Unemployment and Human Capital*

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1 Introduction

The paper explores the interaction between sector-specific human capital accumulation and sector-specific productivity or demand shocks. Our objective is to better understand the determinants of skill and experience premia, the costs of displacement for workers with long job tenure, and the nature of unemployment among such workers. For example, our model suggests why skilled workers can remain unemployed indefinitely even though low wage jobs are readily available and are acceptable to unskilled workers.

We develop a version of the Lucas and Prescott (1974) search model with sector-specific human capital. Competitive firms in each sector hire skilled and unskilled workers to produce an intermediate good. The intermediate goods produced in different sectors are combined by a competitive final goods producer to generate a consumption good using a constant returns to scale technology. Unskilled workers are free to work in any sector, while skilled workers have sector-specific human capital, although they can always leave the sector to become unskilled. Skills are accumulated while working in a sector, with an unskilled worker becoming skilled at a constant rate. Finally, sectoral productivity is continuously buffeted by idiosyncratic shocks. Thus the state of a sector is described by its productivity and the number of skilled workers.

We assume that log productivity follows a random walk, i.e. the growth rate of productivity is independent of its level. We show that equilibrium behavior depends on a single state variable ω, a linear combination of the sector's log productivity and the log of the number of skilled workers (see also Alvarez and Shimer, 2009). The value of being a skilled worker is increasing in ω, as is the ratio of unskilled to skilled workers. An increase in the number

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of skilled workers reduces ω, so this acts as a natural equilibrating mechanism for markets. When ω is high, many unskilled workers enter the labor market. As they accumulate skills, ω falls back down. At the other extreme, skilled workers exit labor markets when ω is too low, again pushing it back towards its average value.

At particularly low values of ω, some skilled workers may be unemployed, collecting leisure instead of a wage. They are willing to do this even though they could always get a job in another labor market. Moreover, they choose not to work in their labor market even though the wage there exceeds the unskilled wage in other labor markets. An outsider who did not understand the structure of the economy might argue that these workers are inactive (have dropped out of the labor force), since they are unwilling to take jobs that similar workers find acceptable. We draw a distinction between inactivity and unemployment by assuming that unemployed workers get less leisure than inactive workers, reflecting their need to stay in contact with their labor market while unemployed.

A typical labor market experience in our model looks as follows: a worker starts off unskilled and moves rapidly between labor markets, working at a low wage. Eventually she becomes skilled in one of the labor markets and experiences a large wage increase. Adverse productivity shocks and an increase in competition from other skilled workers eventually depress her wage, possibly pushing her into unemployment. If she leaves her labor market and becomes unskilled again, her wage falls further, though this is offset by the possibility of large subsequent wage gains. Thus our model makes contact with the large literature documenting the consequences for wages and employment of displacement for long-tenure workers (e.g. Jacobson, LaLonde, and Sullivan, 1993).

This paper is an outgrowth of our earlier research on "rest unemployment." Indeed, in Section 4, we establish that our model is isomorphic to the directed search model in Alvarez and Shimer (2009) if the wage of unskilled workers is always equal to zero. The workers whom we called "search unemployed" in our earlier paper are now relabeled "unskilled." This is important because in our earlier paper, we found that our model had quantitative problems unless search unemployment was extraordinarily unpleasant or lasted an extraordinarily long time. It is much easier to understand why being unskilled is unpleasant and lasts a long time. Our full model here is more general because the wage of unskilled workers is typically positive and depends on labor market conditions, but many of the insights from our earlier work carry over to this framework with this reinterpretation of search unemployment.

Our paper is conceptually similar to Rogerson (2005). He studies a two-period-life overlapping generations version of the Lucas and Prescott (1974) model. If young workers are employed, they are unskilled but accumulate sector-specific skills while working. They are willing to work at a low wage in return for a high wage when they are old and skilled. An adverse shock to a sector may lead some old workers to drop out of the labor force. In particular, they are unwilling to work in another sector because they would earn a low wage but not live to reap the reward from becoming skilled. Relative to this paper, we do not have an overlapping generations structure, but instead have a richer dynamic structure and process for sectoral shocks. We believe both of these assumptions help bring the model closer to the data.

There is one important difference in interpretation between this paper and Rogerson (2005): we label skilled workers who are not employed as "unemployed," while he views them as having dropped out of the labor force. We do this in part to make contact with data on long-term unemployment for older workers. We believe that many skilled workers engage in low-intensity search to make sure that they are aware if labor market conditions improve, and hence would be measured as unemployed by government statistical agencies. In particular, in our model workers always have a chance of returning to work, while the two-period-life assumption in Rogerson (2005) precludes this possibility. Still, we acknowledge that statistical agencies would likely measure some of our "unemployed" workers as having dropped out of the labor force.

This paper is also related to Kambourov and Manovskii (2009), who extend the Lucas and Prescott (1974) model to allow for two levels of sector-specific experience. Much of the difference between these papers lies in their emphasis. Kambourov and Manovskii (2009) follow most of the related literature in assuming that unemployment occurs when workers switch sectors. In contrast, we allow workers to switch sectors instantaneously. We do this for two reasons. First, it makes the model significantly more tractable by eliminating a state variable, the number of unskilled workers in a sector. We believe that the tractability is useful because it allows one to better understand the role of particular parameters for outcomes of interest and, in some cases, to prove general results about the model. Second, we emphasize unemployment for skilled workers who stay in a sector, a possibility that does not arise in Kambourov and Manovskii's work. A consequence of the absence of barriers to mobility is that our model has no predictions for the mobility of unskilled workers. In contrast, the mobility of inexperienced workers plays a key role in Kambourov and Manovskii's analysis, where they argue that an increase in occupational mobility is a key force in understanding the evolution of wage inequality.

Our paper proceeds as follows. In the next section we develop our model. Section 3 characterizes the equilibrium as the solution to a second order differential equation. We then look at three special cases. First, we assume unskilled workers are unproductive and show the isomorphism between this special case and our model in Alvarez and Shimer (2009). Then we assume unskilled workers are perfect substitutes for skilled workers. Finally, we allow for a constant, but finite, elasticity of substitution between the two types of workers.

2 Model

We consider a continuous time, infinite-horizon model. We focus for simplicity on an aggregate steady state and assume markets are complete.

2.1 Intermediate Goods

There is a continuum of intermediate goods indexed by $j \in [0,1]$. Each good is produced in a separate labor market with a constant returns to scale technology that uses only labor. In a typical labor market j at time t, there is a measure $l(j,t)$ unskilled (low skilled) workers and $h(j,t)$ skilled (high skilled) workers. All the unskilled workers and $\tilde{h}(j,t) \le h(j,t)$ of the skilled workers are employed, while the remaining skilled workers are unemployed. Industry output is $x(j,t)F(l(j,t), \tilde{h}(j,t))$. Assume F is linearly homogeneous in its two arguments, with $f(l/\tilde{h}) \equiv F(l,\tilde{h})/\tilde{h}$. Output per skilled worker f is an increasing and concave function. $x(j,t)$ is an idiosyncratic shock that follows a geometric random walk,

$$d \log x(j,t) = \mu_x dt + \sigma_x dz(j,t), \tag{1}$$

where μ_x measures the drift of log productivity, $\sigma_x > 0$ measures the standard deviation, and $z(j,t)$ is a standard Wiener process, independent across labor markets. The price of good j, $p(j,t)$, the wage for unskilled workers in labor market j, $\tilde{w}_l(j,t)$, and the wage for skilled workers in labor market j, $\tilde{w}_h(j,t)$, are determined competitively at each instant t and are expressed in units of the final good.

We assume that labor market j shuts down according to a Poisson process with arrival rate δ, independent across labor markets and independent of labor market j's productivity. When this shock hits, all the workers are forced out of the market. A new labor market, also named j, enters with positive initial productivity x_0, keeping the total measure of labor markets constant. There are no skilled workers in a new labor market. We assume a law of large numbers, so the share of labor markets experiencing any particular sequence of shocks is deterministic.

2.2 Final Goods

A competitive sector combines the intermediate goods into the final good using a constant returns to scale technology

$$Y(t) = \left(\int_0^1 y(j,t)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $y(j,t)$ is the input of good j at time t and $\theta > 0$ is the elasticity of substitution across goods. The final goods sector takes the price of the intermediate goods $\{p(j,t)\}$ as given and chooses $y(j,t)$ to maximize profits. It follows that

$$y(j,t) = \frac{Y(t)}{p(j,t)^{\theta}}. \tag{3}$$

We assume $\theta \neq 1$ throughout the paper. If $\theta = 1$, it is straightforward to verify that productivity shocks would not induce the reallocation of workers across sectors. On the other hand, we allow for the possibility that $\theta < 1$, in which case an increase in productivity induces workers to exit a labor market, or $\theta > 1$, where the relationship between productivity and labor flows are reversed. Although our mathematical results do not depend on which case we examine, we find it plausible that $\theta > 1$ and our language sometimes reflects that restriction.

2.3 Households

There is a representative household consisting of a measure 1 of members. The large household structure allows for full risk sharing within each household, a standard device for studying complete markets allocations.

At each moment in time t, each member of the representative household engages in one of the following mutually exclusive activities:

- $H(t)$ household members are skilled and attached to one of the labor markets. Of these, $\tilde{H}(t)$ are employed, earning a wage $\tilde{w}_h$ but getting no leisure. $H(t) - \tilde{H}(t)$ are unemployed. They do not earn a wage but get leisure $b_u > 0$.

- $L(t)$ household members are unskilled and employed, earning a wage $\tilde{w}_l$ but getting no leisure.

- The remaining household members are inactive. They do not earn a wage but get leisure $b_i > b_u$.

Unskilled household members may costlessly switch between inactivity and employment. Since $b_i > b_u$, inactivity dominates unemployment for unskilled workers. An employed unskilled worker becomes skilled in her labor market according to a Poisson process with arrival rate α, while an inactive worker remains unskilled. Skilled household members may costlessly switch between unemployment and employment within a labor market. If they leave their market, either to work in another market or to become inactive, they become unskilled. We discuss the possibility of relaxing this last assumption in the conclusion.

We can represent the household's preferences via the utility function

$$\int_0^\infty \exp(-\rho t)\big(u(C(t)) + b_i\big(1 - L(t) - H(t)\big) + b_u\big(H(t) - \tilde{H}(t)\big)\big)dt, \tag{4}$$

where $\rho > 0$ is the discount rate, u is increasing, differentiable, strictly concave, and satisfies the Inada conditions $u'(0) = \infty$ and $\lim_{C\to\infty} u'(C) = 0$, and $C(t)$ is the household's consumption of the final good. The household finances its consumption using its labor income. If $u(C) \equiv \log(C)$, the economy exhibits balanced growth, where an increase in the productivity of all sectors raises wages and consumption proportionately but does not affect labor supply. We introduce risk-aversion to allow for this possibility, but do not discuss growth any more in our treatment of the model.

Skilled workers exit their labor market three circumstances: first, they may do so endogenously at any time; second, they must do when their market shuts down, which happens at rate δ; and third, they must do so when they are hit by an idiosyncratic shock, according to a Poisson process with arrival rate q, independent across individuals and independent of their labor market's productivity. We introduce the idiosyncratic "quit" shock q to account for separations that are unrelated to the state of the industry. We can also reinterpret this as a "retirement" shock, with each retiree replaced by an unskilled worker. In this case, the discount rate ρ accounts both for impatience and for retirement.

2.4 Equilibrium

We look for a competitive equilibrium of this economy. At each instant, each household chooses how much to consume and how to allocate its members between employment in each labor market, unemployment in each labor market, and inactivity, in order to maximize utility subject to technological constraints on reallocating members across labor markets, taking as given the stochastic process for wages in each labor market; each final goods producer maximizes profits by choosing inputs taking as given the price for all the intermediate goods; and each intermediate goods producer j maximizes profits by choosing how many skilled and unskilled workers to hire taking as given the wage in its labor market and the price of its good.

Moreover, the demand for skilled and unskilled labor from intermediate goods producers is equal to the supply from households in each labor market; the demand for intermediate goods from the final goods producers is equal to the supply from intermediate goods producers; and the demand for final goods from the households is equal to the supply from the final goods producers.

3 Characterization

We look for a steady state equilibrium where the household maintains constant consumption, obtains a constant income stream, and keeps a positive and constant fraction of its members in employment and inactivity. Under some conditions, some workers will also be unemployed.

Because it is costly to switch labor markets, different markets pay different wages at each point in time. Skilled workers cannot move to arbitrage differences in wages, which means that their wage depends on the conditions in their labor market. This induces differences in the value of being a skilled worker, depending on local labor market conditions. Unskilled workers are indifferent between inactivity and employment in any labor market where they are active. Because the value of becoming skilled depends on labor market conditions, the wage of unskilled workers also depends on conditions. The entry of unskilled workers into the most desirable labor markets puts downward pressure on their wages and, though the skill accumulation process, upward pressure on the number of skilled workers.

3.1 Optimization

The household values its members according to the expected present value of marginal utility that they generate either from leisure or from income. First consider an individual who is permanently inactive. It is immediate from equation (4) that he contributes $\bar{v} \equiv b_i/\rho$ to the household. Since the household may freely shift unskilled workers between employment and inactivity, this must be the value of all unskilled workers. The household can also freely shift skilled workers into inactivity, so this is a lower bound on the value of a skilled worker. The actual value depends on her labor market's condition, which we turn to next.

The state of a labor market j is its productivity $x(j,t)$ and the number of skilled workers in the market $h(j,t)$. We look for an equilibrium where a sufficient statistic for the state of the labor market is simple combination of these state variables,

$$\omega(j,t) = \frac{(\theta - 1)\log x(j,t) - \log h(j,t) + \log Y(t)}{\theta} + \log u'(C(t)), \tag{5}$$

where we include aggregate output $Y(t)$ and aggregate consumption $C(t)$ in this expression for analytical convenience. That is, the value of a skilled worker in market j at time t is $v(\omega(j,t))$.

To establish this claim, let $m(\omega(j,t)) \equiv \tilde{h}(j,t)/h(j,t)$ denote the employment rate of skilled workers in labor market j at time t and $\lambda(\omega(j,t)) \equiv l(j,t)/\tilde{h}(j,t)$ denote the ratio of unskilled to skilled employment in the labor market. We verify that both of these are functions only of ω.

To proceed, observe that industry output is $y(j,t) = x(j,t)m(\omega(j,t))h(j,t)f(\lambda(\omega(j,t)))$. Then the price of output is given from equation (3) as $p(j,t) = (Y(t)/y(j,t))^{1/\theta}$. Workers are paid their marginal revenue product, so the wage of an unskilled worker is

$$\tilde{w}_l(j,t) = p(j,t)x(j,t)f'(\lambda(\omega(j,t)))$$

and the wage of a skilled worker is

$$\tilde{w}_h(j,t) = p(j,t)x(j,t)\big(f(\lambda(\omega(j,t))) - \lambda(\omega(j,t))f'(\lambda(\omega(j,t)))\big).$$

Substituting for $p(j,t)$ and simplifying, both wages depend on labor market conditions only via ω. It is convenient to measure the wage in units of marginal utility, $\tilde{w}u'(C)$. For a labor market in state ω, these solve

$$w_l(\omega) = \exp(\omega)\big(m(\omega)f(\lambda(\omega))\big)^{-\frac{1}{\theta}}f'(\lambda(\omega)), \tag{6}$$

$$w_h(\omega) = \exp(\omega)\big(m(\omega)f(\lambda(\omega))\big)^{-\frac{1}{\theta}}\big(f(\lambda(\omega)) - \lambda(\omega)f'(\lambda(\omega))\big). \tag{7}$$

Of course, to characterize these wages, we must find the employment rate of skilled workers m and the share of unskilled workers λ.

We start by determining $m(\omega)$. Skilled workers have a static option to work or be unemployed, with work dominating unemployment if $w_h(\omega) > b_u$. On the other hand, if $w_h(\omega) < b_u$, no skilled workers are employed, $m(\omega) = 0$. But then equation (7) implies $w_h(\omega) = \infty$, a contradiction. This implies that labor markets can be in one of two conditions. First, all skilled workers are employed, $m(\omega) = 1$, while $w_h(\omega) \geq b_u$. Second, some skilled workers are unemployed, $m(\omega) \in (0,1)$, while skilled workers are indifferent about working, $w_h(\omega) = b_u$. This, together with equation (7), pins down

$$m(\omega) = \min\left\{\frac{1}{f(\lambda(\omega))}\left(\frac{\exp(\omega)\big(f(\lambda(\omega)) - \lambda(\omega)f'(\lambda(\omega))\big)}{b_u}\right)^{\theta}, 1\right\}. \tag{8}$$

Substituting this into equations (6) and (7) gives

$$w_l(\omega) = \frac{f'(\lambda(\omega))}{f(\lambda(\omega)) - \lambda(\omega) f'(\lambda(\omega))} w_h(\omega), \tag{9}$$

$$w_h(\omega) = \max\left\{ b_u, \frac{\exp(\omega)\big(f(\lambda(\omega)) - \lambda(\omega) f'(\lambda(\omega))\big)}{f(\lambda(\omega))^{\frac{1}{\theta}}} \right\}. \tag{10}$$

These depend only on the share of unskilled workers λ and the state ω.

Next we turn to the determination of the share of λ and the value function for skilled workers v. Start with the Bellman equation for an unskilled worker in a market ω. If $\lambda(\omega) > 0$, she must generate value $\bar{v} = b_i/\rho$ for the household. Thus

$$b_i = w_l(\omega) + \alpha(v(\omega) - b_i/\rho), \tag{11}$$

where $v(\omega)$ is the value of a skilled worker in a labor market in state ω. Observe that $w_l(\omega) \geq 0$, which places a bound on $v(\omega)$:

$$\frac{b_i}{\rho} \leq v(\omega) \leq b_i \left(\frac{1}{\alpha} + \frac{1}{\rho} \right). \tag{12}$$

The lower bound comes from the fact that skilled workers are always free to become inactive. As $v(\omega)$ approaches the upper bound, $\lambda(\omega) \to \infty$, which pushes $w_l(\omega)$ towards zero.

We also have a Bellman equation for a skilled worker:

$$\rho v(\omega) = w_h(\omega) + (q+\delta)(b_i/\rho - v(\omega)) + \mu(\omega) v'(\omega) + \tfrac{1}{2}\sigma^2 v''(\omega), \tag{13}$$

where

$$\mu(\omega) = \frac{\theta - 1}{\theta}\mu_x - \frac{\alpha \lambda(\omega) m(\omega) - q}{\theta}, \tag{14}$$

with $m(\omega)$ given in equation (8), and

$$\sigma = \frac{|\theta - 1|}{\theta}\sigma_x. \tag{15}$$

A skilled worker discounts future income at rate ρ. She earns a wage $w_h(\omega)$ if she is employed in the labor market or the equivalent if she is unemployed. She is forced to exit the market at rate $q + \delta$, in which event she experiences a capital loss. Finally, the state of the market changes over time. The drift in ω is caused in part by the exogenous drift in productivity. It is also affected by the endogenous growth of the number of skilled workers, at rate $\alpha l(j,t)/h(j,t) = \alpha\lambda(\omega(j,t))m(\omega(j,t))$, and the exogenous exit of skilled workers at rate q.

Finally, the variance in ω reflects the variance in the stochastic process for productivity.

In addition, skilled workers have the option to leave their market. They do so to keep $\omega \geq \underline{\omega}$, an endogenous threshold. Standard arguments imply value-matching and smooth-pasting conditions at the lower threshold:

$$v(\underline{\omega}) = b_i/\rho \text{ and } v'(\underline{\omega}) = 0. \tag{16}$$

Equations (13) and (16) and the bounds in equation (12) yield a unique solution to the Bellman equation given $\lambda(\omega)$.

To summarize, an equilibrium is described by a threshold $\underline{\omega}$ and two functions: the value function v and the share of unskilled workers λ. These must solve two functional equations: the value function for unskilled workers (equation 11) and the value function for skilled workers (equations 12, 13, and 16).

3.2 Social Planner's Problem

To help characterize the equilibrium, it is useful to prove that the equilibrium allocation is equivalent to the solution to a hypothetical social planner's problem. We consider a benevolent planner who runs one of the labor markets j. The planner's objective is to maximize the consumer's surplus from the production of good j net of the cost of labor inputs. Her labor market initially has productivity $x(j,t)$ and she owns $h(j,t)$ skilled workers. Productivity then follows a geometric Brownian motion. At each subsequent time t', the planner chooses the number of unskilled workers to rent, at cost b_i per worker. Unskilled workers become skilled at rate α, at which time the planner must buy them at the present value of their rent, b_i/ρ. She can also sell her skilled workers at the same price, and she has to sell her skilled workers at rate q.

If at time t' she rents $l(j,t')$ unskilled workers and owns $h(j,t')$ skilled workers, she produces $y(j,t') = x(j,t')F(l(j,t'),\tilde{h})$ units of good j, where $\tilde{h} \in [0, h(j,t')]$ is the number of employed skilled workers. Consumer's surplus is the area under the demand curve in equation (3), $\frac{\theta}{\theta-1}Y(t)^{\frac{1}{\theta}}y(j,t')^{\frac{\theta-1}{\theta}}$. She can also rent back the $h(j,t') - \tilde{h}$ remaining skilled workers for b_u per worker. Thus the planner's period payoff, measured in units of marginal utility, is

$$\max_{\tilde{h}\in[0,h(j,t')]} \Big(\tfrac{\theta}{\theta-1} Y(t)^{\frac{1}{\theta}} \big(x(j,t')F(l(j,t'),\tilde{h})\big)^{\frac{\theta-1}{\theta}} u'(C(t)) + b_u\big(h(j,t') - \tilde{h}\big) - b_i l(j,t') \Big).$$

The first term is consumer's surplus, the second term is the revenue from renting back unemployed skilled workers, while the final term is the cost of renting the unskilled workers.

The number of skilled workers the planner owns is constrained by the same law of motion as in the decentralized economy, $dh(j,t') \leq (\alpha l(j,t') - qh(j,t'))dt$.

The social planner chooses how many unskilled workers to rent at each instant to maximize the sum of profits discounted at rate ρ. She also recognizes that her market shuts down at rate δ, in which event she must sell her skilled workers. Let $V(\tilde{x}, h)$ denote the value of the planner as a function of the current state $(\tilde{x}, h)$, where $\tilde{x}$ is log productivity. In the region of inaction, where $dh(j,t') = (\alpha l(j,t') - qh(j,t'))dt$, this solves

$$\rho V(\tilde{x},h) = \max_{\lambda \geq 0, m \in [0,1]} \Big(\tfrac{\theta}{\theta-1} Y^{\frac{1}{\theta}} \big(\exp(\tilde{x}) m h f(\lambda)\big)^{\frac{\theta-1}{\theta}} u'(C) + b_u h(1-m) - b_i h \lambda m \\ + \delta\big(hb_i/\rho - V(\tilde{x},h)\big) + (\alpha\lambda m - q)h\big(V_h(\tilde{x},h) - b_i/\rho\big) + \mu_x V_x(\tilde{x},h) + \tfrac{1}{2}\sigma_x^2 V_{xx}(\tilde{x},h)\Big), \quad (17)$$

where $m \equiv \tilde{h}/h$ is the fraction of skilled workers put into production and $\lambda \equiv l/\tilde{h}$ is the ratio of unskilled to employed skilled workers. Optimality dictates that the planner fires skilled workers if $V_h(\tilde{x},h) \leq b_i/\rho$. The value function is differentiable, so this implies that whenever the planner reduces the number of skilled workers,

$$V_h(\tilde{x},h) = b_i/\rho \text{ and } V_{hh}(\tilde{x},h) = 0. \quad (18)$$

These are standard smooth pasting and supercontact conditions.

We claim that the marginal value of a skilled worker to the social planner equals the private value of a skilled worker, $V_h(\tilde{x},h) = v(\omega)$, where ω satisfies equation (5). It is immediate to verify that this implies that the value matching and smooth pasting conditions in the decentralized equilibrium line up with the smooth pasting and supercontact conditions in the planner's problem, i.e. equations (16) and (18) are the same. Next, consider the first order condition for λ. Assuming $\lambda > 0$, $V_h = v$ implies

$$\exp(\omega)(mf(\lambda))^{-\frac{1}{\theta}} f'(\lambda) = b_i - \alpha(v(\omega) - b_i/\rho), \quad (19)$$

consistent with equations (6) and (11). Third, look at the first order condition for m:

$$\exp(\omega)(mf(\lambda))^{-\frac{1}{\theta}} f(\lambda) = b_u + \lambda\big(b_i - \alpha(v(\omega) - b_i/\rho)\big).$$

Eliminate the terms multiplying λ using equation (19), or drop the terms completely if $\lambda = 0$. Solving for m gives

$$m = \frac{1}{f(\lambda)} \left(\frac{\exp(\omega)\big(f(\lambda) - \lambda f'(\lambda)\big)}{b_u} \right)^{\theta},$$

assuming this defines $m \leq 1$; otherwise $m = 1$. This is consistent with equation (8).

Finally, look at the envelope condition for h. As a preliminary step, note that $V_h(\tilde{x}, h) = v(\omega)$ implies

$$V_{hh}(\tilde{x}, h) = -\frac{v'(\omega)}{\theta h}, \; V_{xh}(\tilde{x}, h) = \frac{\theta - 1}{\theta} v'(\omega), \text{ and } V_{xxh}(\tilde{x}, h) = \left(\frac{\theta - 1}{\theta}\right)^2 v''(\omega),$$

where we use the functional form of ω in equation (5) to obtain each of these expressions. Then using equation (19), the envelope condition reduces to

$$\rho v(\omega) = m \exp(\omega)(mf(\lambda))^{-\frac{1}{\theta}}\big(f(\lambda) - \lambda f'(\lambda)\big) + (1 - m)b_u \\ - (q + \delta)(v(\omega) - b_i/\rho) + \mu v'(\omega) + \tfrac{1}{2}\sigma^2 v''(\omega),$$

where μ and σ are the drift and standard deviation of ω,

$$\mu \equiv \frac{\theta - 1}{\theta}\mu_x - \frac{\alpha\lambda m - q}{\theta} \text{ and } \sigma^2 \equiv \left(\frac{\theta - 1}{\theta}\right)^2 \sigma_x^2,$$

as in equations (14) and (15). Replace m using equation (8) to get the Bellman equation for skilled workers, equation (13). This verifies that the marginal value of a skilled worker to the social planner is equal to her private value. It also confirms that the planner's optimal policy, $\lambda(\omega)$ and $m(\omega)$, coincides with equilibrium outcomes.

This result is important for two reasons. First, it formally extends standard optimality results to our setup with human capital accumulation. We do not view this as particularly surprising because there are no externalities, incomplete markets, or other reasons why the first welfare theorem would fail in this environment. Second, it implies certain properties about the decentralized equilibrium. The social planner's objective function is concave and she faces a convex constraint set. Therefore the planner's problem has a unique solution. Since the necessary and sufficient first order conditions of the planner's problem coincides with the conditions describing a decentralized equilibrium, the equilibrium is unique as well. In addition, the social planner's value function is concave in the number of skilled workers she has, $V_{hh} \leq 0$. Since $V_h h(\tilde{x}, h) = -v'(\omega)/\theta h$, it follows that the equilibrium value function is nondecreasing, $v'(\omega) \geq 0$. This is useful for characterizing an equilibrium. In summary:

Proposition 1. *The decentralized equilibrium solves a social planner's problem and is unique. The equilibrium value function v is nondecreasing.*

3.3 Aggregation

We now return to the decentralized equilibrium and aggregate individual behavior to compute objects like the unemployment rate. Let g_h denote the stationary density of skilled workers across ω. This must solve a Kolmogorov forward (or Fokker-Planck) equation,

$$\big(q + \delta - \alpha\lambda(\omega)m(\omega)\big)g_h(\omega) = -\mu(\omega)g_h'(\omega) - \mu'(\omega)g_h(\omega) + \frac{\sigma^2}{2}g_h''(\omega) \text{ for all } \omega \geq \underline{\omega}. \quad (20)$$

Note that this implies that g_h is twice continuously differentiable. The left hand side is the difference between the inflow and outflow of skilled workers at ω. Skilled workers exit markets either because of quits or shutdowns at rate $q + \delta$, while unskilled workers become skilled at rate $\alpha\lambda(\omega)m(\omega)$. The right hand side captures how the evolution of ω influences the density. If μ is negative and g_h is increasing, the drift in ω pulls in mass from points with a higher density, raising g_h. If μ is decreasing (and g_h is positive), this effect is exacerbated. In addition, if σ is positive and g_h is convex, the variance of shocks raises the density.

To solve for the density g_h, we require two constants of integration. First, since g_h is a density,

$$\int_{\underline{\omega}}^{\infty} g_h(\omega)d\omega = 1. \quad (21)$$

Second, we relate the level and first derivative of g_h at its lower bound:

$$\frac{\sigma^2}{2}g_h'(\underline{\omega}) - \left(\mu(\underline{\omega}) + \frac{\theta\sigma^2}{2}\right)g_h(\underline{\omega}) = 0. \quad (22)$$

The elasticity of substitution θ appears in this equation because it determines how many skilled workers must exit from depressed markets to regulate ω above $\underline{\omega}$. The exogenous separation rate $q + \delta$ and endogenous entry rate $\alpha\lambda(\underline{\omega})m(\underline{\omega})$ do not appear because the ratio of endogenous to exogenous turnover is infinite in a short time interval for a market at the lower bound. Since by definition there are no markets with smaller ω, $g_h(\underline{\omega})$ is not fed from below, which explains the difference between equations (20) and (22). The second order differential equation (20) and the boundary conditions equations (21) and (22) describe the density of skilled workers across ω completely.

Let $g_l(\omega)$ denote an improper density of unskilled workers across ω. This satisfies

$$g_l(\omega) = \lambda(\omega)m(\omega)g_h(\omega). \quad (23)$$

$\lambda(\omega)m(\omega)$ is the ratio of unskilled to skilled workers at ω, which is equal to the ratio of the densities. This is an improper density because it does not integrate to 1, but rather to the ratio of unskilled workers to skilled workers.

The unemployment rate is

$$u = \frac{\int_{\underline{\omega}}^{\infty}(1 - m(\omega))g_h(\omega)d\omega}{\int_{\underline{\omega}}^{\infty}(g_l(\omega) + g_h(\omega))d\omega}. \tag{24}$$

Skilled workers in markets with conditions ω experience a $1 - m(\omega)$ unemployment rate. Integrating across ω and dividing by the total labor force gives the unemployment rate.

Note that when a new market is created, with productivity x_0 and no skilled workers, it has $\omega = \infty$. However, there will typically be many unskilled workers in such a market, ensuring a rapid increase in the number of skilled workers rises and decline in ω.

4 Unskilled Labor is Unproductive

We start by considering a special case where unskilled labor is unproductive, $F(l, h) = h$ and so $f(\lambda) = 1$ for all λ. It is straightforward to prove that this model is mathematically isomorphic to the "directed search" model in Alvarez and Shimer (2009). That model had no notion of unskilled workers, but instead assumed that it took time to find a job. A worker who searched for a job could not work, but a found a job at rate α and then moved to the labor market of her choice. The isomorphism follows because the unskilled wage is always equal to zero, $w_l(\omega) = 0$, under this production function. In this case, equation (11) pins down the value of a skilled job in any labor market attracting unskilled workers:

$$v(\omega) = \bar{v} \equiv b_i \left(\frac{1}{\rho} + \frac{1}{\alpha}\right).$$

This determines an upper threshold for ω, so ω is a regulated Brownian motion on $[\underline{\omega}, \bar{\omega}]$, with $v(\bar{\omega}) = \bar{v}$ and $v'(\bar{\omega}) = 0$. If ever $\omega > \bar{\omega}$, the market would attract a positive measure of unskilled workers, instantaneously pushing ω back down to $\bar{\omega}$. On the other hand, when $\omega < \bar{\omega}$, $v(\omega) < \bar{v}$ and so unskilled workers are not attracted to the labor market. The dynamics of ω are determined simply by the exit of skilled workers and the stochastic process for productivity.

We find this interesting because Alvarez and Shimer (2009) required an implausibly high search cost in order to replicate the observed behavior of industry wages. Intuitively, observed wages are persistent and dispersed. If the only cost of moving from a low-wage industry to a high-wage industry is the time spent searching, then we need high search costs to keep workers in low-wage industries. The problem is that empirically, searching for a new job only takes a few months. The model in this paper resolves this issue by suggesting that the cost of moving from a low-wage to a high-wage industry is not search, but rather the process of accumulating human capital. This likely takes years in many industries. This much higher cost is consistent with much more disperse and persistent inter-industry wage differentials.

Our reinterpretation here also addresses another issue in Alvarez and Shimer (2009). Empirically, displaced workers with long job tenure suffer significant and long-lasting wage losses (Jacobson, LaLonde, and Sullivan, 1993). In our earlier paper, wages were an increasing function of labor market conditions ω and workers at the point of displacement were paid the lowest wage. Here wages depend both on ω and skill and displaced workers take wage cuts in anticipation of future wage increases. In the extreme version of the model where unskilled labor is unproductive, wages fall to zero for, on average, $1/\alpha$ periods following displacement, potentially addressing the evidence in (Jacobson, LaLonde, and Sullivan, 1993). We will see that this qualitative result carries over to a more general framework where unskilled labor is productive, though less so than skilled labor.

5 Perfect Substitutes

We next turn to the case where skilled and unskilled workers are perfect substitutes, so $F(l, h) \equiv sh + l$ for some $s > 1$. This implies $f(\lambda) = s + \lambda$.

5.1 Analytical Results

The state of a labor market ω determines whether there are unskilled workers, $\lambda(\omega) > 0$, and whether there is unemployment, $m(\omega) < 1$. We start by proving that whenever there are unskilled workers in a labor market, there is no unemployment.

Lemma 1. *Assume skilled and unskilled workers are perfect substitutes and there is a skill premium, $s > 1$. There is no unemployment in markets with unskilled workers. That is, if $\lambda(\omega) > 0$, $m(\omega) = 1$.*

Proof. Suppose for some ω, $m(\omega) < 1$. Then equations (8)–(10) imply $w_l(\omega) = b_u/s$ and $w_h(\omega) = b_u$. If also $\lambda(\omega) > 0$, equation (11) implies

$$b_i = \frac{b_u}{s} + \alpha(v(\omega) - b_i/\rho),$$

which pins down a constant $v(\omega)$, i.e. $v'(\omega) = v''(\omega) = 0$. Equation (13) then implies

$$\rho v(\omega) = b_u + (q + \delta)(b_i/\rho - v(\omega)).$$

Eliminate $v(\omega)$ between these equations to get

$$\frac{b_u}{b_i} = \frac{s(\alpha + q + \delta + \rho)}{\alpha s + q + \delta + \rho} > 1,$$

where the inequality holds because $s > 1$, $\rho > 0$, and the remaining parameters on the right hand side are nonnegative. But this contradicts $b_u \leq b_i$ and completes the proof. □

This implies that labor markets can be in one of three states: they have skilled and unskilled workers and full employment, they have only skilled workers and full employment, or they have only skilled workers and some unemployment. We now show that these states correspond partition the set of ω, with unskilled workers only at the highest values of ω and unemployment only at the lowest values.

Proposition 2. Assume skilled and unskilled workers are perfect substitutes and there is a skill premium, $s > 1$. For all $\omega \geq \underline{\omega}$, the employment rate $m(\omega)$ and the ratio of unskilled-to-skilled workers $\lambda(\omega)$ are nondecreasing. More precisely, there exists a $\hat{\omega} \geq \underline{\omega}$ such that for all $\omega \in [\underline{\omega}, \hat{\omega})$, $m(\omega)$ is strictly increasing, with $m(\omega) = 1$ for all $\omega \geq \hat{\omega}$; and there exists a $\bar{\omega} > \hat{\omega}$ such that for all $\omega > \underline{\omega}$, $\lambda(\omega)$ is strictly increasing, with $\lambda(\omega) = 0$ for all $\omega < \bar{\omega}$.

Proof. Recall from Proposition 1 that the value function is nondecreasing. If $\lambda(\omega) > 0$, Lemma 1 implies $m(\omega) = 1$ and so equations (9)–(11) imply

$$b_i = \frac{\exp(\omega)}{(s + \lambda(\omega))^{\frac{1}{\theta}}} + \alpha(v(\omega) - b_i/\rho). \tag{25}$$

This implicitly defines $\lambda(\omega)$ as an increasing function of ω. This immediately establishes the threshold property. Moreover, the threshold $\bar{\omega}$ satisfies

$$b_i = \frac{\exp(\bar{\omega})}{s^{\frac{1}{\theta}}} + \alpha(v(\bar{\omega}) - b_i/\rho), \tag{26}$$

while for $\omega < \bar{\omega}$, unskilled workers are unwilling to participate in the labor market for any positive value of λ, and so $\lambda(\omega) = 0$.

If $m(\omega) < 1$, Lemma 1 implies $\lambda(\omega) = 0$. Then equation (8) gives $m(\omega) = \exp(\theta\omega)s^{\theta-1}b_u^{-\theta}$, increasing in ω. The threshold $\hat{\omega}$ satisfies $\hat{\omega} = \log b_u - \frac{\theta-1}{\theta}\log s$, assuming this exceeds $\underline{\omega}$. Otherwise, let $\hat{\omega} = \underline{\omega}$ so $m(\omega) = 1$ for all ω, and the result holds trivially. □

5.2 Numerical Example

We use a shooting algorithm to compute the value function $v(\omega)$ and skill ratio $\lambda(\omega)$. Guess a value for $\underline{\omega}$. As long as $\lambda(\omega) = 0$, equation (13) implies

$$\rho v(\omega) = \max\left\{b_u, \exp(\omega) s^{\frac{\theta-1}{\theta}}\right\} + (q+\delta)(b_i/\rho - v(\omega)) + \left(\frac{\theta-1}{\theta}\mu_x + \frac{q}{\theta}\right) v'(\omega) + \tfrac{1}{2}\sigma^2 v''(\omega).$$

We solve this differential equation using the terminal conditions $v(\underline{\omega}) = b_i/\rho$ and $v'(\underline{\omega}) = 0$, stopping at the first value of $\bar{\omega}$ that satisfies equation (26). For $\omega > \bar{\omega}$, solve equation (25) for $\lambda(\omega)$ and substitute this into equation (13) to again obtain an ordinary second order differential equation for v. Solve this using as boundary conditions the requirement that v is continuously differentiable, including at $\bar{\omega}$. Finally, vary $\underline{\omega}$ until we obtain a value function defined for all $\omega \geq \underline{\omega}$ and satisfying equation (12).

To implement this, we develop a numerical example. We think of a time period as a year and set the discount rate to $\rho = 0.05$. The skill premium is $s = 2$, so skilled workers earn twice the wage of unskilled workers in the same labor market. Unskilled workers take ten years on average to become skilled, $\alpha = 0.1$, while skilled workers are forced to leave their market after twenty years on average, $q = 0.05$, possibly representing retirement. We normalize the value of leisure from inactivity to $b_i = 1$ and set $b_u = 0.95$, so unemployment gives almost as much leisure as inactivity. We comment on this choice later. We set the elasticity of substitution between goods to $\theta = 3$, in line with numbers reported in Broda and Weinstein (2006). We set the standard deviation of productivity to $\sigma_x = 0.2$ and the drift in productivity to $\mu_x = -\frac{1}{2}(\theta - 1)\sigma_x^2 = -0.04$. This last assumption ensures that the level of employment in a labor market has no drift in a frictionless version of our model and allows us to focus on the limit as the exogenous breakdown rate of markets converges to zero, $\delta \to 0$ (Alvarez and Shimer, 2009).

We find that skilled workers exit labor markets to keep $\omega \geq \underline{\omega} = -0.882$, there is full employment if $\omega \geq \hat{\omega} = -0.513$, and there are unskilled workers if $\omega > \bar{\omega} = -0.064$. Figure 1 shows the remaining results. The top left panel gives the value function of skilled workers, initially convex and then concave for high values of ω. We find that $\lim_{\omega\to\infty} v(\omega) = 23.13$, far below the bound in equation (12). This is because markets with the highest value of ω attract many unskilled workers, i.e. $\lambda(\omega)$ is high, putting strong downward pressure on ω. We see that in the top right panel, which shows that $\lambda(\omega)$ increases exponentially; indeed, one can prove that it grows asymptotically at rate ω.

The left panel in the middle row shows the cross-sectional distribution of ω both for unskilled and skilled workers. Unskilled workers are located in more productive markets, reflecting the fact that ω increases so sharply, while both densities rapidly fall to zero since

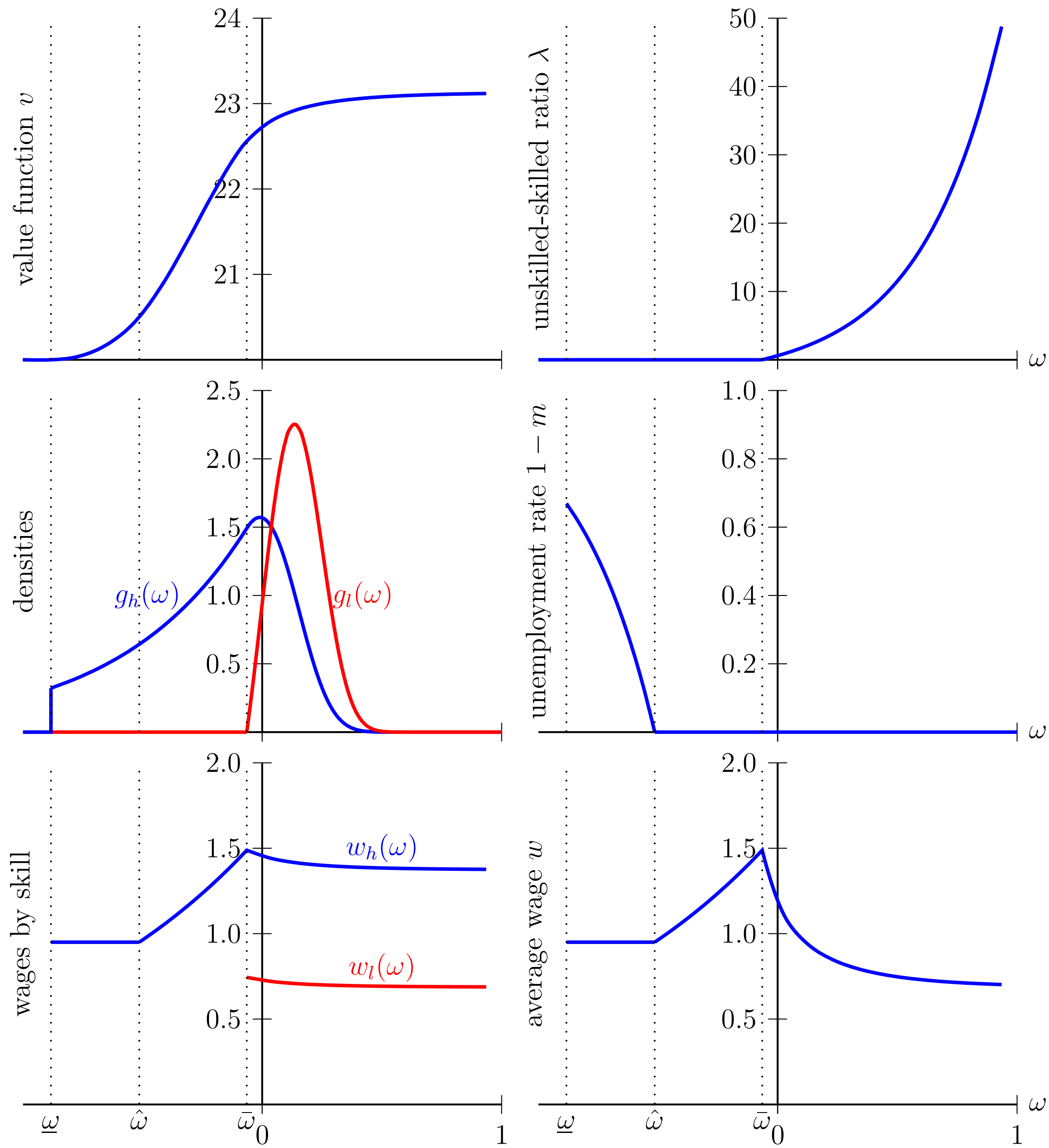

Figure 1: Model simulations with perfect substitutes. The top left shows the value function of skilled workers. The top right shows the ratio of unskilled to skilled workers. The middle left shows the density of skilled workers and the improper density of unskilled workers. The middle right shows the unemployment rate of skilled workers. The bottom left panel shows the wage of unskilled and skilled workers. The bottom right panel shows the industry average wage $w(\omega) \equiv (g_h(\omega)m(\omega)w_h(\omega) + g_l(\omega)w_l(\omega))/(g_h(\omega)m(\omega) + g_l(\omega))$.

markets are pushed away from high values of ω. The right panel in the middle row shows the unemployment rate of skilled workers, which rises above two-thirds at the lower bound $\underline{\omega}$.

Finally, the bottom row shows the pattern of wages. The skilled wage is flat at $b_u = 0.95$ below $\hat{\omega}$ and then increases until $\bar{\omega}$, when unskilled workers start entering the market. Thereafter, their wage starts falling. This reflects arbitrage by unskilled workers. Since the value function $v(\omega)$ is increasing, equation (11) implies that the wage of unskilled workers must be decreasing in markets where they are active. But the skilled wage is just s times the unskilled wage, so the skilled wage is decreasing as well. When we look at the average wage in the industry, this decreases even more rapidly in ω since more productive industries have comparatively more unskilled workers. This figure cautions against using a measure as simple as the average wage, or even skill-specific wages, to gauge the state of the industry.

We can also compute the fraction of workers who are skilled. Unskilled workers become skilled at rate α, while skilled workers exit exogenously at rate q. If there were no endogenous flows out of the skilled labor pool, a fraction $q/(\alpha+q) = 1/3$ of the workers would be unskilled. Endogenous quits boost the unskilled share slightly, to 0.37.

Finally, we compute the unemployment rate. It is 3.9 percent of the total population. We would not expect this model, which abstracts completely from search frictions, to pick up all of the observed unemployment. The important point is that most skilled workers are in labor markets without any unemployment. Once a worker experiences unemployment, her market is likely to have unemployment in the future. If she stays in the market, she earns a low wage. If she eventually decides to leave the market, she loses her skills and suffers a further wage cut, but holds onto the hope that she may again accumulate skills and earn a much higher wage.

6 Imperfect Substitutes

We now turn to the case where skilled and unskilled workers are imperfect substitutes,

$$F(l,h) = \left((sh)^{\frac{\eta-1}{\eta}} + l^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \Rightarrow f(\lambda) = \left(s^{\frac{\eta-1}{\eta}} + \lambda^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}},$$

where $\eta \in (0,\infty)$ measures the elasticity of substitution between the two types of labor. We assume $(s-1)(\eta-1) \geq 0$, i.e. $s \gtreqless 1 \Leftrightarrow \eta \gtreqless 1$, which ensures that if there are equal numbers of unskilled and skilled workers, the skilled wage exceeds the unskilled wage.

6.1 Analytical Results

Imperfect substitutes qualitatively changes the behavior of the model.

Lemma 2. For any finite and constant elasticity of substitution η between unskilled and skilled workers, $\lambda(\omega) > 0$ for all $\omega \geq \underline{\omega}$.

Proof. The skilled wage satisfies $w_h(\omega) \geq b_u$ since otherwise unemployment would dominate work. Then equation (9) implies

$$w_l(\omega) \geq \frac{f'(\lambda(\omega))}{f(\lambda(\omega)) - \lambda(\omega) f'(\lambda(\omega))} b_u.$$

Moreover, $b_i \geq w_l(\omega)$, since an unskilled worker would always be willing to work at a wage above b_i, even if becoming skilled did not lead to a capital gain. With a constant elasticity of substitution between the two types of labor, this implies

$$\frac{f(\lambda(\omega)) - \lambda(\omega) f'(\lambda(\omega))}{f'(\lambda(\omega))} = s^{\frac{\eta-1}{\eta}} \lambda(\omega)^{\frac{1}{\eta}} \geq \frac{b_u}{b_i}.$$

This places a lower bound on $\lambda(\omega)$. Note that since $b_u < b_i$ and $(s-1)(\eta-1) \geq 0$, the lower bound lies between 0 and 1. □

This simplifies our analysis by reducing the number of cases we need to analyze. In particular, equation (11) holds for all ω. We can also prove that λ is nondecreasing:

Proposition 3. For any finite and constant elasticity of substitution η between unskilled and skilled workers, the ratio of unskilled-to-skilled workers $\lambda(\omega)$ is nondecreasing for all $\omega \geq \underline{\omega}$.

Proof. First suppose $m(\omega) = 1$. Then equations (6) and (11) imply

$$b_i = \frac{\exp(\omega) f'(\lambda(\omega))}{f(\lambda(\omega))^{\frac{1}{\theta}}} + \alpha(v(\omega) - b_i/\rho).$$

Since f is increasing and concave, $f'(\lambda)/f(\lambda)^{\frac{1}{\theta}}$ is decreasing in λ. Then since v is nondecreasing, this defines $\lambda(\omega)$ as an increasing function of ω.

Alternatively, if $m(\omega) < 1$, equations (9), (10), and (11) imply

$$b_i = \frac{b_u f'(\lambda(\omega))}{f(\lambda(\omega)) - \lambda(\omega) f'(\lambda(\omega))} + \alpha(v(\omega) - b_i/\rho).$$

Again, $f'(\lambda)/(f(\lambda) - \lambda f'(\lambda))$ is decreasing, so this defines $\lambda(\omega)$ as a nondecreasing function of ω. □

We do not have a proof that $m(\omega)$ is nondecreasing, nor do we have a counterexample.

6.2 Numerical Example

We again solve our model using a shooting algorithm. We find it simpler to solve equation (11) for $\lambda(\omega)$ and use that to rewrite equation (13) as a second order differential equation in λ. Similarly, the smooth-pasting and value matching conditions (16) impose restrictions on $\lambda(\underline{\omega})$ and $\lambda'(\underline{\omega})$. Given an initial condition $\underline{\omega}$, we solve for $\lambda(\omega)$ and back out $v(\omega)$. We again vary $\underline{\omega}$ until we obtain a value function defined for all $\omega \geq \underline{\omega}$ and satisfying condition (12).

We leave all the parameters unchanged from the model with perfect substitutes, but set the elasticity of substitution to $\eta = 5$. We find that the lower bound on ω is $\underline{\omega} = -0.808$ and that unemployment exists in markets with $\omega \leq \hat{\omega} = -0.504$, both slightly higher than with perfect substitutes. This reflects the presence of unskilled workers, pulling down the skilled wage. The unemployment rate is 4.3 percent of the total population, slightly higher than in the case with perfect substitutes. The share of unskilled workers rises a bit to 39 percent of the labor force, reflecting greater endogenous outflows from the high-skill state. To understand the reason for these increases, we look more closely at equilibrium outcomes.

Figure 2 shows that the model's behavior is, for the most part, similar to the case with perfect substitutes (Figure 1). Perhaps the most notable difference is the skilled wage $w_h(\omega)$. In the model with perfect substitutes, this is decreasing in ω when there are unskilled workers in the market, while here it is increasing in ω. An increase in ω, possibly due to an increase in productivity, draws unskilled workers into the labor market, pulling down the unskilled wage in both models. When unskilled and skilled workers are perfect substitutes, this necessarily pulls down the skilled wage. But if they are imperfect substitutes, the increase in the ratio of unskilled to skilled workers raises the wage premium for skilled workers, $w_h(\omega)/w_l(\omega) = s^{\frac{\eta-1}{\eta}} \lambda^{\frac{1}{\eta}}$, actually raising their wage. Indeed, one can prove that for large ω, $w_l(\omega)$ converges to a positive constant, as does $w_h(\omega) \exp(-\theta\omega/\eta)$. That is, the skilled wage grows exponentially with productivity as long as there is any degree of imperfect substitutability.

As in the model with perfect substitutes, the value function asymptotes to a finite bound, $\lim_{\omega \to \infty} v(\omega) = 23.94$. This is higher than in the case of perfect substitutes and convergence to the bound is visibly slower. Both facts are related to the behavior of the skilled wage. Skilled workers in markets with very high ω benefit from a temporary increase in their wage with imperfect substitutes. This is also a cause of the higher unemployment rate in this version of the model. When workers are imperfect substitutes, the option of remaining unemployed and waiting for market conditions to improve is more valuable because the slope of the value function is greater.

Other changes are more modest. The distribution of unskilled workers extends down to $\underline{\omega}$, which pulls the distribution of skilled workers to the left as well. The presence of unskilled workers pulls down the peak in the average wage $w(\omega)$ and it is a smoother function when

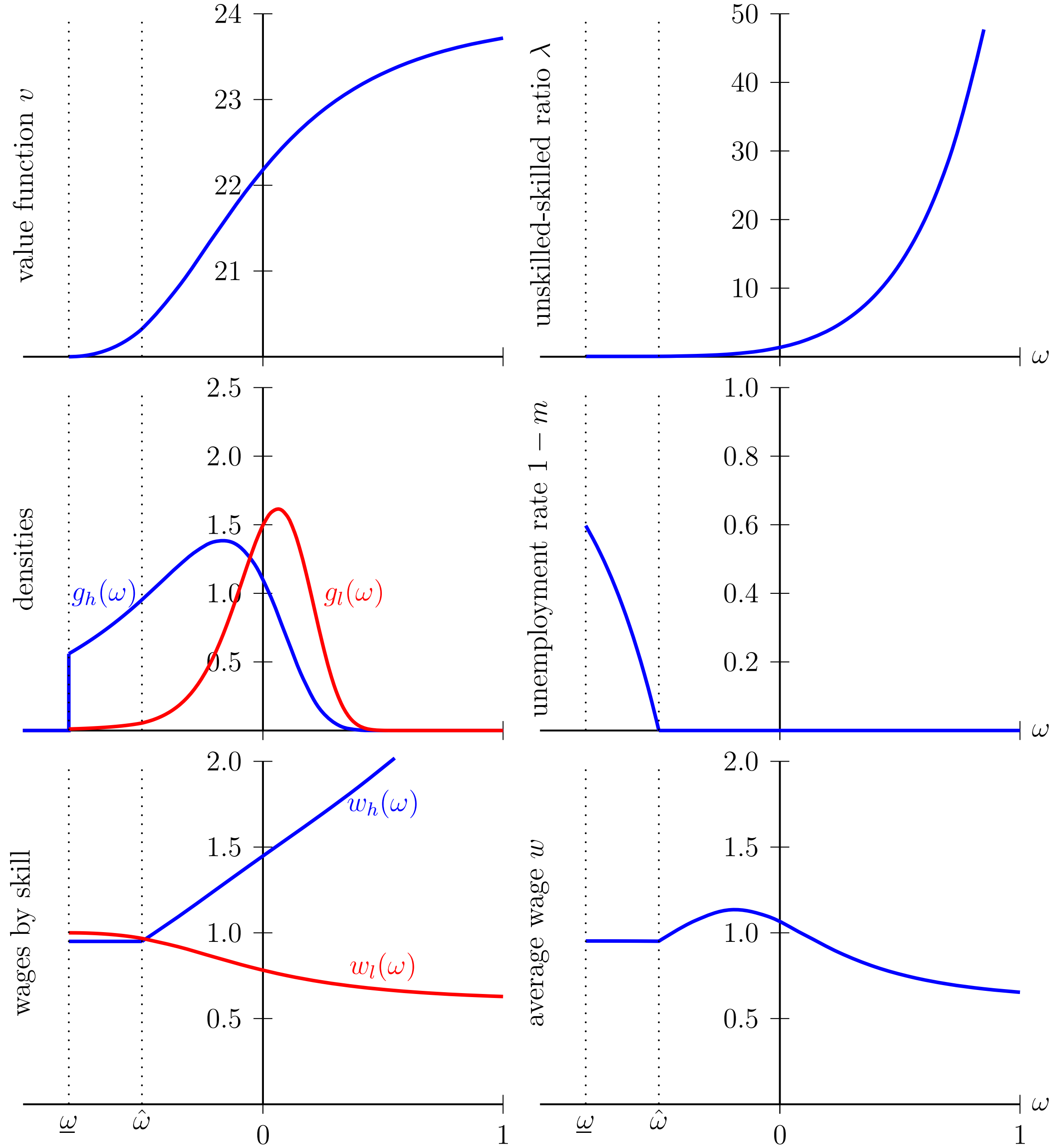

Figure 2: Model simulations with imperfect substitutes. The top left shows the value function of skilled workers. The top right shows the ratio of unskilled to skilled workers. The middle left shows the density of skilled workers and the improper density of unskilled workers. The middle right shows the unemployment rate of skilled workers. The bottom left panel shows the wage of unskilled and skilled workers. The bottom right panel shows the industry average wage $w(\omega) \equiv (g_h(\omega)m(\omega)w_h(\omega) + g_l(\omega)w_l(\omega))/(g_h(\omega)m(\omega) + g_l(\omega))$.

workers are imperfect substitutes. Finally, note that the unskilled wage exceeds the skilled wage for $\omega < -0.489$. Although superficially undesirable, we do not view this as a significant problem with the model. First, only about 1 percent of the unskilled workers are located in labor markets with a negative skill premium. Second, the issue is easily fixed by allowing skilled workers to perform unskilled jobs. Change the production function to

$$F(l,h) = \max_{h_s \in [0,h]} \left((sh_s)^{\frac{\eta-1}{\eta}} + (l + h - h_s)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}},$$

so h_s denotes the number of skilled workers allocated to skilled tasks. If $h > ls^{\eta-1}$, it is optimal to so. This ensures that the skilled wage always weakly exceeds the unskilled wage. It is straightforward to solve the model in this case and verify that it has little effect on our results.

Further reductions in the elasticity of substitution exacerbate our main findings. With $\eta = 2$, the unemployment rate rises to 6.1 percent and the share of skilled falls to 53.9 percent. This reflects an increase in endogenous quits as skilled workers exit labor markets where they have little chance of producing. Of course, the model has other means of increasing the unemployment rate, e.g. by raising the value of unemployment b_u. Again set $\eta = 5$ but increase b_u from 0.95 to 0.99, almost indistinguishable from the value of inactivity. The unemployment rate increases to 11.3 percent, with fewer workers ever exiting unemployment.

7 Conclusion

Although our exploration of this model is preliminary, we feel that we have already learned a few lessons from it. First, the average wage is a poor indicator of the quality of an industry. Indeed, positive productivity shocks may lower the average wage by inducing an inflow of unskilled workers. Instead, the ratio of unskilled-to-skilled workers λ or the skill premium w_h/w_l may be a better indicator, since both are generally increasing functions of ω. Likewise, an industry in a good state ω may tend to shrink over time. As unskilled workers accumulate skills, they reduce ω, which lowers the unskilled-to-skilled ratio, possibly inducing some of the remaining unskilled workers to exit the industry. On the other hand, high ω industries are most likely to grow rapidly as positive productivity shocks induce a large inflow of unskilled workers.

Second, the model suggests that skill accumulation can be a powerful mechanism for keeping workers in industries that have been hit by adverse shocks. Indeed, our model generates a form of unemployment for skilled workers who prefer to retain their human capital, rather than quit for another labor market. We believe that this may be important for understanding the long-term consequences of displacement.

We have assumed throughout our analysis that workers lose their skill when they switch labor markets. It seems straightforward to relax this assumption. First, if workers can become inactive without losing their skills, noone would be unemployed. Thus it is most natural to focus on the case where $b_u = b_i$, so inactivity and unemployment are indistinguishable.[1] Next, since skilled workers always have the option to be unemployed, $w_h(\omega) \geq b_u$. On the other hand, unskilled workers cannot earn more than b_u in any labor market, or all of them would show up. Thus $w_l(\omega) \leq b_u$. But this implies that skilled work in any labor market always weakly dominates unemployment, which always weakly dominates unskilled work in any other labor market. It follows that a skilled worker would never choose to work at a low wage in another labor market if she could not take advantage of the possibility of accumulating skills. Our model therefore offers some explanation for why unemployed workers are unwilling to temporarily take a job that unskilled workers are willing to take.

References

Alvarez, Fernando and Robert Shimer. 2009. "Search and Rest Unemployment." University of Chicago mimeo.

Broda, Christian and David E. Weinstein. 2006. "Globalization and the Gains from Variety." *Quarterly Journal of Economics* 121 (2):541–585.

Jacobson, Louis S., Robert J. LaLonde, and Daniel G. Sullivan. 1993. "Earnings Losses of Displaced Workers." *American Economic Review* 83 (4):685–709.

Kambourov, Gueorgui and Iourii Manovskii. 2009. "Occupational Specificity of Human Capital." *Review of Economic Studies* 76 (2):731–759.

Lucas, Robert E. Jr. and Edward C. Prescott. 1974. "Equilibrium Search and Unemployment." *Journal of Economic Theory* 7 (2):188–209.

Rogerson, Richard. 2005. "Sectoral Shocks, Human Capital, and Displaced Workers." *Review of Economic Dynamics* 8 (1):89–105.

[1] One can prove that in this case, $\underline{\omega} = -\infty$, so skilled workers never voluntarily leave their labor market.