

A Model of International Cities: Implications for Real Exchange Rates

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Abstract

We develop a model of international cities with each city inhabited by two representative agents, one specializing in manufacturing, the other in distribution. The manufactured good is exported from the city of production to all destination cities. The distribution sector represents the physical transformation of all internationally traded goods from the factory gate to the final consumer. We allow the distribution share to vary across goods. Using a panel of micro-prices at the city level, we decompose the long-run cross-sectional variance of LOP deviations into the fraction due to distribution costs, trade costs and a residual. For the median good, we find trade costs account for 50 percent of the variance, distribution costs account for about 10 percent with 40 percent of the variance unexplained. Since the sample of items in the Economist Intelligence Unit survey is heavily skewed toward traded goods, we also decompose the variance based on the median good on an expenditure-weighted basis. Now the tables turn, with distribution accounting for 43 percent and trade costs 36 percent and 21 percent of the variance unexplained.

JEL Classification: E30, F31, D41

Key Words: Real Exchange Rates; Price Dispersion; Law-of-One-Price; Competitive Equilibrium; Cities

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1. Introduction

According to the U.S. National Income and Product Accounts, expenditure by consumers at the retail level is about twice what producers receive for the same goods and services. This difference has come to be called the distribution margin. The distribution margin includes transportation costs from the factory gate to the final point of consumption as well costs and markups at the wholesale and retail stage.

Most existing international models abstract from the distribution sector entirely and focus on the fraction of transportation costs attributable to international shipments. Abstracting from the distribution sector is problematic for three reasons. First, the distribution sector may help us to understand the large and persistent deviations from the Law-of-One-Price and Purchasing Power Parity. Second, the general equilibrium interaction of the distribution sector and the production sector is not well understood. Given the prominent role of the dichotomy between traded and non-traded goods in international finance, this is an important omission. Recent evidence also suggests that information technology and scale economies in distribution have altered the efficiency and markup structure of the distribution sector. These developments may have fundamentally altered price dispersion and dynamics, both across locations within countries and across countries. Third, the distribution sector includes the financial, legal, medical and education sectors which have grown immensely in economic importance over time. Much of this market activity is geographically segmented due either to the arms-length nature of the exchange, public policy decisions or some combination of the two. As these sectors tend to rise in expenditure share with the level of development, their economic importance continues to rise globally.

We have two related goals in this paper. The first is to develop a tractable stochastic general equilibrium model of production and distribution at the microeconomic level of individual goods and services across cities. The second is to use the theory to specify a regression model to estimate microeconomic parameters of a cost function which includes the distribution sector at the level of individual goods and services. This cost function is used to conduct a variance decomposition of prices across cities into a distribution margin, a trade cost margin and a residual, good-by-good. The distribution margin is further parsed into the influences of labor and retail infrastructure costs across cities. The Economist Intelligence Unit retail price data along with supplementary sources for wages at the city level are used in the empirical work. Since the model assumes perfect competition and abstracts from official barriers to trade, the residual in the regression equation is expected to include markups, official barriers to trade and measurement error.

The model assumes that each city is inhabited by two representative agents, a manufacturer and a retailer. The manufacturer produces a single homogeneous good using labor as the only input, the good is shipped to all other cities of the world and deviations in the prices of these traded goods reflect only shipping costs from the factory door to the receiving dock at the retail establishment. The retailer transforms these goods by combining them with her labor and a fixed factor; she may also produce pure services which require no traded inputs at all. The fixed factor is intended to capture fixed retail infrastructure, broadly defined to include land, buildings, equipment and public infrastructure.

The advantage of drilling down to the level of individual goods and services at the city level is that we can learn a great deal about the production structure from the cross-sectional

variance in the data. What distinguishes the manufacturing sector from the distribution sector is intimately related to what distinguishes a personal computer from a haircut. Aggregating the data tends to obscure these differences. For example, if trade costs are symmetric, aggregating across imports and exports has the effect of understating their role. Having cities as the locations allows both greater attention to the spatial dimensions of manufacturing specialization and a more precise measure of the distance between production and consumption locations.

We have two sets of results, one for the sources of LOP variance for the median good, the other for the differences in the sources of variance across goods in the cross-section. For the median good, trade costs account for about 50% of LOP deviations, the distribution margin accounts for about 10% and the remaining 40% is unaccounted for. The relative importance of trade and distribution is fairly stable across sets of locations that include high and local income countries and when comparing within country and cross-border pairs. The absolute level of cross-sectional variance rises when a border is crossed as one would expect or when comparison are made between cities with vastly different wealth levels. One exception is the division of the variance accounted for by the distribution sector into labor and capital infrastructure components. Variance across low income countries is dominated by differences in the infrastructure component, with labor playing a small role. For other countries, the division of the distribution margin across labor and land is similar.

Turning to differences across goods, we find substantial variation at the absolute level of LOP, consistent with Crucini, Telmer and Zachariadis (2005). The structure of the model and methodology allow us to say more about the underlying sources of this heterogeneity. In the international data, the distribution margin accounts for 50% of cross-sectional variance in LOP deviations for the good with the highest distribution share and this fraction falls to a mere 10% as we move to the good with the lowest distribution share. Retail infrastructure accounts for more than 30% of the cross-sectional variance in LOP deviations across Canada and the United States for the good with the highest infrastructure intensity while accounting for virtually none of the variance for the good with the lowest infrastructure intensity.

Our theoretical model is closest to Giri (2007) who adds a good-specific distribution cost onto the baseline Eaton-Kortum structure. In his model distribution services are in fixed proportion to the physical units of the base good as in Burstein, Neves and Rebelo (2003) (BNR) with efficiencies drawn from a distribution with a country-specific mean and common world-wide variance. In contrast, we assume that the technological parameter for distribution inputs is good-specific. Given that this margin is measurable in the NIPA, we view this as a more plausible way to model the distribution sector than the random efficiency approach. Our model shares with Alvarez and Lucas (2007), Atkeson and Burstein (2007), Eaton and Kortum (2002), and Kanda Naknoi (2008) an interest in the role of traditional trade costs. However, to the extent these papers incorporate a distribution sector, it is a common wedge across all goods in the retail basket, which assumes away any cross-sectional variance in price deviations due to the distribution margin. We find this heterogeneity to be essential in our empirical work.

2. The Model

Each city, indexed by j , is inhabited by two representative agents. As is usual in representative agent frameworks, these two agents should be viewed as stand-ins for a large number of atomistic agents of each type, since we will be assuming perfect competition in all factor and final goods markets throughout. One agent specializes in the production of a single traded good, also indexed by j , while the other specializes in retail trade and production of non-traded services. Production in the manufacturing sector is proportional to labor input, the factor of proportionality is a random productivity variable. Retail production requires both labor and infrastructure. Productivity varies across cities in both the traded goods sector and the retail sector.

Traded goods are subject to iceberg transportation costs which are good and destination specific. Final goods and local inputs (retail labor and infrastructure) are not traded beyond the city limits. While hours and consumption are both choice variables, the assumptions we make in the model imply constant hours in all sectors in all locations, reminiscent of the Long and Plosser (1983) multi-sector, closed economy, real business cycle model. Retail infrastructure, including land, capital and equipment, is in fixed supply (denoted K).

The good index, i , distinguishes physical objects from the identities of agents and locations only when needed to avoid confusion. In describing the flow of goods from one location to another, the source is the first subscript and the destination is the second subscript. Thus, X_{sd} refers to the shipment of good X , from city s to city d . Given the assumption that individuals at each location specialization, s also indicates the good and the individual to whom the income flows, while d indicates the expenditure side of the equation. τ_{sd} is the iceberg shipping cost from the source to the destination. Since there are no durable goods or assets in the model, adding time subscripts is innocuous: they are omitted here to focus on the steady-state properties of the model and to simplify notation.

The full solution for quantities and prices is given in the model appendix. This section presents the overall model and parts of the solution relevant for pricing implications, which is the focus of our empirical work.

2.1. Consumers

Agents preferences are log-additive over consumption and leisure:

$$U(C_j^A, L_j^A) \equiv (1 - \theta) \log C_j^A + \theta \log L_j^A, \quad A = m, s. \quad (2.1)$$

C_j^A is aggregate consumption and L_j^A is hours of leisure, for an individual working in city j . There are two individuals in each city, indexed by $A = m, s$; one is engaged in the manufacture of a single good (m) and the other is engaged in retailing and service activities (s).

The consumption aggregate is CES over varieties of manufactured goods produced worldwide:

$$C_j^A = \left(\sum_i^M (\beta_i)^{\frac{1}{\varepsilon}} (C_{ij}^A)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}. \quad (2.2)$$

C_{ij}^A is the consumption of good i in city j by worker type A ; $\varepsilon > 0$ is the elasticity of substitution across goods, β_i is a good specific taste parameter and M is the number of

manufactured goods in existence. M is also the number of cities given our specialization assumption.

The two agents inhabiting city j , maximize utility (2.1) subject to their respective budget constraints:

$$\sum_i P_{ij} C_{ij}^A \leq W_j^A N_j^A + \varphi^A H_j K_j \quad (2.3)$$

where P_{ij} is the price of good i in destination city j . These prices will be the same for all agents in the same location, but differ across locations for reasons described below. Each of the two residents of city j earn labor income from their production activities and split the rental income accruing to the retail infrastructure in their city ($\varphi^m + \varphi^s = 1$), the stock of which is assumed to be fixed at K_j . The unit price of retail infrastructure is denoted H_j .

The consumer's problem may be solved in two stages. In the first stage, the consumer chooses aggregate consumption and leisure, subject to a budget and time allocation constraint. In the second stage, the consumer minimizes expenditure across goods. Here we collapse the problem to a single stage for brevity. The key equations from the solution to the consumer's problem are:

$$C_{ij}^A = \beta_i \left(\frac{P_{ij}}{P_j} \right)^{-\varepsilon} C_j^A \quad (2.4)$$

$$C_j^A = \frac{W_j^A N_j^A + \varphi^A H_j K_j}{P_j} \quad (2.5)$$

$$N_j^A = 1 - \theta - \varphi^A \theta \frac{H_j K_j}{W_j^A} \quad (2.6)$$

$$L_j^A = \theta + \varphi^A \theta \frac{H_j K_j}{W_j^A} \quad (2.7)$$

Aggregate real consumption is nominal consumption deflated by the ideal deflator $P_j \equiv (\sum_i \beta_i (P_{ij})^{1-\varepsilon})^{\frac{1}{1-\varepsilon}}$, which ensures $\sum_i P_{ij} C_{ij} = P_j C_j$ as well as a mapping from price indices to welfare.

The first equation determines consumption demand for a particular good as a function of the relative price of the good paid by the final consumer and that individual's aggregate consumption level. It is important to note that, P_{ij} is the retail price of good i , in city j ; it embodies the cost of local retail services paid to the retailer in addition to the traditional iceberg trade costs of the imported item. The price index, is a weighted average of these retail prices, the closest empirical counterpart would be the CPI index. The second equation is aggregate consumption of an agent, which is equal to her real income. Real income is the sum of nominal wage and rental income, deflated by the price level.

The last two equations determine hours of work and leisure. In the absence of rental income, the two agents would work the same number of hours, independent of their relative wage, due to the offsetting income and substitution effect of wages on effort with Cobb-Douglas preferences. In the presence of rental income the requirement for constant effort in equilibrium is that the ratio of rental income to labor income be constant. Most growth models impose restrictions on tastes and technology to ensure constancy of hours per capita

in the presence of tending productivity.¹

2.2. Manufacturers

The production function for manufactured good, i , is:

$$Y_i = A_i N_i^m \quad (2.8)$$

where A_i is the productivity level and N_i^m is hours of work.

Manufacturers choose labor input to maximize profit:

$$\max_{N_i^m} (Q_{ii} Y_i - W_i^m N_i^m) . \quad (2.9)$$

The manufacturer receives the factor gate price, Q_{ii} , for every unit produced, no matter where the goods end up being sold. Given our assumptions of constant returns to scale, perfect competition and one factor of production, the factory gate price equals the manufacturing wage divided by productivity:

$$Q_{ii} = \frac{W_i^m}{A_i} . \quad (2.10)$$

Given our assumption of specialization, the productivity level in this expression is good and city-specific. The presence of a nation-specific component could easily be incorporated by allowing A_i to have a common factor across cities within a country.

Retailers in each city purchase the manufactured goods and pay a proportional shipping cost. Thus the retail purchase price is the factory-gate price marked up by a proportional shipping cost:

$$Q_{ij} = (1 + \tau_{ij}) Q_{ii} = (1 + \tau_{ij}) W_i^m / A_i \quad (2.11)$$

where $\tau_{ij} > 0$ is the net transportation cost from city i to j . Q_{ij} is the retail purchase price in the destination city. The local manufacturing plant is close enough to the city to ignore local transportation costs so that $\tau_{ii} = 0$. The destination price of the manufactured good depends: i) positively on both the manufacturing wage and the trade cost; and ii) negatively on manufacturing productivity.

2.3. Retailers

The retailer in each city optimally chooses how much of each manufactured good to purchase from various cities of the world. The retailer transforms these goods using a fraction of her time endowment and some amount of the local retail infrastructure. The retailer then sells the resulting composite good in the local retail market. The production function for good i , sold in city j is:

$$R_{ij} = ((B_j N_{ij}^s)^{\gamma_i} (K_{ij})^{1-\gamma_i})^{1-\alpha_i} (G_{ij})^{\alpha_i} \quad (2.12)$$

G_{ij} is the amount of the manufactured good (or input) imported from city i , by a retailer in city j . N_{ij}^s is the fraction of the retailer's time endowment allocated to the transformation of

¹Details of these restrictions in the context of the one sector stochastic growth model may be found in King, Plosser and Rebelo (1988).

imported good i for local consumption in city j and K_{ij} is the amount of retail infrastructure allocated to retail good i in city j . B_j is labor-augmenting productivity specific to the city (equivalently, the retailer), common to all goods sold there.²

While the production function is restricted to be common to all locations, it is very flexible across goods. It captures pure labor services (e.g., baby-sitting services) with α_i equal to zero and γ_i equal to one; internet purchases (e.g., Amazon.com book purchases), α_i equal to one, and all points in between.

The retailer in city, j , maximizes profits by choosing the quantity of the traded input, G_{ij} , to import and hours and retail infrastructure to allocate to each activity:

$$\max_{N_{ij}^s, K_{ij}, G_{ij}} (P_{ij}R_{ij} - W_j^s N_{ij}^s - H_j K_{ij} - Q_{ij}G_{ij}) \quad (2.13)$$

At the optimum, the unit price equals marginal cost. Given constant returns to scale and three factors of production, the retail price of good i sold in location j , is a Cobb-Douglas aggregate of the price (inclusive of trade cost) that the retailer paid to acquire the traded input, Q_{ij} , the retailer's market wage, W_j^s , and the price of retail infrastructure, H_j :

$$P_{ij} = MC_{ij}^s = \phi_i \left((W_j^s/B_j)^{\gamma_i} (H_j)^{(1-\gamma_i)} \right)^{(1-\alpha_i)} (Q_{ij})^{\alpha_i} \quad (2.14)$$

$$\phi_i^{-1} \equiv \alpha_i^{\alpha_i} \left((1-\alpha_i) (\gamma_i)^{\gamma_i} (1-\gamma_i)^{(1-\gamma_i)} \right)^{(1-\alpha_i)} \quad (2.15)$$

The retail price in city j is rising in input prices and falling in retail productivity, B_j .

2.4. Equilibrium

The appendix contains the tedious algebra necessary to arrive at the equilibrium allocations discussed in this section. In the remainder of the paper the consumption aggregator is restricted to Cobb-Douglas to arrive at closed form solutions.

In the global general equilibrium, all the optimality conditions of partial equilibrium must hold for consumers, retailers and manufacturers. In addition, the supply of each good must equal its total demand, including the resources lost to iceberg shipping costs.

$$Y_i = \sum_j G_{ij}(1 + \tau_{ij}) \quad (2.16)$$

$$= \sum_j G_{ij} + \sum_j G_{ij}\tau_{ij} \quad (2.17)$$

$$= G_i + T_i \quad (2.18)$$

In words: the production of good i , Y_i , is exhausted between the global demand for that good by retailers aggregated across destinations indexed by j , G_i , and physical losses due to iceberg costs, T_i .

²In principle one could add good-specific productivity of retailers to account for different levels of competency across goods, but we lack productivity data to operationalize this idea.

Each individual has a fixed amount of time to devote to hours of work or leisure, here, normalized to unity. The time constraints for the manufacturer and the retailers are:

$$L_j^m + N_j^m = 1 \quad (2.19)$$

$$L_j^s + \sum_i N_{ij}^s = 1 . \quad (2.20)$$

The summation in the second time constraint reflects the fact that the retailer must divide her time across the M different retailing activities. The notation implicitly sets the number of goods at the retail level equal to the number of goods in the manufacturing sector. Nothing we derive requires this: we could have some activities that use no traded inputs at all in which case the number of retail goods would exceed the number of manufacturing goods by the number of pure services produced in each city.

The city's retail infrastructure is exhausted across uses:

$$K_j = \sum_i K_{ij} .$$

3. The data

Our focus is retail price dispersion across international cities at the microeconomic and macroeconomic level. The data source for prices is the Economist Intelligence Unit (EIU) worldwide retail price survey. The survey spans 123 cities, located in 79 countries. Most of the cities are national capitals. The larger number of cities than countries is due to the fact that the survey also includes multiple cities in a few countries. Noteworthy are the 16 U.S. cities included in the survey; the next largest number of cities surveyed equals 5 in Australia, China and Germany. Up to data availability for particular years and cities, the number of goods and services priced is 301. The available sample is 1990 to 2005. We work with time-averaged logarithms of LOP deviations across bilateral city-pairs:

$$q_{ijk} = T^{-1} \sum_t q_{ijk,t} = T^{-1} \sum_t \ln(S_{jk,t} P_{ij,t} / P_{ik,t}) . \quad (3.1)$$

Figure 1 plots four kernel estimates of $q_{ijk,t}$, in 2005. Two of the distributions are confined to city pairs lying within the U.S. We focus on U.S. city pairs because the U.S. is the country with the largest number of cities surveyed. The distributions using a broader set of intranational pairs is not materially different. The other two distributions are confined to city pairs that span a national border. We see a clear ranking of price dispersion, with U.S. traded goods being least dispersed, U.S. non-traded goods next, followed by the same ranking of international traded and non-traded goods.

The EIU survey offers little in the way of wage data. Supplemental wage data at the country level come from the International Labor Organization (ILO) survey of occupational and sectoral wages and at the city level from the Union Bank of Switzerland (UBS) survey. The ILO data are averages for countries. They span 49 sectors, 162 occupations and 137 countries. The sample period is annual from 1983 to 2003. The complete list of these sectors, occupations, and countries is found in Oostendorp (2005). In the raw ILO data, the most common period is the month, followed by the hour, but some countries report weekly pay,

others give daily rates for some occupations, and so on. In order to have a comparable wage data across countries, the standardized version of ILO survey by Oostendorp (2005) is used: in cases in which the wage data are reported as hourly or daily, then these wages were made (roughly) comparable with monthly wages by multiplication by 160 and 20 respectively. In order to have the maximum panel size of wage data that are comparable across countries, the monthly wages in US dollars that have been obtained by country-specific and uniform calibration in Oostendorp (2005) are used.

Wage data at the city level is more appropriate given the EIU retail price data is city based and the intent of the model. International cities were surveyed by the UBS in 2006. These are hourly wages in US dollars, spanning occupations in 71 international cities, 60 of which are also surveyed by the EIU. Among the 60 EIU cities there two cities from Brazil, Canada, China, France, Italy, Spain and Switzerland; three cities from Germany, and 4 cities from the U.S. The hourly wages have been obtained by dividing the income per year in each occupation by the city level hours of work in a year, where the latter we collected by a survey, also conducted by the UBS.

Bureau of Labor Statistics (BLS) city wage data from the Occupational Employment Statistics (OES) Survey in 2006 are used to complement UBS data. These wage data are hourly wages in US dollars for the same 16 US cities found in EIU retail price survey. The combination of UBS and BLS wage data, then, provides wage data for 72 EIU cities, comprised of 16 from the BLS and the remainder (non-U.S. cities) from the UBS. Within these 72 EIU cities, in terms of intranational cities, we have two cities from Brazil, Canada, China, France, Italy, Spain and Switzerland; three cities from Germany, and 16 cities from the US.³

In a preliminary part of the analysis, the BLS city wage data are used for broader wage dispersion analysis. These data cover 2 industries, namely production and sales, for 400 cities (on average) within the U.S. in terms of hourly wages from 1999 to 2006.

A number of trade-offs present themselves in terms of the model focus and the available data. Country-level wage data is generally available for more time periods, but fewer locations than city-level wage data. Since the model is explicitly constructed to mimic city level aggregation and steady-state features, ideally one would want long time series at the city level. Unfortunately these are simply not available. These trade-offs are discussed as they arise below.

Land prices and rents are even more difficult to come by than are wages and prices. We use two entries from the EIU survey data: "Typical annual gross rent for top-quality units, 2,000 square meters, suitable for warehousing or factory use" and "Typical annual gross rent for a 1,000 square meter unit in a Class A building in a prime location."

The other two pieces of information are sectoral estimates of the traded-input shares, α_i which are calculated from a combination of U.S. NIPA data and input-output tables. The NIPA data extend to 57 sectors, while the input-output data span 33 sectors. The NIPA shares are computed as the value the producer receives relative to the value consumers pay for the output of a particular sector. The distribution margin, $1 - \alpha_i$, includes transportation

³In an earlier version of this paper we used PWT per capita annual income data covering the annual period from 1990 to 2004 to proxy for real wages. These data span all 79 EIU countries. The results were qualitatively similar to those reported here.

costs, retail and distribution costs and markups. Sectors involving arms-length transactions, such as medical services are recorded in the NIPA as though the producer and consumer valuation is equal. While this is literally true in some cases, this accounting fails to distinguish local inputs and traded inputs used in the production of services. For these sectors we use the input-output tables to determine the distribution share.

These sectoral parameter measures are used in conjunction with good-level parameters estimated using a regression framework discussed below. Finally, the greater circle distance between cities in the EIU sample is used to estimate the trade cost component of the LOP deviations at the retail level. In the future, these trade cost estimates will be compared to measures constructed by Hummels (2000) and others.

4. Microeconomic sources of long-run variation in wages

In the model, wage deviations arise across the retail and manufacturing sectors and across cities. The amount of labor income accruing to the manufacturer relative to the retailer in city j is,

$$\frac{N_j^m W_j^m}{N_j^s W_j^s} = \frac{\sum_i \alpha_i \beta_i}{\sum_i (1 - \alpha_i) \gamma_i \beta_i} \quad (4.1)$$

Which is intuitive: the numerator is an expenditure share weighted average of labor's share of manufacturing and the denominator is the counterpart in retailing. The appearance of the parameter γ_i in the denominator accounts for the fact that retail production involves some retail infrastructure, unless $\gamma_i = 1$, in which case production is labor-only. Note, also, that the ratio is the same in all cities.

As the primary interest is wage variation in order to account for cost variation across cities, we would like to understand the wage ratio and effort ratios separately. The equilibrium relative *sectoral* wage is given by:

$$\frac{W_j^m}{W_j^s} = \frac{\varphi^m (1 - \theta) - N_j^s}{\varphi^s (1 - \theta) - N_j^m}$$

Thus, given fixed shares of rental income across agents in the city, relative wages and relative hours move inversely as one would expect. The appendix shows that the equilibrium effort levels are:

$$\begin{aligned} N_j^m &= \frac{(1 - \phi_0) (1 - \theta)}{(1 - \phi_0) + \varphi^m \theta (\phi_0 - \phi_1)} \\ N_j^s &= \frac{\phi_1 (1 - \theta)}{\phi_1 + \varphi^s \theta (\phi_0 - \phi_1)} \end{aligned}$$

where $\phi_0 \equiv \sum_i (1 - \alpha_i) \beta_i$ and $\phi_1 \equiv \sum_i (1 - \alpha_i) \gamma_i \beta_i$. Effort in either sector is declining in the share of rental income allocated to the agent, and declining in the preference for leisure (θ), as one would expect.

Substituting these expressions into the wage ratio eventually leads to the following expression for relative wages:

$$\frac{W_j^m}{W_j^s} = \frac{\varphi^s (1 - \phi_0) + \varphi^A \mu}{\varphi^m \phi_1 + (1 - \varphi^A) \mu}$$

$\mu = \theta (\phi_0 - \phi_1)$.

As the retail sector becomes more labor intensive (thus rental income falls), $(\phi_0 - \phi_1)$ converges to zero and the mode reverts to the labor-only version with a common fraction of available hours worked by both agents, equal to $(1 - \theta)$ and the sectoral wage ratio converges to:

$$\frac{W_j^m}{W_j^s} = \frac{1 - \phi_0}{\phi_0} = \frac{\sum_i \alpha_i \beta_i}{\sum_i (1 - \alpha_i) \beta_i}$$

Turning to wage differences *across* cities things are much simpler even in the general case:

$$\frac{W_j^s}{W_k^s} = \frac{\alpha_j \beta_j}{\alpha_k \beta_k} \quad (4.2)$$

$$\frac{W_j^m}{W_k^m} = \frac{\alpha_j \beta_j}{\alpha_k \beta_k} . \quad (4.3)$$

The cross-city wage differential is the same in both sectors and is determined by the product of the taste and technology parameters in the two locations being compared. The intuition for this result is as follows. Consider, first, the special case in which all goods use traded inputs in the same proportion, $\alpha_j = \alpha$. The bilateral wage differential then favors the location whose goods are most preferred, β_j versus β_k , a demand-side effect. Next, consider the case in which there is no taste bias across goods $\beta_j = \beta$; then wages are highest for producers of manufactures with the greatest amount of traded input at the distribution stage. Essentially, this determines the productivity of manufacturing effort in delivering a unit of final consumption.

Wage data is available by occupation or sector of employment. Our model focuses on the distinction between goods and services, suggesting the production sector definition is more appropriate. However, we use both labor classifications as a robustness check.

The more comprehensive of the sources used is the ILO survey of wage levels across *countries*. These data span 49 sectors, 162 occupations and 137 countries. The sample period is annual from 1983 to 2003.⁴ Because the model is intended to be based on city-level data, the preferred measure is wage data from the UBS that span 14 occupations and 71 international cities for the year 2006.

According to the model, if the retail sector uses only labor and traded goods, the ratio of manufacturing wages to service wages provides an estimate of the overall scale of the distribution sector $W_j^m/W_j^s = \kappa/(1 - \kappa)$, $\kappa = \sum_i \beta_i \alpha_i$. Since we lack consumption expenditure shares at the present time, we associate this with the distribution share alone, using the symmetric taste version of the model, κ reduces $\bar{\alpha}$. A direct way to measure the overall size of the distribution sector is to use U.S. NIPA data and input-output data. Crucini and Shintani (2008) do exactly this and find $\kappa = 0.57$. The advantage of their calculation is that it is based on expenditure weighting of sectoral α 's.

Table 1 reports the sectoral wage ratio averaged across locations as well as the implied value for κ . It turns out that the direct and indirect (model-based) estimates are equal when U.S. wages in production sector relative to the sales sector are used. The wage ratio in the international data is consistent with a value of 0.52. While this is a modest difference from

⁴Useful technical documentation is found in Remco H. Oostendorp (2005).

the U.S. value, the implied manufacturing wage premium is much smaller: a factor of 5 smaller. It could be that relative productivity differences are the cause. Another possibility is that the U.S. and international data differ due to somewhat different classification systems for the sectors.

As the theory is a two-sector model, any sectoral variation in wages at a particular city is attributable to the ratio of wages in the manufacturing sector to those in the service sector. Variation in wages across sub-sectors are abstracted from entirely. Thus, it is important for the theory that wages differ significantly across locations and less so across sectors other than the two in the model. And this is what is found.

Table 2 conducts a variance decomposition by sector and country for time-averaged wages in the case of the ILO survey and an analogous decomposition by sector and city for wages in 2006 for the UBS survey data. Since the answer may depend on the set of sectors and locations used, we consider three location groups and allow for many sectors throughout. The location groups are the entire world, the OECD and the LDC.

Based on the ILO wage data: locations account for between 72 and 85 percent of the cross-sectional variation in wages, sectoral differences account for less than 10 percent. The relative importance of location is somewhat lower when the data is organized by occupation, with location effects dropping to between 38 and 65 percent. Most of difference is not attributed to a pure sectoral component, but rather an interaction of location and sector. The UBS tell a similar story to the ILO for location effects, with the occupation effect rising in contribution due to a lower interaction with location compared to the ILO.

In sum, location is a key component of wage dispersion with the precise fractions depending somewhat on the set of locations examined and the precise definition of wage categories.

5. Microeconomic sources of long-run variation in real exchange rates

We turn, now, to the main focus, price dispersion. In the model, prices consumers actually pay may differ from factory gate prices for two reasons. The first is the trade cost to import the good from the foreign production location. The second is the value added by the retailer. To simplify the notation, all international prices have been converted to common currency units (it does not matter which numeraire is chosen). The ratio of the price of good i in city j relative to k , based on the theory is:

$$\frac{P_{ij}}{P_{ik}} = \left(\frac{W_j^s/B_j}{W_k^s/B_k} \right)^{\gamma_i(1-\alpha_i)} \left(\frac{H_j}{H_k} \right)^{(1-\gamma_i)(1-\alpha_i)} \left(\frac{Q_{ij}}{Q_{ik}} \right)^{\alpha_i}.$$

Noting that the first term reduces to the ratio of trade costs from the single source of good i to each of the destinations, j and k and taking logs, defines the Law-of-One-Price deviation across a city pair:

$$\begin{aligned} q_{ijk} &= (1 - \alpha_i) [\gamma_i \omega_{jk} + (1 - \gamma_i) h_{jk}] + \alpha_i \tau_{ijk} \\ &= \rho_i \omega_{jk} + \chi_i h_{jk} + \alpha_i \tau_{ijk}. \end{aligned} \tag{5.1}$$

The retail margin is the first term in square braces and is a weighted average of the productivity-adjusted wage and the rental price differential faced by the retailer in each

city. The second term is the relative trade cost. The weights attached to the relative input prices in the retail sector depend on γ_i . The entire retail component gets weighted by its overall share in the production of the final good, $(1 - \alpha_i)$. The last line, used to specify our regression approach, expresses the relationship in terms of the three key cost ratios, retail wages, rental prices and trade costs.

The aggregate real exchange rate relevant for the theory is immediate since the consumption aggregator is Cobb-Douglas (i.e., $\varepsilon = 1$):

$$\begin{aligned} q_{jk} &= \rho\omega_{jk} + \chi h_{jk} + \sum_i \beta_i \alpha_i \tau_{ijk} \\ \rho &\equiv \sum_i \beta_i \gamma_i (1 - \alpha_i), \chi \equiv \sum_i \beta_i (1 - \gamma_i) (1 - \alpha_i) \end{aligned} \quad (5.2)$$

The aggregate real exchange rate has a number of interesting features. The distribution sector components of PPP deviations are driven by the same wage and rental ratios as the LOP deviations, they are just evaluated at the expenditure-weighted means of the LOP deviations. The trade cost component, however is likely to average out across goods since the τ_{ijk} are expected to vary in sign across goods.

5.1. Regression specification

This section conducts a variance decomposition of retail prices into the channels described by the equilibrium model. Adding a measurement error term to the theoretical equation for the LOP deviation, gives:

$$q_{ijk} = \rho_i \omega_{jk} + \chi_i h_{jk} + \alpha_i \tau_{ijk} + \varepsilon_{ijk} \quad (5.3)$$

Data on retail prices, wages and rent, are available, but no data on retail productivity or trade costs exist for this cross-section of locations. The raw wage ratios are used in place of ω_{jk} and a two-stage estimation approach is used to infer the impact of trade costs.

The first-stage regression is:

$$q_{ijk} = \hat{\rho}_{1i} \omega_{jk} + \hat{\chi}_{1i} h_{jk} + \hat{\theta}_{ijk}. \quad (5.4)$$

where $\hat{\theta}_{ijk}$ is an estimated residual, which, according to the model, is the LOP deviation in the traded component of cost. In practice it will incorporate other sources of deviations as well. In an attempt to purge these other factors from the pure trade cost component, the estimated residuals are projected on bilateral distance. To accomplish this, define the direction-of-trade indicator function:

$$\mathbf{I}_{ijk} = \begin{cases} 1 & \text{if } \hat{\theta}_{ijk} > 0 \\ -1 & \text{if } \hat{\theta}_{ijk} < 0 \end{cases} \quad (5.5)$$

where $\hat{\theta}_{ijk} = q_{ijk} - \hat{\rho}_{1i} \omega_{jk} + \hat{\chi}_{1i} h_{jk}$ from the first-stage regression.

Consider, now, the more elaborate equation for stage two:

$$q_{ijk} = \hat{\rho}_{2i} \omega_{jk} + \hat{\chi}_{2i} h_{jk} + \hat{\varsigma}_i \mathbf{I}_{ijk} d_{jk} + \hat{\varepsilon}_{ijk} \quad (5.6)$$

$$\hat{\rho}_{2i} = (1 - \alpha_i) \gamma_i \quad (5.7)$$

$$\hat{\chi}_{2i} = (1 - \alpha_i) (1 - \gamma_i) \quad (5.8)$$

$$\hat{\varsigma}_i = \alpha_i \delta_i \quad (5.9)$$

with the trade cost replaced by $\mathbf{I}_{ijk}\delta_i d_{jk}$. The indicator function ensures the sign of the implied trade cost is consistent with the sign of the residual estimated in stage one. The greatest circle distance between locations j and k is the empirical counterpart to d_{jk} with goods having a different trade cost elasticity with respect to distance, δ_i . The benefit of projecting the prices on wages, rents, and the indicator function multiplying distance is that we relegate any sources of variation in retail prices not correlated with wages, rental prices or distance to the error term. This gives us more confidence that the wage, rental, and trade cost components are capturing what the model says they should.

The model seems best suited to describe the long-run properties of real exchange rates since we abstract from nominal exchange rate variation and sticky prices. While we have a long panel of EIU retail price data from which to construct time-averages and target long-run dispersion, we lack city-level panel data on wages. Moreover, the argument could be made for estimating the parameters with a single cross-section. Our benchmark estimation and variance decomposition uses time-average data as available (i.e., for q_{ijk} and h_{jk}) and wage data for a single cross-section in 2006. Wage data from the UBS is used for cities outside of the U.S. and from the BLS in the case of U.S. cities. Preliminary experimentation with alternatives does not seem to alter the main thrust of the results.

We see in Table 3, that the empirical model captures the majority of long-run variation in retail prices across locations for all groupings of the data. The range of variance accounted for is between 70% and 90% for the median good when pooling all international cities or just those in North America. The quality of the fit of the model is excellent over much of the distribution across goods. The lowest quartile for the R^2 is a very respectable 0.67 (the OECD cross-border pairs). In summary, the empirical model fits well across sub-panels of locations and goods ranging from haircuts to personal computers.

5.2. Variance Decomposition

Interesting and more relevant than explanatory power for the theory is the magnitude and source of variation in the underlying price data. The theoretical model attributes price dispersion to distribution margins, trade costs and potentially their covariance. Using the estimated equations motivated by our theory, we are able to provide a cross-sectional variance decomposition analysis according to the following equation:

$$var_{jk}(q_{ijk}) = \hat{\rho}_{2i}^2 var_{jk}(\omega_{jk}) + \hat{\chi}_{2i}^2 var_{jk}(h_{jk}) + (\hat{\alpha}_i \hat{\delta}_i)^2 var_{jk}[\hat{\mathbf{I}}_{ijk} d_{jk}] + var_{jk}[\hat{\varepsilon}_{ijk}] + \text{cov terms}$$

Consider a good which uses no traded inputs at the retail level ($\alpha_i = 0$). The prediction simplifies reduces to:

$$var_{jk}(q_{ijk}) = \hat{\gamma}_{2i}^2 var_{jk}(\omega_{jk}) + (1 - \hat{\gamma}_{2i})^2 var_{jk}(h_{jk}) + \text{cov terms}$$

and price dispersion is entirely due to retail costs associated with wage and rental price dispersion, $var_{jk}(\omega_{jk})$ and $var_{jk}(h_{jk})$, respectively. These numbers naturally depend on the locations pooled in the estimation and larger variation is found in both wages and prices internationally than internationally. Borders matter.

Consider the opposite end of the continuum, a good with no retail costs at all (e.g., a good available on the internet that trades up to a shipping cost everywhere in the world).

Now the expression for the predicted price dispersion reduces to:

$$var_{jk}(q_{ijk}) = \widehat{\delta}_i^2 var_{jk} \left[\widehat{\mathbf{I}}_{ijk} d_{jk} \right]$$

This is an intriguing expression. The coefficient out front is the elasticity of trade cost with respect to distance (recall, the empirical model assumes a log-linear proportional trade cost function as is typical in the gravity literature). The variance of distance is a function of the set of locations under examination. As bilateral distance become less symmetric (less equal), trade cost matters more for price deviations.

Beginning with the absolute variances, in the first column of Table 4, the lowest retail price dispersion exists within countries, at about 0.06. Keep in mind that the U.S. has the largest number of intranational pairs and therefore dominates the intranational samples throughout the table. International price dispersion is highest when we pool the LDC locations (0.42) and lowest when we focus on North America (0.07).

For the median good, distribution costs account for between 5 and 20 percent of overall price dispersion. The wage component tends to account for more of this dispersion than the rental component. An exception is the LDC group where the rental component accounts for 12.6 percent of the dispersion, compared to only 2.5 percent for wages. Trade costs dominate the picture throughout the table, accounting for as much as 60 percent of the price dispersion for cross-border OECD pairs, to a lower, but still very substantial, 36.1 percent across the Canada-U.S. border. Approximately 30% of the variance is left unaccounted for by the model. This variation could be due to a combination of markup variation, official barriers to trade or measurement error. The covariance across effects is typically less than 5 percent. The bottom line of the analysis of the median good are that trade costs dominate independent of the location or border crossing and that distribution margins are important enough not to ignore.

Variation across goods within the cross-section, is interesting. Figure 2 shows the variance decomposition at the individual good level as a function of the traded input share, α_i . To make these easier to read we have smoothed the profiles by taking centered moving averages of the variance share across 10 goods. For the good with the lowest traded input share (roughly 0.4), wage dispersion accounts for about 45% of price dispersion. As we move to goods with the highest traded input share (roughly 0.97), wage dispersion accounts for almost none of the price dispersion. Of course if this good had literally no non-traded inputs the contribution would necessarily be exactly zero. The OECD group tells a similar story with about 30 percent of price dispersion accounted for by wage dispersion at one end of the continuum of goods and less than 10 percent contributed for goods embodying mostly traded inputs. The Canada-U.S. pairs have a lower contribution from wage dispersion as we would expect given the similar wage levels of the two countries, the contribution of this component also declines as α rises, though not as smoothly as the other groups. In most cases, the falling contribution of wage differences is associated with a rising role for trade costs. The intranational pairs show less heterogeneity in the proportion of variance explained by various components as the trade share of final good production varies. Partly this reflects the lower variance of wages and rent across cities within countries. Nonetheless, the contribution of distribution is not negligible for the intranational pairs either.

Figure 3 displays the same variance decomposition by good plotted against the labor

share of total retail cost, γ_i . We see the dramatic effect of this parameter on the split of distribution margin variance across labor and rent in the case of Canada and the U.S. as well as the the world. As we move across goods based on this parameter, the contribution of rent goes from zero to about 40% in the Canada-U.S. and from zero to about 20% in the world grouping. The contribution of wage dispersion tends to follow the same pattern in reverse, maintaining the total share of price dispersion due to distribution costs. The OECD is anomalous in the sense that the distribution share contributes about 10% without much variation across goods until we reach very high labor intensities in distribution. However, the variation appears to come at the expense of the residual. Turning to the intranational pairs, the contribution of wage dispersion is rising in its production share as one would expect.

The results for the median good in the EIU cross-section seem to downplay the role of distribution costs relative to trade costs. Given the dramatic differences in how the variance decomposition plays out across goods, the natural question that arises is how representative the EIU sample is of the CPI basket. A second issue is the extent to which the estimated distribution share matches up with the direct measures in the NIPA data.

Regarding the second issue, the average estimated value of the distribution share across goods we are able to use in the estimation is 0.2. This value is significantly below, 0.5, the average we get when we merge our micro-data with the U.S. NIPA and use the sectoral values of the distribution share from that source. Moreover, the difference is not due to a few outliers, only 9 out of 160 good are the estimate values above the NIPA counterparts. This suggests that our good-level estimates of the distribution shares are downward biased.

To account for this estimation bias and make the results relevant for aggregate consumption, we recompute our variance decomposition using goods with distribution shares in the neighborhood of $\alpha = 0.5$. Recall, this is the expenditure weighted average of the distribution share found in the U.S. NIPA data. What we do is average the decomposition results across 5 goods on either side of this value. Table 5 reports these findings. We see that the contribution of the distribution margin is much more significant. Wage dispersion alone now accounts for more than one-third of retail price dispersion when all cross-border pairs of cities are pooled (world). The role of wages for the OECD and LDC groupings is more limited suggesting the city pairs that straddle high and low income countries are the reason for the much elevated wage component. It is interesting to note that for the Canada-U.S. pairs, wage dispersion plays a significant role as well. Keep in mind that the absolute dispersion in prices is about 5 times higher for the world group than the Canada-U.S. group, thus the significant role of wage dispersion is partly due to the relatively limited amount of total variation in long-run relative prices across the Canada-U.S. border, compared to broader international comparisons.

6. Conclusions

Consumers face prices that are to a varying degree, location-specific. Our model of production and distribution across cities shows how these differences are shaped by the distances separating cities due to trade costs, the good-specific share of retail distribution and its division between local labor and rental costs. While we found trade costs dominated distribution costs by a factor of 5 to 1 for the median good in the Economist Intelligence Unit micro-

sample, their relative contribution varies greatly across goods. For final goods that involve mostly non-traded inputs, distribution margins dominate trade costs. Given that most of the goods in the EIU have high traded input content, these unweighted average understate the role of distribution margins in the aggregate consumption basket. Using the aggregate distribution share and estimates of the variance decomposition for individual goods with that share, distribution costs actually dominate trade costs.

In future work we will undertake an analysis of PPP using our model and empirical methodology. We expect the distribution margin will dominate trade costs in this case as well. These findings point to the importance of incorporating a distribution sector into existing international trade and macroeconomic models.

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Table 1. Mean sectoral wage differentials

	W_j^m/W_j^s	Implied κ
World Manufacture to Sales (ILO)	1.07	0.52
U.S. Production to Sales (BLS)	1.34	0.57

Note: For details on the data sources, see the data appendix.

Table 2. Variance of wage differentials across sectors and locations

	Industry (ILO)			Occupation (ILO)		
	Location	Sector	Error	Location	Sector	Error
World						
Proportion of variance	0.85	0.04	0.10	0.65	0.01	0.34
Observations	46	19		136	113	
OECD						
Proportion of variance	0.84	0.06	0.10	0.64	0.03	0.33
Observations	27	12		26	113	
LDC						
Proportion of variance	0.72	0.08	0.20	0.38	0.04	0.58
Observations	36	19		109	113	
				Occupation (UBS)		
World						
Proportion of variance				0.66	0.19	0.15
Observations				56	14	
OECD						
Proportion of variance				0.51	0.31	0.18
Observations				32	14	
LDC						
Proportion of variance				0.48	0.29	0.23
Observations				24	14	

Notes: A panel has been selected such that the total number of observations is maximized.

Table 3 - Explanatory power

	First quartile	Median	Third quartile
Panel A: International cities, cross-border pairs			
CANADA-US	0.83	0.90	0.94
OECD	0.67	0.71	0.74
LDC	0.70	0.73	0.75
WORLD	0.69	0.72	0.75
Panel B: Intranational cities, no border			
CANADA-US	0.72	0.77	0.81
OECD	0.71	0.76	0.79
WORLD	0.70	0.75	0.79
LDC	0.70	0.73	0.75

Table 4 - Variance Decomposition (median across goods, $\alpha = 0.8$)

	Total	Fraction of variance account for by:				
		Wages	Land Prices	Trade cost	Error	Covariance
Panel A: International cities, cross-border pairs						
CANADA-US	0.07	10.1	7.8	36.1	12.8	15.5
OECD	0.25	2.3	1.7	60.0	29.4	0.8
LDC	0.42	2.5	12.6	53.9	27.2	0.4
WORLD	0.38	7.7	3.4	50.7	28.1	5.1
Panel B: Intranational cities, no border						
CANADA-US	0.06	4.8	5.9	53.8	24.8	2.3
OECD	0.06	5.1	3.7	51.4	26.5	3.0
LDC	—	—	—	—	—	—
WORLD	0.07	5.2	4.9	55.8	26.1	1.7

Table 5 - Variance Decomposition (aggregate $\alpha = 0.5$)

	Total	Fraction of variance account for by:				
		Wages	Land Prices	Trade cost	Error	Covariance
	Panel A: International cities, cross-border pairs					
CANADA-US	0.10	31.8	16.6	38.5	13.1	
OECD	0.36	10.7	6.2	54.6	28.5	
LDC	0.75	15.2	16.7	45.7	22.4	
WORLD	0.66	36.4	6.8	36.0	20.9	
	Panel B: Intranational cities, no border					
CANADA-US	0.12	10.0	8.5	57.2	24.3	
OECD	0.13	6.6	11.0	56.0	26.3	
LDC						
WORLD	0.10	8.6	6.3	59.7	25.4	

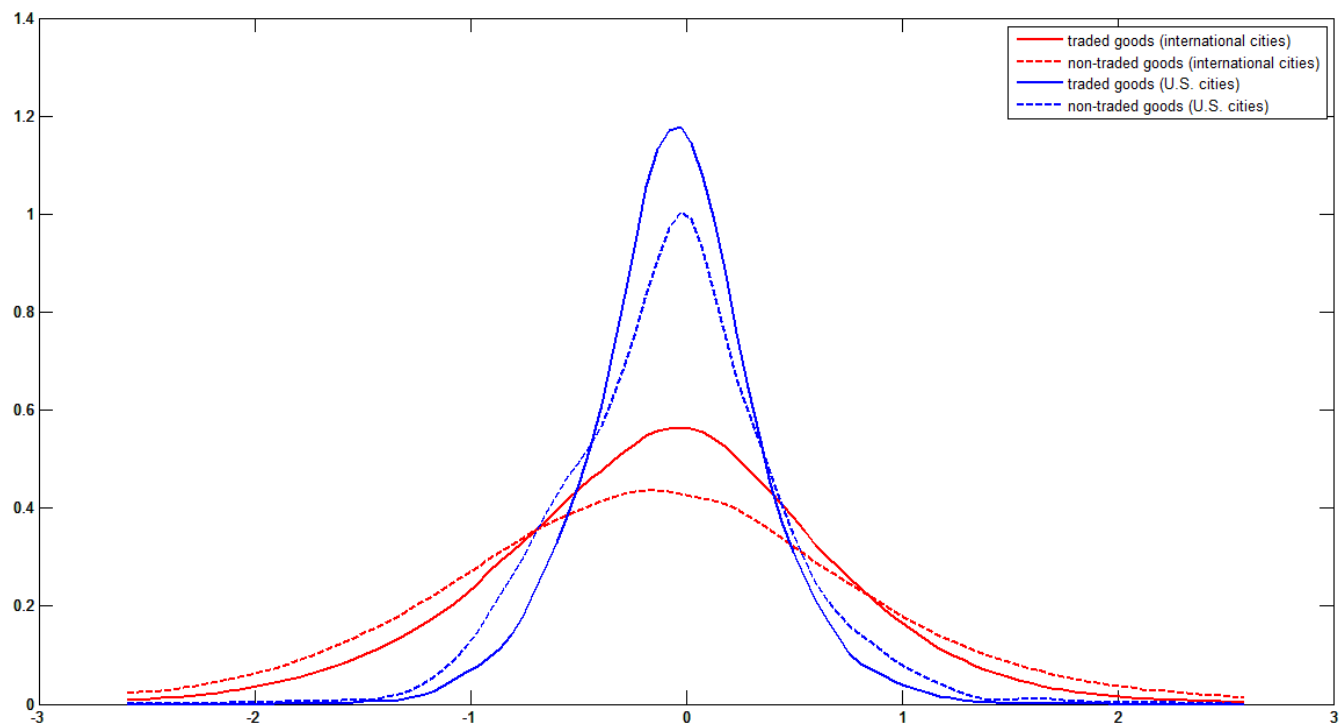


Figure 1. Kernel density estimates of price distributions. Densities ranked from less dispersed to most dispersed: U.S. traded goods, U.S. non-traded goods, international traded goods, international non-traded goods.

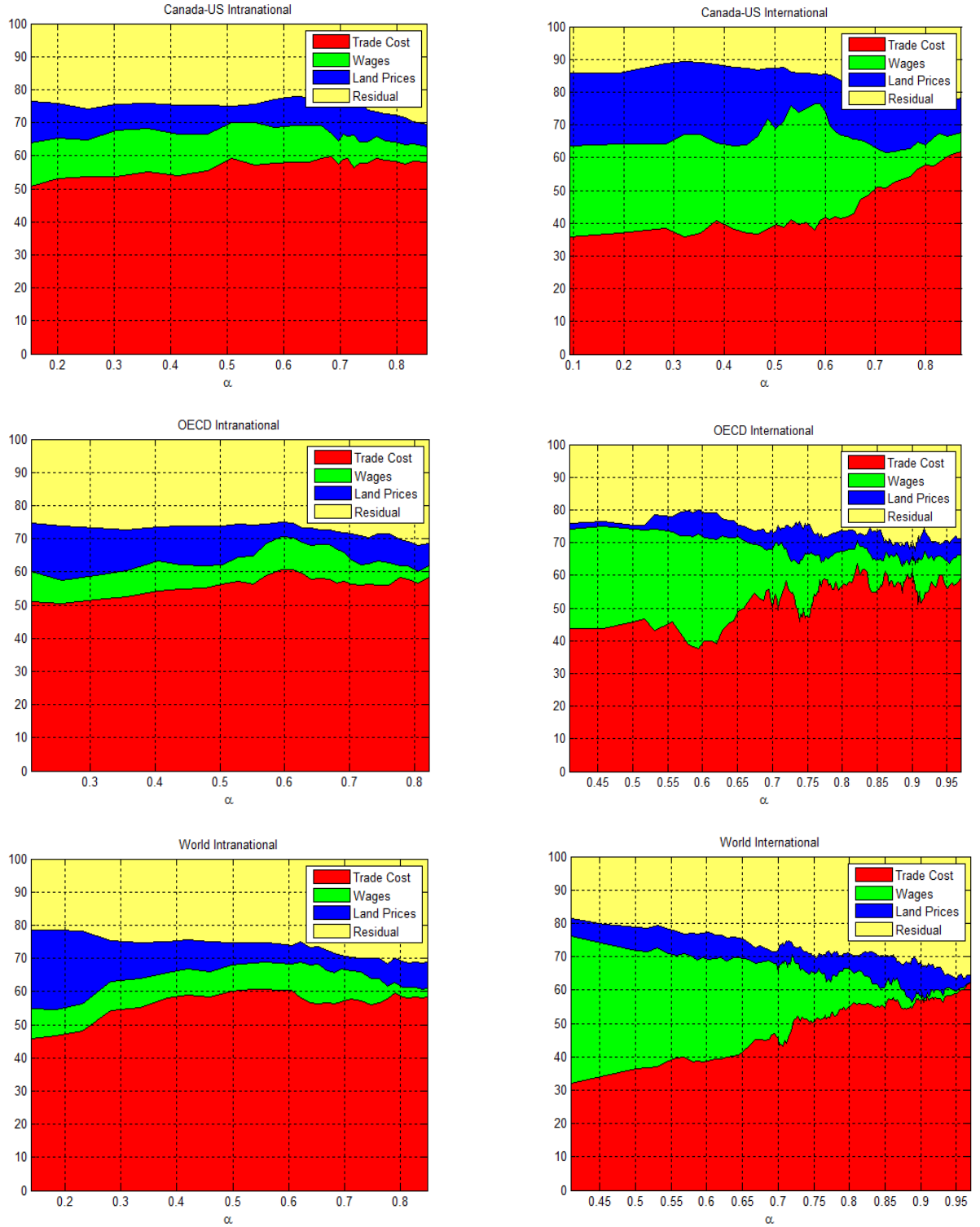


Figure 2. Variance decomposition of lprice dispersion as a function of α_i the share of traded input costs (x-axis)

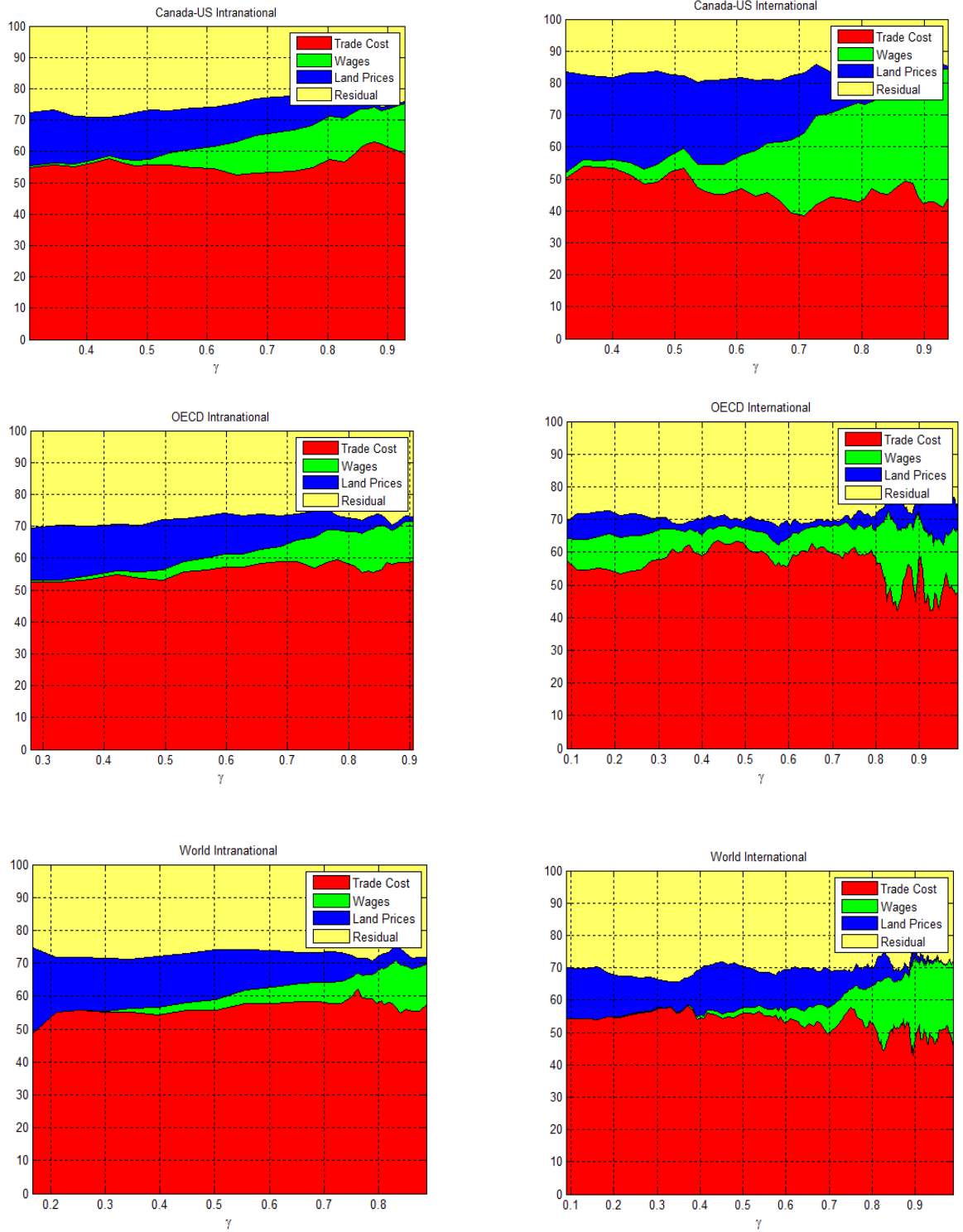


Figure 3. Variance decomposition of price dispersion as a function of γ_i the labor share of non-traded input costs (x-axis)

8. Model Appendix

This appendix presents the function forms of the model, the first-order conditions and details for the model solution.

8.1. Function forms

$$U(C_j, N_j) \equiv (1 - \theta) \log(C_j) + \theta \log L_j \quad (8.1)$$

$$C_j = \left(\sum_i^M (\beta_i)^{\frac{1}{\varepsilon}} (C_{ij})^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (8.2)$$

$$P_j \equiv \left(\sum_i \beta_i (P_{ij})^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \quad (8.3)$$

$$Y_j = A_j N_j^m \quad (8.4)$$

$$R_{ij} = (G_{ij})^{\alpha_i} ((B_j N_{ij}^s)^{\gamma_i} (K_{ij})^{1-\gamma_i})^{1-\alpha_i} \quad (8.5)$$

8.2. Constraints

$$\begin{aligned} L_j + N_j &= 1 \\ \sum_i P_{ij} C_{ij} &= P_j C_j \\ \sum_i P_{ij} C_{ij}^m &\leq W_j^m N_j^m + \varphi H_j K_j \\ \sum_i P_{ij} C_{ij}^s &\leq W_j^s N_j^s + (1 - \varphi) H_j K_j \end{aligned}$$

where $\varphi \in (0, 1)$ is the capital income share received by the manufacturer, $(1 - \varphi)$ is the capital income share received by the retailer, H_j is the price of capital, and K_j is the amount of capital.

8.3. Consumer and producer problems

$$\max_{C_j} \{ (1 - \theta) \log(C_j) + \theta \log L_j + \lambda_j [W_j^m (1 - L_j) + \varphi H_j K_j - P_j C_j] \} \quad (8.6)$$

$$\max_{C_j} \{ (1 - \theta) \log(C_j) + \theta \log L_j + \lambda_j [W_j^s (1 - L_j) + (1 - \varphi) H_j K_j - P_j C_j] \} \quad (8.7)$$

$$\max_{N_j^m} \{ Q_{jj} A_j N_j^m - W_j^m N_j^m \} \quad (8.8)$$

$$\max_{G_{ij}, N_j^s} \{ P_{ij} (G_{ij})^{\alpha_i} ((B_j N_{ij}^s)^{\gamma_i} (K_{ij})^{1-\gamma_i})^{1-\alpha_i} - Q_{ij} G_{ij} - W_j^s N_j^s - H_j K_{ij} \} \quad (8.9)$$

8.4. Efficiency conditions

$$C_{ij}^A = \beta_i \left(\frac{P_{ij}}{P_j} \right)^{-\varepsilon} C_j^A \quad (8.10)$$

$$C_j^A = \frac{W_j^A (1 - \theta)}{P_j} (1 - N_j^A) \quad (8.11)$$

$$N_j^m = 1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m} \quad (8.12)$$

$$N_j^s = 1 - \theta - \frac{(1 - \varphi) \theta H_j K_j}{W_j^s} \quad (8.13)$$

$$L_j^m = \theta + \frac{\varphi \theta H_j K_j}{W_j^m} \quad (8.14)$$

$$L_j^s = \theta + \frac{(1 - \varphi) \theta H_j K_j}{W_j^s} \quad (8.15)$$

$$(8.16)$$

$$N_{ij}^s = \frac{(1 - \alpha_i) \gamma_i}{\alpha_i} \frac{Q_{ij}}{W_j^s} R_{ij} \left(\frac{B_j Q_{ij} (1 - \alpha_i) \gamma_i}{W_j^s \alpha_i} \right)^{(\alpha_i - 1) \gamma_i} \left(\frac{Q_{ij} (1 - \alpha_i) (1 - \gamma_i)}{H_j \alpha_i} \right)^{(\alpha_i - 1)(1 - \gamma_i)} \quad (8.17)$$

$$G_{ij} = R_{ij} \left(\frac{B_j Q_{ij} (1 - \alpha_i) \gamma_i}{W_j^s \alpha_i} \right)^{(\alpha_i - 1) \gamma_i} \left(\frac{Q_{ij} (1 - \alpha_i) (1 - \gamma_i)}{H_j \alpha_i} \right)^{(\alpha_i - 1)(1 - \gamma_i)} \quad (8.18)$$

$$K_{ij} = \frac{(1 - \alpha_i) (1 - \gamma_i)}{\alpha_i} \frac{Q_{ij}}{H_j} R_{ij} \left(\frac{B_j Q_{ij} (1 - \alpha_i) \gamma_i}{W_j^s \alpha_i} \right)^{(\alpha_i - 1) \gamma_i} \left(\frac{Q_{ij} (1 - \alpha_i) (1 - \gamma_i)}{H_j \alpha_i} \right)^{(\alpha_i - 1)(1 - \gamma_i)} \quad (8.19)$$

$$Q_{jj} = MC_j = \frac{W_j^m}{A_j} \quad (8.20)$$

$$P_{ij} = MC_{ij}^s = \frac{(Q_{ij})^{\alpha_i} \left(\left(\frac{W_j^s}{B_j} \right)^{\gamma_i} (H_j)^{(1 - \gamma_i)} \right)^{(1 - \alpha_i)}}{\alpha_i^{\alpha_i} \left((1 - \alpha_i) (\gamma_i)^{\gamma_i} (1 - \gamma_i)^{(1 - \gamma_i)} \right)^{(1 - \alpha_i)}} \quad (8.21)$$

8.5. Price relationships

$$Q_{ji} = (1 + \tau_{ji}) Q_{jj} \quad (8.22)$$

8.6. The retail firm

$$N_j^s = 1 - \theta - \frac{(1 - \varphi) \theta H_j K_j}{W_j^s} = \sum_i N_{ij}^s = \sum_i \left\{ \frac{(1 - \alpha_i) \gamma_i}{\alpha_i} \frac{Q_{ij}}{W_j^s} R_{ij} \left(\frac{B_j Q_{ij} (1 - \alpha_i) \gamma_i}{W_j^s \alpha_i} \right)^{(\alpha_i - 1) \gamma_i} \times \left(\frac{Q_{ij} (1 - \alpha_i) (1 - \gamma_i)}{H_j \alpha_i} \right)^{(\alpha_i - 1)(1 - \gamma_i)} \right\}$$

$$G_{ij} = R_{ij} \left(\frac{B_j Q_{ij} (1 - \alpha_i) \gamma_i}{W_j^s \alpha_i} \right)^{(\alpha_i - 1) \gamma_i} \left(\frac{Q_{ij} (1 - \alpha_i) (1 - \gamma_i)}{H_j \alpha_i} \right)^{(\alpha_i - 1)(1 - \gamma_i)}$$

8.7. General equilibrium

8.7.1. Manufacturing Labor Market

The labor supply of the manufacturer is used in the manufacturing process, which implies:

$$\frac{Y_j}{A_j} = 1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m} \quad (8.23)$$

8.7.2. Goods Market

In the global general equilibrium all the conditions of partial equilibrium must hold. However we also require that the supply of each good equals the demand for each good. This is where the treatment of trade costs becomes crucial. We will assume that trade costs are of the iceberg variety, so the physical resource constraint for good j must satisfy:

$$Y_j = \sum_i G_{ji} (1 + \tau_{ij}) \quad (8.24)$$

In words: the units produced equal the demand of traded inputs of retailers at the destinations plus a fraction lost to iceberg costs. The aggregate fraction lost will depend on the equilibrium allocations since the loss along any bilateral trade route is proportional to the volume of trade along that branch:

$$\frac{T_j}{Y_j} = \frac{\sum_i G_{ji} \tau_{ij}}{\sum_i G_{ji} (1 + \tau_{ij})} \quad (8.25)$$

Returning to our global equilibrium, we substitute the optimal traded input choices of the retailers into the resource constraint to arrive at:

$$Y_j = \sum_i R_{ji} \left(\frac{B_i Q_{ji} (1 - \alpha_j) \gamma_j}{W_i^s \alpha_j} \right)^{(\alpha_j - 1) \gamma_j} \left(\frac{Q_{ji} (1 - \alpha_j) (1 - \gamma_j)}{H_i \alpha_j} \right)^{(\alpha_j - 1) (1 - \gamma_j)} (1 + \tau_{ji})$$

Recall 8.23:

$$\frac{Y_j}{A_j} = 1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m}$$

Combining these last two we get:

$$\begin{aligned} & \sum_i R_{ji} \left(\frac{B_i Q_{ji} (1 - \alpha_j) \gamma_j}{W_i^s \alpha_j} \right)^{(\alpha_j - 1) \gamma_j} \left(\frac{Q_{ji} (1 - \alpha_j) (1 - \gamma_j)}{H_i \alpha_j} \right)^{(\alpha_j - 1) (1 - \gamma_j)} (1 + \tau_{ji}) \\ &= A_j \left(1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m} \right) \end{aligned} \quad (8.26) \quad (8.27)$$

The equilibrium of the retailer implies:

$$R_{ij} = C_{ij}^m + C_{ij}^s = \beta_i \left(\frac{P_{ij}}{P_j} \right)^{-\varepsilon} (C_j^m + C_j^s)$$

Assuming that $\varepsilon = 1$ (for the rest of the text), we have:

$$R_{ij} = C_{ij}^m + C_{ij}^s = \frac{\beta_i}{P_{ij}} (P_j C_j^m + P_j C_j^s) = \frac{\beta_i}{P_{ij}} (N_j^m W_j^m + N_j^s W_j^s + H_j K_j)$$

$$R_{ji} = \frac{\beta_j}{P_{ji}} (N_i^m W_i^m + N_i^s W_i^s + H_i K_i)$$

which says that the total income (sales) of the retailer from good i is equal to the share of that good in the budget of the region. Thus, we have

$$\begin{aligned} & \sum_i R_{ji} \left(\frac{B_i Q_{ji} (1 - \alpha_j) \gamma_j}{W_i^s \alpha_j} \right)^{(\alpha_j - 1) \gamma_j} \left(\frac{Q_{ji} (1 - \alpha_j) (1 - \gamma_j)}{H_i \alpha_j} \right)^{(\alpha_j - 1) (1 - \gamma_j)} (1 + \tau_{ji}) \\ &= A_j \left(1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m} \right) \\ & \sum_i \left\{ \frac{\beta_j}{P_{ji}} (N_i^m W_i^m + N_i^s W_i^s + H_i K_i) \left(\frac{B_i Q_{ji} (1 - \alpha_j) \gamma_j}{W_i^s \alpha_j} \right)^{(\alpha_j - 1) \gamma_j} \right. \\ & \quad \left. \times \left(\frac{Q_{ji} (1 - \alpha_j) (1 - \gamma_j)}{H_i \alpha_j} \right)^{(\alpha_j - 1) (1 - \gamma_j)} (1 + \tau_{ji}) \right\} = A_j \left(1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m} \right) \end{aligned}$$

Recall the price set by the retailer:

$$P_{ij} = \frac{(Q_{ij})^{\alpha_i} \left(\left(\frac{W_j^s}{B_j} \right)^{\gamma_i} (H_j)^{(1 - \gamma_i)} \right)^{(1 - \alpha_i)}}{\alpha_i^{\alpha_i} \left((1 - \alpha_i) (\gamma_i)^{\gamma_i} (1 - \gamma_i)^{(1 - \gamma_i)} \right)^{(1 - \alpha_i)}}$$

which is to say:

$$P_{ji} = \frac{(Q_{ji})^{\alpha_j} \left(\frac{W_i^s}{B_i} \right)^{(1 - \alpha_j) \gamma_j} (H_i)^{(1 - \alpha_j) (1 - \gamma_j)}}{\alpha_j^{\alpha_j} \left((1 - \alpha_j) \gamma_j \right)^{(1 - \alpha_j) \gamma_j} \left((1 - \alpha_j) (1 - \gamma_j) \right)^{(1 - \alpha_j) (1 - \gamma_j)}}$$

Thus,

$$\begin{aligned} & \sum_i \left\{ \frac{\frac{\beta_j}{(Q_{ji})^{\alpha_j} \left(\frac{W_i^s}{B_i} \right)^{(1 - \alpha_j) \gamma_j} (H_i)^{(1 - \alpha_j) (1 - \gamma_j)}} (N_i^m W_i^m + N_i^s W_i^s + H_i K_i)}{\alpha_j^{\alpha_j} \left((1 - \alpha_j) \gamma_j \right)^{(1 - \alpha_j) \gamma_j} \left((1 - \alpha_j) (1 - \gamma_j) \right)^{(1 - \alpha_j) (1 - \gamma_j)}}} \right. \\ & \quad \left. \times \left(\frac{B_i Q_{ji} (1 - \alpha_j) \gamma_j}{W_i^s \alpha_j} \right)^{(\alpha_j - 1) \gamma_j} \left(\frac{Q_{ji} (1 - \alpha_j) (1 - \gamma_j)}{H_i \alpha_j} \right)^{(\alpha_j - 1) (1 - \gamma_j)} (1 + \tau_{ji}) \right\} \\ &= A_j \left(1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m} \right) \\ & \sum_i \frac{\beta_j}{Q_{ji}} (N_i^m W_i^m + N_i^s W_i^s + H_i K_i) \alpha_j (1 + \tau_{ji}) = A_j \left(1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m} \right) \end{aligned}$$

By using $Q_{jj} = \frac{W_j^m}{A_j}$ and $Q_{ji} = (1 + \tau_{ji}) Q_{jj}$, we can write

$$\begin{aligned} \sum_i \frac{\beta_j}{(1 + \tau_{ji}) W_j^m} (N_i^m W_i^m + N_i^s W_i^s + H_i K_i) \alpha_j (1 + \tau_{ji}) &= \left(1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m}\right) \\ \alpha_j \beta_j \sum_i (N_i^m W_i^m + N_i^s W_i^s + H_i K_i) &= W_j^m \left(1 - \theta - \frac{\varphi \theta H_j K_j}{W_j^m}\right) \\ \alpha_j \beta_j \sum_i (N_i^m W_i^m + N_i^s W_i^s + H_i K_i) &= W_j^m N_j^m \end{aligned} \quad (8.28)$$

This is the first equation for the relation between $N^m W^m$, $N^s W^s$, and HK .

8.7.3. Retailing Labor Market

We have the following condition for the retailing labor market equilibrium:

$$\begin{aligned} N_j^s &= \sum_i N_{ij}^s = \sum_i \left\{ \frac{(1-\alpha_i)\gamma_i}{\alpha_i} \frac{Q_{ij}}{W_j^s} R_{ij} \left(\frac{B_j Q_{ij}}{W_j^s} \frac{(1-\alpha_i)\gamma_i}{\alpha_i} \right)^{(\alpha_i-1)\gamma} \right. \\ &\quad \left. \times \left(\frac{Q_{ij}}{H_j} \frac{(1-\alpha_i)(1-\gamma_i)}{\alpha_i} \right)^{(\alpha_i-1)(1-\gamma_i)} \right\} \\ N_j^s &= \sum_i \left\{ \frac{(1-\alpha_i)\gamma_i}{\alpha_i} \frac{Q_{ij}}{W_j^s} \frac{\beta_i (N_j^m W_j^m + N_j^s W_j^s + H_j K_j)}{(Q_{ij})^{\alpha_i} \left(\frac{W_j^s}{B_j} \right)^{(1-\alpha_i)\gamma_i} (H_j)^{(1-\alpha_i)(1-\gamma_i)}} \right. \\ &\quad \left. \times \left(\frac{B_j Q_{ij}}{W_j^s} \frac{(1-\alpha_i)\gamma_i}{\alpha_i} \right)^{(\alpha_i-1)\gamma_i} \left(\frac{Q_{ij}}{H_j} \frac{(1-\alpha_i)(1-\gamma_i)}{\alpha_i} \right)^{(\alpha_i-1)(1-\gamma_i)} \right\} \\ N_j^s W_j^s &= (N_j^m W_j^m + N_j^s W_j^s + H_j K_j) \sum_i (1 - \alpha_i) \gamma_i \beta_i \end{aligned} \quad (8.29)$$

This is the second equation for the relation between $N^m W^m$, $N^s W^s$, and HK .

8.7.4. Capital Market

We have the following condition for the capital market equilibrium:

$$\begin{aligned} K_j &= \sum_i K_{ij} = \sum_i \left\{ \frac{(1-\alpha_i)(1-\gamma_i)}{\alpha_i} \frac{Q_{ij}}{H_j} R_{ij} \left(\frac{B_j Q_{ij}}{W_j^s} \frac{(1-\alpha_i)\gamma_i}{\alpha_i} \right)^{(\alpha_i-1)\gamma} \right. \\ &\quad \left. \times \left(\frac{Q_{ij}}{H_j} \frac{(1-\alpha_i)(1-\gamma_i)}{\alpha_i} \right)^{(\alpha_i-1)(1-\gamma_i)} \right\} \\ H_j K_j &= (N_j^m W_j^m + N_j^s W_j^s + H_j K_j) \sum_i (1 - \alpha_i) (1 - \gamma_i) \beta_i \end{aligned} \quad (8.30)$$

This is the third equation for the relation between $N^m W^m$, $N^s W^s$, and HK .

8.7.5. Implications for Wages, Rents, Wage Income, and Capital Income

Recall 8.28, 8.29, 8.30, which are:

$$\begin{aligned} N_j^m W_j^m &= \alpha_j \beta_j \sum_i (N_i^m W_i^m + N_i^s W_i^s + H_i K_i) \\ N_j^s W_j^s &= (N_j^m W_j^m + N_j^s W_j^s + H_j K_j) \sum_i (1 - \alpha_i) \gamma_i \beta_i \\ H_j K_j &= (N_j^m W_j^m + N_j^s W_j^s + H_j K_j) \sum_i (1 - \alpha_i) (1 - \gamma_i) \beta_i \end{aligned}$$

Combine 8.29 and 8.30 to get:

$$\frac{H_j K_j}{N_j^s W_j^s} = \frac{\sum_i (1 - \alpha_i) (1 - \gamma_i) \beta_i}{\sum_i (1 - \alpha_i) \gamma_i \beta_i} \quad (8.31)$$

and

$$(H_j K_j + N_j^s W_j^s) = N_j^m W_j^m \frac{\sum_i (1 - \alpha_i) \beta_i}{(1 - \sum_i (1 - \alpha_i) \beta_i)} \quad (8.32)$$

and thus

$$\frac{N_j^s W_j^s}{N_j^m W_j^m} = \frac{\sum_i (1 - \alpha_i) \gamma_i \beta_i}{(1 - \sum_i (1 - \alpha_i) \beta_i)} \quad (8.33)$$

which show that the sectoral wage incomes and capital incomes are all proportional to each other within each city.

Recall the individual optimality condition for the retailer:

$$\begin{aligned} N_j^s &= 1 - \theta - \frac{(1 - \varphi) \theta H_j K_j}{W_j^s} \\ N_j^s W_j^s &= W_j^s (1 - \theta) - (1 - \varphi) \theta H_j K_j \end{aligned}$$

Combine this with 8.31 to get:

$$\begin{aligned} H_j K_j &= \frac{(N_j^s W_j^s) \sum_i (1 - \alpha_i) (1 - \gamma_i) \beta_i}{\sum_i (1 - \alpha_i) \gamma_i \beta_i} \\ N_j^s W_j^s \left(1 + \frac{(1 - \varphi) \theta \sum_i (1 - \alpha_i) (1 - \gamma_i) \beta_i}{\sum_i (1 - \alpha_i) \gamma_i \beta_i} \right) &= W_j^s (1 - \theta) \\ N_j^s &= \frac{(1 - \theta) \sum_i (1 - \alpha_i) \gamma_i \beta_i}{(\sum_i (1 - \alpha_i) \gamma_i \beta_i) + (1 - \varphi) \theta (\sum_i (1 - \alpha_i) (1 - \gamma_i) \beta_i)} \end{aligned} \quad (8.34)$$

which shows that N_j^s is constant across regions. In a special case in which the share of capital is equal to zero in the retail production function (i.e., $\gamma_i = 0$), or in which the share of capital income received by the retailer is equal to zero (i.e., $\varphi = 1$), we have $N_j^s = (1 - \theta)$.

Combine 8.33 with 8.28 to get:

$$\frac{\frac{N_j^s W_j^s (1 - \sum_i (1 - \alpha_i) \beta_i)}{\sum_i (1 - \alpha_i) \gamma_i \beta_i}}{\frac{N_k^s W_k^s (1 - \sum_i (1 - \alpha_i) \beta_i)}{\sum_i (1 - \alpha_i) \gamma_i \beta_i}} = \frac{N_j^m W_j^m}{N_k^m W_k^m} = \frac{\alpha_j \beta_j}{\alpha_k \beta_k} = \frac{N_j^s W_j^s}{N_k^s W_k^s}$$

which show that the manufacturing wage income and the retailing wage income are proportional across cities. It is implied that:

$$\frac{W_j^s}{W_k^s} = \frac{\alpha_j \beta_j}{\alpha_k \beta_k} \quad (8.35)$$

since N_j^s is constant across regions. If we also use 8.31, we obtain:

$$\frac{H_j K_j}{H_k K_k} = \frac{\alpha_j \beta_j}{\alpha_k \beta_k} \quad (8.36)$$

where K_A is the capital stock in city $A = j, k$.

Recall the individual optimality conditions for both the retailer and the manufacturer:

$$\begin{aligned} W_j^m N_j^m &= W_j^m (1 - \theta) - \varphi \theta H_j K_j \\ W_j^s N_j^s &= W_j^s (1 - \theta) - (1 - \varphi) \theta H_j K_j \end{aligned}$$

These conditions can be combined to obtain:

$$\frac{W_j^m}{W_j^s} = \frac{\varphi (1 - \theta) - \varphi N_j^s}{(1 - \varphi) (1 - \theta) - (1 - \varphi) N_j^m}$$

Recall 8.33:

$$\frac{N_j^m W_j^m}{N_j^s W_j^s} = \frac{(1 - \sum_i (1 - \alpha_i) \beta_i)}{\sum_i (1 - \alpha_i) \gamma_i \beta_i}$$

Combine the last two expressions to get:

$$\begin{aligned} \frac{N_j^m W_j^m}{N_j^s W_j^s} &= \frac{N_j^m}{N_j^s} \frac{\varphi (1 - \theta) - \varphi N_j^s}{(1 - \varphi) (1 - \theta) - (1 - \varphi) N_j^m} = \frac{(1 - \sum_i (1 - \alpha_i) \beta_i)}{\sum_i (1 - \alpha_i) \gamma_i \beta_i} \\ &= N_j^m \left(\frac{\varphi (1 - \theta)}{N_j^s} - \varphi \right) \sum_i (1 - \alpha_i) \gamma_i \beta_i \\ &= (1 - \varphi) (1 - \theta) \left(1 - \sum_i (1 - \alpha_i) \beta_i \right) - (1 - \varphi) N_j^m \left(1 - \sum_i (1 - \alpha_i) \beta_i \right) \end{aligned}$$

which can be combined with 8.34 (i.e., N_j^s) to obtain:

$$\begin{aligned} N_j^m &= (1 - \theta) - \frac{\varphi \theta (\phi_0 - \phi_1) (1 - \theta)}{\varphi \theta (\phi_0 - \phi_1) + (1 - \phi_0)} \\ &= (1 - \theta) \text{ when } \varphi = 0 \text{ or } \gamma_i = 1 \end{aligned}$$

where $\phi_0 \equiv \sum_i (1 - \alpha_i) \beta_i$, $\phi_1 \equiv \sum_i (1 - \alpha_i) \gamma_i \beta_i$, $\phi_0 - \phi_1 = \sum_i (1 - \alpha_i) (1 - \gamma_i) \beta_i$. $\phi_2 \equiv \sum_i \alpha_i \beta_i = 1 - \phi_0$. This shows that N_j^m is constant and equal across cities. The level of effort is equal to the leisure share of expenditure, $(1 - \theta)$ when either rental income is zero for the

manufacturer ($\varphi = 0$) or when retail production is labor-only $\gamma_i = 1$. Effort is declining in asset income.

$$\begin{aligned}
\frac{N_j^m}{(1-\theta)} &= 1 - \frac{\varphi\theta(\phi_0 - \phi_1)}{\varphi\theta(\phi_0 - \phi_1) + (1 - \phi_0)} \\
\frac{\partial \frac{N_j^m}{(1-\theta)}}{\partial \varphi} &= -\theta(\phi_0 - \phi_1)d^{-1} + \varphi\theta(\phi_0 - \phi_1)d^{-2} \\
&= \theta(\phi_0 - \phi_1)d^{-1}[-1 + \varphi d^{-1}] \\
\text{sign } \frac{\partial \frac{N_j^m}{(1-\theta)}}{\partial \varphi} &= \text{sign } [-1 + \varphi d^{-1}] \text{ since } \theta(\phi_0 - \phi_1)d^{-1} > 0 \\
\varphi &< d
\end{aligned}$$

$$\begin{aligned}
N_j^m &= \frac{(1 - \phi_0)(1 - \theta)}{(1 - \phi_0) + \varphi\theta(\phi_0 - \phi_1)} \\
N_j^s &= \frac{\phi_1(1 - \theta)}{\phi_1 + (1 - \varphi)\theta(\phi_0 - \phi_1)}
\end{aligned}$$

$$\frac{W_j^m}{W_j^s} = \frac{(1 - \phi_0) + \varphi\theta(\phi_0 - \phi_1)}{(1 - \varphi)(1 - \theta)(1 - \phi_0) + \varphi\theta(\phi_0 - \phi_1) - (1 - \varphi)\phi_1 + (1 - \varphi)\theta(\phi_0 - \phi_1)} \frac{(1 - \varphi)\theta(\phi_0 - \phi_1)}{(1 - \varphi)\phi_1 + (1 - \varphi)\theta(\phi_0 - \phi_1)}$$

8.8. Implications for Price Ratios across Cities

Recall the retail price of good i in city j and city k :

$$\begin{aligned}
P_{ij} &= \frac{(Q_{ij})^{\alpha_i} \left(\frac{W_j^s}{B_j}\right)^{(1-\alpha_i)\gamma_i} (H_j)^{(1-\alpha_i)(1-\gamma_i)}}{\alpha_i^{\alpha_i} ((1 - \alpha_i)\gamma_i)^{(1-\alpha_i)\gamma_i} ((1 - \alpha_i)(1 - \gamma_i))^{(1-\alpha_i)(1-\gamma_i)}} \\
P_{ik} &= \frac{(Q_{ik})^{\alpha_i} \left(\frac{W_k^s}{B_k}\right)^{(1-\alpha_i)\gamma_i} (H_k)^{(1-\alpha_i)(1-\gamma_i)}}{\alpha_i^{\alpha_i} ((1 - \alpha_i)\gamma_i)^{(1-\alpha_i)\gamma_i} ((1 - \alpha_i)(1 - \gamma_i))^{(1-\alpha_i)(1-\gamma_i)}}
\end{aligned}$$

Take their ratio to get:

$$\frac{P_{ij}}{P_{ik}} = \frac{(Q_{ij})^{\alpha_i} \left(\frac{W_j^s}{B_j}\right)^{(1-\alpha_i)\gamma_i} (H_j)^{(1-\alpha_i)(1-\gamma_i)}}{(Q_{ik})^{\alpha_i} \left(\frac{W_k^s}{B_k}\right)^{(1-\alpha_i)\gamma_i} (H_k)^{(1-\alpha_i)(1-\gamma_i)}}$$

By using $Q_{ij} = (1 + \tau_{ij})Q_{ii}$, we can write the ratio of the price of good i across regions j and k as follows:

$$\frac{P_{ij}}{P_{ik}} = \frac{((1 + \tau_{ij}))^{\alpha_i} \left(\frac{W_j^s}{B_j}\right)^{(1-\alpha_i)\gamma_i} (H_j)^{(1-\alpha_i)(1-\gamma_i)}}{((1 + \tau_{ik}))^{\alpha_i} \left(\frac{W_k^s}{B_k}\right)^{(1-\alpha_i)\gamma_i} (H_k)^{(1-\alpha_i)(1-\gamma_i)}}$$

By using 8.35 and 8.36, the analytical solution for the price ratios can be written as:

$$\frac{P_{ij}}{P_{ik}} = \left(\left(\frac{B_k}{B_j} \right)^{\gamma_i} \frac{\alpha_j \beta_j}{\alpha_k \beta_k} \right)^{(1-\alpha_i)} \left(\frac{1 + \tau_{ij}}{1 + \tau_{ik}} \right)^{\alpha_i} \left(\frac{K_k}{K_j} \right)^{(1-\alpha_i)(1-\gamma_i)}$$

where K_j is the total amount of capital in city j .

9. Estimation Appendix

The derivation of the variance decomposition of equation 5.6 can be written as follows:

$$\begin{aligned} var_{jk} [E_t (q_{ijk,t})] &= var_{jk} [(1 - \hat{\alpha}_i) \hat{\gamma}_i E_t (\omega_{jk,t})] + var_{jk} [(1 - \hat{\alpha}_i) (1 - \hat{\gamma}_i) E_t (h_{jk,t})] \\ &\quad + var_{jk} [\hat{\alpha}_i E_t (\hat{\mathbf{I}}_{ijk,t} \hat{\delta}_i d_{jk})] + var_{jk} [E_t (\hat{\varepsilon}_{ijk,t})] \\ &\quad + 2cov ((1 - \hat{\alpha}_i) \hat{\gamma}_i E_t (\omega_{jk,t}), (1 - \hat{\alpha}_i) (1 - \hat{\gamma}_i) E_t (h_{jk,t})) \\ &\quad + 2cov ((1 - \hat{\alpha}_i) \hat{\gamma}_i E_t (\omega_{jk,t}), \hat{\alpha}_i E_t (\hat{\mathbf{I}}_{ijk,t} \hat{\delta}_i d_{jk})) \\ &\quad + 2cov ((1 - \hat{\alpha}_i) (1 - \hat{\gamma}_i) E_t (h_{jk,t}), \hat{\alpha}_i E_t (\hat{\mathbf{I}}_{ijk,t} \hat{\delta}_i d_{jk})) \end{aligned}$$

where $\hat{\alpha}_i$'s, $\hat{\gamma}_i$'s, $\hat{\mathbf{I}}_{ijk}$'s, $\hat{\delta}_i$'s and $\hat{\varepsilon}_{ijk}$'s are all estimated values for the relevant variables. Note that the covariance terms including $E_t (\hat{\varepsilon}_{ijk,t})$ are equal to zero by OLS regression.