returns from training makes an agent’s human capital represented by the labor quality random. The labor quality of an agent \( i \) at date \( t+j \) depends on the undepreciated level of inherited average aggregate labor quality and the human capital accumulated in the previous periods via training, the first and second terms respectively in the following equation

\[
x_i(t+j) = (1 - \delta_h) x(t) + \sum_{m=0}^{j-1} h_i(t+m+1)
= (1 - \delta_h) x(t) + \sum_{m=0}^{j-1} \{ \gamma(t+m) \tau_i(t+m) \phi \epsilon_i(t+m+1) + \eta_i(t+m+1) \}
\]

where the average aggregate labor quality inherited when young, \( x(t) \), depreciates at rate \( \delta_h \).

\(^{12}\)Empirical studies, a cross country study by Pereira and Martins (2002), and a study by Christainsen, Joensen and Nielsen (2006) using Danish data, show that a positive risk-return trade-off exists in human capital investment.
The labor quality of an agent has the following mean and variance

\[ Ex_i^t(t + j) = (1 - \delta_h)^j x(t) + \sum_{m=0}^{j-1} \gamma(t + m) \tau_i^j(t + m)^\psi \]  

(6)

\[ Vx_i^t(t + j) = \sum_{m=0}^{j-1} \{[\gamma(t + m) \tau_i^j(t + m)^\psi]^2 \sigma_x^2(t + m + 1) + \sigma^2_\epsilon(t + m + 1) \}. \]  

(7)

In this paper labor income uncertainty arises due to uncertain labor quality as a result of risky returns from training.\(^{13}\) Krebs (2003) incorporates human capital risk by assuming that there are idiosyncratic shocks to the stock of human capital. In Krebs (2003), human capital investment is modelled as a portfolio allocation problem. In this paper I study human capital investments using Ben-Porath’s (1967) production function for human capital with time allocated to training as input in the production technology.\(^{14}\)

Human capital investment in this paper requires agents to give up labor income early in the life-cycle in order to generate higher efficiency units in the future. This investment increases the level of expected future labor income in all subsequent periods.

Market incompleteness that arises from idiosyncratic uninsurable risks make the wealth distribution a relevant state making these problems intractable. I use CARA-normal specification for preferences and risk in order to make an individual’s risk-taking decision independent of wealth. I also assume that today’s training decision does not interact with labor quality and only inherited human capital depreciates over the life-cycle to ensure that both training and asset holding decisions are independent of wealth. As a result, each agent within a generation makes identical training and asset holding decisions. However agents are still heterogeneous within a generation in labor quality, labor earnings and consumption due

\(^{13}\)We abstract from labor income uncertainty arising because of frictions in the labor market. See, for example, Gomes, Greenwood and Rebelo (2001), Costain and Reiter (2005), Hall (2006), Rudanko (2007) among others.

\(^{14}\)We use a one-input training technology. In a richer multi-input specification, returns to training could depend on an agent’s ability, the level of human and physical capital in the previous periods.
to different realizations of the two shocks. Across generation heterogeneity is primarily due to the life-cycle features of the model.

The production technology of the final good is standard. There are two inputs, capital and labor and the production process exhibits constant returns to scale with the following specification

\[ Y(t) = AK(t)^\alpha L(t)^{1-\alpha}, \]  

where \( K(t) \) is the aggregate capital stock and \( L(t) \) is total labor supply measured in efficiency units at date \( t \). The intensive form representation of the training technology is standard, \( y(t) = Ak(t)^\alpha \), where \( y(t) \) is the output per efficiency unit and \( k(t) \) is the capital per efficiency unit. I suppress superscript \( i \) since all the agents within a generation are identical in their endogenous decisions. Total labor supply at date \( t \), measured in efficiency units, is given by

\[ L(t) = \sum_{j=0}^{J_R-1} \{ (1 - \tau_{t-j}(t))x + x_{t-j}(t) \}. \]  

Inputs are hired in competitive markets and are therefore paid their marginal products. Hence the wage rate per efficiency unit is \( w(t) = (1 - \alpha)Ak(t)^\alpha \) and the rental rate of capital is \( r(t) = \alpha Ak(t)^{\alpha-1} \). Let \( \delta_k \) be the rate at which capital depreciates, then \( R(t) = r(t+1) + 1 - \delta_k \).

At the aggregate level there is no uncertainty. We can write the aggregate market clearing condition for physical capital as

\[ L(t)k(t) = \sum_{j=1}^{T-1} s_{t-j}(t-1). \]  

The law of motion of aggregate average skills is described by the following equation

\[ x(t) = \frac{E \sum_{j=0}^{J_R-1} x_{t-j}(t)}{J_R}. \]  

At date \( t \), the average labor quality of any generation born in period \( t - j \) is \( Ex_{t-j}(t) \), for \( j = 0, 1, \ldots J_R - 1 \). Averaging over all the generations gives the average aggregate labor quality at date \( t \) which is inherited by the agents born at date \( t \).
3 Some intuition

In this section I briefly describe some intuition coming from the simplest two-period case before considering the extended model described in the previous section. Let $T = 2$ so that the model collapses to a two-period overlapping generations model. For simplicity there is no retirement and since agents live for two periods, they hold assets and train only in the first period. We consider this case to illustrate the qualitative implications of our model. We also drop the agent specific superscript for clarity. CARA-normal specification allows us to get closed form solution for an individual’s decisions. With closed form solutions we can clearly see how risk impacts these decisions.

3.1 Individual decisions

For $T = 2$, equation (1) simplifies and the utility function of an agent born at date $t$ is given by the following equation

$$U = -\frac{1}{\alpha} \exp[-ac(t)] - \beta \frac{1}{\alpha} \exp[-ac(t+1)].$$  \hspace{1cm} (12)

The budget constraint in the first and second period of the life-cycle are $c_{t}(t) = w(t)\{(1 - \tau_{t}(t))x + x_{t}(t)\} - s_{t}(t)$ and $c_{t}(t+1) = w(t+1)\{x + x_{t}(t+1)\} + R(t)s_{t}(t)$ respectively. Labor quality when old is given by

$$x_{t}(t+1) = (1 - \delta_{t})x(t) + \gamma(t)\tau_{t}(t)^{\phi}e(t+1) + \eta_{t}(t+1).$$  \hspace{1cm} (13)

Since we use CARA preferences and assume that the shocks are normally distributed, we can rewrite the agent’s problem as

$$U = -\frac{1}{\alpha} \exp[-ac(t)] - \beta \frac{1}{\alpha} \exp[-a\{Ec(t+1) - \frac{a}{2} Vc(t+1)\}],$$  \hspace{1cm} (14)

where the expected value of date $t + 1$ consumption is

$$Ec_{t}(t+1) = w(t+1)\{x + (1 - \delta_{t})x(t) + \gamma(t)\tau_{t}(t)^{\phi}\} + R(t)s_{t}(t)$$  \hspace{1cm} (15)

and the variance is

$$Vc_{t}(t+1) = w(t+1)^{2}\{(\gamma(t)\tau_{t}(t)^{\phi})^{2}\sigma_{e}^{2} + \sigma_{\eta}^{2}\}.$$  \hspace{1cm} (16)
Given initial inheritance of labor quality $x(t)$, wages $w(t)$, and $w(t+1)$ and interest rate $R(t)$, maximizing the agent’s utility with respect to $c_t(t)$, $c_{t+1}(t)$, and $\tau_t(t)$ gives the following first order conditions

$$Ec_{t+1}(t) - c_t(t) = \frac{1}{a} \ln(\beta R(t)) + \frac{a}{2}Vc_t(t+1)$$  \hspace{1cm} (17)

$$w(t)x_\overline{\gamma} = \varphi w(t+1)\gamma(t)\tau_t(t)^{\phi-1} - \frac{\gamma(t)^2\tau_t(t)^{2\phi-1}}{R(t)}$$  \hspace{1cm} (18)

These two equations collapse to the complete markets benchmark when $\sigma_\epsilon = 0$ and $\sigma_\eta = 0$. Relative to the complete markets case, the Euler equation (17) that determines the optimal level of real asset holding has an extra term on the right hand side, $\frac{a}{2}Vc_t(t+1)$, the precautionary savings. Similarly, an extra term appears on the right hand side of equation (18) because markets are incomplete. When markets are complete, at the optimal level of training the marginal cost of acquiring training, the left hand side of equation (18) equals the present value of the future marginal benefit, the first term on the right hand side of this equation. When markets are incomplete, however, agents have to be compensated for the risk they face when they invest in training. Therefore, the second term on the right hand side is the risk premium on human capital investment. It is increasing in specialization risk $\sigma_\epsilon^2$. Interestingly, from the two equations we see that while optimal real asset holding is affected by both types of risk, the training decision, even when markets are incomplete, is not directly affected by the career risk. This suggests that individual decisions could differ depending on whether the agents face specialization risk, career risk or both, similar to the findings of Angeletos and Calvet (2006).

To explore this further, let the elasticity of the training technology $\phi$ equal 1/2. This allows us to get explicit solutions for the optimal level of real asset holding and training of a young agent.\(^{15}\) Using these solutions we can clearly see the role of risk and how it enters an individual’s decisions

\(^{15}\)In the human capital literature the estimated elasticity lies in the range (0.5, 0.9). See Browning, Hansen and Heckman (1999) Table 2.3 and 2.4 for further details. We intend to vary $\phi$ while investigating the robustness of our results.
in this simple two-period model with incomplete markets.

\[
s_t(t) = \frac{1}{1 + R(t)} \left( \frac{1}{a} \ln(\beta R(t)) + w(t) \{(1 - \tau_t(t))x + x(t)\} - w(t + 1) \{x + (1 - \delta_h)x(t) + \gamma(t)\tau_t(t)^{0.5} + \frac{a}{2}V_t c_t(t + 1)\}\right) \tag{19}
\]

\[
\tau_t(t) = \left( \frac{w(t + 1)\gamma(t)}{2R(t)w(t)x + aw(t + 1)^2\gamma(t)^2\sigma_z^2} \right)^2. \tag{20}
\]

Consider the case where \(\sigma_e = 0\), and \(\sigma_\eta > 0\). Based on the comparative static results for this case where changes in the time allocated to training only influence the mean return from training but not the variance, we see that the training decision is in fact unaffected by the career risk. Real asset holding is however increasing in this risk because of the precautionary motive. Now let \(\sigma_e > 0\), and \(\sigma_\eta = 0\). In this case, since altering the decision to train impacts the variance of the return from training, agents train less when risk increases. As before, the direct effect of risk on real asset holding is positive for precautionary reasons. However, in this case risk also impacts real asset holding indirectly via training decisions. If the following inequality holds, then the saving decision is increasing in risk

\[
-1 \leq \frac{0.5w(t + 1)\gamma(t)\tau_t(t)^{-0.5}}{w(t)x} - \frac{\frac{1}{2}w(t + 1)^2\gamma(t)^2\sigma_z^2}{w(t)x} \tag{21}
\]

From equation (18) we know that the right hand side of the above inequality equals the gross interest rate on the risk-free asset which is greater than 1 implying that this inequality holds.

## 4 Quantitative analysis

In the previous section we argued that the qualitative effect of risk, the specialization risk versus the career risk, on an individual’s decisions could potentially be very different. This section explores the general equilibrium consequences of market incompleteness on decisions and the aggregate
economy in various calibrated stationary equilibria of our model. It also investigates whether these qualitative results are in fact quantitatively significant.

Our calibration strategy, similar to Aiyagari (1994), is to first choose parameters such that they match some targets in the US data under the complete markets benchmark. We use the life-cycle features of our model to calibrate the two types of risks. We consider three calibrated versions of our model that primarily differ along one dimension, the calibration of risk, which is chosen to endogenously generate labor income variance that matches data.\footnote{The only exception is the Aiyagari-like economy that differs in the calibration of other parameters as well since we want to abstract from human capital accumulation in order to match key features of Aiyagari’s model.} The three cases are the Aiyagari-like economy, the career risk only economy, and the baseline calibration economy. In spite of the fact that all of these economies endogenously generate variance in labor income that matches the relevant data, the aggregate implications are strikingly different across these economies.

4.1 Calibration of complete market benchmark

In our calibration, the model period is eight years and agents live for 64 periods from age 16 to 80.\footnote{In the final version of the paper, a period will be one year.} In the last two periods, at the age of 64, agents stop participating in the labor market and they retire. An agent trains for 6 years between the ages 16–32, the first two periods in this eight period model. We use standard values of $\alpha$, $\beta$ and $\delta_k$. To match the share of physical capital in income we use $\alpha = 0.4$. The annual discount factor $\beta$ is 0.975 and the annual depreciation rate of physical capital is 8 percent. We choose the depreciation rate of human capital as 4 percent such that initially when agents are training, labor quality increases but after they stop allocating time to training the labor quality decreases gradually till they exit the labor market.\footnote{We intend to verify whether our results are robust to changes in the depreciation rate of human capital.} The other five parameters $A$, $a$,$x_0$, $\gamma(t)$ and $\gamma(t + 1)$...
are chosen to match the following targets in the U.S. data. Our first goal is to make sure that the economy has a reasonable level of physical capital. Following Rios-Rull (1996), we target capital-output ratio at 2.94 and consumption-output ratio at 0.748. We target the gross interest rate on the risk free asset to be 1.03 percent. Our second goal is to ensure that in equilibrium there is an appropriate level of education that matches the U.S. data. An average American spends 16 years in school, 6 years in primary education, starting at the age of 6, 6 years in secondary education starting at age 12 and 4 years in post-secondary education.\textsuperscript{19} Since our model starts at age 16, this suggests that agents would train for 6 years, two years in secondary and four years in post-secondary education. We calibrate the time devoted to training in each period to match the enrollment rate for different age groups. According to the Current Population Survey (Table S1401) on school enrollment, 95.7 percent of the Americans between the age 15 – 17, 72.5 percent from the age 18 – 19, 39.4 percent from 20 – 24 and 12 percent between the age 24 – 34 are enrolled in school. In our eight period model we target 5.5 years of training between the ages 16 to 24 and about half year from 24 to 32.\textsuperscript{20} Overall, individuals must spend 6 years in school between the age 16 – 32. The relative risk aversion implied by our model lies in the range (1, 3). Given that we have more targets relative to free parameters, our calibration is not precise but it is very close to the targets.

4.1.1 Two types of risks

In the second step of our calibration, we use the life-cycle features of our model to match the variance of labor earnings over the life-cycle which is primarily due to age effects to the U.S. data.\textsuperscript{21} In our model since there is no

\textsuperscript{19}This is according to the estimates of the United Nation Educational, Scientific and Cultural Organization’s (UNESCO) Global Education Digest 2004.

\textsuperscript{20}A calculation based on CPS would suggest that agents would train for approximately 5 years between the age 16-24 and 1 year between the age 24-32.

\textsuperscript{21}We follow Aiyagari (1994) in calibrating our model. Our targets for physical and human capital suggest that the data are generated under complete markets. However, when we calibrate risk, we assume that the data are generated under incomplete markets. One advantage of this procedure is that the complete markets case is identical across the economies.
Table 1: Calibration of Risk

<table>
<thead>
<tr>
<th>Specialization risk</th>
<th>Career risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>( { \sigma_{e(t+j)} }_{j=1}^{5} )</td>
<td>( { \sigma_{\eta(t+j)} }_{j=1}^{5} )</td>
</tr>
<tr>
<td>Aiyagari-like</td>
<td>{0}_{j=1}^{5} {0.08, 0.03, 0.04, 0.04, 0.04}</td>
</tr>
<tr>
<td>Career risk only</td>
<td>{0}_{j=1}^{5} {0.08, 0.03, 0.03, 0.06, 0.03}</td>
</tr>
<tr>
<td>Baseline calibration</td>
<td>{1.41, 1.84, 0, 0, 0} {0, 0.01, 0.02, 0.02, 0.02}</td>
</tr>
</tbody>
</table>

Table 1: Calibrated values of the standard deviation of the two shocks under incomplete markets in different economies.

growth and at age 16 all agents are ex ante identical, variance in labor earnings is entirely due to age effects. But in the data there may be numerous other reasons why dispersion in labor earnings increases over the life-cycle, for example, time effects, cohort effects, differences in initial conditions, human capital risk, and other employment related risks. For our analysis, Huggett, Ventura and Yaron’s (2006) paper provides a close approximation of the age effects of the variance in labor earnings. In addition, in our model there are no other sources of within generation heterogeneity except for the heterogeneity due to the idiosyncratic human capital shocks. Therefore the variance of labor earnings over the life-cycle which is due to age effects must be only due to these shocks. Huggett, Ventura and Yaron investigate different sources of labor income dispersion and conclude that with endogenous training. Ideally, calibration of our model with market incompleteness must match the US data. However, as will be apparent from Tables 2 and 3, the calibration strategy used here does not matter for two out of the three economies that we consider in this paper since market incompleteness has a very small and quantitatively insignificant impact. However in the specialization risk economy, the incomplete markets case differs substantially from the complete markets benchmark. In the robustness section of the paper, our goal is to address this issue.

22They use earnings data from PSID 1969-2004 family files. They assume that the earnings data is generated by three factors, cohort, time and age effects. Cohort effects are the effects that impact all the agents born in a particular year, time effects impact all the generations alive at a particular date. Due to the linear relationship between time and age, the age effects cannot be isolated easily. As a result they isolate the age effects by either controlling for time or controlling for cohort. In our analysis, we use the regression results where they control for time. See Figure 2 in Huggett, Ventura and Yaron (2007).
approximately 1/3 of the variance in lifetime earnings due to age effects is actually explained by idiosyncratic human capital risks. As a result, we choose the variance of the two idiosyncratic human capital shocks such that the variance in the labor earnings generated by our model under market incompleteness is in fact 1/3 of the variance in labor earnings in the data.

In our quantitative analysis, we consider three calibrated economies, the Aiyagari-like economy, the career risk only economy, and the baseline calibration economy. Table 1 gives the underlying standard deviation of the two shocks, \( \sigma_{(t+j)} \) and \( \sigma_{(t+j)} \), that are used in the calibration of the incomplete markets version of these economies. The standard deviation varies over the life-cycle of an agent but it is identical for agents within a generation at all times. Notice that in this model there is no risk in the first period of life as agents have not yet invested in training and in the last two periods after they retire. Figure 1 illustrates how this calibration allows us to match earnings dispersion generated by our model with Huggett, Ventura and Yaron’s measure of the variance of labor earnings which is due to idiosyncratic human capital shocks. All the economies, except the specialization risk only economy, can be calibrated to match the data. Specialization risk interacts with the time allocated to education, and agents invest in training only in the first two periods of their life-cycle. Therefore in the

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23 Identification of idiosyncratic human capital shocks in Huggett, Ventura, and Yaron (2007) relies on the fact that closer to retirement agents do not invest in human capital. During this period when there is no investment in human capital, changes in the wage rate, product of the rental rate and human capital, are entirely due to changes in the rental rate or the human capital shocks. Using this identification strategy, Huggett, Ventura, and Yaron estimate the standard deviation of the human capital shocks.

24 In our model, human capital investment refers to time the spent in education and hence agents train for 6 years after the age of 16. Human capital risk is both multiplicative and additive in our framework. In Huggett, Ventura and Yaron (2007) human capital shocks enter multiplicatively in the law of motion of human capital. For these shocks to play a role over most of the life-cycle, they assume that agents invest time in training until they are close to the retirement age. Therefore the time allocation problem, the trade-off between training and working exists for most part of the life-cycle, an unrealistic implication of their model. Therefore, we consider that the risks identified in Huggett, Ventura and Yaron (2007) are in fact a proxy for both the risks that the agents face over their life-cycle, career and specialization risk.
specialization risk only economy, shocks cannot be calibrated to match the increasing variance of labor earnings over the life-cycle.\(^\text{25}\)

### 4.2 Results

Following the calibration procedure described above, we study stationary equilibria of calibrated economies to determine the quantitative macroeconomic consequences of market incompleteness in a general equilibrium life-cycle model.

#### 4.2.1 Aiyagari-like economy

We construct a version of Aiyagari (1994) in a life-cycle context, Aiyagari-like economy, in order to determine the conditions under which the predictions based on our model will be consistent with the quantitative findings of the

\(^{25}\)The specification of risk in the specialization risk only economy is the following \(\{\sigma_{\varepsilon}(t+j)\}^{j=5}_{j=1} = \{1.732\}^{j=5}_{j=1} \) and \(\{\sigma_{\eta}(t+j)\}^{j=5}_{j=1} = \{0\}^{j=5}_{j=1}.\)
existing literature that studies the effect of uninsurable idiosyncratic labor income risk on the aggregate economy. Aiyagari studied uninsurable labor endowment risk in a model with no human capital where agents live forever. This is a life-cycle model, and unlike Aiyagari there are no borrowing constraints and no wealth effects. To collapse our model with human capital accumulation to an Aiyagari-like economy, we set $\gamma(t)$ and $\gamma(t + 1)$ equal to zero such that agents do not invest in training. Due to lack of education, specialization risk does not play any role in this calibration. We endow the agents with an average steady state labor quality from our complete markets benchmark, $\hat{x}$ in Table 2, panel B, column 1. We set $\delta_h = 0$. The rest of the parameters are calibrated to meet the targets mentioned in the previous section.

Panel A of Table 2 reports the results from the Aiyagari-like economy. Both qualitative and quantitative results in the Aiyagari-like economy are similar to Aiyagari (1994). When markets are incomplete the savings rate and output increases in the Aiyagari-like economy. In addition, the quantitative implications of idiosyncratic risk are very small. In Aiyagari’s (1994) paper, the increase in the savings rate (which equals $\delta_k/y$) was in the range $\{0.06, 7.33\}$, expressed in percentage points. In our Aiyagari-like economy, the increase in savings rate is 0.11 percentage points. The increase in the savings rate in turn causes output to increase in both economies. Another consequence of increased savings under market incompleteness relative to complete markets benchmark is that the net interest rate (in percentage points) declines in Aiyagari and the range of this decline is given by $\{0.02, 2.88\}$. The comparable number is 0.035 in the Aiyagari-like econ-

\footnote{Aiyagari used CRRA preferences with three different values of relative risk aversion coefficient $\{1, 3, 5\}$. The discount factor was 0.96. The production technology was Cobb-Douglas with capital share as 0.36 and the annual depreciation rate of capital was 8%. Logarithm of labor endowment shock followed an AR(1) process. Values of the autoregressive coefficient and the coefficient of variation were chosen based on different studies. He reports his results based on different combinations of relative risk aversion, coefficient of variation and serial correlation. See Table II in Aiyagari (1994). In reporting the range, we do not consider the case where the net return on capital is negative, the last entry in Table II.}
Table 2: Comparing quantitative macroeconomic effects of an Aiyagari-like economy with a career risk only economy. In both economies uncertainty is only due to career shocks.

4.2.2 Career risk only economy

Can market incompleteness in a model where agents allocate time towards training when young also produce small and quantitatively insignificant results like Aiyagari? Consider the career risk only economy that differs from the Aiyagari-like economy along two dimensions, agents endogenously decide how much to train and human capital depreciates to maintain a constant level of labor quality in the stationary equilibrium. In this economy labor supply is not inelastic in the first two periods of the life-cycle when instead of working in the labor market, agents allocate time towards training.

Table 2, panel B presents results for the career risk only economy. Relative to the complete markets benchmark, incomplete markets have very small and quantitatively insignificant effects, even smaller than the Aiyagari-
like economy. One reason could be that similar to the Aiyagari-like economy, agents in this case can self-insure by holding precautionary savings but unlike before they can also mitigate the effect of risk by altering their training decisions. The savings rate increases by 0.02 percentage points, lower relative to both Aiyagari and the Aiyagari-like economy. As before, output is higher when markets are incomplete. A very striking result in the career risk only economy is that under incomplete markets there is overaccumulation of human capital, though quantitatively insignificant but in compliance with Angeletos and Calvet (2006). These results demonstrate that allowing agents to optimally decide how much time to allocate to training when returns from training are uncertain and uninsurable is not enough to generate large quantitative effects. Moreover, when individuals face career risk, the risk where changes in training only affect the mean return but not the variance of the return from training, market incompleteness has very small effects.

Our next specification of the incomplete markets incorporates specialization risk as well.

4.2.3 Baseline calibration economy

In the incomplete markets case of our baseline calibration, we assume that agents face both risks. Since the empirical literature provides no guidance on how to assign weights to these shocks, we assign these weights in a way that attributes almost all the variability in labor earnings to specialization risk early in the life-cycle and later in life, the additional risk that agents face is entirely due to the career shocks. The reason behind such an extreme calibration is that in this economy we want to explore the role of specialization risk while still matching the data on life-cycle labor income variability. From Figure 1 it is obvious that specialization risk alone cannot match the data and hence we combine it with career risk.

As mentioned before, the two economies with endogenous training de-

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27 Labor quality-output ratio also increases, albeit very marginally.
Table 3: Aggregate effects of market incompleteness in the baseline calibration economy.

<table>
<thead>
<tr>
<th></th>
<th>Complete markets</th>
<th>Incomplete markets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{x}$</td>
<td>0.154</td>
<td>0.088</td>
</tr>
<tr>
<td>$\hat{k}$</td>
<td>1.275</td>
<td>1.381</td>
</tr>
<tr>
<td>$\check{K}/\check{Y}$</td>
<td>0.503</td>
<td>0.528</td>
</tr>
<tr>
<td>$\check{C}/\check{Y}$</td>
<td>0.755</td>
<td>0.742</td>
</tr>
<tr>
<td>$\check{L}/\check{Y}$</td>
<td>0.395</td>
<td>0.382</td>
</tr>
<tr>
<td>$\check{Y}$</td>
<td>4.272</td>
<td>3.366</td>
</tr>
<tr>
<td>$\check{R}$</td>
<td>1.034</td>
<td>1.030</td>
</tr>
<tr>
<td>$\hat{\omega}$</td>
<td>1.521</td>
<td>1.575</td>
</tr>
</tbody>
</table>

Table 3: Aggregate effects of market incompleteness in the baseline calibration economy.

...
In this model, investment in training influences the future stream of income. The net present value of the future returns from first period training is given by the following expression

$$\sum_{j=1}^{5} \beta^j \left\{ \hat{\phi} \hat{\omega} \gamma(t) \tau_f(t)^{\theta-1} - \frac{\delta}{2} 2 \hat{\phi} \hat{\omega}^2 \gamma(t)^2 \tau_f(t)^2 \sigma_e(t+1)^2 \right\} - \hat{\omega} \chi. \quad (22)$$

The first term is the discounted value of the return from training in the first period of the life-cycle, 0.495 under the baseline calibration, the second term is the risk premium and the last term in the above expression is the marginal cost of acquiring training which equals 0.085. Thus the net present value of the future returns from first period training is 0.167 in the stationary equilibrium of the baseline calibration economy. Holding the risk-free asset on the other hand gives a one period return and for this economy the present value of the capital income from the first period asset is 0.034. Thus the present value of net returns from training in the first period is 5 times the capital income from the first period asset holdings. Similarly, the present value of the net returns from training in the second period is 1.5 times the capital income from the second period asset holdings.

Quantitative analysis in Krebs (2003) suggests that for his baseline calibration, investment in human capital is 4.2 percentage points (of GDP) lower. The comparable number for this paper is 23 percentage points.\textsuperscript{28} This indicates that not accounting for an agent’s time allocation problem along with the specialization risk in the returns to training can lead to a substantial misstatement of the impact of risk on human capital investment. Investment in physical capital in Krebs is 3.95 percentage points (of GDP) higher relative to complete markets benchmark. In our baseline calibration economy, investment in physical capital relative to output, the savings rate,

\textsuperscript{28}Investment in human capital in this model is determined by the time devoted to training. In the first period, time devoted to training relative to output ($\hat{\tau}_0 / \hat{y}$) decreases by 22.7 percentage points. The decrease is lower in the second period, 0.5 percentage points. Thus the overall decrease in investment in human capital relative to output is 23 percentage points.
increases by 1.20 percentage points, an increase which is 1/3 of the increase in Krebs. Therefore it appears that in our model agents self insure but more by altering their decision to train and less via the traditional precautionary savings channel.

In the quantitative analysis we compared three different economies, the Aiyagari-like economy, the career risk only economy and the baseline calibration economy. Such a comparison is convincing in that all three economies endogenously generate the same amount of labor income variance over the life-cycle even when they have extremely different macroeconomic implications. In the Aiyagari-like economy and in the career risk only economy, incomplete markets have very small quantitative effects. In the baseline calibration economy with higher weight on specialization risk, however, market incompleteness leads to a dramatic decrease in labor quality and all other aggregate variables. Thus depending on the weights of the two shocks, the quantitative implications of incomplete markets could be as high as our baseline case or as low as the case with only career shocks. Unless empirical studies isolate the relative weights of these shocks in the data, it is hard to take a stand on the exact role of market incompleteness.  

4.3 Life-cycle features of the model

In this section we take a closer look at the underlying training and asset holding decisions and how these decisions get influenced by the two types of risk relative to the complete markets benchmark. All agents within a generation make identical training and asset holding decisions in this model. Within generation heterogeneity with respect to labor earnings and consumption permits us to explore the distribution of earnings and consumption across different generations. By stationarity, the distribution of earnings and consumption over an agent’s life-cycle is also identical to the

\[29\] More work needs to be done in order to ensure that the results presented here are robust to changes in the depreciation rate, the elasticity parameter \(\phi\) in the production technology for human capital. In addition, I intend to recalibrate the model such that the incomplete markets version of the model matches data.
Table 4: Fraction of one unit of time devoted to training in the first two periods. No time is allocated to training thereafter.

cross sectional distribution of earnings and consumption at any point in time.

In the case when there is no uncertainty, agents spend 6 years in training, 5.5 years between the age 16 to 24 and half a year between 24 to 32. When labor income uncertainty is due to career risk, the time allocated to training does not change much relative to the complete markets benchmark. However, in the baseline calibration where specialization risk plays a central role, agents train for 2 years, 1.6 and 0.4 years in first and second periods respectively. Relative to the complete markets benchmark, training is merely 1/3 which is why labor quality is dramatically lower in the stationary equilibrium of the baseline calibration economy. As noted earlier this is a striking result. Table 4 reports time allocated to training as a fraction of one unit of time in the first two periods, where each period corresponds to eight years, of the life-cycle.

Figure 2 shows that the real asset holding as a fraction of the total income increases over the life-cycle, irrespective of whether markets are complete or not. One reason is that agents accumulate human capital when young and therefore their asset holdings are relatively low in the earlier part of their life-cycle. In addition, we also see that the asset holding decision gets distorted by risk, a common result in this literature. Agents tend to save more when markets are incomplete for precautionary purposes. Consequently, in both economies, the career risk only economy and

---

\[ \hat{\tau}_0 = \frac{0.683}{8} = 0.085 \]  
\[ \hat{\tau}_1 = \frac{0.066}{8} = 0.00825 \]

---

\[ \hat{\tau}_0 = \frac{0.684}{8} = 0.085 \]  
\[ \hat{\tau}_1 = \frac{0.067}{8} = 0.008375 \]

---

\[ \hat{\tau}_0 = \frac{0.202}{8} = 0.02525 \]  
\[ \hat{\tau}_1 = \frac{0.054}{8} = 0.00675 \]

---

Table 4. Time in Training

<table>
<thead>
<tr>
<th>complete markets</th>
<th>incomplete markets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Career risk only economy</td>
<td>Baseline calibration economy</td>
</tr>
<tr>
<td>$\hat{\tau}_0$</td>
<td>0.683</td>
</tr>
<tr>
<td>$\hat{\tau}_1$</td>
<td>0.066</td>
</tr>
</tbody>
</table>

---

\[ \hat{\tau}_0 = \frac{0.683}{8} = 0.085 \]  
\[ \hat{\tau}_1 = \frac{0.066}{8} = 0.00825 \]

---

\[ \hat{\tau}_0 = \frac{0.684}{8} = 0.085 \]  
\[ \hat{\tau}_1 = \frac{0.067}{8} = 0.008375 \]

---

\[ \hat{\tau}_0 = \frac{0.202}{8} = 0.02525 \]  
\[ \hat{\tau}_1 = \frac{0.054}{8} = 0.00675 \]
the baseline calibration economy, real assets are higher relative to the case when markets are complete. Therefore, both decisions, the decision to train less and the decision to save more allow agents to self-insure against uninsurable idiosyncratic human capital risk.

Due to heterogeneity in labor quality, our model generates a distribution of labor earnings over the life-cycle even though agents make identical training decisions. The solid line in the left graph of Figure 3 is the age effects in mean earnings according to the estimates of Huggett, Ventura and Yaron (2007). Following Huggett, Ventura and Yaron (2007), we scale the mean labor income such that at the end of an agent’s work life mean labor income is 100. We see that the mean labor income generated by our general equilibrium model has the familiar hump shape which is seen in the data. However, in our model the hump occurs in the earlier periods of the life-cycle, close to age 30, but in the data labor earnings peak when agents are in their mid 40s. One reason for this mismatch could be that in our model we do not allow labor quality to increase due to reasons other than schooling, like on-the-job training, learning by doing and other mechanisms that
Figure 3: The left graph plots mean labor income and the right plots the variance of logarithm of the labor income over the life cycle in the different economies.

improve labor quality over the entire working life of an agent. Not surprisingly, the variance of the logarithm of labor earnings, the right graph in Figure 3, matches data in all the different economies that we consider since we calibrated the risk to match this aspect of the data.

Our model does not match the moments of life-cycle consumption data very well. In our specification the two shocks have a normal distribution and consumption is residual. This suggests that in our quantitative analysis there can be some instances under which consumption is negative. The bands in the left graph of Figure 4 for mean consumption contain 95% of the values suggesting that the shocks in our model are sufficiently small to preclude high incidences of negative consumption. Mean age-consumption in our model has an upward slope, a common implication of life-cycle models with additively separable utility function defined over consumption alone, even though agents face uncertain labor earnings. Bullard and Feigenbaum (2007) argue that inclusion of both consumption and leisure in the utility function would produce a hump-shaped consumption along with a U-shaped leisure profile, given a hump-shaped life-cycle productivity profile. Comparing the consumption inequality over the life-cycle generated by our model to Deaton and Paxson’s (1994) measure of consumption in-
equality in the data, we find that our model generates higher consumption volatility relative to data even when we account for just 1/3 of the variance in labor earnings. Storesletten, Telmer and Yaron (2004) find that without a social security system, variance in consumption is roughly 20% higher relative to data. In this model there is no form of social insurance and that could partly explain high consumption variance. Another reason that I get high consumption variance is that an agent’s endogenous decisions are unaffected by the actual realization of the two shocks. This causes consumption to be residual and therefore more volatile. Apart from being more volatile, the variance of consumption generated by the model is concave whereas in the data it is somewhat linear. In this model model when agents are young, they accumulate human capital which is risky and their holdings of risk-free real assets are relatively low. As a result shocks to training have a larger impact on consumption in the early periods of the life-cycle. Over time as holdings of real risk-free asset increases, human capital shocks tend to have a lower impact. Overall, in both the economies, the life-cycle consumption implications of our model do not match data very well. By incorporating leisure, allowing for wealth effects and introducing some form of social insurance, we could potentially correct the implications of our model for consumption.

Even though the aggregate implications of these two economies, the career risk only economy and the baseline calibration economy, are undoubtedly very different, the life-cycle features are somewhat similar. The two economies endogenously generate the same amount of labor income variability over the life-cycle. Other life-cycle features, such as mean labor income, mean consumption and the variance of consumption over the life-cycle are also similar across these two economies. One reason could be that these divergent decisions occur only in the early part of the life-cycle and their influence gets smoothed over many periods thus understating the

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31 Deaton and Paxson (1994) use consumption data from the Consumption expenditure Survey (CEX), 1980-1990. They remove the cohort effects from the data when measuring the age-profile of consumption inequality over the life-cycle. See Deaton and Paxson (1994) for further details.
differences in these differing endogenous decisions in the two economies.

5 Conclusion

This paper studies the impact of uninsurable idiosyncratic human capital risk, using a Ben-Porath version of production function for human capital, in a general equilibrium life-cycle model. Our results indicate the nature of risk is crucial in determining the quantitative implications of market incompleteness for the aggregate economy. Calibrated stationary equilibrium of our model with only career risk has properties similar to Aiyagari (1994) reconfirming the long held belief that the quantitative implications of uninsurable idiosyncratic risk are inconsequential. However, our baseline calibration clearly illustrates that market incompleteness could have large, quantitatively significant, macroeconomic implications. Stationary equilibrium under this case is severely distorted relative to the complete markets benchmark.

Based on our quantitative analysis, we conclude that in order to comment on the macroeconomic effect of uninsurable idiosyncratic human capital risk, empirical studies based on micro data would need to determine the relative weights of these shocks in data. Even though the life-cycle features generated by our model under different calibrations of risk are not very different, the decision to train is particularly dissimilar in these
economies. This dissimilarity in the level of education is primarily due to specialization risk. Empirical cross country studies estimating risks associated with different types of education that require varying degrees of specialization could however be indicative of the level of specialization risk that exists in these countries. Such studies could thereby alert us about the likely extent of distortion in these countries due to uninsurable idiosyncratic specialization risk.

References


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