Supplementary Note to Accompany

Social Desirability of Free Entry: A Bilateral Oligopoly Analysis

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This supplementary note consists of two sections. Section 1 presents details related to the final point made in Section 3 of the paper. Section 2 presents details on the alternative bargaining set-up mentioned in the last paragraph of Conclusions.

1 Details related to the final point made in Section 3

In this section we present details of the analysis of the alternative model discussed in the last five paragraphs of Section 3 as a final point to the section.

Consider a Stage 3 subgame in which \( S \equiv S(M, N) \) successful matches were formed in Stage 2. We assume that the disagreement point is \((0, 0)\), and suppose Assumption 1 in the paper holds with \( S \) replacing \( M \). The outcome of the second-stage bargaining is \( S \) pairs of input prices and quantities \((\hat{r}_s, \hat{q}_s)\) \((s = 1, 2, ..., S)\) which solves the following maximization problem:

\[
\max_{\{r_s, q_s\}} \left\{ (P(q_s + \sum_{t \neq s} \hat{q}_t) - r_s)q_s \right\}^{\beta} \left[(r_s - c)q_s\right]^{1-\beta}, \tag{1}
\]

subject to \((P(q_s + \sum_{t \neq s} \hat{q}_t) - r_s)q_s \geq 0\) and \((r_s - c)q_s \geq 0\). Given any \( S \geq 1 \), there exists a unique bargaining outcome characterized by \( \hat{r}_1 = ... = \hat{r}_S = \hat{r} \) and \( \hat{q}_1 = ... = \hat{q}_S = \hat{q} \), where \( \hat{r} (> 0) \) and \( \hat{q} (> 0) \) are uniquely determined by (2) and (3) below:

\[
\hat{q} = -\frac{P(S\hat{q}) - c}{P'(S\hat{q})}, \tag{2}
\]

\[
\hat{r} = (1 - \beta)P(S\hat{q}) + \beta c. \tag{3}
\]

Using (2) and (3) we now compute expected profits. Consider a downstream firm which was successful in Stage 2 in finding an upstream partner. The profit of such a downstream firm in the equilibrium of the Stage 3 subgame is \( (P(S\hat{q}) - \hat{r})\hat{q} = \beta(P(S\hat{q}) - c)\hat{q} \). Similarly, the profit of an upstream firm which found a downstream partner is \((\hat{r} - c)\hat{q} \equiv (1 - \beta)(P(S\hat{q}) - c)\hat{q} \). Then, taking the matching probabilities into account, each downstream entrant’s post-entry expected profit is \( \tilde{\pi}_D(M, N) \equiv \pi_D(M, N) \) while each upstream entrant’s post-entry expected profit is \( \pi_U(M, N) \equiv \tilde{\pi}_U(M, N) - K \). In the equilibrium of the entire game, each downstream entrant’s profit is \( \pi_D(M, N) \equiv \tilde{\pi}_D(M, N) \) while each upstream entrant’s profit is \( \pi_U(M, N) \equiv \tilde{\pi}_U(M, N) - KU \).

In what follows we proceed by treating \( M, N \), and \( S \) as continuous variables. We find that the Stage 3 aggregate profit, \( S(P(S\hat{q}) - c)\hat{q} \), is strictly decreasing in \( S \). This, along with the property that \( S(M, N) \) is strictly increasing in \( M \) and \( N \), implies that \( \tilde{\pi}_D(M, N) \)
is strictly decreasing in $M$ and $N$ for all $M > 1$ and $N > 1$, provided $S(M,N) > 1$. The same holds for $\bar{\pi}_U(M,N)$. For meaningful analysis of the Stage 3 subgame we need $S \geq 1$. We assume that for some $M (\geq 1)$ and $N (\geq 1)$, $S(M,N) \geq 1$ holds and also $K$ and $K_U$ are low enough so that enough upstream and downstream firms enter in Stage 1 which in turn ensures that $S \geq 1$.

The free entry number of downstream firms and upstream firms respectively are $M_f (\geq 1)$ and $N_f (\geq 1)$ such that $\pi_D(M_f,N_f) = 0$ and $\pi_U(M_f,N_f) = 0$. We find that a unique pair $(M_f,N_f)$ exists.

Now consider another three-stage game which is identical to the one we have analyzed so far except for the downstream entry process. There is free entry in the upstream sector but entry in the downstream is regulated by the social planner. Stages 2 and 3, i.e. the stages involving search and bargaining respectively, remain the same as before. In Stage 1, potential upstream firms make entry decisions taking the social planner’s choice of $M$ as given while the social planner selects the number of downstream firms to maximize total surplus (to be defined below), assuming that $N$ upstream firms enter.

Consider a Stage 2 subgame in which $M$ downstream firms and $N$ upstream firms have entered in the previous stage, and let $W(M,N)$ denote the total surplus in the outcome of the third-stage bargaining. We have that

$$W(M,N) = \int_0^{S\hat{q}} P(y)dy - cS\hat{q} - MK - NK_U,$$

(4)

where $S(\equiv S(M,N))$ denotes the number of successful matches and hence

$$\frac{\partial W(M,N)}{\partial M} = (P(S\hat{q}) - c)(\hat{q} + S \frac{\partial \hat{q}}{\partial S}) \frac{\partial S}{\partial M} - K.$$

In this alternative game, the equilibrium number of downstream and upstream firms is a pair $(M^*, N_f^*)$ such that $\frac{\partial W(M^*,N_f^*)}{\partial M} = 0$ and $\pi_U(M^*,N_f^*) = 0$. We say that insufficient entry into the downstream sector occurs if $S(M_f,N_f) \geq 1$ and $M_f < M^*$ holds.\footnote{In the paper we stated that insufficient entry occurs if and only if $1 \leq M_f < M^*$ holds. Here $S(M_f,N_f) \geq 1 \Rightarrow M_f \geq 1$. Thus $S(M_f,N_f) \geq 1$ and $M_f < M^*$ together imply that $1 \leq M_f < M^*$.}

In what follows, we present an example in which insufficient entry occurs even in the presence of the business-stealing effect. We say that the business-stealing effect is present at $(M,N) = (\hat{M},\hat{N})$ if $\frac{\partial \bar{q}}{\partial M} |_{(M,N)=(\hat{M},\hat{N})} < 0$ and $\frac{\partial \bar{q}}{\partial N} |_{(M,N)=(\hat{M},\hat{N})} < 0$. Since $\frac{\partial S}{\partial M} > 0$ and $\frac{\partial S}{\partial N} > 0$ by assumption, the business-stealing effect is present at $(M,N) = (\hat{M},\hat{N})$ if and only if $\frac{\partial \hat{q}}{\partial S} |_{S=S(\hat{M},\hat{N})} < 0$.\footnote{In the paper we stated that insufficient entry occurs if and only if $1 \leq M_f < M^*$ holds. Here $S(M_f,N_f) \geq 1 \Rightarrow M_f \geq 1$. Thus $S(M_f,N_f) \geq 1$ and $M_f < M^*$ together imply that $1 \leq M_f < M^*$.}
Assume that demand is linear, i.e., \( P(Q) = a - Q \) where \( a > 0 \) and consider the matching function \( S(M, N) = \frac{MN}{M+N} \). It is easy to check that this matching function satisfies all the properties that are assumed to be satisfied. We find that \( \hat{q} = \frac{1}{S+1} = \frac{M+N}{MN+M+N} \) and \( P(S\hat{q}) - c = \frac{(M+N)(a-c)}{M+N+M+N} \). Since \( \frac{\partial M}{\partial S} = -\frac{1}{(S+1)^2} < 0 \), the business-stealing effect is present for all \( (M, N) \). Using \( S(M, N) = \frac{MN}{M+N} \) and the expressions for \( \hat{q} \) and \( P(S\hat{q}) - c \) from above, we find that

\[
\begin{align*}
\pi_D(M, N) &= \beta S(M, N)[P(S(M, N)\hat{q}) - \tilde{c}\hat{q}] - K = \frac{\beta N(M + N)}{(MN + M + N)^2} - K, \\
\pi_U(M, N) &= \frac{(1 - \beta)S(M, N)[P(S(M, N)\hat{q}) - \tilde{c}\hat{q}]}{N} - K_U = \frac{(1 - \beta)M(M + N)}{(MN + M + N)^2} - K_U, \\
\frac{\partial W(M, N)}{\partial M} &= (P(S\hat{q}) - c)\hat{q}(1 + \frac{S}{\hat{q}} \frac{\partial \hat{q}}{\partial S}) \frac{\partial S}{\partial M} - K = \frac{N^2(M + N)}{(MN + M + N)^3} - K.
\end{align*}
\]

Now let \( a = 1, c = 0, K = \frac{1}{256}, K_U = \frac{11}{192} \) and \( \beta = \frac{1}{12} \). Solving \( \pi_D(M, N) = 0 \) and \( \pi_U(M, N) = 0 \) yields \( M_f = 4, N_f = 12 \) and \( S(M_f, N_f) = 3 \) while solving \( \frac{\partial W(M, N)}{\partial M} = 0 \) and \( \pi_U(M, N) = 0 \) yields \( M^* = 6, N^*_f = 6 \). Observe that \( M_f = 4 < 6 = M^* \). This implies that there exists a range of parameterizations in which insufficient entry occurs in the downstream sector even if there is free entry in the upstream sector.

2. An Alternative Bargaining Set-up

In this subsection we consider an alternative bargaining set-up mentioned in the last paragraph of Conclusions and show that insufficient entry can occur in this set-up.

Suppose that at Stage 2 each pair of downstream and upstream firm \( i \) bargains over the input price \( r_i \) only, and then at Stage 3 \( M \) downstream firms compete against each other by choosing a quantity taking the input prices \( r \equiv (r_1, r_2, ..., r_M) \) as given. We consider the class of inverse demand functions with constant elasticity of slope\(^2\) (i.e. \( \frac{P''(Q)}{P'(Q)} = E \) is constant for all \( Q \), and continue to assume that Assumption 1 holds.\(^3\))

In Stage 3, each downstream firm \( i \) (\( i = 1, 2, ..., M \)) chooses \( q_i \) to maximize \( (P(q_i + \sum_j q_j) - r_i)q_i \), taking \( q_j \) (\( j \neq i \)) and \( r = (r_1, r_2, ..., r_M) \) as given.

\(^2\)This class of inverse demand functions include constant elasticity demand functions, \( P(Q) = a - Q^b \) (\( a > 0, b > 0 \)), and semilog demand functions.

\(^3\)The proof analogous to the one presented in the paper would not work, because the equilibrium quantity and \( M^* \) are both affected by \( \beta \) in this alternative set-up.
Claim A: For a given $M \geq 1$ and $r \equiv (r_1, r_2, \ldots, r_M)$, the stage 3 subgame has an equilibrium outcome in which $q_i > 0$ for all $i = (1, 2, \ldots, M)$, if and only if the following conditions hold: (i) there exists a unique $Q^* > 0$ such that $MP(Q^*) - \sum_j r_j^i + Q^* P'(Q^*) = 0$ and (ii) $q_i^*(r) \equiv -\frac{P'(Q^*)}{P'(Q^*)} > 0$ for all $i$. In the equilibrium $q_i = q_i^*(r)$ holds for all $i$.

Proof: Suppose, for a given $M \geq 1$ and $r$, the stage 3 subgame has an equilibrium outcome in which $q_i > 0$ for all $i$. Then, given $r$, $q_i$ maximizes $[P(q_i + \sum_{j \neq i} q_j) - r_i]q_i$, which implies that the following first-order conditions must hold for all $i = 1, 2, \ldots, M$:

$$P(q_i + \sum_{j \neq i} q_j) - r_i + P'(q_i + \sum_{j \neq i} q_j)q_i = 0.$$  \hspace{1cm} (5)

Adding all $M$ equations yields $MP(\sum q_j) - \sum_j r_j + P'(\sum q_j)(\sum_j q_j) = 0$. Let $f(Q) \equiv MP(Q) - \sum_j r_j + QP'(Q)$. We have $\lim_{Q \to -\infty} f(Q) < 0$ because $P_\infty = 0$, and $f'(Q) = (M-1)P'(Q) + [2P'(Q) + QP''(Q)] < 0$ for all $Q > 0$ by Assumption 1-(ii) and $P'(Q) < 0$ for all $Q > 0$. Assumption 1-(i) then implies that $\lim_{Q \to 0} f(Q) = MP_0 - \sum_j r_j > 0$ must hold. Since $f(Q)$ is continuous for all $Q > 0$, there exists a unique $Q^*$ $(> 0)$ such that $MP(Q^*) - \sum_j r_j + Q^* P'(Q^*) = 0$. This in turn implies that in the equilibrium $q_i = q_i^*(r) > 0$ must hold for all $i$. Now suppose that the conditions (i) and (ii) stated in the claim hold. Note $P(q_i^*(r) + \sum_{j \neq i} q_j^*(r)) - r_i + P'(q_i^*(r) + \sum_{j \neq i} q_j^*(r))q_i^*(r) = 0$, which implies that given $q_j = q_j^*(r)$ for all $j \neq i$, $(\frac{d}{dt})[P(q_i + \sum_j q_j^*(r)) - r_i]q_i = 0$ at $q_i = q_i^*(r)$. We have $(\frac{d}{dt})[P(q_i + \sum_{j \neq i} q_j^*(r)) - r_i]q_i = 2P'(q_i + \sum_{j \neq i} q_j^*(r)) + P''(q_i + \sum_{j \neq i} q_j^*(r))q_i$. If $P''(q_i + \sum_{j \neq i} q_j^*(r))q_i < 0$ we then have that $(\frac{d}{dt})[P(q_i + \sum_{j \neq i} q_j^*(r)) - r_i]q_i < 0$. Suppose $P''(q_i + \sum_{j \neq i} q_j^*(r))q_i > 0$. We then have $(\frac{d}{dt})[P(q_i + \sum_{j \neq i} q_j^*(r)) - r_i]q_i < 2P'(q_i + \sum_{j \neq i} q_j^*(r)) + P''(q_i + \sum_{j \neq i} q_j^*(r))q_i < 0$, where the last inequality holds because of Assumption 1-(ii). Thus we have $(\frac{d}{dt})[P(q_i + \sum_{j \neq i} q_j^*(r)) - r_i]q_i < 0$ for all $q_i > 0$, which in turn implies that $q_i = q_i^*(r) > 0$ is firm $i$’s best response given $q_j = q_j^*(r)$ for all $j \neq i$. Q.E.D.

Given Claim A, the outcome of the second-stage bargaining is the set of input prices $(\hat{r}_1, \hat{r}_2, \ldots, \hat{r}_M)$ which satisfies the following condition: $r_i = \hat{r}_i$ is the Nash Solution to the bargaining problem between downstream firm $i$ and its supplier, given that both expect $\hat{r}_j$ $(j \neq i)$ to be agreed upon between downstream firm $j$ and its supplier. We again assume that the disagreement point is $(0, 0)$. Then, $r_i = \hat{r}_i$ solves the following maximization problem:

$$\max_{\{r_i\}} \left( P(\sum_{j=1}^M q_j^*(r)) - r_i \right) q_i^*(r)^\beta [(r_i - c)q_i^*(r)]^{1-\beta},$$

where $r_i \geq 0$ and $r_j = \hat{r}_j$ for all $j \neq i$.  


Claim B: Suppose $E > -1$ holds. For a given $M \geq 1$, the stage 2 subgame has a unique interior equilibrium outcome in which each pair $i$ chooses $(r_i, q_i) = (\hat{r}, \hat{q})$, if and only if (2) and (3) below uniquely determine $\hat{q} > 0$ and $\hat{r} > 0$:

$$\frac{P(M\hat{q}) - \hat{r}}{\hat{r} - c} = \beta + 1 - \frac{M(1 + \beta + E)}{M(M + 1 + E)},$$

(6)

$$\hat{q} = -\frac{P(M\hat{q}) - \hat{r}}{P'(M\hat{q})}.$$  (7)

Proof: Suppose that for a given $M \geq 1$, the stage 2 subgame has a unique interior equilibrium outcome in which each pair $i$ chooses $(r_i, q_i) = (\hat{r}, \hat{q})$. Let $\hat{f}$ denote an $M$-element vector with each element $\hat{r}$. Since the equilibrium is interior, Claim A implies that there exists unique $Q^*(\hat{f}) > 0$ such that $M(P(Q^*(\hat{f})) - \hat{r}) + Q^*(\hat{f})P'(Q^*(\hat{f})) = 0$, and that $\hat{q} = q^*(\hat{f}) = -\frac{P(Q^*(\hat{f}) - \hat{r})}{P'(Q^*(\hat{f}))}$. This in turn implies $M\hat{q} = Q^*(\hat{r})$, and hence $\hat{q} = -\frac{P(M\hat{q}) - \hat{r}}{P'(M\hat{q})}$. Let $(r_i, \hat{f}_{-i})$ denote an $M$-element vector with $i$-th element $r_i$, and all other elements $\hat{r}$. We then have that given $r_j = \hat{r}$ for all $j \neq i$, $r_i = \hat{r}$ maximizes $S_i(r_i, \hat{f}_{-i})$, where

$$S_i(r_i, \hat{f}_{-i}) \equiv [(P(\sum_{j=1}^{M} q_j^*(r_i, \hat{f}_{-i})) - r_i)q_i^*(r_i, \hat{f}_{-i})]^\beta [(r_i - c)q_i^*(r_i, \hat{f}_{-i})]^{1-\beta},$$

(8)

subject to $(P(\sum_{j=1}^{M} q_j^*(r_i, \hat{f}_{-i})) - r_i)q_i^*(r_i, \hat{f}_{-i}) \geq 0$ and $(r_i - c)q_i^*(r_i, \hat{f}_{-i}) \geq 0$. It suffices to consider $(r_i, \hat{f}_{-i})$ such that $S_i(r_i, \hat{f}_{-i}) > 0$, which implies $P(\sum_{j=1}^{M} q_j^*(r_i, \hat{f}_{-i})) > r_i > c$ and $q_i^*(r_i, \hat{f}_{-i}) \geq 0$. We have $S_i'(r_i, \hat{f}_{-i}) \geq S_i''(r_i, \hat{f}_{-i}) > 0 \iff \ln S_i(r_i, \hat{f}_{-i}) \geq \ln S_i'(r_i, \hat{f}_{-i}) > 0$. Hence maximizing $S_i(r_i, \hat{f}_{-i})$ is equivalent to maximizing $\ln S_i(r_i, \hat{f}_{-i}) \equiv \beta \ln[P(\sum_{j=1}^{M} q_j^*(r_i, \hat{f}_{-i})) - r_i]q_i^*(r_i, \hat{f}_{-i}) + (1 - \beta) \ln[(r_i - c)q_i^*(r_i, \hat{f}_{-i})]$. Substituting $q_i^*(r_i, \hat{f}_{-i}) = -\frac{P(Q^*(r_i, \hat{f}_{-i})) - r_i}{P'(Q^*(r_i, \hat{f}_{-i}))} \ln S_i(r_i, \hat{f}_{-i})$, where $Q^*(r_i, \hat{f}_{-i}) \equiv \sum_{j=1}^{M} q_j^*(r_i, \hat{f}_{-i})$, and simplifying yields

$$\ln S_i(r_i, \hat{f}_{-i}) = (1 + \beta) \ln(P(Q^*(r_i, \hat{f}_{-i})) - r_i) + (1 - \beta) \ln(r_i - c) - \ln(-P'(Q^*(r_i, \hat{f}_{-i}))).$$

(9)

Then the following first order conditions must hold at $r_i = \hat{r}$ for all $i$:

$$(1 + \beta) \frac{P'(Q^*(r_i, \hat{f}_{-i}))}{(P(Q^*(r_i, \hat{f}_{-i})) - r_i)} \frac{dQ^*(r_i, \hat{f}_{-i})}{dr_i} + 1 - \beta - \frac{P''(Q^*(r_i, \hat{f}_{-i}))}{-P'(Q^*(r_i, \hat{f}_{-i}))} = 0.$$  (10)

Substituting $\frac{dQ^*(r_i, \hat{f}_{-i})}{dr_i} = \frac{1}{(M + 1)P'(Q^*(r_i, \hat{f}_{-i}))+Q^*(r_i, \hat{f}_{-i})P''(Q^*(r_i, \hat{f}_{-i}))}$ and $r_i = \hat{r}$ in (10) and simplifying gives

$$-\frac{(1 + \beta)(M + E)}{(P(Q^*(\hat{f})) - \hat{r})(M + 1 + E)} + \frac{E}{M(P(Q^*(\hat{f})) - \hat{r})(M + 1 + E)} + \frac{1 - \beta}{\hat{r} - c} = 0,$$

(11)
where $E \equiv \frac{Q''(Q)}{P'(Q)}$ is constant. Noting that $(\hat{r}, \hat{r}_{-i}) \equiv \hat{r}$ and $Q^*(\hat{r}) \equiv \sum_j q_j^*(\hat{r}) = M\hat{q}$, simplifying (11) further and rearranging yield equation (6). Hence equations (6) and (7) must hold. We have that LHS of (6) is strictly decreasing in $\hat{r}$ since \[ \frac{d}{dr}(\frac{P(M\hat{q})-\hat{r}}{\hat{r}-c}) = -\frac{2M(M+E)-E}{(1-\beta)M(M+E)(\hat{r}-c)} < 0 \] while the RHS of (6) is constant. Hence (6) uniquely determines $\hat{r}$ further, and rearranging yield equation (6). Hence equations (6) and (7) must hold.

Note that inequality (13) can be rewritten as $(P(M\hat{q})-c)M\left(\frac{d\hat{q}}{dM} + \frac{d\hat{q}}{\hat{r}}\frac{d\hat{r}}{dM}\right) + (\hat{r} - c)(\hat{q} + M\frac{d\hat{q}}{dM}) > 0$, given that $\frac{d\hat{q}}{dM} \equiv \frac{\partial\hat{q}}{\partial M} + \frac{\partial\hat{q}}{\partial \hat{r}}\frac{d\hat{r}}{dM}$. Here, the second term $\frac{\partial\hat{q}}{\partial \hat{r}}\frac{d\hat{r}}{dM}$ arises because the number

\[ \frac{d\hat{q}}{dM} \]

\[ \frac{\partial\hat{q}}{\partial \hat{r}}\frac{d\hat{r}}{dM} \]

Under this alternative bargaining set-up $\pi_D(M)$ can be increasing in $M$ for some $M$ as in, e.g., Naylor (2002). This implies that the solution to $\pi_D(M) = 0$ is not necessarily unique. Given this, here we define $M_f$ to be the maximum $M'$ ($\geq 1$) satisfying $\pi_D(M') = 0$, so that, if $M_f < M^*$, then $M' < M^*$ holds for all $M'$ satisfying $\pi_D(M') = 0$. That is, this definition of $M_f$ is in a sense the most “conservative” for demonstrating the occurrence of insufficient entry.
of downstream firms $M$ affects the input price through second-period bargaining, and
the input price $\hat{r}$ in turn affects the quantity chosen in Stage 3 under downstream Cournot
competition. If upstream firms have no bargaining strength (i.e. if $\beta = 1$) so that $\hat{r} = c$ and
$\frac{\partial \hat{r}}{\partial M} = 0$, this inequality cannot hold under the presence of the business-stealing effect (i.e.
if $\frac{\partial q_j}{\partial M}|_{M=M^*} < 0$). However, there is a possibility of insufficient entry as long as upstream
firms have a positive bargaining strength so that $\hat{r} > c$ holds.

In what follows we present an example in which condition (13) holds despite the presence
of the business-stealing effect at $M = M^*$. Let $P(Q) = a - Q^b$ where $a > 0$ and $b > 0$,
and normalize $c = 0$. We have $P'(Q) = -bQ^{b-1}$, $P''(Q) = -b(b - 1)Q^{b-2}$. Then $P_0 = a$
and $\lim_{Q \to 0} P(Q) + QP'(Q) = \lim_{Q \to 0} a - (1 + b)Q^b = a$. Hence Assumption 1(i) is satisfied.
Also we have $2P'(Q) + QP''(Q) = -b(1 + b)Q^{b-1} < 0$ for all $Q > 0$, and hence Assumption
1-(ii) is also satisfied. For the class of demand functions considered here $E = -\frac{QP''(Q)}{P'(Q)} =
\frac{a}{b} - 1 > -1$ holds. Under this specification, solving the system of equations (6) and (7) yields
$\hat{r} = \frac{M^{b(1-\beta)}}{(Mb+1)+(M-1)b+M+M\beta)}$ and $q = \frac{(M(a-\hat{r}))^\frac{1}{b}}{M+\hat{r}}$, where $\hat{r} > 0$ and $\hat{q} > 0$ hold for all $M \geq 1$
(because $a > \hat{r}$ holds). Claim B then implies that the stage 2 subgame given by any $M \geq 1$
has a unique interior equilibrium outcome in which each pair $i$ chooses $(r_i, q_i) = (\hat{r}, \hat{q})$.

Since $\frac{\partial q_j}{\partial M} = -\frac{(M(1-b))(a-\hat{r})}{(M+b)^{1+\frac{a}{b}}M^2 - \frac{a}{b}} < 0$ for all $M \geq 1$, the business-stealing effect is present
in the Stage 2 equilibrium for all $M \geq 1$. Now consider the following parameterization:
$a = 10$, $b = 0.5$, $\beta = 0.25$ and $K = 34.33$. We find that $M^* = 2.5 > 1$, and at $M = M^* = 2.5$,
$\hat{r} = 1.22$, $\hat{q} = 21.42$, $P(M\hat{q}) = 2.68$, and $\frac{\partial q_j}{\partial M} = -3.44$. Substituting these values we find that
$(P(M\hat{q}) - \hat{r})M^{\frac{\partial q_j}{\partial M}} + (\hat{r} - c)(\hat{q} + M^{\frac{\partial q_j}{\partial M}}) = 2.99 > 0$. We also find that $\pi_D(1) = 28.17 > 0$
which implies that $M_f > 1$. Hence $M^* > M_f > 1$ holds; that is, insufficient entry occurs in
the presence of the business-stealing effect under this parameterization.

\footnote{Note that in the proof of Claim B we implicitly assumed that $q_j(r_i, \hat{r}_{-i}) > 0$ for all $r_i \geq 0$ and for all $j$. We checked that under the specification considered here, this condition holds for all $M \geq 1$.}