The Value of Information in a Principal-Agent Model with Moral Hazard and Ex Ante Contracting*

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Abstract

We examine a principal-agent model with moral hazard in which, after contracting, the principal receives a signal correlated with the technology. We call this ex ante contracting and examine the value of information both when the principal has private information and when information is public. We show that: (i) the principal prefers private information to no information; (ii) the principal prefers public to private information if and only if the principal finds it optimal to implement different actions; (iii) the value of information is non-monotonic with both private and public information; and (iv) the value of information may be larger or smaller with public than with private information. An important consequence of the second result is that the value of public information may be negative.

KEYWORDS: *Moral Hazard, Ex Ante Contracting, Informed Principal, Technology, Value of Information.* JEL Classification: D82, D86.

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1 Introduction

A standard paradigm in contract theory is the principal-agent model with moral hazard. A risk neutral principal hires a risk averse agent to undertake an action that determines the outcome through a stochastic process. Because the action is unobservable to the principal, the principal pays the agent a wage conditional upon the realized outcome; the incentive scheme must both provide the agent at least his reservation utility and impose risk on the agent in order to induce him to take the appropriate action.

Several studies have extended this basic model to allow the agent to have private information (Holmström, 1979; Myerson, 1982; Sobel, 1993), and others have extended it to allow the principal to have private information (Myerson, 1983; Maskin and Tirole, 1992; Inderst, 2001; Chade and Silvers, 2002) at the contracting date. Still others have compared situations where one or both players receive information after the contract has been offered (Demski and Sappington, 1986; Lewis and Sappington, 1997; Lizzeri et al, 2002; Nafziger, 2008; Ederer, 2008). However, this assumption may be too restrictive. For instance, when a firm hires a manager to head operations in a new market, the firm may not have information about local demand and the importance of managerial effort. Also in the ROTC, cadets enlist and are committed to the corps. Their assignment upon graduation depends upon the needs of the military four years after enlistment, needs that were uncertain when the cadet enlisted.[1] Finally, government procurement contracts often involve the winning firm having to modify the project as new information is received.

We examine a principal-agent model with moral hazard in which the principal receives a signal that can be imperfectly correlated with one of two technologies that the principal has. The technology is the set of conditional probability densities over outcomes for each action. It affects the costs to implement an action and the resulting revenues. We begin by examining the situation in which the principal has private information about the technology. With this setup, we determine the consequences of better information for the actions implemented and equilibrium payoffs.

Importantly, if the principal has private information, then the contracting game becomes a signaling game, where the contract selection now has the additional purpose of signaling the principal's private information to the agent. With ex ante contracting, the signaling has effects at two stages, in the contracting stage when the principal essentially selects the space of feasible contracts (i.e., the signal space), and then the announcement stage when the principal selects a contract from that space (i.e., the signal).

We show that: (i) if the principal implements a different action for each signal correlated with the technology, then the principal prefers to have imperfect information compared to null about the technology, and the principal prefers public to private information; (ii) when the principal implements the same action for each signal, the value of information is non-negative if the principal

[1] During their training periods, they select preferences of job assignment (e.g., aviation and military intelligence) and undergo interim evaluations. Moreover, pay depends upon job and location; promotion and career prospects depend upon these and upon how hard the cadet works.

has private information but may be negative if information is public; and (iii) when the principal implements a different action for each signal, the value of information may be greater with private or with public information. The second result implies that public information can have negative value when both the principal and agent have no information, even if it induces the principal to implement different actions for either signal.

Our work adds to the growing literatures on the timing of information and the value of information. The timing and symmetry of information together determine whether the principal can insure against the possibility of being a low type and whether or not the agent can hold pessimistic beliefs about the principal's type. Consequently, they affect the tradeoff between the action that the principal implements and the cost of providing both utility and incentives to the agent. Regarding the timing of information, by receiving information sooner, a principal can tailor the action that the principal induces the agent to take, to the state of the world. However, this often makes it more costly to induce the agent to undertake any action, particularly if the information is public (by which we mean both the principal and the agent receive the signal).

Regarding the symmetry of information, in contrast to the model we examine, suppose that the principal has private information at the contracting date. Myerson (1983) showed that the principal does best by withholding the private information until after the agent either accepts or rejects the contract; nevertheless, as Maskin and Tirole (1992) showed, a privately informed principal may not attain her complete information payoff. In contrast with ex post contracting, the principal can do better than the complete information payoff if the principal has private information.

Gjesdal (1982, Proposition 3) showed that the value of information for the principal can be negative. This arises when the agent's utility is convex in the action so that imposing risk on the agent (through an incentive scheme that pays a random wage given the outcome) can implement a higher action at the same cost. Gjesdal considered information that is related to the action the agent took; i.e., better information yields greater control over the agent's action. In contrast, when the information is related to the principal's technology – i.e., better information yields greater certainty about the state of the world – implementing the same action may become more or less expensive with better information. The convexity is in the cost to implement an action.

As to the value of public information and moral hazard, when the principal has private information, the agent may be able to hold pessimistic beliefs that preclude certain contracts from being offered. As such, the principal generally prefers public information. For example, Mezzetti and Tsoulouhas (2000) examined a situation in which the principal's private information is the disutility of effort, and showed that a high type of informed principal would benefit if the agent gathered information. In contrast, our results indicate that the principal may be worse off if the principal's private information were to become common knowledge.

Intuitively, better information reduces risk-sharing possibilities and, when information is private, increases both the expected cost to implement an action and the cost to separate, but allows the principal to implement actions that are closer to first-best. Additionally, better information may lower the principal's profit by preventing her from implementing a higher action when she receives a signal correlated with the low type. With ex ante contracting, the principal is able to trade off providing utility in the different signals of the technology, but if information is public, faces a riskier gamble of the cost of providing incentives. Thus, the value of information may be positive or negative, and may be larger or smaller with public than with private information.

The paper is structured as follows: In Section 2, the model is laid out. The results for the situation when the principal has private information are presented in Section 3. In Section 4, we examine the results when information is public. Before concluding, in Section 5 we discuss the results and the implications for the value of information. We relegate to the appendix a characterization of the equilibria and the equilibrium contracts, and all proofs.

2 Model

We consider a principal-agent model with moral hazard in which the principal receives information after offering the agent a contract. The first three subsections provide the players' preferences, describe the technologies, and characterize the principal's information structure. After describing the timing of the game, the next two subsections describe the contracts and constraints that must be satisfied. The last subsection provides the program and optimal contract.

2.1 The Players

The agent is a risk-averse expected-utility maximizer with additively separable von Neumann-Morgenstern utility function over income and effort, given by $U(I,a) = V(I) - a$, with $V'(I) > 0, V''(I) < 0$; $\exists \underline{I}$ such that $lim_{I \downarrow \underline{I}} V'(I) = \infty$. Define $h \equiv V^{-1}(\cdot)$. The agent chooses an action a_m from a set $A = \{a_1, \ldots, a_M\}$ where $0 < a_1 < a_2 < \ldots < a_M < \infty$ and $M \geq 2$. Through a stochastic process, the action chosen determines an outcome q_n from a set $\{q_1, \ldots, q_N\}$ where $0 < q_1 < q_2 < \ldots < q_N < \infty$ and $N \geq 2$.

The principal is risk-neutral and has one of two technologies. The principal offers the agent a contract and, if he accepts it, then the principal receives a signal that is correlated with her technology.[2]

2.2 The Technologies

Let $\pi_n(a_m)$ denote the conditional probability that the outcome is q_n given that the agent chose a_m. A *technology* is a matrix whose elements are $\pi_n(a_m)$.

The principal is endowed with one of two technologies, Π_1 or Π_2. Let $\lambda_0 \in (0,1)$ be the prior probability that the principal has Π_1. Without loss of generality, let Π_1 be the preferred technology if information is public and perfect. For example, Π_1 could first-order stochastic dominate

[2] Throughout, we use male pronouns to refer to the agent and female pronouns to refer to the principal.

(FOSD) Π_2, or Π_1 could be generated from Π_2 by a mean-preserving spread in the likelihood ratio distribution function (see Kim, 1995).[3]

Define $\Pi_\lambda = \lambda\Pi_1 + (1-\lambda)\Pi_2$; each of its elements, $\pi_{\lambda n}(a_m)$, is the conditional probability that outcome q_n is realized given that the agent chose a_m and the beliefs about the principal's technology are λ. Π_λ denotes the principal who is believed to have Π_1 with probability λ.

2.3 Information

The principal observes a signal, z_k, of her technology. Let $Z = \{z_1, z_2\}$ be the signal space. She receives z_k with probability $\zeta \geq \frac{1}{2}$ if she has Π_k. The probability of receiving z_1 is $prob(z_1) = \lambda_0\zeta + (1-\lambda_0)(1-\zeta)$ and similarly, $prob(z_2) = \lambda_0(1-\zeta) + (1-\lambda_0)\zeta$. By Bayes' rule, $\lambda(z_1) = \frac{\lambda_0\zeta}{prob(z_1)}$ is the probability that the principal has Π_1 conditional upon observing z_1, and $\lambda(z_2) = \frac{\lambda_0(1-\zeta)}{prob(z_2)}$ is the probability that the principal has Π_1 conditional upon observing z_2.

An *information structure* is $\begin{bmatrix} \zeta & 1-\zeta \\ 1-\zeta & \zeta \end{bmatrix}$ where the rows correspond to Π_1 and Π_2 and the columns correspond to z_1 and z_2. When ζ increases, we will say that the information structure is *better*.[4] When $\zeta = \frac{1}{2}$, information is *null*, when $\zeta = 1$, information is *perfect*, and when $\frac{1}{2} < \zeta < 1$, information is *imperfect*. We consider two environments that differ in the symmetry of information. *Public information* means that both the principal and the agent observe z_k. *Private information* is where only the principal observes z_k. Together then, we examine $\zeta \in (\frac{1}{2}, 1]$ which we call *Private Information* or *Public Information*. At $\zeta = \frac{1}{2}$, both situations become *Null Information*.[5]

2.4 The Timing of the Game

For a given environment, the timing of the ex ante contracting game is as follows. Nature chooses a technology. Next, the principal offers an ex ante contract (to be defined below) to the agent; the agent chooses whether to accept or reject the contract – if the agent rejects it, then the game ends and he receives $\bar{U}$ while the principal receives 0. If the agent accepts the contract, then Nature sends a signal z_k to the principal according to the technology and the information structure.[6] If information is private, the principal announces, possibly not truthfully, z_k to the agent. Finally,

[3]We can think of the FOSD ranking in an employer-employee context as Π_1 corresponding to a more productive employee – for any level of effort, employee Π_1 is more likely to realize a greater output than is Π_2. It is not uncommon that a worker does not know his own productivity, but the employer can become justifiably more certain of this through interviews and interim evaluations.

[4]In statistical decision theory, when one information structure is derived from another by multiplication with a stochastic matrix (a matrix whose elements are all strictly within the unit interval and with column sum unity), the latter is said to be more informative in the sense of Blackwell than the former. Our simplified information structure implies a restriction of the stochastic matrices, in that they also have row sum unity.

[5]Whether one or both players receive the signal, since it is entirely uninformative, their posteriors equal the priors, so that the situation will be as if information is public.

[6]In ex post contracting, the principal offers the agent the contract *after* Nature sends the signal to the principal about her technology and the agent observes the event associated with that signal and the chosen information structure.

the agent chooses an action and Nature chooses an outcome according to the technology and the action choice, and payoffs are made.

The ex ante contract that the principal offers specifies a payment contingent upon both the type that the principal will announce and the outcome. After observing z_k, the principal updates her beliefs. The agent updates his beliefs, after observing z_k if information is public, or upon learning the principal's announcement if information is private.

2.5 The Announcement-Contingent Contracts and Constraints

For a belief λ of the principal's technology, $I_{\lambda n}$ is an outcome-contingent payment from the principal to the agent, with $I_{\lambda n} \in \Re \ \forall n \in \{1, \ldots, N\}$. For clarity, we may write $I_{\lambda n}(a_m)$ for the wage corresponding to the different actions implemented.

The principal announces z_k, which specifies that the outcome-contingent wages will be $\{I_{\lambda 1}, \ldots, I_{\lambda N}\}$, which we denote by I_λ (or by $I_\lambda(a_m)$ where necessary for clarity). We call I_λ the *announcement-contingent contract*.

An announcement-contingent contract is *incentive compatible for the agent* if it induces the agent to choose the action that the principal wants to implement. If the agent believes that the principal is Π_λ, then $a \in \underset{a \in A}{argmax} \sum_{n=1}^{N} \pi_{\lambda n}(a) \ V(I_n) - a$ or

$$(1) \qquad \sum_{n=1}^{N} [\pi_{\lambda n}(a_m) - \pi_{\lambda n}(a_{\tilde{m}})] V(I_{\lambda n}(a_m)) \geq a_m - a_{\tilde{m}} \qquad \forall \tilde{m} \neq m.$$

If the agent believes that the principal is Π_λ, denote the incentive compatibility constraint corresponding to implementing a_m by $IC(\lambda, a_m, a_{\tilde{m}})$.

If the principal implements $a_m > a_1$, the announcement-contingent contract must satisfy (1). If she implements a_1, she does so with the constant wage $\bar{I} = h(\bar{U} + a_1)$.

To save on notation, we will write Π_{z_k} for $\Pi_{\lambda(z_k)}$, π_{z_k} for $\pi_{\lambda(z_k)}$, and I_{z_k} for $I_{\lambda(z_k)}$, except where necessary for clarity. We will often be computing costs ($C_{\lambda(z_k)}$) and benefits ($B_{\lambda(z_k)}$) of the different types for different possible contracts. Similarly, $C_{z_k}(a_m)$ and $B_{z_k}(a_m)$ are the expected costs and benefits for Π_{z_k} from implementing a_m.

Then, $C_{z_k}(I_{z_{k'}})(a_m))$ is the cost for Π_{z_k} to implement a_m with $I_{z_{k'}}$: $C_{z_k}(I_{z_{k'}}(a_m)) = \sum_{n=1}^{N} \pi_{z_k n}(a_m) I_{z_{k'} n}$. Note that $\lambda(z_k) C_1(I_{z_{k'}}(a_m)) + (1 - \lambda(z_k)) C_2(I_{z_{k'}}(a_m)) = C_{z_k}(I_{z_{k'}}(a_m))$.

$B_{z_k}(a_m)$ is the benefit (revenue) for Π_{z_k} from implementing a_m: $B_{z_k}(a_m) = \sum_{n=1}^{N} \pi_{z_k n}(a_m) q_n$. The principal's profit from implementing a_1 is $B_{z_k}(a_1) - \bar{I}$.

2.6 The Ex Ante Contract and Constraints

Denote by $I = \{I_{z_1}, I_{z_2}\}$ the *ex ante contract* where I_{z_k} is the announcement-contingent contract if the principal announces z_k.

Note that the offer of the ex ante contract does not inform the agent about the principal's type. The ex ante contract is *incentive compatible for the principal* if, for each z_k, the principal has the incentive to truthfully announce z_k. $\Pi_{z_{k'}}$ announces truthfully if and only if

$$B_{z_{k'}}(a_m(z_{k'})) - C_{z_{k'}}(I_{z_{k'}}(a_m(z_{k'}))) \geq B_{z_{k'}}(a_m(z_k)) - C_{z_{k'}}(I_{z_k}(a_m(z_k))),$$

where $I_{z_k}(a_m(z_k))$ is incentive compatible for the agent if offered by Π_{z_k}. Denote this constraint by $PIC_{z_k z_{k'}}$. An ex ante contract is *incentive compatible* if it is both incentive compatible for the principal and each announcement-contingent contract is incentive compatible for the agent.

The interim expected utility of an announcement-contingent contract that implements a_m is denoted by $u_k = EU(I_{z_k}) = \sum_{n=1}^{N} \pi_{z_k n}(a_m)V(I_{z_k n}) - a_m$.

By making the acceptance/rejection decision prior to learning anything about the principal's type, each announcement-contingent contract need not yield expected utility $\bar{U}$, but rather the ex ante contract needs to satisfy individual rationality. In designing it, the principal can trade off lower utility (and cost) from one announcement for higher utility (and cost) from another announcement. An ex ante contract is *individually rational* if it satisfies the following inequality:

$$prob(z_1)EU(I_{z_1}) + prob(z_2)EU(I_{z_2}) \geq \bar{U}. \tag{2}$$

2.7 The Program and Optimal Contract

We call a pair of actions that the principal implements, $\{a_m(z_1), a_m(z_2)\}$, an *action profile.*

For a given action profile, the principal's program is to select the $\{I_{z_1}, I_{z_2}\}$ that solves the following program:

$$\underset{I_{z_1}, I_{z_2}}{Min} \sum_{z_k \in Z} prob(z_k) \sum_{n=1}^{N} \pi_{z_k n}(a_m(z_k)) I_{z_k n} \tag{3}$$

$$s.t.\ (1), (2), and\ PIC_{z_1 z_2}.$$

Then the principal selects the action profile that yields the greatest profit; i.e.,

$$\{a_m(z_1), a_m(z_2))\} \in \underset{A \times A}{argmax} \sum_{z_k \in Z} prob(z_k)(B_{z_k}(a_m(z_k)) - C_{z_k}(I_{z_k})).$$

Let $a_m^*(z_k)$ denote the action that the principal implements in the least-cost action profile if she receives z_k, each announcement-contingent contract yields the agent $\bar{U}$, and $PIC_{z_1 z_2}$ does not bind. Let $I_{z_k}^*(a_m(z_k))$ denote this least-cost announcement-contingent contract.

For any level of utility u_k, denote by $I_{z_k}^{**}(a_m)$ the least-cost announcement-contingent contract that yields u_k and implements a_m, and $PIC_{z_1 z_2}$ does not bind. Then, the solution to (3) is an action profile $\{a_m^{**}(z_1), a_m^{**}(z_2)\}$ with announcement-contingent contracts $I_{z_k}^{**}(a_m^{**}(z_k))$.

Let $u_k^{**} = \sum_{n=1}^{N} \pi_{z_k n}(a_m^{**}(z_k))V(I_{z_k n}^{**}) - a_m^{**}(z_k)$ denote the agent's expected utility from the announcement-contingent contract and let $c_{k'k}^{**} = \sum_{n=1}^{N} \pi_{z_{k'} n}(a_m^{**}(z_k))I_{z_k n}^{**}(a_m^{**}(z_k))$ denote $\Pi_{z_{k'}}$'s expected cost if she offers $I_{z_k}^{**}(a_m^{**}(z_k))$.

We assume that there is no natural separation and consider situations in which the lower type would mimic the higher type if the higher type were to announce the public information contract,[7] so that the principal's incentive compatibility constraint binds in the ex ante contract. Let $\hat{I}_{z_1}$ represent the announcement-contingent contract that is least-cost for Π_{z_1} among the set of announcement-contingent contracts that yield the agent $\bar{U}$, and that satisfy (1) and $PIC_{z_1 z_2}$. $\hat{I}_{z_1}$ clearly cannot satisfy the same incentive compatibility constraints with strict equality that $I_{z_1}^{*}$ satisfies (recall that $\lambda(z_1) > \lambda(z_2)$).

Let $\hat{\hat{I}}_{z_1}(a_m)$ denote the announcement-contingent contract that satisfies (1) and yields the agent utility u_1^{**} if the agent's beliefs are $\lambda(z_1)$ and also satisfies $PIC_{z_1 z_2}$.

Because the principal solves a minimization problem and, at the contracting date, she has no private information, the agent does not update his beliefs upon receiving the contract offer; thus, the ex ante contract yields unique payoffs. It is possible that the principal is indifferent between two ex ante contracts, but then they would yield her the same profit, and, as Lemma 1 below shows, the agent is also indifferent.

The characteristics of the possible equilibrium ex ante contracts are relegated to the appendix.

3 Privately Informed Principal

In this section we explore the consequences for the principal when she has private information. By first examining the principal's choice when $\zeta = \frac{1}{2}$, and then when $\zeta > \frac{1}{2}$, we are able to characterize the possible payoff functions. An increase in ζ from $\frac{1}{2}$ has two effects: it improves information and introduces an asymmetry of information. In the next section, we perform the same analysis for the situation when information is public, allowing us to disentangle the effect of improving the information from the effect of an induced asymmetry of information.

In a principal-agent model with a privately informed principal and common values, Maskin and Tirole (1992) showed that the agent may hold pessimistic beliefs that prevent the principal from realizing her complete information payoff.[8] That is, the principal may do worse when she has private information than when information is public. In our current setting, the principal offers an ex ante contract prior to receiving any information about her technology, and so the agent cannot hold pessimistic beliefs upon receiving the ex ante contract. When the principal announces her type, the agent can hold beliefs that would induce him to implement an action other than that

[7] Because the technology does not satisfy the single-crossing property, it is possible that the higher type would want to mimic the lower type.

[8] In their model, there exist equilibria in which the principal offers a contract that is more expensive than that with complete information. The agent's out-of-equilibrium beliefs for contracts that yield her her complete information payoff are such that the principal is believed to be the low type; this prevents the principal from offering this contract since the agent would reject it.

which the principal intends. That the agent's beliefs are synonymous with the principal's at the contracting date allows the principal to do at least as well with private information as with null, and therefore public, information. We establish this in our first proposition below. That the agent's beliefs may differ from the principal's after learning her announcement implies that the principal may prefer public to private information. We establish this in Proposition 3 in the next section.

In order to derive the first result, we first show that the agent is not better off with ex ante contracting in any environment since if an ex ante contract ever yielded more than $\bar{U}$, the principal could lower the wage associated with the minimum outcome in each announcement-contingent contract in such a way as to maintain incentive compatibility of the principal and also lower her costs, while being individually rational.

Throughout, assume that the minimum wage from each announcement-contingent contract is strictly greater than $\underline{I}$. Proofs of all lemmas and propositions are in Appendix 7.2.

Lemma 1 *Agent's Utility*

For any environment, every equilibrium ex ante contract yields the agent exactly $\bar{U}$.

Suppose that one announcement-contingent contract, I_{z_k}, had a minimum wage equal to $\underline{I}$. Then this announcement-contingent contract could provide more than $\bar{U}$ at its optimum because in order to satisfy incentive compatibility, the wage structure yields an expected utility greater than $\bar{U}$. The agent may still receive only $\bar{U}$, however, if the other announcement-contingent contract can be adjusted to provide less than $\bar{U}$. Only if in this other announcement-contingent contract, the minimum wage also equals $\underline{I}$ or if $PIC_{z_1 z_2}$ binds, would the agent receive more than $\bar{U}$ from the equilibrium ex ante contract.

In order to better understand this lemma and the subsequent results, it is helpful to think of contracting in principal-agent models with moral hazard as utility provision subject to the agent's incentive compatibility constraints. For a given action profile, the principal wants to minimize the cost to provide utility; with ex ante contracting, the principal compares the marginal costs of utility provision, μ_k (see (6) in Appendix 7.2), for each announcement-contingent contract and adjusts the utility provided by each inversely to its marginal cost.

If the principal implements a constant action profile, then she pools if this is feasible. It is feasible if the agent is ignorant. In such situations, the principal is not hurt by receiving private information after contracting, as Proposition 1 shows.

Proposition 1 *Private vs. Null Information*

The principal weakly prefers private information to null information; if the optimal (equilibrium) action profile is not constant, the preference is strict.

For example, consider a firm that wants to transfer a manager, who is currently contracted with the firm, to head operations in a new market. It will pay to learn how valuable managerial effort is, if it would want the manager to exert different levels of effort (e.g., if the returns depend upon the sensitivity of demand), even if the manager would not know how valuable his effort is. In the next section, we show that the firm would gain even more if the manager would learn this. However, when the firm already has imperfect (but not null) and private information, the value to the firm of a better information structure (hence, better information) may be negative.

In the situations of Proposition 1, the principal does not lose by having private information because the same contracts to implement an action profile are still feasible. Unlike with ex post contracting,[9] the agent cannot hold pessimistic beliefs when he makes his acceptance/rejection decision. If the principal has private information, then the same contract still induces the agent to work hard. That is, there are two crucial factors that allow the principal to insure herself (against the possibility of having a lower technology) by trading off utility and incentive provisions – information arriving after the contracting date and only the principal receiving information.

Gjesdal (1982) defined two ways that better information can benefit the principal: (i) it can allow better risk-sharing and so have marginal insurance value; and (ii) it can induce the principal to implement a higher action, and so have marginal incentive informativeness. When the principal does not alter the action profile she implements, the latter vanishes.[10] If the principal already has imperfect and private information, then the marginal insurance value of better information depends upon two factors: the changes to the probabilities of receiving each signal and the marginal costs of utility provision from each signal. The analysis following Lemma 2 below shows that the value of information may be positive or negative.

Before stating this result, we clarify the nature of the risk that the principal faces. Because she faces a risk with negative expected return, she will gain by insuring herself if she can; i.e., the principal reduces her cost of providing the agent utility by offering a pooling contract. As Lemma 1 shows, the agent receives the same expected utility from any ex ante contract. Before the contracting date, if information is public, the principal faces the risk of receiving z_1 and implementing an action at a lower cost, or receiving z_2 and implementing the same action at a higher cost.[11] Even though the principal is risk neutral, this risk makes her worse off since the cost reduction (from receiving z_1) is smaller than the cost increase (from receiving z_2). In order to see this, it is worthwhile relating the costs of public information contracts to the notion of a mean-utility preserving increase in risk (see Diamond and Stiglitz (1974)).

Before the contracting date, when information is public, the agent faces a gamble over both the contract and the outcomes. This gamble is a spread of the contract over only the outcomes, which is the pooling contract that he could receive if the agent has no information. Suppose that the cost to implement a_m with Π_1 is less than that with Π_2; e.g., Π_1 is derived from Π_2 by a mean-preserving spread in the likelihood ratio distribution function. Since the agent has no information, any pooling

[9]In ex post contracting, the principal offers the contract after she receives the signal z_k. As such, each contract she offers must provide the agent expected utility $\bar{U}$.

[10]As in the proof of Proposition 1, simple algebra shows that $prob(z_1|\zeta)B_{(z_1|\zeta)}(a_m(z_1)) + prob(z_2|\zeta)B_{(z_2|\zeta)}(a_m(z_2))$ is constant with respect to ζ.

[11]If the technologies are related as in Kim (1995) or by a Blackwell ordering, then $C_{z_1}(I^*_{z_1}(a_m)) < C_{z_2}(I^*_{z_2}(a_m))$ holds for all a_m. All we need is that these costs are unequal for the action under consideration (see also Grossman and Hart, 1983, Proposition 13).

contract can induce the agent to undertake a certain action with an incentive scheme that is not as high-powered as when the agent believes she has Π_2, but is more high-powered as when he believes she has Π_1. The agent's risk aversion implies that the bonus is increasing and convex (the contract is becoming more high-powered at an increasing rate) as the likelihood ratio distribution functions become more similar to those of Π_2. The next lemma establishes this point.

Lemma 2 *Expected Cost: Null vs Public Information*

Let λ, λ' be probabilities that the principal has Π_1, with $(1-\lambda') < \lambda < \lambda'$. Let $p_{\lambda'} = \frac{\lambda+\lambda'-1}{2\lambda'-1}$.[12] *The expected cost of public information contracts for $\Pi_{\lambda'}$ and $\Pi_{(1-\lambda')}$ with probabilities $p_{\lambda'}$ and $(1-p_{\lambda'})$ is greater than the cost of a pooling contract, I^*_{λ}.*

Through a series of mean-utility preserving spreads, one can generate a $(1-p_{\lambda'}) : p_{\lambda'}$ gamble on $I^{**}_{(1-\lambda')}$ and $I^{**}_{\lambda'}$, from I^{**}_{λ}. If $Y \stackrel{d}{=} x + \epsilon$ where $E[\epsilon \mid x] = 0$, then the distribution induced by Y is a mean-preserving spread of the distribution induced by x. In our context, the utilities $V(I^{**}_{(1-\lambda')n})$ and $V(I^{**}_{\lambda' n})$, are the utility $V(I^{**}_{\lambda n})$ plus noise that has zero conditional expected mean. This generates a spread of the distribution induced by $I_\lambda(a_m)$ and $\pi_\lambda(a_m)$. Note as well that the $(1-p_{\lambda'}) : p_{\lambda'}$ gamble on $I^{**}_{(1-\lambda')}$ and $I^{**}_{\lambda'}$ yields u^{**}_k as does I^{**}_{λ}. Thus, both conditions of being a mean-utility preserving increase in risk are satisfied. Then, the agent's risk aversion implies that the expected cost of $I^{**}_{(1-\lambda')}$ and $I^{**}_{\lambda'}$ exceeds the expected cost of the announcement-contingent contract I^{**}_{λ}.

The lemma implies two inequalities. First, the agent's risk aversion implies that the $(1-p_{\lambda'}) : p_{\lambda'}$ gamble on $I^{**}_{(1-\lambda')} : I^{**}_{\lambda'}$ is more costly due to participation. That is,

$$(4) \qquad p_{\lambda'} C_{\lambda'}(I^{**}_{\lambda'}(a_m)) + (1-p_{\lambda'}) C_{(1-\lambda')}(I^{**}_{(1-\lambda')}(a_m)) > C_\lambda(I^{**}_\lambda(a_m)).$$

In particular, for $\lambda' = 1$, (4) becomes $\lambda_0 C_1(I^{**}_1(a_m)) + (1-\lambda_0) C_0(I^{**}_0(a_m)) > C_{\lambda_0}(I^{**}_{\lambda_0}(a_m))$. The left-hand side equals the principal's ex ante expected cost of implementing a_m if the principal and agent have perfect information.

As ζ increases, the principal increases $u^{**}_{z_1}$ and decreases $u^{**}_{z_2}$; (4) shows that cost increases even if the probabilities of receiving either signal are constant ($\lambda_0 = 0.5$). The incentive compatibility constraints become easier to satisfy for Π_{z_1} but more difficult to satisfy for Π_{z_2}. These generate a less (respectively, more) high-powered incentive scheme, but the latter effect dominates the former effect.

Second, without loss of generality, the separating equilibria that can exist have Π_{z_2} choose $I^{**}_{z_2}$ and Π_{z_1} choose an announcement-contingent contract that dissuades mimicking. This means that she cannot offer $I^{**}_{z_1}$ as part of the ex ante contract unless information is public. The least-cost separating announcement-contingent contract is $\hat{\hat{I}}_{z_1}$, which necessarily costs her more than $I^{**}_{z_1}$ costs. Therefore,

[12] As $\lambda' > \lambda > 1-\lambda'$, $p_{\lambda'} \in (0,1)$ and $p_{\lambda'} \pi_{\lambda' n}(a_m) + (1-p_{\lambda'}) \pi_{(1-\lambda')n}(a_m) = \pi_{\lambda n}(a_m)$.

(5) $\lambda_0 C_{z_1}(\hat{\hat{I}}_{z_1}) + (1-\lambda_0)C_{z_2}(I_{z_2}^{**}) > \lambda_0 C_{z_1}(I_1^{**}) + (1-\lambda_0)C_{z_2}(I_{z_2}^{**}) >$

$$C_{\lambda_0}(I_{\lambda_0}^*) = \lambda_0 C_{z_1}(I_{\lambda_0}^*) + (1-\lambda_0)C_{z_2}(I_{\lambda_0}^*).$$

All other pooling announcement-contingent contracts cost the principal more than $I_{\lambda_0}^*$ costs, and all other separating announcement-contingent contracts for Π_{z_1} cost her more than $\hat{\hat{I}}_{z_1}$ costs.

We are now ready to state our second main result. In ex post contracting (see Silvers, 2008), the principal would not voluntarily receive better information when she has private information unless it would result in her implementing the more profitable contract more often. In ex ante contracting, however, the principal prefers better information.

Proposition 2 *Value of Better Private Information*

Consider the Private Information *environment. For $1 \geq \zeta > \zeta'$, if the principal implements a constant action profile with ζ', then her equilibrium profit with ζ is weakly greater than that with ζ'.*

Given ζ', the optimal contract if the principal implements the same action for either signal is a pooling contract $I_{\lambda_0}^*(a_m)$. With ζ, she can always still offer $I_{\lambda_0}^*(a_m)$ for z_1 and for z_2, but she may do better if she implements different actions for the two signals.

We have considered what happens to the principal's payoff function when she implements a constant action profile. Suppose instead that the principal were to implement different actions given z_1 versus z_2 with ζ. Better information may make her worse off. This could follow because an increase in ζ causes a spread of $\lambda(z_k)$ and the cost of public information contracts is convex in the sense of Lemma 2. Moreover, the separation cost may also increase if the principal incentive compatibility constraint becomes more difficult to satisfy after the increase in ζ since Π_{z_2} would have more to gain, and therefore would mimic some contracts that she would not with the smaller ζ.

In this situation, when the principal implements different actions given z_1 versus z_2, she must offer a separating ex ante contract. To see how the cost of separation may increase, first observe that a decrease in $\lambda(z_2)$ increases the marginal cost of utility provision from I_{z_2}. If there were no possibility of Π_{z_2} selecting I_{z_1}, then the marginal cost of utility provision from I_{z_1} would decrease, but the average may still increase. Second, because the principal's incentive compatibility constraint is more difficult to satisfy, the marginal cost of utility provision decreases less, or may even increase. Third, an increase in $\lambda(z_1)$ and a decrease in $\lambda(z_2)$ also each lower the signaling cost, which is smaller as $|\frac{\pi_{z_1 n}(a_m)}{\pi_{z_2 n}(a_m)} - 1|$ increases.

The payoff function from the following example is depicted in Figure 1, where the solid dot indicates the principal's payoff with null information. As ζ increases, when the marginal incentive informativeness first exceeds the marginal insurance value (if this is negative) plus the signaling cost, the principal implements a non-constant action profile. Denote the smallest value of ζ at which the

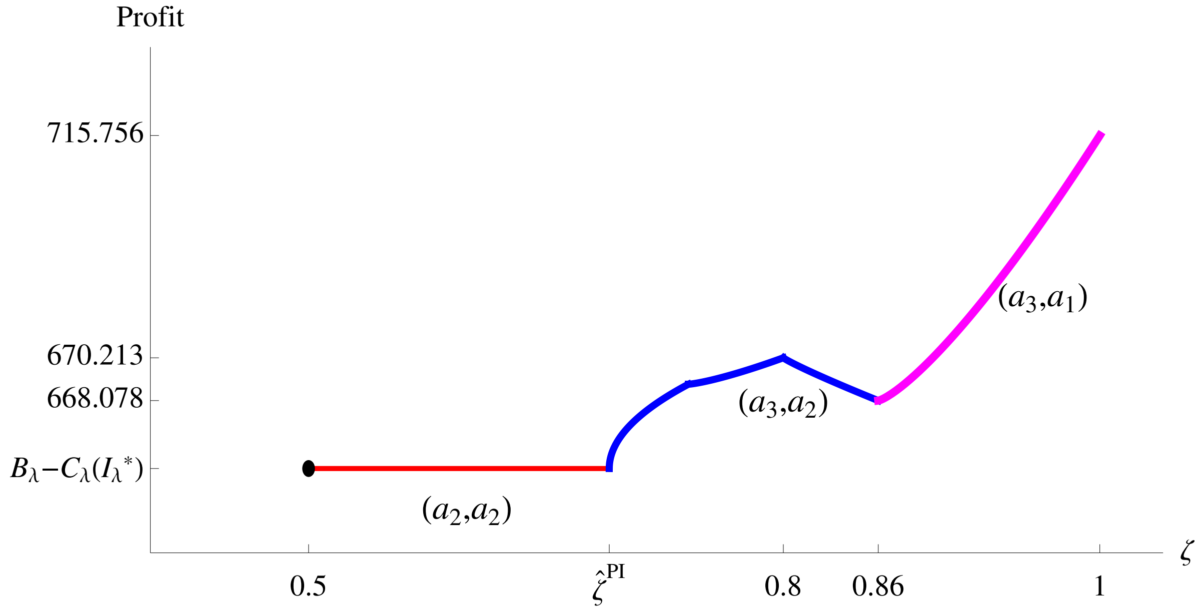

Figure 1: Principal's Payoff Function With Private Information: Principal Pools for $\zeta \in (\frac{1}{2}, \hat{\zeta}^{PI})$

principal is indifferent between implementing a constant action profile and a non-constant action profile, by $\hat{\zeta}^{PI}$ (0.69 in the example). Also, the value of information is the slope of this function, except at $\zeta = \frac{1}{2}$ – recall that for $\zeta = \frac{1}{2}$, an increase in ζ not only improves the information, but also establishes an asymmetry of information.

To highlight the non-monotonicity of the principal's expected profit and the changes in the action profile, consider the following example.

Example 1 *Non-Monotonic Value of Private Information*

There are three possible actions and three possible outcomes. The agent has log utility, disutility of effort $a \in \{0, 1, 3\}$*, and reservation utility* $\bar{U} = 1$*. The revenues are* $q = \{450, 700, 1100\}$*. Let the technologies be given by* $\Pi_2 = \begin{bmatrix} .6 & .3 & .1 \\ .5 & .3 & .2 \\ .4 & .35 & .25 \end{bmatrix}$ *and* $\Pi_1 = \begin{bmatrix} .7 & .25 & .05 \\ .4 & .3 & .3 \\ .1 & .3 & .6 \end{bmatrix}$ *with* $\lambda = 0.6$*. If the principal pools to implement* a_2*, her expected profit is 664.973, which is greater than pooling to implement* a_3 *and receiving 471.577 or* a_1 *and receiving 560.282.*

To complete the characterization of the principal's payoff function, consider the possibility that the principal implements a constant action profile at $\zeta = \frac{1}{2}$ but a non-constant action profile for $\zeta > \frac{1}{2}$. This could arise since for such ζ, the expected costs of such a separating ex ante contract and of the pooling contract are independent of the expected revenues from each action profile. Nevertheless, with null information, separation is impossible, yielding a discontinuity in the payoff function.

Finally, note that it is possible for the principal to implement a constant action profile for some

$\zeta > \hat{\zeta}^{PI}$, since the expected revenues change at a constant rate for any action profile, but the cost difference between alternative ex ante contracts may rise or fall with ζ.

In other words, if the principal implements the same action given z_1 or z_2 with ζ', the marginal incentive informativeness is positive. Even though both the marginal insurance value is negative and possibly exacerbated by an increase to ζ, the principal would not have to pay more in expectation, unless the value due to marginal incentive informativeness exceeds the increase in expected cost. Alternatively, if she were to implement different actions given z_1 or z_2 with ζ', then the marginal insurance value of an increase to ζ can be positive or negative – in part depending on whether the cost of separating decreases or increases.

4 Public Information

The previous section considered situations in which only the principal receives the signal. Before the contracting date, Lemma 2 shows that the principal faces a gamble that is declining in expected value as the principal receives better information because the agent is risk averse. In order to understand why the agent not having information is critical, we examine the impact of information being public. If information is public, the principal will be precluded from offering the pooling contract, and thus from completely insuring herself against the realization of z_2. This will certainly make her worse off if she implements a constant action profile. She will still be able to trade off utility provision, but not incentive provision.

However, if she implements a non-constant action profile, then she does best by not having to pay a signaling cost and there is no loss from being unable to trade off incentive provision since this is impossible when she separates. Proposition 3 below shows this, namely, that public information has negative value for the principal if (but not only if) she implements a constant action profile.

Two main differences arise with respect to private information. First, for ζ slightly larger than $\frac{1}{2}$, the cost of separation to different actions may exceed the revenue gains, so that the principal implements a constant action profile. However, because information is now public, the principal is unable to offer the pooling contract – the agent would not choose a_m if the principal announced z_2. This means that she has less profit than when she has private information.

Second, as ζ increases, the principal implements a non-constant action profile at a lower value of ζ than when she has private information because the cost of her separating contracts is lower – but the expected revenues are the same as when she has private information. Additionally, for any value of ζ for which the principal finds it beneficial to separate when she has private information, she certainly finds it beneficial to separate when information is public, and she earns more profit because she does not have to satisfy the principal's incentive compatibility constraint. That is, $\hat{\zeta}^{SI} < \hat{\zeta}^{PI}$, where $\hat{\zeta}^{SI} > \frac{1}{2}$ is the smallest value at which the principal is indifferent between implementing a constant versus a non-constant action profile. Note also that the principal may implement a constant action profile for some $\zeta > \hat{\zeta}^{SI}$.

We have then the following proposition:

Proposition 3 *Public vs. Private Information*

(a) If the principal implements a constant action profile with ζ, then her equilibrium profit is less than that with $\zeta = \frac{1}{2}$ which is equal to that when she has private information.

(b) If the principal implements a non-constant action profile with ζ when she has private information, then her equilibrium profit with public information is greater than that with private information.

The proposition says, essentially, that the principal (i) would pay to keep her private information private if she would implement a constant action profile; (ii) would pay to acquire private information if she would change one or both actions that she implements; and (iii) pay even more to share this information with the agent.

We are now ready to examine the impact of private versus public information about the technology. By comparing the values of the payoff functions, we know when the principal prefers private or public information; by comparing the slopes of the payoff functions, we know when the principal is willing to pay more for better information.

Consider the following example:

Example 2 *Non-Monotonic Value of Public Information*

In the example from the previous section, if $\zeta = .69$, implementing $\{a_3, a_2\}$ yields profit of 686.163, which exceeds her profit with private information (665.135). Similarly, with perfect information, she still implements $\{a_3, a_1\}$ but her profit increases from 715.756 with private information to 762.003 with public information.

The two payoff functions from the examples are shown in Figure 2. Note that as in the privately informed principal situation, it is also possible for the principal to implement a non-constant action profile for $\zeta \in (\frac{1}{2}, \hat{\zeta}^{SI}]$ – the graph would have a similar jump discontinuity, but be greater.

For $\zeta > \hat{\zeta}^{PI}$, though the principal prefers public to private information, the value of a marginal increase in ζ may be larger with private information; this is indeterminate since the signaling cost may increase or decrease. Moreover, comparing the payoff functions for $\zeta \in [\frac{1}{2}, \hat{\zeta}^{SI}]$, the principal prefers null to public information.

That is, the value of public information may be negative for ζ near $\frac{1}{2}$. For example, in a government procurement contract, the government receives a signal that is relevant to the effort level it wants to implement. If it would want to implement different effort levels depending upon the information, the government does better by credibly sharing its information with the firm; however, it may still do worse than if the signal had been entirely uninformative.

Recall that a marginal increase in ζ when $\zeta = \frac{1}{2}$ and the principal has private information may not affect the payoff for the principal. Previously, we were unable to determine the directions and magnitudes of each effect – that due to better information for the principal versus that due to the induced asymmetry of information. When information is public, the payoff declines if the principal

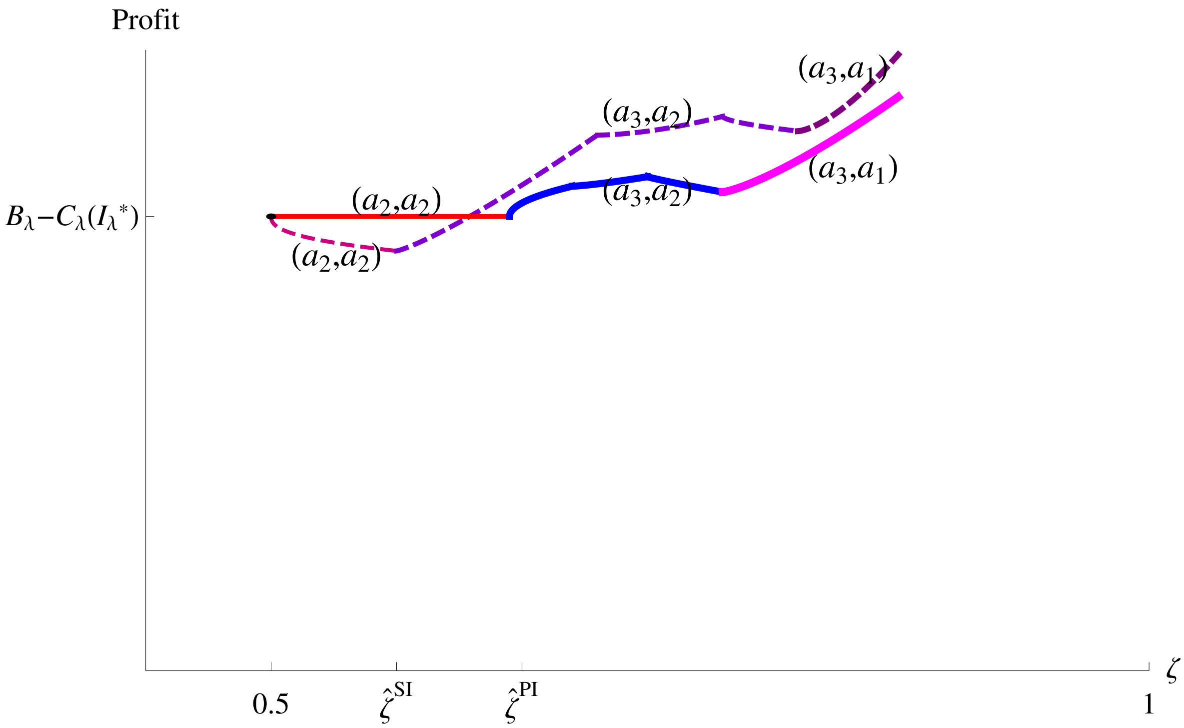

Figure 2: Principal's Payoff Function With Public (Dashed) vs. Private (Solid) Information

implements a constant action profile. Therefore, we can disentangle the two effects and state that having private information is beneficial if the principal implements a constant action profile in the sense that it negates this loss, but if the principal implements a non-constant action profile, having private information may be harmful.

Intuitively, the symmetry of information may be helpful when the principal would want to separate (she implements a non-constant action profile); whereas, the symmetry is harmful when she would want to pool since it precludes her from doing so. If with better information she still wants to pool, the marginal incentive informativeness vanishes; if she cannot pool, as when information is public, then the marginal insurance value is negative.

5 Discussion

In this section, we examine more closely the non-monotonicity in the value of information, and then the importance of commitment and the timing of information.

Consider first public information. It can be shown that the value of information is strictly positive even if the prinicpal implements the same action profile provided that either $\lambda_0 = \frac{1}{2}$ or Π_{z_2} implements a_1.

An increase in ζ has two effects on expected cost: the frequency with which each signal is received changes and the contracts that are offered given each signal also change. Moreover, the contracts are altered for two reasons: the cost to provide incentives for the agent to choose $a_m(z_1)$ declines while the cost to provide incentives for the agent to choose $a_m(z_2)$ rises; and, as a consequence, the principal adjusts the amounts of utility provided given either signal.

If $\lambda = \frac{1}{2}$ so that the probabilities of receiving either signal are constant with respect to ζ, then the value of information is monotonically decreasing as long as the principal implements the same action profile. If Π_{z_2} implements a_1, then the marginal cost for Π_{z_2} to provide a given level of utility is unaffected by an increase in ζ since $I_{z_2} = h(u_{z_2})$. Thus, for $\lambda < \frac{1}{2}$, marginal cost is monotonically decreasing so that the value of information is positive. For $\lambda > \frac{1}{2}$, better information lowers the principal's cost by reducing the marginal cost of utility provision given z_1, but raises her cost by increasing the probability of the more expensive contract $(I_{z_1}(a_m))$ being chosen.

Now consider private information. Better information both reduces the marginal signaling cost for Π_{z_1} to increase the cost of Π_{z_2} mimicking by one dollar, but also increases the amount of signaling that Π_{z_1} must perform. The first effect arises because the likelihood ratios given by $\frac{\pi_{z_1 n}(a_m(z_1))}{\pi_{z_2 n}(a_m(z_2))}$ diverge from unity. The second effect arises because $C_{z_2}(u_{z_2}) - C_{z_2}(u_{z_1})$ may increase. Thus, the net impact is ambiguous; i.e, the value of information may be larger or smaller with public than with private information.

We have assumed that the agent's utility is additively separable. If utility were multiplicative, the proofs of all propositions would still go through for two primary reasons. They relied upon the ability of the principal to trade off utility provision, and her desire to do so due to the agent's risk aversion.

Gjesdal (1982, Propositions 2 and 3) showed that better information has positive value if the agent's utility is additively separable, but need not if utility is multiplicative. In our model, the potential non-monotonicity in the value of information arises with additively separable or multiplicative utility. Moreover, better information does not induce the principal to implement a lower action. In contrast, in our model, with ζ' the principal may be able to implement an action given z_2 that with better information $(\zeta > \zeta')$, she is unable to implement. Because $\lambda(z_2 \mid \zeta) < \lambda(z_2 \mid \zeta')$, the principal's cost to implement the same action increases, inducing her to implement a lower action. This also means that, compared to perfect information, the principal with ζ' is implementing a higher action.

Example 3 *Non-Monotonic Value of Information with Multiplicative Utility*

Continuing from the example presented above, make the following two changes: $u(w, a) = log(w)/a$ *and let* $a = \{1, 2, 4\}$. *The principal can receive expected profit 664.973 from pooling on* a_2, *and does this for* $\zeta \in [0.5, 0.69)$. *For* $\zeta \geq 0.69$, *the principal separates, implementing* $\{a_2, a_3\}$ *until* $\zeta \geq 0.86$. *Her profit is rising with* ζ *to 670.593 at* $\zeta = 0.84$ *and then is falling with* ζ. *For* $\zeta \geq 0.86$, *the principal implements* $\{a_1, a_3\}$.

We now turn to the importance of commitment. The contracts that the principal selects after receiving the signal yield the agent different expected utilities. Since their mean is $\bar{U}$ and the agent receives this in equilibrium, one announcement-contingent contract necessarily provides the agent less utility than his reservation utility. Thus, an important requirement is that the agent, upon accepting the contract, be committed. Without the ability to commit, the situation is as in ex post contracting. As such, our results can also be seen to add to the potential losses to the principal and agent when the agent cannot commit.

Additionally, the ex ante contract is renegotiation-proof because the principal cannot gain except by providing less utility to the agent. Each announcement-contingent contract minimizes the cost to implement the desired action (for brevity, denoted by a_k^{**}) while providing the agent expected utility u_k^{**}. The principal and agent would only renegotiate if the principal could provide at least as much utility and implement a different action that yields greater expected profit. However, the action specified in the ex ante contract is optimal.

To see this, suppose that the principal considers implementing $a_k' > a_k^{**}$. Let $C(a;u)$ denote the cost for the principal to implement a with a contract that provides the agent expected utility u. Because providing u_k^{**} and implementing a_k^{**} is optimal in the ex ante contract, $C(a_k';u_k^{**}) - C(a_k^{**};u_k^{**}) \geq B(a_k') - B(a_k^{**})$. For any $u' > u_k^{**}$, $C(a_k';u') > C(a_k';u_k^{**})$. Thus,

$$B(a_k^{**}) - C(a_k^{**};u_k^{**}) \geq B(a_k') - C(a_k';u_k^{**}) > B(a_k') - C(a_k';u')$$

A similar analysis shows that the principal providing the agent more utility and implementing a lower action strictly reduces profit.

There are some situations in which the agent is unable to commit to the contract, such as employment and insurance contracts. In these situations, principals sometimes induce commitment, through for example, instilling a worker with specific human capital or granting repeat customers discounts.

Finally, there are other examples in which firms pay to contract before receiving information. A firm that hires a professional on retainer is consistent with this. The professional is obligated to work when and as the firm directs, and no particular case that arises is known with certainty at the time they signed the contract.

6 Conclusion

We have analyzed a principal-agent model with moral hazard in which the principal has one of two technologies; the technology affects the returns to the agent's action and the cost to implement the action. The principal receives a signal correlated with her technology. We examined the consequences when information is private versus public, and when the signal is imperfectly correlated versus entirely uninformative. We then contrasted the value of information in each situation. If information arrives before the agent chooses his action, it affects the cost to provide incentives differently as the amounts of utility provided are changed; if the principal has private information, these effects are further altered.

Having private, as opposed to having null, information, also allows the principal to implement a different action profile. We have shown that the trade-off, between implementing actions closer to first-best and reduced gains from risk-sharing, is sensitive to both the symmetry of information and the timing of information relative to the contract offer.

Specifically, we have shown that when the principal receives an informative signal, she may prefer public information only if she implements a non-constant action profile; otherwise, public information makes it more expensive in expectation to implement an action, because the marginal insurance value is negative. Consequently, the value of information can be positive or negative; i.e., information can benefit or harm the principal.

We have also shown that, with ex ante contracting unlike with ex post contracting, a principal is certainly made better off by having private information compared to having null information. Still, the value of information can be positive or negative, and may be larger or smaller than with public information because there are two opposing effects: better information reduces signaling costs, but raises the amount by which one type of principal must increase the cost in order to deter the other type of principal from mimicking. In contrast to ex post contracting, the agent does not gain when the principal has private information; he still receives exactly his reservation utility.

Additionally, with ex ante contracting, both the principal, and the principal and agent together, prefer null to public (including complete) information if and only if the principal implements a constant action profile. This is similar to ex post contracting, except that there the implementation of a non-constant action profile is not sufficient for complete information to be preferred to null information. Contrasting the same situation across the two timings of information shows that the principal prefers ex ante to ex post contracting.

Finally, the actions that the principal implements can be distorted either up or down; these distortions can be for either signal realization, and between private and public information, or between ex ante and ex post contracting.

One possible avenue to explore is to determine the properties of the technology that yield the non-monotonicity. This could be clarified by extending the model to other types of private information that affect the costs to implement an action, such as the agent's disutility of effort or reservation utility. This would help determine, with ex ante contracting, which consequences to private information about the technology are due to the existence of private information versus the type of private information. Additionally, it may be of interest to examine situations in which the agent has, or may come to have, private information only after contracting and how this may influence his action choice.

7 Appendix

7.1 Equilibria and Equilibrium Contracts

A Perfect Bayesian Equilibrium (PBE) must specify: (i) an ex ante contract for the principal to offer; (ii) whether the agent accepts or rejects the ex ante contract; (iii) an announcement by the principal of a signal; (iv) an action for the agent to take; and (v) beliefs both on and off the equilibrium path.

Both the principal and the agent have a common prior, λ_0, that the principal has Π_1. The principal forms her posterior beliefs after receiving z_k. If the principal has private information, then the agent forms posterior beliefs after hearing the principal's announcement. If information is public, then the agent forms his posterior beliefs after observing the signal that Nature sent to the principal. These beliefs follow Bayes' rule. Finally, beliefs for all other possible contracts and possible announcements must be specified, though they are not restricted.

In ex ante contracting, the principal offers a contract that is a menu of announcement-contingent contracts, each of which specifies a wage for each possible outcome. When information is private, each announcement-contingent contract must be incentive compatible for the principal.

The agent's acceptance/rejection decision depends upon whether the contract is individually rational, which requires that (2) holds. The agent's action choice is the argmax of his expected utility, so that he chooses a_m if and only if (1) holds. Note that incentive compatibility depends on the agent's posterior belief, which is a function of z_k and, if information is private, the principal's incentives to announce truthfully.

A principal of a certain type may deviate by selecting a different announcement-contingent contract. In each equilibrium, beliefs that the agent holds for all contracts not offered in equilibrium, such that no type gains by deviating to such a contract, must be specified. The out-of-equilibrium beliefs can be any function mapping from the space of feasible contracts into $[0, 1]$; the beliefs partition the space of feasible contracts both into those that are individually rational and those that are not, and into those that are incentive compatible and those that are not. The ex ante contract may be separating or pooling. An ex ante contract is pooling if the announcement-contingent contracts are identical.

- *Public Information*

  The principal can offer the ex ante contract consisting of $I_1^*(a_m(z_1))$ and $I_2^*(a_m(z_2))$ but may do better by trading off utility provision. Note that $I_{\lambda_0}^*$ is not feasible since it does not implement a_m when the principal is Π_{z_2}. The optimal contract is denoted $\{I_1^{**}(a_m(z_1)), I_2^{**}(a_m(z_2))\}$.

- *Private Information*

  Pooling and separating contracts are both possible. Because the principal has private information, the ex ante contract must induce the principal to truthfully announce her signal.

  The ex ante contract consisting of $\hat{I}_1(a_m(z_1))$ and $I_2^*(a_m(z_2))$ is a possible ex ante contract, but the principal may do better by trading off utility provision.

  If the principal implements the same action given z_1 or z_2, then, by (4), the principal offers $I_{\lambda_0}^*$ whether she announces Π_{z_1} or Π_{z_2}; else, she offers a least-cost separating contract denoted $\{\hat{\hat{I}}_{z_1}(a_m(z_1)), I_{z_2}^{**}(a_m(z_2))\}$. This maximizes

$$\lambda_0\Big(B_{z_1}(a_m(z_1)) - C_{z_1}(\hat{\hat{I}}_{z_1}(a_m(z_1)))\Big) + (1-\lambda_0)\Big(B_{z_2}(a_m(z_2)) - C_{z_2}(I_{z_2}^{**}(a_m(z_2)))\Big),$$

where $\hat{\hat{I}}_{z_1}(a_m(z_1))$ satisfies (1) for Π_{z_1}, $I^{**}_{z_2}(a_m(z_2))$ satisfies (1) for Π_{z_2}, and together they satisfy (2) and $PIC_{z_1 z_2}$.

- *Null Information*

  Because the principal and agent learn nothing, the ex ante contract consists only of $I^{*}_{\lambda_0}(a_m)$.

7.2 Proofs

Proof of Lemma 1.

Because the ex ante contract would not be renegotiated, the solution to (3) is equivalent to the solutions to the following programs, for both k:

$$
\begin{aligned}
&\underset{I_{z_k n}}{Min}\ \ prob(z_k)\sum_{n=1}^{N}\pi_{z_k n}(a^{**}_m(z_k))I_{z_k n}\\
(6)\qquad &s.t.\ \ \sum_{n=1}^{N}\pi_{z_k n}(a^{**}_m(z_k))V(I_{z_k n}) - a^{**}_m(z_k) \geq u^{**}_k,\\
&IC(z_k, a_m, a_{\tilde{m}}) \quad \forall a_{\tilde{m}} \neq a_m, \text{ and}\\
&B_{z_{k'}}(a^{**}_m(z_{k'})) - C_{z_{k'}}(I^{**}_{z_{k'}}(a^{**}_m(z_{k'}))) \geq B_{z_{k'}}(a^{**}_m(z_k)) - c^{**}_{k'k} \quad \forall k' \neq k.
\end{aligned}
$$

To see this, form the Lagrangian from the program for both possible z_k and add them together.[13] The value of these summed Lagrangians is the same as the value of the Lagrangian in the initial program (3). Moreover, if μ_k is the Lagrange multiplier on the individual rationality constraint in (6) and μ is the Lagrange multiplier on the individual rationality constraint in (3), then $\mu_k = \mu$.

The program given by (6) for each possible signal shows that as long as the μ_k are not equal the principal can adjust an announcement-contingent contract to reduce her cost and reduce the agent's utility. This fraction, equal to μ at the optimal solution to (3), is the marginal cost of utility provision at the current ex ante contract. If the agent receives more than $\bar{U}$ from some ex ante contract, then the principal can adjust one or more announcement-contingent contracts by lowering the wages, and thereby the expected utility from that announcement-contingent contract and thus of the ex ante contract. This is possible as long as the minimum wage associated with each announcement-contingent contract is greater than $\underline{I}$. ■

Proof of Proposition 1.

Simple algebra shows that expected revenues are the same – that is, $B_{\lambda_0}(a_m) = prob(z_1)B_{z_1}(a_m) + prob(z_2)B_{z_2}(a_m)$. Thus, we only need to focus on the costs.

In *Private Information*, if the principal implements the same action, then her cost is necessarily $C_{\lambda_0}(I^{*}_{\lambda_0})$, the same as it is in *Null Information*.

[13] By assumption, the last constraint does not bind for Π_{z_2}.

If better information induces her to implement different actions, then

$$prob(z_1)\Big(B_{z_1}(a_m(z_1)) - C_{z_1}(\hat{I}_{z_1}(a_m(z_1)))\Big) + prob(z_2)\Big(B_{z_2}(a_m(z_2)) - C_{z_2}(I^*_{z_2}(a_m(z_2)))\Big) > B_{\lambda_0}(a_{\tilde{m}}) - C_{\lambda_0}(I^*_{\lambda_0}(a_{\tilde{m}}))$$

where $a_m(z_1) \neq a_m(z_2)$ and $a_{\tilde{m}}$ may equal $a_m(z_1)$ or $a_m(z_2)$ or neither. $\{\hat{I}_{z_1}(a_m(z_1)), I^*_{z_2}(a_m(z_2))\}$ is feasible; however, the principal can do better with $\{\hat{\hat{I}}_{z_1}(a_m(z_1)), I^{**}_{z_2}(a_m(z_2)),\}$. ■

Proof of Lemma 2.

Note that $\sum_{n=1}^N \pi_{\lambda' n}(a_m)V(I^*_{\lambda' n}) - a_m = \bar{U}$ and $\sum_{n=1}^N \pi_{(1-\lambda')n}(a_m)V(I^*_{(1-\lambda')n}) - a_m = \bar{U}$, so that

$$p_{\lambda'}\Big(\sum_{n=1}^N \pi_{\lambda' n}(a_m)V(I^*_{\lambda' n}) - a_m\Big) + (1-p_{\lambda'})\Big(\sum_{n=1}^N \pi_{(1-\lambda')n}(a_m)V(I^*_{(1-\lambda')n}) - a_m\Big) = \sum_{n=1}^N p_{\lambda'}\pi_{\lambda' n}(a_m)V(I^*_{\lambda' n}) - p_{\lambda'}a_m + \sum_{n=1}^N (1-p_{\lambda'})\pi_{(1-\lambda')n}(a_m)V(I^*_{(1-\lambda')n}) - (1-p_{\lambda'})a_m = \bar{U}$$

In addition, $\forall \tilde{m} \neq m$, $\sum_{n=1}^N(\pi_{\lambda' n}(a_m) - \pi_{\lambda' n}(a_{\tilde{m}}))V(I^*_{\lambda' n}) \geq a_m - a_{\tilde{m}}$ and $\sum_{n=1}^N(\pi_{(1-\lambda')n}(a_m) - \pi_{(1-\lambda')n}(a_{\tilde{m}}))V(I^*_{(1-\lambda')n}) \geq a_m - a_{\tilde{m}}$, so that

$$p_{\lambda'}\Big(\sum_{n=1}^N(\pi_{\lambda' n}(a_m) - \pi_{\lambda' n}(a_{\tilde{m}}))V(I^*_{\lambda' n})\Big) + (1-p_{\lambda'})\Big(\sum_{n=1}^N(\pi_{(1-\lambda')n}(a_m) - \pi_{(1-\lambda')n}(a_{\tilde{m}}))V(I^*_{(1-\lambda')n})\Big) \geq p_{\lambda'}(a_m - a_{\tilde{m}}) + (1-p_{\lambda'})(a_m - a_{\tilde{m}}) = a_m - a_{\tilde{m}}$$

Thus, an announcement-contingent contract that gives the agent $I^*_{\lambda' n}$ with probability $p_{\lambda'}\pi_{\lambda' n}(a_m)$ and $I^*_{(1-\lambda')n}$ with probability $(1-p_{\lambda'})\pi_{(1-\lambda')n}(a_m)$, when summed over all possible outcomes, would give the agent exactly his reservation utility and would implement a_m. But, I^*_λ is the least-cost contract among those that satisfy individual rationality and incentive compatibility if the agent believes the principal is Π_λ. ■

Proof of Proposition 2.

As the principal implements a_m both given z_2 and given z_1 with ζ', $\{I^*_{\lambda_0}, I^*_{\lambda_0}\}$ is the equilibrium ex ante contract since this is the least-expensive contract that implements a_m for either signal and provides the agent $\bar{U}$.

Let $\zeta > \zeta'$. If with ζ, the principal implements the same action profile, then she does so with $\{I^*_{\lambda_0}, I^*_{\lambda_0}\}$ and earns the same profit. If she implements a different action profile, then her profit is:

$$prob(z_1)\Big(B_{(z_1|\zeta)}(a_m(z_1)) - C_{(z_1|\zeta)}(\hat{I}_{(z_1|\zeta)}(a_m(z_1)))\Big) + prob(z_2)\Big(B_{(z_2|\zeta)}(a_m(z_2)) - C_{(z_2|\zeta)}(I^*_{(z_2|\zeta)}(a_m(z_2)))\Big) > B_\lambda(a_m) - C_\lambda(I^*_\lambda(a_m))$$

She is necessarily better off since the same ex ante contract is feasible. ■

Proof of Proposition 3.

Consider part (a). As with the first part of the proof of Proposition 1, the principal's equilibrium profits in *Null Information* and in *Private Information* are equal. As (4) shows, a spread of $\lambda(z_2)$ and $\lambda(z_1)$ to 0 and 1 increases the principal's cost from implementing a given action. Each ex ante contract provides the same utility, but the ex ante contract in *Public Information* is a mean-utility preserving increase in risk of the ex ante contract in *Null Information.* That is, the agent would be indifferent between getting $I^*_{\lambda_0 n}$ with probability $\pi_{\lambda_0 n}(a_m)$, and having the gamble $I^*_{z_1 n}$ with probability $\lambda_0 \pi_{z_1 n}(a_m)$ and $I^*_{z_2 n}$ with probability $(1-\lambda_0)\pi_{z_2 n}(a_m)$. The agent's risk aversion implies that this second gamble is more costly than the first. □

For part (b), because the principal implements different actions after she receives z_1 versus z_2 in *Private Information*, the ex ante contract is necessarily separating. In *Public Information*, she could implement the same actions and thereby receive the same revenues, but at a lower cost since I_{z_1} and I_{z_2} do not have to satisfy the additional principal incentive compatibility constraint. If she implements a different action profile in *Public Information* than she does in *Private Information*, then she earns even greater profit.

Finally, since she could implement the same action in *Private Information* and earn the same profit as in *Null Information* but she chooses to implement different actions, she must earn greater profit in the former than in the latter. ■

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