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Net Foreign Assets, Productivity and Real Exchange Rates in Constrained Economies*

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Abstract

Empirical evidence suggests that real exchange rates (RER) behave differently in developed and developing countries. We develop an exogenous 2-sector growth model in which RER determination depends on the country’s capacity to borrow from international capital markets. The country faces a constraint on capital inflows. With high domestic savings, the country converges to the world per capita income and RER only depends on productivity spread between sectors (Balassa-Samuelson effect). If the constraint is too tight and/or domestic savings too low, RER depends on both net foreign assets (transfer effect) and productivity. We then analyze the empirical implications of the model and find that, in accordance with the theory, RER is mainly driven by productivity and net foreign assets in constrained countries and exclusively by productivity in unconstrained countries.

JEL Classification: E39; F32; F41.

Keywords: Real exchange rate; capital inflows constraint; overlapping generations.

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1 Introduction

A recurrent question in International Macroeconomics concerns the main long-run determinants of real exchange rates (RER). There is, however, no consensus yet on this question. Among the most often quoted determinants we can find productivity, terms of trade and net foreign assets (NFA) [see Chinn (2006)]. Empirical evidence suggests that these determinants change significantly as we vary periods and countries considered.

The empirical literature on the Balassa-Samuelson (BS) effect shows that RER appreciation may be related to productivity growth but not systematically. It seems to have special relevance for countries like Japan, some OECD countries [Canzoneri et al. (1999)] and transition economies [Égert et al. 2003, 2006]. Ito et al. (1999) show that RER and growth are positively correlated in Japan, Korea, Taiwan, Hong Kong whereas the correlation remains negative for Indonesia, Thailand, Malaysia, Philippines and China. Hong Kong, Taiwan and Singapore combine a high growth rate and a small appreciation. For other Asian countries except China, Singapore, Taiwan and Thailand, Chinn (2000a) finds that productivity explains RER only when public spending and oil prices are taken into account. Chinn (2000b), using panel data, finds that the RER requires around 5 years to converge to the level predicted by BS. In a recent paper, Bergin et al (2006) also report that the BS effect is not stable through time, but it appears to have become more important in recent years. The fit of the standard BS theory to explain RER changes seems to be very poor and largely country- and period-specific.

Recently, in line with the theory that emphasises the role of foreign assets for equilibrium RERs, Lane and Milesi-Ferretti (2004) have developed a model that highlights the transfer effect - which relates RER to NFA. Using a database that covers 64 industrial and less developed countries between 1970 and 1998, they show that a rise in NFA appreciates the RER, especially for countries that have low income, low openness or foreign exchange restrictions. The theoretical model they present links international payment to RER through an adjustment of labor supply\(^1\). However, they fall short of explaining why developing countries that face a constraint on capital inflows experiment a higher transfer effect than others.

Linking together this diverse set of results, our study contributes to this literature in two

\(^{1}\)Obstfeld and Rogoff (1995), Galstyan (2007) also develop models in which international payments affects the relative price of the non-traded good through a labor supply adjustment.
ways. First, it presents a model that reconciles these empirical findings in which RER determination depends on the country’s capacity to borrow from international capital markets. Second, it tests whether the behavior of the data is consistent with the main results of the model focusing on whether the long- and short-run relationships between the RER and its main determinants depend on the financial constraints faced by countries.

Although not our primary focus, the paper is also related to the growth and convergence literature. Our model stresses that a RER appreciation may help a developing country catching-up with the world per capita income. Indeed, because of the way the constraint is specified, a RER appreciation attracts capital flows\(^2\) and may fill in a lack of domestic savings that accelerates growth temporarily or increases permanently long-run income per capita. Some models with constraints on capital inflows have already been developed in the exogenous growth literature [Barro, Mankiw and Sala-i-Martin (1995), Lane (2001)]. They exhibit a common property: the constraint on capital inflows slows down economic convergence but does not stop it. The convergence speed they obtained is empirically plausible [Mankiw, Romer and Weil (1992)] while it was not the case in closed economy models. In the long-run, however, per capita income of the developing country systematically converges to the world per capita income: the developing country is no longer constrained in steady state. In contrast, by making use of overlapping generations, our model allows the steady state to be constrained or unconstrained. The credit constraint we impose can not only slow down absolute convergence but also prevent it from occurring even in the long-run. This model then predicts convergence clubs rather than absolute convergence: a developing country with lack of domestic savings or unstable institutions may not converge even in the long-run - since it reaches a constrained steady state with lower income per capita.\(^3\)

We use an overlapping generations setting of a constrained economy initially developed by Obstfeld and Rogoff [1996] in which we introduce two production sectors [Gente, 2006] a non-traded and a traded sector. We assume that the amount the country can borrow on the international capital market is an exogenous fraction of per-capita income. This fraction

\(^2\)In Rodrick (2007), capital inflows are related to traded inputs and a real appreciation also increases capital inflows as in our model.

\(^3\)Bosworth and Collins (2003), amongst others, have provided evidence in favor of such type of convergence and against absolute convergence. See also Durlauf et al (2005) for further discussion.
represents the trust of foreign investors about local institutions, creditworthiness, and the ease of cross-border financial transactions. If the constraint is not too tight - or if there are high domestic savings - the constrained economy will become unconstrained in the long-run. Otherwise, if investors are not confident - or there are low domestic savings - the developing country will converge to the constrained steady state. The RER behaviour differs widely between those two kinds of steady states.

In the unconstrained steady state, the RER will exclusively be determined by the Balassa-Samuelson effect. Conversely, in the constrained steady state, the RER will depend on supply and demand of non-traded goods. A productivity shock operates through a demand effect and not only through the Balassa-Samuelson effect. In the same way, an international transfer from abroad will appreciate the RER whereas this is not the case in the unconstrained steady state. This transfer effect is higher in less open economies. This is consistent with Lane and Milesi-Ferretti (2004) empirical results. Assuming perfect mobility of factors between sectors, total output depends on both capital intensity and RER. A RER appreciation as well as an increase in savings relaxes the constraint and may promote convergence. Then a positive transfer from abroad may increase permanently income per capita. Indeed, through a RER appreciation, the transfer effect loosens the constraint. It then increases capital inflows and helps the country converging to a higher long-run per capita income, at the unconstrained steady state.

These implications of the model for the RER behavior are then tested using a simple econometric model based on Lane and Milesi-Ferretti (2004). We estimate separate RER models for financially constrained and unconstrained economies, selected using the Chinn and Ito (2007) measure of external financial openness. The findings are supportive of the implications of the model in the long-run, with the RER driven only by productivity in financially open countries and by both productivity and NFA for countries with restricted access to international finance.

The paper is organised as follows. Section 2 presents the theory model. Section 3 analyses the steady state solution of the model for constrained and unconstrained economies. Section 4 presents the econometric evidence, and Section 5 provides some conclusions.
2 The model

The model is a variant of the small open economy overlapping generations model of Obstfeld and Rogoff (1996) in which we introduce two production sectors: a tradable sector and a non-tradable sector. In this setting, the real exchange rate $R$ denotes the relative price of non-tradable to tradable goods. The country faces a constraint on capital inflows

$$B_{t+1} \geq -\eta N_t y_t$$

where $B_{t+1}$ denotes the NFA of the domestic country in terms of traded goods, and $\eta > 0$ is the proportion of total income ($N_t y_t$) the domestic country can borrow, where $N_t$ is total population and $y_t$ is per capita income. The $\eta$ parameter reflects the ease of access the country has to international capital flows and may be related to institutional features such as restrictions to capital and current account transactions\textsuperscript{4}. The smaller $\eta$ the more constrained the country is to capital inflows. In the model we present below, agents live for two periods and only work in their first period of life.

2.1 Individuals

The economy consists of a sequence of individuals who live for two periods. In the second period of her life, each individual gives birth to $1 + n$ others so that the per period rate of population growth is $n$. At time $t$, each generation consists of $N_t$ identical individuals who make decisions concerning consumption and savings.

The intertemporal preferences of an individual belonging to generation $t$ are represented by

$$U (c_t, d_{t+1}) = \beta \ln c_t + (1 - \beta) \ln d_{t+1}$$

where $c_t$ and $d_{t+1}$ are respectively composite consumption when adult and composite consumption when old; $\beta \in (0,1)$ denotes individuals’ thrift.

Let $x = c, d$ denote individual consumption at each period of life, and $x_N$ and $x_T$ be, respectively, the spending allocated into non-traded and traded goods. Instantaneous preferences

\textsuperscript{4}Our purpose in this paper is not to explain the $\eta$ parameter or estimate it, but to emphasize the role that this constraint can play in an open economy model.
are defined according to a Cobb-Douglas aggregator:

\[ u(x_T, x_N) = x_T^\alpha x_N^{1-\alpha}, \quad 0 < \alpha < 1 \]  

(3)

Following Obstfeld and Rogoff (1996), the small economy faces a constraint on capital inflows (Equation 1). The consequence of this assumption is that the domestic return on capital may be higher than the world return. During the first period of life, individuals offer labor inelastically and allocate their net earnings \( w_t \) among consumption spending \( \pi_t c_t \) and savings

\[ \pi_t c_t + (1 + n) k_{t+1} + (1 + n) b_{t+1} = w_t \]  

(4)

where \( k_{t+1} \) is total capital stock per young agent in terms of traded good prices \( k_{t+1} = K_{t+1}/N_{t+1} \), and \( b_{t+1} \) net foreign assets per young agent \( b_{t+1} \equiv B_{t+1}/N_{t+1} \). The price of the tradable good is normalized at unity. We denote the composite consumption good by \( x \equiv x_T^\alpha x_N^{1-\alpha} \) with \( x = c, d \) to specify the same preferences among the two goods for both periods and \( \pi_t \) the consumer price index. Hence, national savings can be held into two forms, capital stock and net foreign assets. Since the returns on these stocks are different, the agents choose both the amount of their savings \( s_t \) and their allocation between the two assets \( k_{t+1} \) and \( b_{t+1} \). To take into account this arbitrage, we assume, following Obstfeld and Rogoff (1996), that the constraint on capital inflows holds at the microeconomic level. This assumption means that banks cannot lend more than \( \eta_y t \) to each individual at the world market interest rate \( \bar{r} \). Agents know both the world and domestic returns on capital. A spread between these two returns is a new potential source of income for them: they can borrow from the world market to lend on the domestic market and realize a capital gain.

When old, individuals are retired and consume the proceeds of their savings according to

\[ \pi_{t+1} d_{t+1} = \left(1 + r_{t+1}^d \right) (1 + n) k_{t+1} + (1 + \bar{r}) (1 + n) b_{t+1} \]  

(5)

The domestic return on capital is the market interest rate \( r_{t+1}^d \) whereas the world return \( \bar{r} \) is fixed according to the small open economy assumption.

The maximization program of an individual born in period \( t \) is solved in two steps. First, the individual chooses \( \pi_t c_t \) and \( b_{t+1} \) to maximize life-cycle utility (2) under the budget constraints (4), (5) and the capital inflows constraint

\[ b_{t+1} \geq -\frac{\eta_y t}{1 + n} \]  

(6)
Second, he shares his consumption spending ($\pi x$) between the two goods $x_N$ and $x_T$ to maximize instantaneous utility (3) under the spending constraint $\pi x = Rx_N + x_T$. Hence, the allocation of total consumption spending between the two goods at each period is

$$x_T = \alpha \pi x$$

$$Rx_N = (1 - \alpha) \pi x$$

where the price index is $\pi = \phi(\alpha) R^{1-\alpha}$, with $\phi(\alpha) \equiv \alpha^{-\alpha} (1 - \alpha)^{\alpha-1}$.

### 2.2 Production Sectors

Investment transforms instantaneously a unit of tradable good into a unit of installed capital: $K_{t+1} = I_t$ and capital fully depreciates after one period ($\delta = 1$). The representative firm produces in the two sectors, the traded ($T$) and the non-traded ($N$) sector.

$$\max_{I_t,K_{Tt},K_{Nt}} F(K_{Tt},L_{Tt}) + RH(K_{Nt},L_{Nt}) - wL_t - I_t$$

s.t. $K_{t+1} = I_t$

$$K_{Tt} + K_{Nt} = K_t$$

(7)

$$L_{Tt} + L_{Nt} = L_t$$

(8)

with $L_t$ being total labor supply, and $K_i$ and $L_i$ the amount of capital stock and labor supply used in sector $i = T,N$ respectively. $F(\cdot)$ and $H(\cdot)$ are the traded and non-traded sector production functions. Dropping time indices, optimal allocation of factors is given by

$$a_T f'(k_T) = a_N R h'(k_N)$$

(9)

$$a_T \left[ f(k_T) - k_T f'(k_T) \right] = a_N R \left[ h(k_N) - k_N h'(k_N) \right]$$

(10)

where $k_i \equiv K_i/(l_i L)$ is capital intensity, and the share of labor used in sector $i$ is $l_i = L_i/L$, $i = T,N$. The intensive form production functions are $F(k_T,1) \equiv f(k_T)$, $H(k_N,1) \equiv h(k_N)$. Finally, $a_i$ is the total factor productivity level of sector $i = T,N$. According to (9) and (10), $k_N$ and $k_T$ depend only on RER whereas the allocation of labor depends both on capital intensity
and the RER. Hence, \( k_N \equiv k_N (a_T, a_N, R) \) and \( k_T \equiv k_T (a_T, a_N, R) \), while \( l_N \equiv l_N (a_T, a_N, k, R) \) and \( l_T \equiv l_T (a_T, a_N, k, R) \). From (7), (8), (9) and (10), the optimal factor allocation implies

\[
\begin{align*}
\frac{\partial k_N}{\partial R} &= \frac{a_T f}{R^2 a_N h'' (k_N - k_T)} \\
\frac{\partial k_T}{\partial R} &= \frac{R a_N h}{f a_T (k_N - k_T)}
\end{align*}
\]

Similarly, \( \partial l_N / \partial k \leq 0 \) if \( k_N \leq k_T \) and \( \partial l_N / \partial R > 0 \). When the tradable sector is capital intensive, a real appreciation leads to an increase in both capital intensities \( k_N \) and \( k_T \) whereas labor moves from the traded to the non-traded sector. These factor movements reflect that a real appreciation makes the non-tradable sector more attractive. Assuming perfect intersectoral mobility, the returns on capital \( r^d = a_T f (k_T (a_T, a_N, R)) - 1 \equiv r^d (a_T, a_N, R) \) and labor \( w = a_T [f (k_T (a_T, a_N, R)) - f' (k_T (a_T, a_N, R)) k_T (a_T, a_N, R)] \equiv w (a_T, a_N, R) \) only depend on the RER (\( R \)) and productivity. A RER appreciation, profitable to the non-traded sector which is labor intensive, will increase wage and reduce the domestic interest rate. An exogenous rise in traded (non-traded) sector productivity increases (decreases) domestic interest rates and reduces (increases) wages when the traded sector is capital intensive. Unless otherwise slated, we will omit the productivity terms \( (a_i) \) when there is no productivity change so that: \( k_T \equiv k_T (R) \), \( k_N \equiv k_N (R) \), \( l_N \equiv l_N (k, R) \), \( r^d \equiv r^d (k, R) \), \( w \equiv w (k, R) \).

Per capita total income depends on both the RER and per capita capital stock: \( y \equiv (1 + r^d (R)) k + w \equiv y (R, k) \) with

\[
\begin{align*}
\frac{\partial y}{\partial k} &= 1 + r^d \\
\frac{\partial y}{\partial R} &= R h (k_N (R)) l_N (k, R)
\end{align*}
\]

A RER appreciation and a rise in per capita capital stock both exert a positive effect on total income.

2.3 The temporary equilibrium in the constrained case

We will focus on the case where the capital inflows constraint binds, at least initially, with a capital intensive traded sector. This creates a gap between domestic and world returns on capital. This gap - in a similar way as a risk premium\(^5\) - reflects the fact that many developing

\(^5\)Similar results could potentially be obtained by considering country-risk.
countries do not have full access to international capital markets: the return on domestic capital \( r_{t+1}^d \) must be higher than the world market interest rate \( \bar{r} \) to offset the perceived risky return due to, for instance, a restrictive capital account regime.

Hence, young and old agents’ consumption functions are

\[
\pi_t c_t = \beta \left[ w_t - \frac{r_{t+1}^d - \bar{r}}{1 + r_{t+1}^d} (1 + n) b_{t+1} \right] \tag{14}
\]

\[
\pi_{t+1} d_{t+1} = (1 - \beta) \left[ (1 + r_{t+1}^d) w_t - (r_{t+1}^d - \bar{r}) (1 + n) b_{t+1} \right] \tag{15}
\]

Individuals consume a proportion \( \beta \) of their life-cycle income during the first period of life and the remaining in the second. Life-cycle income consists of:

- the wage \( w \)
- the capital gain agents may realize borrowing at world rate \( \bar{r} \) to invest in domestic capital whose return \( r^d \) is higher than \( \bar{r} \).

The period-\( t \) temporary equilibrium conditions are

(i) **Capital market equilibrium.** Given the optimal intersectoral factor allocation \( k_T (R) \) and \( k_N (R) \), net foreign assets per capita are given by

\[
b_{t+1} = -\frac{\eta y (R_t, k_t)}{1 + n} \tag{16}
\]

Let \( \Gamma (R_{t+1}) \equiv \eta \left[ r^d (R_{t+1}) - \bar{r} \right] \left[ 1 + r^d (R_{t+1}) \right]^{-1} \) be the *arbitrage premium* which depends on the interest rate gap between domestic and world capital markets and on proportion \( \eta \) of the income agents can borrow. The higher \( \Gamma \) the higher the capital gain agents realize. Therefore, capital per worker is

\[
k_{t+1} = \left[ 1 - \beta + \eta - \beta \Gamma (R_{t+1}) \right] \frac{w (R_t)}{1 + n} + \left[ \eta - \beta \Gamma (R_{t+1}) \right] \frac{1 + r^d (R_t)}{1 + n} k_t \tag{17}
\]

(ii) **Labor market equilibrium.** The inelastic labor supply \( N_t \) is equal to the labor demand \( L_t \). Given the capital market equilibrium, the wage \( w \) equalizing labor supply and demand is defined by

\[
w (R_t) \equiv f (k_T (R_t)) - k_T (R_t) f' (k_T (R_t)) \tag{18}
\]

(iii) **Non-tradable goods market equilibrium.** There are \( N_t \) young agents and \( N_{t-1} \) old agents. Hence, the equilibrium on the non tradable goods market is

\[
(1 - \alpha) (N_t \pi_t c_t + N_{t-1} \pi_t d_t) = R_t Y_N (R_t, k_t) \tag{19}
\]
with $Y_N (R_t, k_t) \equiv I_N (R_t, k_t) N_t h (k_N (R_t))$. Consumption spending is given by equations (14) and (15).

Equation (17) describes the allocation of saving between both assets. It offers a first dynamic relationship between the RER and capital intensity. Using (16), (18) and (19), with consumption spending given by (14) and (15), we get a second dynamic relationship between $R$ and $k$.

The intuition behind the dynamics is the following. In such a constrained economy, the amount the country can borrow on world market is limited to a $\eta/(1 + n)$ fraction of the global income. In this 2-sector 2-factor model, global income does not only depend on capital intensity but also depends on RER. A RER appreciation - or an increase in capital intensity - increases total income in terms of traded good and then loosens the constraint. The country can borrow more, increases its capital stock and total output, loosening the constraint again. This mechanism will help the country converging to an unconstrained steady state if non-traded consumption is sufficiently high and if the constraint is not too tight (if $\eta$ not too small). Otherwise the country will remain constrained in the long-run\(^6\).

In the existing literature, there are constrained economy models like Barro, Mankiw and Sala-i-Martin (1995) or Lane (2001) that focused on the convergence speed issue. Indeed, in those constrained economy models the country systematically converges to an unconstrained steady state and the point is to know at what speed. In our model, the country may converge in the long-run to a steady state that could either be constrained or unconstrained\(^7\). Hence, the important question here is to study the relationship between NFA and RER in both types of equilibrium.

\(^6\)When the tradable sector is labor intensive, it is the RER depreciation that helps the country converge to an unconstrained steady state. We do not focus on this case in what follows because it is less frequently observed and corresponds to a preliminary stage of development [Ito, Isard and Symanski (1999)].

\(^7\)This is due to the presence of overlapping generations and the fact that there is no need for the time preference rate to equalize the world interest rate.
3 Steady state

There are two kinds of steady state: a constrained one and an unconstrained one. However, these two steady states do not exist simultaneously. The country may converge to an unconstrained steady state if the constraint is not too severe (high \(\eta\)), domestic saving is high (low \(\beta\)) or if agents consume enough non-traded goods (low \(\alpha\)). The relationship between RER, NFA and productivity depends on the kind of steady state the economy converges to.

3.1 Constrained or unconstrained?

We now aim at determining the threshold level of the constraint, \(\tilde{\eta}\) such that if \(\eta < \tilde{\eta}\), the country will remain constrained in the long-run (see Appendix B). We will then proceed into three stages. First, we describe the constrained steady state. Second, we describe the unconstrained steady state Then we determine the threshold level of the constraint \(\tilde{\eta}\).

3.1.1 A constrained steady state

The constrained steady state is denoted by a *. If the country remains constrained even in the long-run, the steady state \((k^*, R^*)\) is defined by the following system

\[
k^* = \frac{w(R^*)}{1 + n} \left( \frac{1 - \beta}{1 - (\eta - \beta \Gamma(R^*))} \right) \frac{1 + r^d(R^*)}{1 + n}
\]

\[
\left[ \beta + \frac{1 + r^d(R^*)}{1 + n} (1 - \beta) \right] \left[ w(R^*) + \frac{r^d(R^*) - \bar{r}}{1 + r^d(R^*)} \eta y(R^*, k^*) \right] = \frac{y_N(R^*, k^*) R^*}{1 - \alpha}
\]

Equation (20) gives the long-run allocation of saving. In this constrained economy, capital per capita \(k\) is financed by domestic saving plus capital inflows. Equation (21) is the long-run non-traded good market clearing condition. Both long-run capital intensity and RER are determined by those two conditions. Then, the constraint gives net foreign assets: \(b^* = -\eta y(R^*, k^*) / (1 + n)\).

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*We can show, using a simple Cobb-Douglas example, that, when the unconstrained steady state exists, the constraint is no longer respected and then the constrained steady state does not exist.

9To guarantee potential existence, we assume that \(\eta < (1 + n) / (1 + \bar{r})\).
3.1.2 An unconstrained steady state

An overbar denotes the unconstrained steady state. It is the standard steady state that occurs in a two-sector two-factor small open economy model. The developing country has a perfect access to the international capital market so that

\[
r^d(\bar{R}) = \bar{r} \tag{22}
\]

That is, domestic return on capital converges to the world one. Equation (22) determines the long-run RER that depends only on the world interest rate\(^{10}\). The long-run RER determines the wage and hence the demand for non-traded goods (left hand side of equation (23)). Domestic capital \(\bar{k}\) clears the non-traded good market

\[
\left[\beta + \frac{(1 + \bar{r})(1 - \beta)}{1 + n}\right] w(\bar{R}) = \frac{y_N(\bar{R}, \bar{k})}{1 - \alpha} \bar{R} \tag{23}
\]

Finally, the net foreign assets fill the gap between domestic capital \(\bar{k}\) and domestic saving

\[
\bar{b} = \frac{w(\bar{R})}{1 + n} (1 - \beta) - \bar{k} \tag{24}
\]

In this unconstrained steady state, the Balassa-Samuelson analysis holds since RER only depends on the supply side of the model.

3.1.3 The threshold level

The level of the constraint, \(\eta\), is exogenous and could be interpreted as the penalty imposed by international investors to a country because of lack of creditworthiness and institutional restrictions to financial flows. We take this penalty as given and determine whether this \(\eta\)—penalty is severe enough to allow the developing country to converge to the unconstrained steady state. We focus on the case where \(k_0 < \bar{k}\). Let \(\tilde{\eta}\) be the threshold level of the constraint such that

- when \(\eta \geq \tilde{\eta}\), the country converges to the unconstrained steady state and we recover the standard small open economy setting
- when \(\eta < \tilde{\eta}\), the country converges to the constrained steady state and remains constrained in the long-run.

\(^{10}\)The RER is also determined eventually here by the productivity spread between sectors as well: \(r^d(a_N, a_T, R) = \bar{r}\) with \(\partial r^d/\partial a_N < 0, \partial r^d/\partial a_T > 0\) when the traded sector is capital intensive.
A special case of this model would be $\eta = 0$ where the country would be so constrained that net foreign assets would be zero. This case would correspond to a closed economy setting.

A rise (drop) in $\tilde{\eta}$ makes the convergence to the (unconstrained) constrained steady state more likely to occur. We can characterize the threshold level $\tilde{\eta}$ in a simple Cobb-Douglas case.

**Example: The Cobb-Douglas case.** We assume Cobb-Douglas technologies in both sectors.

Let the long-run propensity to consume the non-traded good be

$$\Psi = (1 - \alpha) \left[ \beta + (1 - \beta) \frac{(1 + \bar{r})}{(1 + n)} \right]$$

After a bit of algebra (see Appendix B) we can show that

$$\tilde{\eta} = \frac{1 + n}{1 + \bar{r}} \left[ \nu + \Psi (\rho - \nu) - (1 - \beta) (1 - \nu) \right] \frac{1 + \Psi (\rho - \nu)}{1 + \Psi (\rho - \nu)}$$

Where $\rho$ and $\nu$ are the elasticities of output with respect to capital in the traded and non-traded sectors respectively. We assume that the total propensity to consume the non-traded good $\Psi$ is lower than unity and that the traded sector is capital intensive. This means that $1 + \Psi (\rho - \nu) > 0$. This implies that a rise in $\Psi$ promotes convergence to the unconstrained steady state\(^{11}\). The intuition behind this result is simply that a rise in non-traded goods consumption tends to appreciate the RER. This RER appreciation relaxes the constraint and helps the country reaching the unconstrained steady state. In the same way, the threshold level $\tilde{\eta}$ depends on $n$ and $\beta$ since population growth and time preference influence both propensity to consume $\Psi$ and savings.\(^{12}\)

**Calibration.** We assume that half of the consumption is spent on non-traded goods. Assuming that each generation lives for 25 years, the world interest factor is $1 + \bar{r} = 0.37$ which means that the world real interest rate is about 1.25% per year, and $n = 0.6$ corresponds to a rate of population growth of 1.9%. In accordance with Beine et al. (2001), let $\beta = 0.6$ to have a domestic rate of time preference of around 3.56%. Using those figures, the threshold level is $\tilde{\eta} = 0.129$. Figure 1 represents the long-run equilibrium. The constrained steady state is represented for $\eta = 0.1$ and the unconstrained steady state for $\eta = 0.2$. In the constrained steady state, the domestic real interest rate exceeds the world interest rate

---

\(^{11}\)Since $\eta < (1 + n) / (1 + \bar{r})$, we have $\partial \tilde{\eta} / \partial \Psi < 0$.

\(^{12}\)For instance, we have that $\partial \tilde{\eta} / \partial n > 0$. 

---

13
and more resources are allocated to the production of the traded good. Since domestic interest rate and RER are negatively related, the RER is lower in the constrained steady state than in the unconstrained one. We choose the elasticities of output with respect to capital per capita for the two sectors to match empirical evidence: 40% of total output is traded with 37% of labor being employed in that sector [Mahbub Morshed and Turnovsky (2004)]. With $\nu = 0.4$ and $\rho = 0.2$ the traded sector is capital intensive. Then, we have $l^*_T = 39.24\%$ and $\bar{l}_T = 37.70\%$ whereas $y^*_T/y^* = 43.51\%$ and $\bar{y}_T/\bar{y} = 41.62\%$ (See Table 1). In the constrained steady state, production factors are over-allocated in the traded sector.

![Figure 1]

[Table 1]

### 3.2 NFA and RER

The relationship between NFA and RER, the so-called transfer effect, will depend on the nature of the steady state the economy converges to. Let $T_t$ denote a transfer received from abroad. Then capital market equilibrium becomes

$$s_t + T_t = (1 + n)[b_{t+1} + k_{t+1}]$$

We can consider $T$ as a gift from foreigners. Like savings, this gift will be used for asset accumulation.

#### 3.2.1 Unconstrained steady state

Long-run equilibrium is given by equations (23) and (22). The introduction of the transfer changes NFA accumulation

$$\bar{b} = T + \frac{w(\bar{R})}{1 + n} (1 - \beta) - \bar{k}$$

(25)

A transfer will increase NFA. Since RER is exclusively determined by productivity and world interest rates: there is no transfer effect and NFA do not affect the RER.
3.2.2 Constrained steady state

In the constrained steady state, long-run equilibrium is given by equations (20) and (21) and the constraint still binds

\[ b^* = -\frac{\eta y (R^*, k^*)}{1 + n} \]

The introduction of the transfer \( T \) changes equation (20) that becomes

\[ k^* = \frac{w(R^*)}{1 + n} \left( (1 - \beta) + \eta - \beta \Gamma (R^*) + \frac{T}{1 + n} \right) \]

A transfer will have two kinds of effects

(i) a direct effect: a rise in \( T \) increases capital intensity. Since the non-traded sector is labor intensive, this rise in capital reduces non-traded output and leads to a RER appreciation.

(ii) an indirect effect: a transfer increases total production and loosens the constraint. As a result, capital stock increases more and this reinforces the RER appreciation. The higher \( \eta \) - the more the country is allowed to borrow on international markets - the higher the RER appreciation.

As in Lane and Milesi-Ferretti (2004), the transfer effect increases with the size of the non-traded sector: the less open (low \( \alpha \)) the country, the higher the direct effect. However, our model shows analytically that the transfer effect depends also on the country’s access to external borrowing. It holds only in the constrained economy case, that is, when \( \eta < \tilde{\eta} \). Conversely, when \( \eta \geq \tilde{\eta} \) the transfer effect does not hold.

**Calibration: The transfer effect** An international transfer can be considered as an exogenous increase in savings. Figure 2 depicts the consequences of a transfer on steady state. The transfer shifts the (CM) curve to the North. In the unconstrained steady state, it does not affect domestic interest rate -because the equilibrium lies at the intersection between (WIR) and (CM). In the constrained steady state, the domestic interest rate increases and more resources are allocated to the production of the non-traded good. It follows that in this case, RER appreciates whereas RER is not affected by the transfer in the unconstrained steady state\(^{13}\).

\[ [\text{Figure 2}] \]

\(^{13}\) The transfer also relaxes the constraint moving the equilibrium to the West part of the Figure. If the transfer is high enough, it can help the constrained economy reaching an unconstrained steady state.
3.3 RER and Productivity

In this 2x2 model global output increases not only with capital intensity but also with RER appreciation [See equations (12) and (13)]. However, the relationship between RER and output still depends on the nature of the steady state the economy converges to.

3.3.1 Unconstrained steady state

The long-run equilibrium is given by equations (23) and (24). The RER is exclusively determined by the world interest rate and productivity spread between sectors according to

\[ r^d (a_T, a_N, R) = \bar{r} \]  

with \( \partial r^d / \partial a_T > 0 \). An increase in traded productivity will directly generate a RER appreciation (Balassa-Samuelson effect). Then, this RER appreciation leads to a rise in output: the higher the saving rate and population growth, the higher the output rise.

3.3.2 Constrained steady state

RER and capital intensity clear the non-traded goods market and the long-run equilibrium is given by

\[ b^* = -\frac{\eta y (a_T, R^*, k^*)}{1 + n} \]

\[ k^* = \frac{w (a_T, R^*) (1 - \beta) + \eta - \beta \Gamma (a_T, R^*)}{1 - (\eta - \beta \Gamma (a_T, R^*))} \frac{1 + r^d (a_T, R^*)}{1 + n} \]

with \( \partial w / \partial a_T < 0 \) and \( \partial r^d / \partial a_T > 0 \). The constraint always binds so that NFA are determined by output. The Balassa-Samuelson theory does not hold here in the sense that equation (22) no longer applies. The RER does not only depend on productivity and world interest rate but instead results from the interaction between demand and supply of non-traded output. A rise in traded goods productivity \( a_T \) leads to changes in both demand and supply of non-traded goods and will generate:

(i) an ambiguous effect on non-traded goods demand\(^{14}\) due to a rise in domestic return on capital combined with a wage decrease.

\(^{14}\)With a simulation exercise, we can show that, in the large majority of cases, demand for non-traded goods will decrease.
(ii) a decrease in non-traded output

(iii) an ambiguous effect on global output

The third effect will affect the country’s capacity to borrow on international markets. A rise in total output will relax the constraint, increase capital stock, and decrease non-traded output. Conversely, a decrease in total output will tighten the constraint, reducing capital stock and increasing non-traded output. Since this third effect is ambiguous, the relationship between traded productivity and RER is difficult to characterize in this constrained steady state. For economies with high rate of time preference and/or not allowed to borrow enough on international markets, the Balassa-Samuelson effect may be reversed: a rise in traded productivity may lead to a RER depreciation. Otherwise (high $\beta$ and/or high $\eta$), the RER still appreciates as in the unconstrained case but operating here through a demand effect and not only through a productivity, channel as in the unconstrained case. It is hence possible that productivity can have an ambiguous effect on the RER for financially constrained countries.

4 Econometric tests

We now present empirical evidence on the determinants of the RER in order to analyze the main conclusions from the theory model presented in the previous section. We first estimate the long-run (steady state) solution of the model by analyzing the long-run cointegration vector for the RER and its main determinants and split the sample according to the degree of restrictiveness to foreign capital. We then estimate a system Error Correction Model (ECM) where we can analyze causal relationships between the variables. It is worth noting that, far from being a direct test of the model, we aim at analyzing whether its main conclusions are reflected in the behavior of the RER data. Throughout our analysis, we make use of the Lane and Milesi-Ferretti (2004, 2007) dataset, which we match with the Chinn-Ito (2007) index of financial openness ($KOPEN_t$) as will be explained below. Our data set comprises 55 countries for the period 1970-1998.

We estimate an equilibrium (log) real effective exchange rate ($LREER_t$) equation where we consider relative productivity (proxied by log-relative per-capita income $LYD_t$) and net foreign assets as a percentage of GDP ($NFA_t$) as the main steady state determinants of the RER. This specification is derived from equations (26) (20) (21) that determined long-run RER. Following Lane and Milesi-Ferretti (2004), we also consider the (log) terms of trade ($LTT_t$) as a control
variable. The equation takes the form:

\[ \text{LREER}_{it} = \theta_1 \text{LYD}_{it} + \theta_2 N\text{FA}_{it} + \theta_3 \text{LTT}_{it} + \varepsilon_{it} \] (27)

where \( \varepsilon_{it} \) is a random iid normally distributed error and \( \theta_j \) are the estimated long-run coefficients.

We estimated the equation using a panel modified version of the Dynamic-OLS estimator. Following Phillips and Loretan (1991), we augmented the regression not only with leads and lags of the variables, but also with lags of the residuals. This estimator, which behaves equivalently to a Maximum Likelihood estimator, is not only efficient but ensures that the parameters are normally distributed. In our application, given the yearly nature of the data, we used one lag (and lead) augmentation\(^\text{15}\).

Given our interest on the impact of external financial access on the determination of the RER, we need to split the sample into constrained and unconstrained economies according to an observable indicator. We pay special attention to this issue as it is central to interpreting the results of the empirical model. In principle, a variety of variables could be used as indicators of the degree of financial access of countries in international markets. One such variable is the level of income. However, this is a very imperfect proxy for financial access as we can have rich countries with restrictive capital account practices and vice versa. This would affect the sample splitting exercise precisely for countries close to the splitting threshold. Another such measure is the ratio of NFA to GDP. This is a \textit{de facto} measure that shows the exposure of a country to capital flows. However, this measure also presents an important disadvantage. Our interest is on whether the country is \textit{a priori} restricted by the international capital markets. Countries that are unconstrained could have very different levels of external indebtedness depending on whether their relative prices and domestic interest rates are close to the world ones. We could hence observe countries with low NFA to GDP ratios due to either fundamental economic reasons or financial constraints.

Another variable that could be used to separate constrained and unconstrained countries is the real interest rate. A clear implication of financial constraints, as discussed in Section 2.3, is that it introduces a wedge between domestic and world returns to capital. However, reliable data on nominal interest rates and inflation for instruments with similar maturities is not available\(^\text{15}\).

\(^{15}\text{The results from using a DOLS estimator as proposed in Kao and Chiang (2001) are very similar.}\)
for our sample of countries.\textsuperscript{16} The problems with these data are multiple. For many countries, data is simply not reported or is available only for a few years of the sample (or, in some cases, discontinued). This makes it impossible to compare real rates across countries for similar sample periods. Another important problem is that real interest rates for some developing countries are extremely volatile due to processes of hyperinflation.

For this reason, we prefer to use alternative measures of access to international capital flows based on \textit{de jure} classifications. One such measure is the capital openness index ($KOPEN_t$) developed by Chinn and Ito (2007). This is an index that measures the extent of openness in capital account transactions of an economy. It is based on the first standardized principal component of a series of binary variables accounting for the presence of multiple exchange rates, capital account transaction restrictions, current account transactions restrictions and the appropriation of export proceeds. Low values indicate high capital account restrictions and hence constraints on the access to international finance. The higher the $KOPEN_t$ index, the more open the country is to international capital flows. Quinn and Toyoda (2008) discuss different measures of capital account liberalization and also develop their own indexes based on Quinn (1997). Both of their measures are highly correlated with $KOPEN_t$ and produce similar country rankings. Given that $KOPEN_t$ comprises a larger number of countries matching the Lane and Milesi-Ferretti dataset, we prefer to use $KOPEN_t$ for our sample splitting.

In order to obtain an optimal sample split, we used the methodology developed by Hansen (2000) based on threshold estimation. It would be desirable to obtain a sample split for the whole panel data. However, threshold sample splitting techniques for nonstationary variables are not available in the literature. For this reason we applied the sample splitting method on a cross-sectional estimate where we regress the first difference of $LREER$ on the first difference of the independent variables. In order to avoid problems of initial and final anomalous observations, the first difference is defined as the difference between the average value of the variable for 1985-1998 and the average for 1970-1984. The threshold variable used is the average value of $KOPEN_t$ throughout the sample period for each country.\textsuperscript{17} It is worth noting, though, that

\textsuperscript{16}Reliable data are usually available precisely after periods of internal and external financial liberalisation, which would obviously introduce a sample selection bias if we were to use only countries where comparable real interest rate data is available.

\textsuperscript{17}We also used the initial value of $KOPEN_t$ as a threshold variable. However, it was only marginally significant
splitting the sample arbitrarily between countries with positive and negative average $KOPEN_t$ values gave similar results. The results from the sample splitting tests are reported in Table 2\textsuperscript{18}.

\textbf{Table 2}

We report the value of the threshold of $KOPEN_t$ for sample splitting, the LM test for the null of no threshold (no sample split) and its bootstrapped p-value, and the number of countries in each regime. These show that $KOPEN_t$ is a significant variable for sample splitting and the split point is at a value of -0.53. Figure 3 shows the recursive Likelihood Ratio test as a function of the threshold, variable, which is minimized at -0.53. The list of countries in each group is reported in Appendix A.\textsuperscript{19}

\textbf{Figure 3}

We then proceed to the estimation of the long-run cointegration vector for both sub-samples of countries. All the variables were checked for stationarity and the I(1) specification could not be rejected. In testing for unit roots, we used the IPS and Maddala-Wu panel unit root tests. We then proceeded to test for cointegration using $LREER_t$, $NFA_t$, and $LYD_t$ as dependent variables in the cointegration vector. Table 3 presents the results from the group-ADF test and delivered a very uneven split between countries. Using the average value of $KOPEN_t$ for 1970-1984, though, resulted in very similar results to those reported in the paper in terms of the countries comprising the group of constrained and unconstrained economies.

\textsuperscript{18}Despite their obvious disadvantages, we also experimented with other variables. We used $NFA_t$ and per capita income $y_{pc}$. The results from the LM tests for the existence of a threshold could not reject the null of no threshold.

\textsuperscript{19}In order to further our previous point about the use of real interest rates, we calculated sample averages and standard deviations of real interest rates for the countries and periods available from IMF’s IFS database. These real rates are not reliable as they include different instruments and maturities, they were only available for 41 countries and, for many of them, they do not run for the whole sample period. The data, available on request, shows that the average real rate for the group of constrained economies is much higher than that of unconstrained ones classified using the $KOPEN_t$ index. If we drop Argentina and Brazil from the sample, due to their unusually high nominal and real interest rates during some hyperinflation periods, the real interest rate continues to be higher than in the unconstrained group, albeit the difference is much smaller. However, the average standard deviation (standard deviation for each country averaged across countries within the group) is much higher regardless of whether we drop Argentina and Brazil from the sample. This indicates that the classification based on $KOPEN_t$ may also be correlated with the level and volatility of real interest rates in this group of countries.
of Pedroni (1999)\textsuperscript{20} and shows that only the equations using $LREER_t$ as dependent variable constitute long-run equilibrium relations.

[Table 3]

The results for the cointegration vectors are reported in Tables 4 to 6.

[Table 4]

[Table 5]

[Table 6]

The results for the whole sample show a positive relationship of the three variables considered with the RER. This is similar to the results in Lane and Milesi-Ferreti (2004), but in our case relative income appears to have an insignificant effect. The results, though, are not directly comparable due to differences in the sample of countries. The results from the sample splitting show that for unconstrained economies only $LYD$ and $LTT$ appear as significant long-run determinants of the RER. Conversely, for constrained economies, $NFA$ becomes strongly significant, and $LYD$ appears to have a negative impact on the RER. This lends support to the theoretical hypothesis that NFA is only important for constrained economies and YD is the main driver of RER for unconstrained economies (see Subsections 3.2 and 3.3). In the model, the transfer effect (positive effect from NFA to RER) holds only in constrained economies whereas the Balassa-Samuelson effect (positive effect of YD on RER) always holds in unconstrained economies. For the more constrained economies, the Balassa-Samuelson effect may be reversed, with a rise in YD causing a drop in RER. The long-run estimated vectors lend support for these hypotheses.

Using the estimated long-run vectors, we then proceed to estimate error correction models (ECMs) for each of the variables involved, treating $LTT$ as strictly exogenous. Given the well-known bias of the OLS estimator for dynamic stationary panels, we estimated the ECMs by system GMM. We used as instruments the second and third lags of the variables and the lagged level\textsuperscript{21}. The results are reported in Tables 7 to 9, where we show the coefficients and their p-values and a J-test for over-identifying restrictions. In all cases the J-test indicated that the

\textsuperscript{20}We also checked for cointegration using the panel ADF test and the group and panel PP tests proposed in Pedroni (1999). The results are invariant to extracting group means to account for cross-sectional correlation.

\textsuperscript{21}The results are invariant to the inclusion of the lag level.
system is overidentified. The coefficient on the ECM indicates whether or not the cointegration vector is a long-run attractor and hence can be used to test for causality as is the case with the rest of the short-run coefficients.

[Table 7]

[Table 8]

[Table 9]

The results for unconstrained economies (Table 8) indicate that, in the long-run, only the RER adjusts to the equilibrium relationship with its determinants, indicating that these are weakly exogenous and supporting the existence of just one cointegration relationship. The adjustment speed is found to be low, with 16% per year. In the short run, relative income causes the RER and NFA causes relative income. Finally, NFA also appears to be positively affected by \( LYD \) indicating a two-way causality between these variables in the short-run. From the point of view of the theory model, this is partially supportive since in unconstrained economies (i) RER is caused by productivity; (ii) NFA are caused by income and savings; (iii) Income is caused by capital intensity, and capital intensity results from domestic savings and NFA.

For constrained economies (Table 9) the error correction coefficients display very similar values to those of the unconstrained ones. This again indicates that the determinants of the RER are weakly exogenous. In the short run, we find that NFA have a positive impact on relative income and the RER a negative one. Income also appears to cause NFA in the short run with a positive sign.

We analyzed further the results to check for the possibility that they are sensitive to the inclusion of some countries in the sample. The robustness of our findings was tested using a cross-validation approach. We assume a function:

\[
\Phi_{it}(x_{it}, z_{it}, \delta) = u_{it} \quad i \in [1, N], t \in [1, T]
\]

(28)

where \( x_{it} \) is the vector of endogenous variables; \( z_{it} \) is the vector of exogenous variables; \( \Phi_{it} \) is a vector function representing the estimated cointegration functional form or the EC model; \( \delta \) are the estimated parameters, and \( u_{it} \) is the error term. Denote by \( \hat{\delta}(i-1) \) the estimate of \( \delta \) obtained when we omit country \( i \in [1, N] \) from the sample. In this case, \( \hat{\delta}(i-1) \) is the cross-validated
estimate of δ when we omit country \( i = 1, \ldots, N \). This procedure allows us to check for correct statistical inference, especially when the number of cross-sections is small.

We analyzed the robustness of all the long-run estimated coefficients in both panels as well as the robustness of the error correction term in the ECM. The estimates remained remarkably stable throughout, which further confirms the advantages of our sample split as no further parameter instability arising from the cross-section appears to be present in the model. Figures 4-1 and 4-2 present the estimated coefficient (denoted by a diamond) for the panel when we drop each one of the countries plus-minus one standard error (denoted by the vertical bar).

[Figure 4-1]  
[Figure 4-2]

For reasons of space we only present the coefficients of the long-run vector for \( NFA_{it} \) and \( YD_{it} \), but the other coefficients, including the one for the ECM, also show the same stable pattern. We can easily see that dropping any of the countries does not produce a substantial change in the estimated coefficients and their significance. Any changes are of a small order of magnitude, both statistically and economically, and do not change the conclusions regarding the determinants of the RER in financially constrained and unconstrained economies.

5 Conclusion

Empirical evidence suggests that significant real exchange rate (RER) determinants change as we vary periods and countries considered. We develop an overlapping generations two-factor two-sector model of a small open economy in which the way RER is determined varies with the country’s capacity to borrow on international markets. We assume that the country faces a constraint on capital inflows. A special feature of the model, and in contrast to the existing literature, is that the steady state can either be constrained or unconstrained. The way capital, net foreign assets (NFA) and real exchange rate are determined depends on the nature of the steady state. In the unconstrained steady state, the RER only depends on productivity spread between sectors - a Balassa-Samuelson effect. In the constrained steady state, the RER does not only depend on productivity but also on the determinants of savings and net foreign assets (transfer effect).
We then study the implications of the model using econometric evidence on the determinants of the RER for financially constrained and unconstrained economies. We split the sample endogenously using the Chinn and Ito (2007) capital account openness variable as an index of external financial constraints. Our results lend support for the theoretical implications of the model: the RER appears to be mainly driven by productivity and NFA in countries that face foreign exchange restrictions and exclusively by productivity in countries that have perfect access to the international capital market.
APPENDIX

A  Samples

We split the whole sample into the two following sub-sample:

- Low Kaopen: France, Norway, Greece, Iceland, Ireland, Portugal, Spain, Turkish, South Africa, Argentina, Brazil, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Paraguay, Peru, Jamaica, Trinidad and Tobago, Sri Lanka, Korea, Philippines, Thailand, Algeria, Cote d’Ivoire, Mauritius, Morocco, Tunisia, India, Pakistan.

- High Kaopen: USA, UK, Austria, Belgium, Denmark, Germany, Netherlands, Sweden, Switzerland, Canada, Japan, Australia, New Zealand, Bolivia, Mexico, Panama, Uruguay, Hong Kong, Malaysia, Singapore, Saudi Arabia.

B  Calculation of $\tilde{\eta}$

Under what conditions will the domestic interest rate $r^d$ converge to the world one? Will domestic saving be enough to drive $k^*$ to $\bar{k}$ given the borrowing constraint?

Let $\tilde{k}^*$ denote the steady state capital per capita obtained in the constrained steady state (see equation (20) ) in which $R = \bar{R}$

$$\tilde{k}^* = \frac{w(\bar{R}) (1 - \beta + \eta)}{1 + \frac{1}{\beta (1 + n)}}$$

(29)

since $\Gamma (\bar{R}) = 0$. Then the critical level of the constrained $\tilde{\eta}$ is such that $\tilde{k}^* = \bar{k}$. Let the production functions be

$$f(k_T) = k_T^\nu, \quad 0 < \nu < 1$$

(30)

$$h(k_N) = k_N^\rho, \quad 0 < \rho < 1$$

(31)

Let $\Psi = (1 - \alpha) [\beta + (1 - \beta) (1 + \bar{r}) / (1 + n)]$ denote the aggregate propensity to consume the non-traded good. Then, using $k = (1 - l_N) k_T + l_N k_N$, the convenient Cobb-Douglas specifica-
tion and equations (23) and (29) the critical level of the constraint $\tilde{\eta}$ is given by

$$\tilde{\eta} = \frac{1+n}{1+r} \left[ \nu + \Psi (\rho - \nu) \right] - (1 - \beta) \left( 1 - \nu \right) \frac{1}{1 + \Psi (\rho - \nu)}$$

(32)

We can also calculate $\tilde{\eta}$ when there is a transfer in the long-run. The temporary equilibrium in the constrained case becomes, with the transfer,

$$k_{t+1} = \left[ 1 - \beta + \eta - \beta \Gamma (R_{t+1}) \right] \frac{w(R_t)}{1+n} + (\eta - \beta \Gamma (R_{t+1})) \frac{1 + r^d(R_t)}{1+n} k_t + \frac{T}{1+n}$$

with $T$ a constant per-period per-capita transfer.

The constrained steady state is

$$k^* = \frac{1}{1+n} \left[ 1 - \beta + \eta - \beta \Gamma (R^*) \right] \frac{w(R^*)}{1-n} + \frac{1+r^d(R^*)}{1+n} \left( \eta - \beta \Gamma (R^*) \right)$$

which gives

$$\tilde{k}^* = \frac{1}{1+n} \left[ 1 - \beta + \eta \right] \frac{w(R^*)}{1-n} + \frac{T^*}{1+n}$$

The unconstrained steady state capital stock $\bar{k}$ is unchanged and then we can calculate $\tilde{\eta}$ such that $\tilde{k}^* = \bar{k}$

$$\tilde{\eta} = \frac{1+n}{1+r} \left[ \nu + \Psi (\rho - \nu) \right] - T^* \left( \frac{1+\nu}{1+\nu} \right)^{\frac{\nu}{1+\nu}} - (1 - \beta) \left( 1 - \nu \right)$$

$$\frac{1}{(\rho - \nu) \Psi + 1}$$
References


Tables

Table 1: Calibration

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<thead>
<tr>
<th></th>
<th>Constrained steady state</th>
<th>Unconstrained steady state</th>
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<tbody>
<tr>
<td>$k_{t}/k_{r}$</td>
<td>0.7547</td>
<td>0.7643</td>
</tr>
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<td>$l_{r}$</td>
<td>0.3924</td>
<td>0.3770</td>
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<tr>
<td>$y_{r}/y$</td>
<td>0.4351</td>
<td>0.4162</td>
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<td>$R$</td>
<td>0.5898</td>
<td>0.6053</td>
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Table 2: Sample splitting results using KOPEN index and sample averages of variables

<table>
<thead>
<tr>
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<th>Threshold estimate: -0.53</th>
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<tr>
<td>95% Confidence Interval</td>
<td>[-0.75 -0.06]</td>
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<td>LM test</td>
<td>11.62</td>
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<td>Bootstrapped p-value</td>
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<td>No. countries Regime 1 (KOPEN&gt;threshold)</td>
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</tr>
<tr>
<td>No. countries Regime 2 (KOPEN&lt;threshold)</td>
<td>33</td>
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</table>

Notes: The table shows the results from the Hansen (2000) threshold method for sample splitting. The threshold variable is KOPEN from Ito and Chinn (2007). The bootstrapped CVs were obtained using 1,500 bootstrap draws.

Table 3: Panel cointegration tests, Group-ADF statistics

<table>
<thead>
<tr>
<th>Dep. Variable</th>
<th>Whole sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>LREER$_t$</td>
<td>-2.63***</td>
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<tr>
<td>LYD$_t$</td>
<td>-0.51</td>
</tr>
<tr>
<td>NFA$_t$</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Panel A (KOPEN>-0.53)

<table>
<thead>
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<th>Dep. Variable</th>
<th>Panel A (KOPEN&gt;-0.53)</th>
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</thead>
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<td>LREER$_t$</td>
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<tr>
<td>LYD$_t$</td>
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<td>NFA$_t$</td>
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Panel B (KOPEN<-0.53)

<table>
<thead>
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<th>Dep. Variable</th>
<th>Panel B (KOPEN&lt;-0.53)</th>
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<tbody>
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<td>LREER$_t$</td>
<td>-1.97**</td>
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<td>LYD$_t$</td>
<td>-0.74</td>
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<tr>
<td>NFA$_t$</td>
<td>-1.58</td>
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Note: (***) and (*) show rejection of the null hypothesis of no cointegration at 1%, 5% and 10% statistical level respectively.
Table 4: Long-run cointegration vector. Whole sample.

<table>
<thead>
<tr>
<th>Dependent variable: LREER</th>
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</thead>
<tbody>
<tr>
<td>Variable</td>
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<td>LYD</td>
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<td>NFA</td>
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<td>LTT</td>
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<tr>
<td>R²</td>
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<td>Obs.</td>
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Table 5: Long-run cointegration vector. Unconstrained economies.

<table>
<thead>
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<th>Dependent variable: LREER</th>
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<tbody>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>LYD</td>
</tr>
<tr>
<td>NFA</td>
</tr>
<tr>
<td>LTT</td>
</tr>
<tr>
<td>R²</td>
</tr>
<tr>
<td>Obs.</td>
</tr>
</tbody>
</table>

Table 6: Long-run cointegration vector. Constrained economies.

<table>
<thead>
<tr>
<th>Dependent variable: LREER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>LYD</td>
</tr>
<tr>
<td>NFA</td>
</tr>
<tr>
<td>LTT</td>
</tr>
<tr>
<td>R²</td>
</tr>
<tr>
<td>Obs.</td>
</tr>
</tbody>
</table>

Table 7: GMM estimation of the system ECM model. Whole sample.

<table>
<thead>
<tr>
<th>Eq for ΔLREERₜ₋₁</th>
<th>Eq for ΔLYDₜ₋₁</th>
<th>Eq for ΔNFAₜ₋₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.284 [.004]</td>
<td>-0.107 [.015]</td>
<td>-0.015 [.330]</td>
</tr>
<tr>
<td>0.027 [.265]</td>
<td>0.011 [.685]</td>
<td>0.026 [.001]</td>
</tr>
<tr>
<td>0.122 [.164]</td>
<td>-0.108 [.162]</td>
<td>0.565 [.000]</td>
</tr>
<tr>
<td>-0.044 [.321]</td>
<td>0.559 [.001]</td>
<td>0.048 [.007]</td>
</tr>
<tr>
<td>-0.151 [.000]</td>
<td>0.018 [.191]</td>
<td>0.003 [.617]</td>
</tr>
</tbody>
</table>

| R² | 0.071 | 0.153 | 0.055 |
| J-test [p-val] | 18.87 [0.22] |

Notes: numbers in brackets [ ] are p-values of the coefficients. The J-test is a test for over-identification restrictions for the system.
Table 8: GMM estimation of the system ECM model. Unconstrained economies.

<table>
<thead>
<tr>
<th></th>
<th>Eq for $\Delta LREER$</th>
<th>Eq for $\Delta LYT$</th>
<th>Eq for $\Delta NFA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta LREER_{t-1}$</td>
<td>-0.060 [.629]</td>
<td>-0.041 [.350]</td>
<td>0.001 [.954]</td>
</tr>
<tr>
<td>$\Delta LYT_{t-1}$</td>
<td>0.074 [.009]</td>
<td>-0.007 [.844]</td>
<td>0.011 [.328]</td>
</tr>
<tr>
<td>$\Delta LYT_{t-1}$</td>
<td>0.412 [.001]</td>
<td>0.028 [.781]</td>
<td>0.344 [.004]</td>
</tr>
<tr>
<td>$\Delta NFA_{t-1}$</td>
<td>0.072 [.095]</td>
<td>0.512 [.005]</td>
<td>0.029 [.292]</td>
</tr>
<tr>
<td>$ECM_{t-1}$</td>
<td>-0.159 [.000]</td>
<td>-0.009 [.754]</td>
<td>0.002 [.872]</td>
</tr>
</tbody>
</table>

| R²                | 0.099                  | 0.179                | 0.092                |

J-test [p-val] 18.99 [0.21]

Note: see Table 4.

Table 9: GMM estimation of the system ECM model. Constrained economies.

<table>
<thead>
<tr>
<th></th>
<th>Eq for $\Delta LREER$</th>
<th>Eq for $\Delta LYT$</th>
<th>Eq for $\Delta NFA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta LREER_{t-1}$</td>
<td>0.357 [.007]</td>
<td>-0.215 [.000]</td>
<td>-0.012 [.531]</td>
</tr>
<tr>
<td>$\Delta LYT_{t-1}$</td>
<td>0.004 [.912]</td>
<td>-0.048 [.118]</td>
<td>0.029 [.008]</td>
</tr>
<tr>
<td>$\Delta LYT_{t-1}$</td>
<td>0.024 [.829]</td>
<td>-0.292 [.005]</td>
<td>0.663 [.000]</td>
</tr>
<tr>
<td>$\Delta NFA_{t-1}$</td>
<td>-0.039 [.540]</td>
<td>0.922 [.000]</td>
<td>0.043 [.084]</td>
</tr>
<tr>
<td>$ECM_{t-1}$</td>
<td>-0.159 [.000]</td>
<td>0.027 [.128]</td>
<td>0.004 [.616]</td>
</tr>
</tbody>
</table>

| R²                | 0.049                  | 0.126                | 0.045                |

J-test [p-val] 15.95 [0.38]

Note: see Table 5.
Figures

Figure 1: Calibration of Steady-State

The loci (CM) and (NTM) depict, respectively, the capital market and non-traded good market equilibria. The locus (WIR) is the world interest factor. If (CM) intersects (NTM) below (WIR), the steady state is the unconstrained one.

\[ \eta = 0.2 \]

Unconstrained steady state

\[ \eta = 0.1 \]

Constrained steady state

Figure 2: The transfer effect

The loci (CM0) and (NTM) depict, respectively, the initial capital market and non-traded good market equilibria. The locus (WIR) is the world interest factor. If (CM0) intersects (NTM) below (WIR), the steady state is the unconstrained one.

\[ \nu = 0.4, \alpha = 0.2, \beta = 0.6, \gamma = 0.5, \rho = 0.37, \sigma = 0.6 \]
Figure 3: Confidence interval for the threshold test

Confidence Interval Construction for Threshold

Figure 4: sensitivity analysis.

Figure 4-1: Beta coefficients sensitivity analysis, high KOPEN
Figure 4-2: Beta coefficients sensitivity analysis, low KOPEN