AUSTRALIAN
SCHOOL OF BUSINESS ™
THE UNIVERSITY OF NEW SOUTH WALES

www.business.unsw.edu.au

School of Economics
Australian School of Business
UNSW Sydney NSW 2052 Australia

http://www.economics.unsw.edu.au

Econometric Analysis of Structural Systems with Permanent and Transitory Shocks

Adrian R. Pagan and M. Hashem Pesaran

School of Economics Discussion Paper: 2008/04

ISSN 1323-8949
ISBN 978 0 7334 2610 0

Econometric Analysis of Structural Systems with Permanent and Transitory Shocks *

A.R. Pagan

Queensland University of Technology and University of New South Wales

M. Hashem Pesaran

Faculty of Economics and CIMF, Cambridge University, and USC

January 14, 2006, This Version February 4, 2008

Abstract

This paper considers the implications of the permanent/transitory decomposition of shocks for identification of structural models in the general case where the model might contain more than one permanent structural shock. It provides a simple and intuitive generalization of the influential work of Blanchard and Quah (1989), and shows that structural equations with known permanent shocks can not contain error correction terms, thereby freeing up the latter to be used as instruments in estimating their parameters. The approach is illustrated by a re-examination of the identification schemes used by Wickens and Motto (2001), Shapiro and Watson (1988), King, Plosser, Stock, Watson (1991), Gali (1992, 1999) and Fisher (2006).

JEL Classifications: C30, C32, E10

Key Words: Permanent shocks, structural identification, error correction models, IS-LM models.

*We are grateful to two anonymous referees and the Co-editor (Tim Cogley) for their helpful comments on an earlier version of the paper which was entitled On Econometric Analysis of Structural Systems with Permanent and Transitory Shocks and Exogenous Variables. The research of the first author was supported by ARC Grant DP0449659.

1 Introduction

The fact that macroeconomic variables are often integrated rather than covariance stationary has been increasingly accepted and has affected the design of models describing them. Moreover, variables often seem to be co-integrated, and this has led to the specification of models so as to reflect such a phenomenon e.g. DSGE models which feature a single technology shock need to make it an integrated variable to ensure that variables such as output and consumption are co-integrated. An implication of a co-integrated system is that the shocks to it will be both permanent and transitory and methods to reconstruct such shocks using a VAR are now well known.

Shocks are now regarded as the driving forces of macro-economic systems. Often we wish to attach "names" to the shocks in order to deliver some economic content to our explanations of the evolution of variables, either on average or over particular historical episodes. Thus we increasingly see reference to "technology shocks", "preference shocks", "risk premium shocks", "mark-up shocks". Such shocks are often referred to as structural. They are fundamentally *unobservable* and cannot be identified without reference to an economic model.

Putting aside the "naming" issue, the effects of structural shocks on the evolution of the macroeconomy can be either permanent or transitory. This raises the question of whether the knowledge that certain structural equations are assumed to have permanent shocks, while others have transitory shocks, can aid in their identification. This paper therefore sets out to explore this question and to show exactly what identifying information is provided by such knowledge.

There has been work on this before. For example, the body of research initiated by Blanchard and Quah (1989) stipulated that there were demand and supply shocks, with the latter having a permanent effect on output and the former a transitory effect. Their approach was to work with a two variable structural system, making one of the structural shocks permanent and the other transitory. Generalizations of this approach involve either adding more permanent shocks (and equations) into the system or allowing for some co-integration between the integrated system variables. An example of the former is Fisher (2006) and of the latter would be Gali (1992). Gali (1999) is another application in which it is assumed that there are two permanent shocks in a five variable system with two co-integrating relations between the $I(1)$ variables of the system. Intermediate to these examples are papers that are not specific about which structural equations the permanent shocks are in e.g. King et al. (1990) and Gonzalo and Ng (2001), but we provide a re-interpretation of King et al. using our framework that does allow one to discuss their procedure. One could do this for Gonzalo and Ng as well. Moreover, we do not specifically deal with cases where the permanent shocks are identified by making the system recursive, as this involves stronger *a priori* information than is often needed for identification, although our framework is capable of incorporating such identifying assumptions. Juselius (2006, section 13.6) also presents a discussion of situations in which permanent and transitory shocks appear in a system.

In Section 2 of the paper the structural system to be studied is set out and it is argued that identification would be enhanced if we knew the parameter values (the loadings) attached to the error correction (EC) terms in the structural equations. Section 3 then shows that these parameters will be zero for those structural equations which are known to have a permanent shock. Recognition of this frees up the (lagged) EC terms to be used as instruments in estimating the parameters of the structural equation. We also consider the implications of the presence of permanent structural shocks for the remaining structural equations and show the form of these and what would be needed to identify them.

Section 3.4 turns to the case where the permanent shocks are associated with observable exogenous variables. Wickens and Motto (2001) argued that co-integration in such systems could produce identifying information. We analyze this proposition and find that, in general, it is incorrect. Exogeneity does help in producing identification, as for example in a small open economy which features foreign variables, but it is not the degree of integration or co-integration that matters.[1]

Section 4 looks at some examples of our approach. Firstly, we look at a simple production function application and Blanchard and Quah's (1990) study since these are the simplest applications of the ideas. Secondly, we re-visit the model used in Wickens and Motto (2001). Thirdly, we show how our framework applies to the study of Shapiro and Watson (1988). Then, we examine Gali (1992), Gali (1999) and Fisher (2006). We show that the first of Gali's papers fails to use all the information available from his assumptions and that his recourse to short-run restrictions to identify the shocks of the model is largely unnecessary. Finally, we examine King et al. (1991)

[1] See Garratt, Lee, Pesaran and Shin (2006, Ch. 6) for further details.

and show that they seem to make stronger assumptions than are needed to identify the structural equations which have permanent shocks. This is just a small sampling of the literature. There are many similar papers in the literature that utilize combinations of short and long-run restrictions e.g. Peersman (2005). The thrust of our paper for this literature is that the implications of long-run restrictions should be fully exploited before short-run restrictions are invoked. Often this does not seem to have been the case. Section 6 ends the paper with some concluding remarks.

2 Preliminary Analysis

Suppose we have a Structural VAR(2) system in n $I(1)$ variables of the form[2]

$$\mathbf{A}_0\mathbf{z}_t = \mathbf{A}_1\mathbf{z}_{t-1} + \mathbf{A}_2\mathbf{z}_{t-2} + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\mathbf{A}_i$ are $n \times n$ matrices of unknown coefficients, $\mathbf{A}_0$ is non-singular, and $\boldsymbol{\varepsilon}_t$ is an $n \times 1$ vector of structural shocks with mean zero and a positive definite covariance matrix $\boldsymbol{\Sigma}$. In applications where it is deemed necessary for the structural shocks to form an orthogonal set both sides of the above equation can be multiplied by the Cholesky factor of $\boldsymbol{\Sigma}$ before proceeding with the analysis. Therefore, without loss of generality in what follows we set $\boldsymbol{\Sigma} = \mathbf{I}_n$, an identity matrix of order n. Also to ensure that $\mathbf{z}_t$ does not contain $I(2)$ variables we shall assume that all the eigenvalues of $\mathbf{A}_0^{-1}\mathbf{A}_2$ lie inside the unit circle.

The above Structural VAR (SVAR) specification can be transformed to

$$\mathbf{A}_0\Delta\mathbf{z}_t = -\mathbf{A}(1)\mathbf{z}_{t-1} - \mathbf{A}_2\Delta\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{2}$$

where $\mathbf{A}(1) = \mathbf{A}_0 - \mathbf{A}_1 - \mathbf{A}_2$, with the associated reduced form model given by

$$\begin{aligned}\Delta\mathbf{z}_t &= -\mathbf{A}_0^{-1}\mathbf{A}(1)\mathbf{z}_{t-1} - \mathbf{A}_0^{-1}\mathbf{A}_2\Delta\mathbf{z}_{t-1} + \mathbf{A}_0^{-1}\boldsymbol{\varepsilon}_t, & (3)\\ &= -\boldsymbol{\Pi}\mathbf{z}_{t-1} + \boldsymbol{\Psi}\Delta\mathbf{z}_{t-1} + \mathbf{e}_t. & (4)\end{aligned}$$

Now suppose that there are $r < n$ co-integrating relations in this system, so that $\boldsymbol{\Pi}$ is rank deficient and $\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $n \times r$ full column rank matrices. Then

$$\Delta\mathbf{z}_t = -\ \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{z}_{t-1} + \boldsymbol{\Psi}\Delta\mathbf{z}_{t-1} + \mathbf{e}_t, \tag{5}$$

[2] Our results readily apply to higher order VARs.

$$\mathbf{\Psi} = -\,\mathbf{A}_0^{-1}\mathbf{A}_2, \tag{6}$$

and

$$\mathbf{A}_0\Delta\mathbf{z}_t = -\,\boldsymbol{\alpha}^*\boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{A}_2\Delta\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{7}$$

is a Structural Vector Error Correction Model (SVECM), where $\boldsymbol{\alpha}^* = \mathbf{A}_0\boldsymbol{\alpha}$.

The central task in SVECM (and SVAR) systems is to estimate the n^2 coefficients of $\mathbf{A}_0$, n of which can be fixed by suitable normalization restrictions. The remaining $n(n-1)$ coefficients need to be identified by means of *a priori* restrictions inspired by economic reasoning. A number of different identification schemes are possible depending on the nature of the available *a priori* information. Each identification scheme produces a set of instruments for $\Delta\mathbf{z}_t$ and so enables the estimation of the unknown parameters in $\mathbf{A}_0$. It is clear from (7) that, if one or more elements of $\boldsymbol{\alpha}^*$ are known and we are able to estimate $\boldsymbol{\beta}$ consistently, then $\boldsymbol{\beta}'\mathbf{z}_{t-1}$ can be used as instruments. It is this feature that will be exploited in what follows.

3 Some Structural Shocks are Permanent

3.1 Implications for the EC Terms in the Structural VECM

Consider now the structural errors of interest $\boldsymbol{\varepsilon}_t$ in (7), and suppose that the first $n-r$ shocks in $\boldsymbol{\varepsilon}_t$, denoted by $\boldsymbol{\varepsilon}_{1t}$, are known to be permanent and the remaining r shocks, $\boldsymbol{\varepsilon}_{2t}$, are transitory. Such a decomposition is possible since it is assumed that there are r co-integrating relations amongst the n, $I(1)$ variables in $\mathbf{z}_t$. (see, for example, Lütkepohl (2005, Ch. 9)) To explore the implications for the identification of the structural shocks provided by a permanent/transitory decomposition, we consider the following common trends representation of (5) (see, for example, Johansen (1995, Theorem 4.2))

$$\mathbf{z}_t = \mathbf{z}_0 + \mathbf{F}\sum_{j=1}^{t}\mathbf{e}_j + \sum_{i=0}^{\infty}\mathbf{F}_i^*\mathbf{e}_{t-i}, \tag{8}$$

where $\mathbf{F} = \boldsymbol{\beta}_\perp\left[\boldsymbol{\alpha}'_\perp(\mathbf{I}_n - \mathbf{\Psi})\boldsymbol{\beta}_\perp\right]^{-1}\boldsymbol{\alpha}'_\perp$, with $\boldsymbol{\alpha}'_\perp\boldsymbol{\alpha} = \mathbf{0}$ and $\boldsymbol{\beta}'\boldsymbol{\beta}_\perp = \mathbf{0}$, so that ($\mathbf{\Psi}$ is defined by (6))

$$\mathbf{F}\boldsymbol{\alpha} = \mathbf{0}_{n\times r}, \text{ and } \boldsymbol{\beta}'\mathbf{F} = \mathbf{0}_{r\times n}. \tag{9}$$

Hence $\mathbf{F}$ is a rank deficient matrix with rank equal to the dimension of $\mathbf{z}_t$ minus the number of the co-integrating relations, namely $n-r$. Note also that the number of co-integrating relations uniquely determine the number of transitory shocks in the underlying VAR model. Thus the shocks to a co-integrating VAR model with r co-integrating relations can be decomposed to exactly r transitory and $n-r$ different permanent shocks.

Writing the above common trends representation in terms of the structural shocks we have

$$\begin{aligned}\mathbf{z}_t &= \mathbf{z}_0 + \mathbf{F}\sum_{j=1}^{t}\mathbf{A}_0^{-1}\boldsymbol{\varepsilon}_j + \sum_{i=0}^{\infty}\mathbf{F}_i^*\mathbf{e}_{t-i}, \\ &= \mathbf{z}_0 + \mathbf{F}\mathbf{A}_0^{-1}\begin{pmatrix}\sum_{j=1}^{t}\boldsymbol{\varepsilon}_{1j} \\ \sum_{j=1}^{t}\boldsymbol{\varepsilon}_{2j}\end{pmatrix} + \sum_{j=0}^{\infty}\mathbf{F}_j^*\mathbf{e}_{t-j}.\end{aligned}$$

Hence, in order for $\boldsymbol{\varepsilon}_{2j}$ to have only transitory effects we must have

$$\mathbf{F}\mathbf{A}_0^{-1}\begin{pmatrix}\mathbf{0}_{(n-r)\times r} \\ \mathbf{I}_r\end{pmatrix} = \mathbf{0}. \qquad (10)$$

These restrictions are necessary and sufficient and apply irrespective of whether the transitory shocks are correlated or not. In order to derive the implications of the above restrictions for the structural parameters, particularly the factor loadings, $\boldsymbol{\alpha}^*$, we first note that since the rank of $\mathbf{F}$ is $n-r$ it can be written as $\mathbf{F} = \mathbf{P}^{-1}\boldsymbol{\Lambda}\mathbf{P}$, where $\mathbf{P}$ is an $n\times n$ non-singular matrix, and

$$\boldsymbol{\Lambda} = \begin{pmatrix}\boldsymbol{\Lambda}_{11} & \mathbf{0}_{(n-r)\times r} \\ \mathbf{0}_{r\times(n-r)} & \mathbf{0}_{r\times r}\end{pmatrix},$$

where $\boldsymbol{\Lambda}_{11}$ is a non-singular $(n-r)\times(n-r)$ matrix. Using this representation of $\mathbf{F}$ and partitioning the matrices $\mathbf{P}$ and $\mathbf{A}_0^{-1}$ accordingly, the restrictions in (10) can be written as

$$\mathbf{P}_{11}\mathbf{A}_0^{12} + \mathbf{P}_{12}\mathbf{A}_0^{22} = \mathbf{0}, \qquad (11)$$

where $\mathbf{P}_{11}, \mathbf{P}_{12}$, $\mathbf{A}_0^{12}$, and $\mathbf{A}_0^{22}$ are the partitioned matrices relating to the first $n-r$ rows of $\mathbf{P}$ and $\mathbf{A}_0^{-1}$, respectively. Similarly, the condition $\mathbf{F}\boldsymbol{\alpha} = \mathbf{0}$ in (9) can be written as

$$\mathbf{P}_{11}\boldsymbol{\alpha}_1 + \mathbf{P}_{12}\boldsymbol{\alpha}_2 = \mathbf{0}, \qquad (12)$$

where $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_1', \boldsymbol{\alpha}_2')'$, with $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ being respectively $(n-r)\times r$ and $r\times r$ matrices of reduced form factor loadings.

To obtain restrictions on the structural factor loadings, we first write $\mathbf{A}_0^{-1}\boldsymbol{\alpha}^* = \boldsymbol{\alpha}$ as

$$\begin{aligned} \mathbf{A}_0^{11}\boldsymbol{\alpha}_1^* + \mathbf{A}_0^{12}\boldsymbol{\alpha}_2^* &= \boldsymbol{\alpha}_1, \\ \mathbf{A}_0^{21}\boldsymbol{\alpha}_1^* + \mathbf{A}_0^{22}\boldsymbol{\alpha}_2^* &= \boldsymbol{\alpha}_2, \end{aligned}$$

and observe that $\mathbf{A}_0^{11}$ and $\mathbf{A}_0^{22}$ are $(n-r)\times(n-r)$ and $r\times r$ non-singular matrices, respectively. Solving out $\boldsymbol{\alpha}_2^*$ from the above two equations yields

$$\left[\mathbf{A}_0^{11} - \mathbf{A}_0^{12}\left(\mathbf{A}_0^{22}\right)^{-1}\mathbf{A}_0^{21}\right]\boldsymbol{\alpha}_1^* = \boldsymbol{\alpha}_1 - \mathbf{A}_0^{12}\left(\mathbf{A}_0^{22}\right)^{-1}\boldsymbol{\alpha}_2. \tag{13}$$

Now using (11) produces

$$\mathbf{P}_{12} = -\mathbf{P}_{11}\mathbf{A}_0^{12}\left(\mathbf{A}_0^{22}\right)^{-1},$$

and, substituting this result into (12), we obtain

$$\mathbf{P}_{11}\left[\boldsymbol{\alpha}_1 - \mathbf{A}_0^{12}\left(\mathbf{A}_0^{22}\right)^{-1}\boldsymbol{\alpha}_2\right] = \mathbf{0}.$$

Consequently, since $\mathbf{P}_{11}$ is a non-singular matrix, then $\boldsymbol{\alpha}_1 = \mathbf{A}_0^{12}\left(\mathbf{A}_0^{22}\right)^{-1}\boldsymbol{\alpha}_2$. Using this result in (13), and noting that $\mathbf{A}_0^{11} - \mathbf{A}_0^{12}\left(\mathbf{A}_0^{22}\right)^{-1}\mathbf{A}_0^{21} = \mathbf{A}_{0,11}^{-1}$ is a non-singular matrix, we must have $\boldsymbol{\alpha}_1^* = \mathbf{0}_{(n-r)\times r}$, namely *the structural equations for which there are known permanent shocks must have no error correction terms present in them*, thereby freeing up the latter to be used as instruments in estimating their parameters. More specifically, the identification of the first $n-r$ structural shocks as permanent imposes $r(n-r)$ restrictions on the structural parameters. Also $\boldsymbol{\alpha}_2^* = \left(\mathbf{A}_0^{22}\right)^{-1}\boldsymbol{\alpha}_2$ which is an $r\times r$ matrix of unknown coefficients which is not restricted by the transitory/permanent shock decomposition.

The restrictions $\boldsymbol{\alpha}_1^* = \mathbf{0}_{(n-r)\times r}$ can be exploited by noting that the r lagged error correction terms, $\boldsymbol{\beta}'\mathbf{z}_{t-1}$, are available to be used as instruments for estimating the structural parameters of the first $n-r$ equations in (7).[3]

[3] Note that we do not need to identify β. The IV estimator using the lagged EC terms is invariant to any non-singular transform of them and so β only needs to be set up to a non-singular transform.

More specifically, under $\boldsymbol{\alpha}_1^* = \mathbf{0}_{(n-r)\times r}$ the first $n-r$ equations can be written as

$$\mathbf{A}_{11}^0\Delta\mathbf{z}_{1t} + \mathbf{A}_{12}^0\Delta\mathbf{z}_{2t} = -\mathbf{A}_{11}^2\Delta\mathbf{z}_{1,t-1} - \mathbf{A}_{12}^2\Delta\mathbf{z}_{2,t-1} + \boldsymbol{\varepsilon}_{1t}, \qquad (14)$$

and it is clear that the $r\times 1$ error correction terms, $\boldsymbol{\xi}_{t-1} = \boldsymbol{\beta}'\mathbf{z}_{t-1}$, that do not appear in these equations but are included in the remaining r equations of (7)

$$\mathbf{A}_{21}^0\Delta\mathbf{z}_{1t} + \mathbf{A}_{22}^0\Delta\mathbf{z}_{2t} = -\boldsymbol{\alpha}_2^*\boldsymbol{\xi}_{t-1} - \mathbf{A}_{21}^2\Delta\mathbf{z}_{1,t-1} - \mathbf{A}_{22}^2\Delta\mathbf{z}_{2,t-1} + \boldsymbol{\varepsilon}_{2t}, \qquad (15)$$

can be used as instruments for the $n-r$ equations in (14). These instruments are clearly uncorrelated with the error terms $\boldsymbol{\varepsilon}_{1t}$, whilst at the same time being correlated with $\Delta\mathbf{z}_{1t}$ and $\Delta\mathbf{z}_{2t}$ since $\boldsymbol{\alpha}_2^*$ is a non-singular matrix. Note also that since instrumental variable estimators are unaffected by non-singular transformations of the instruments, for the purpose of estimating the structural parameters of the first $n-r$ equations ($\mathbf{A}_{11}^0$ and $\mathbf{A}_{12}^0$) the error correction terms, $\boldsymbol{\xi}_{t-1}$ (or $\boldsymbol{\beta}$), need only be identified up to a non-singular transformation.

3.2 Implications for the Structure of the SVECM

To appreciate the arguments above and to see that there are some extra restrictions placed upon the SVECM, we need to step back to the original structural system variables and to be explicit about the $(n-r)$ $I(1)$ shocks that are driving the system. Eliminating these $I(1)$ shocks will then produce the SVECM in (7). To this end, denote the original system as

$$\tilde{\mathbf{A}}_0\mathbf{z}_t = \tilde{\mathbf{A}}_1\mathbf{z}_{t-1} + \tilde{\mathbf{A}}_2\mathbf{z}_{t-2} + \mathbf{u}_t,$$

and partition it according to the first $n-r$ equations, which have the $I(1)$ structural shocks, and the r remaining equations that have $I(0)$ shocks i.e. $\Delta\mathbf{u}_{1t} = \boldsymbol{\varepsilon}_{1t}$ and $\mathbf{u}_{2t} = \boldsymbol{\varepsilon}_{2t}$. Then

$$\begin{aligned}\tilde{\mathbf{A}}_0^1\mathbf{z}_t &= \tilde{\mathbf{A}}_1^1\mathbf{z}_{t-1} + \tilde{\mathbf{A}}_2^1\mathbf{z}_{t-2} + \mathbf{u}_{1t}\\ \tilde{\mathbf{A}}_0^2\mathbf{z}_t &= \tilde{\mathbf{A}}_1^2\mathbf{z}_{t-1} + \tilde{\mathbf{A}}_2^2\mathbf{z}_{t-2} + \mathbf{u}_{2t}\end{aligned}$$

Now the first $n-r$ equations can be differenced to remove the $I(1)$ nature of the errors, giving

$$\tilde{\mathbf{A}}_0^1\boldsymbol{\Delta}\mathbf{z}_t = \tilde{\mathbf{A}}_1^1\boldsymbol{\Delta}\mathbf{z}_{t-1} + \tilde{\mathbf{A}}_2^1\boldsymbol{\Delta}\mathbf{z}_{t-2} + \boldsymbol{\varepsilon}_{1t} \qquad (16)$$

and the lack of EC terms in this set of equations is a consequence of the fact that they represent non-cointegrating relations. This makes apparent what the origin of the result in the previous sub-section is.

Turning to the second set of r equations, we reformulate them as

$$\tilde{\mathbf{A}}_0^2 \boldsymbol{\Delta} \mathbf{z}_t = -\tilde{\mathbf{A}}_2(1)\mathbf{z}_{t-1} - \tilde{\mathbf{A}}_2^2 \boldsymbol{\Delta} \mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_{2t} \tag{17}$$

where $\tilde{\mathbf{A}}_2(1) = \tilde{\mathbf{A}}_0^2 - \tilde{\mathbf{A}}_1^2 - \tilde{\mathbf{A}}_2^2$. Now there must be r co-integrating relations in $\tilde{\mathbf{A}}_2(1)\mathbf{z}_{t-1}$ since all the remaining terms in (17) are $I(0)$, namely we must have $\tilde{\mathbf{A}}_2(1) = \boldsymbol{\alpha}_2^* \boldsymbol{\beta}'$, with $\boldsymbol{\alpha}_2^*$ being a non-singular $r \times r$ matrix. Hence

$$\tilde{\mathbf{A}}_0^2 \boldsymbol{\Delta} \mathbf{z}_t = -\boldsymbol{\alpha}_2^* \boldsymbol{\beta}' \mathbf{z}_{t-1} - \tilde{\mathbf{A}}_2^2 \boldsymbol{\Delta} \mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_{2t}. \tag{18}$$

Making the identification $\tilde{\mathbf{A}}_j^i = \mathbf{A}_j^i$ we see that (16) and (18) are the SVECM in (7). Thus, lagged EC terms can only be used as instruments for structural identification of the equations in (18) if $\boldsymbol{\alpha}_2^*$ can be fixed *a priori*.

A case that will be mentioned later is when $\boldsymbol{\alpha}_2^* = \mathbf{I}_r$. Given that $\boldsymbol{\alpha}_2^* = \mathbf{A}_0^2 \boldsymbol{\alpha}$, and $\boldsymbol{\alpha}$ can be estimated without knowing the simultaneous structure, linear restrictions would be imposed upon the coefficients of the contemporaneous endogenous variables in the last r equations of the system. Note that this restriction cannot be tested. To do that we would need instruments for Δz_t.

3.3 A Common Reformulation of the SVECM

The result given in section 3.1, namely that $\boldsymbol{\alpha}_1^* = \mathbf{0}_{(n-r)\times r}$ is the main outcome of the paper and can be used to clarify a number of applications in the literature. We shall consider some of these applications in more detail below.[4] However, most of these studies do not work with the SVECM directly but rather with an SVAR system composed of $n - r$ elements from $\Delta \mathbf{z}_t$ and the r EC terms, $\boldsymbol{\xi}_t = \boldsymbol{\beta}' \mathbf{z}_t$. Our first task therefore is to relate the two systems and to determine what restrictions are placed upon the SVAR by the fact that $\boldsymbol{\alpha}_1^* = \mathbf{0}$.

[4] An important feature of these applications is the assumption that the error correction terms, $\boldsymbol{\xi}_t = \boldsymbol{\beta}' \mathbf{z}_t$, are known (or that β can be estimated super-consistently). Examples of when the vectors are known would be the purchasing power parity condition and the "constancy" of "great ratios" such as consumption to output.

To this end consider (7) and let $\mathbf{z}_t = (\mathbf{z}'_{1t}, \mathbf{z}'_{2t})'$, where $\mathbf{z}_{1t}$ is $(n-r)\times 1$ and $\mathbf{z}_{2t}$ is $r \times 1$ while $\boldsymbol{\alpha}^{*\prime} = (\boldsymbol{\alpha}_1^{*\prime}, \boldsymbol{\alpha}_2^{*\prime})$ and $\boldsymbol{\beta}' = (\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2)$ are $r \times n$ matrices of full row rank. Then defining $\mathbf{w}_t = (\Delta\mathbf{z}'_{1t}, \mathbf{z}'_t\beta)'$, we need to be able to express $\mathbf{w}_t$ as an SVAR(2) of the form

$$\mathbf{B}_0\mathbf{w}_t = \mathbf{B}_1\mathbf{w}_{t-1} + \mathbf{B}_2\mathbf{w}_{t-2} + \boldsymbol{\varepsilon}_t. \tag{19}$$

But

$$\boldsymbol{\xi}_t = \boldsymbol{\beta}'\mathbf{z}_t = \boldsymbol{\beta}'_1\mathbf{z}_{1t} + \boldsymbol{\beta}'_2\mathbf{z}_{2t},$$

or, in differences,

$$\Delta\boldsymbol{\xi}_t = \boldsymbol{\beta}'\Delta\mathbf{z}_t = \boldsymbol{\beta}'_1\Delta\mathbf{z}_{1t} + \boldsymbol{\beta}'_2\Delta\mathbf{z}_{2t}.$$

Suppose that the $r \times r$ matrix $\boldsymbol{\beta}'_2$ is non-singular, then

$$\Delta\mathbf{z}_{2t} = \boldsymbol{\beta}_2'^{-1}\left(\Delta\boldsymbol{\xi}_t - \boldsymbol{\beta}'_1\Delta\mathbf{z}_{1t}\right).$$

Using this result in (7) to eliminate $\Delta\mathbf{z}_{2t}$ in the first $n-r$ equations produces

$$\begin{aligned}\mathbf{A}_{11}^0\Delta\mathbf{z}_{1t} + \mathbf{A}_{12}^0\boldsymbol{\beta}_2'^{-1}\left(\Delta\boldsymbol{\xi}_t - \boldsymbol{\beta}'_1\Delta\mathbf{z}_{1t}\right) &= -\,\boldsymbol{\alpha}_1^*\boldsymbol{\xi}_{t-1} - \mathbf{A}_{11}^2\Delta\mathbf{z}_{1,t-1} \\ &\quad -\mathbf{A}_{12}^2\boldsymbol{\beta}_2'^{-1}\left(\Delta\boldsymbol{\xi}_{t-1} - \boldsymbol{\beta}'_1\Delta\mathbf{z}_{1,t-1}\right) + \boldsymbol{\varepsilon}_{1t},\end{aligned}$$

or

$$\begin{aligned}\left(\mathbf{A}_{11}^0 - \mathbf{A}_{12}^0\boldsymbol{\beta}_2'^{-1}\boldsymbol{\beta}'_1\right)\Delta\mathbf{z}_{1t} + \mathbf{A}_{12}^0\boldsymbol{\beta}_2'^{-1}\Delta\boldsymbol{\xi}_t &= -\,\boldsymbol{\alpha}_1^*\boldsymbol{\xi}_{t-1} - \left(\mathbf{A}_{11}^2 - \mathbf{A}_{12}^2\boldsymbol{\beta}_2'^{-1}\boldsymbol{\beta}'_1\right)\Delta\mathbf{z}_{1,t-1} \\ &\quad -\mathbf{A}_{12}^2\boldsymbol{\beta}_2'^{-1}\Delta\boldsymbol{\xi}_{t-1} + \boldsymbol{\varepsilon}_{1t}.\end{aligned} \tag{20}$$

Similarly the remaining r equations in (7) can be re-written as

$$\begin{aligned}\left(\mathbf{A}_{21}^0 - \mathbf{A}_{22}^0\boldsymbol{\beta}_2'^{-1}\boldsymbol{\beta}'_1\right)\Delta\mathbf{z}_{1t} + \mathbf{A}_{22}^0\boldsymbol{\beta}_2'^{-1}\Delta\boldsymbol{\xi}_t &= -\,\boldsymbol{\alpha}_2^*\boldsymbol{\xi}_{t-1} - \left(\mathbf{A}_{21}^2 - \mathbf{A}_{22}^2\boldsymbol{\beta}_2'^{-1}\boldsymbol{\beta}'_1\right)\Delta\mathbf{z}_{1,t-1} \\ &\quad -\mathbf{A}_{22}^2\boldsymbol{\beta}_2'^{-1}\Delta\boldsymbol{\xi}_{t-1} + \boldsymbol{\varepsilon}_{2t}.\end{aligned} \tag{21}$$

Partitioning the SVAR (19) into a form conformable to the partition of $\mathbf{w}_t$ we have

$$\begin{aligned}\mathbf{B}_{11}^0\Delta\mathbf{z}_{1t} + \mathbf{B}_{12}^0\boldsymbol{\xi}_t &= \mathbf{B}_{11}^1\Delta\mathbf{z}_{1,t-1} + \mathbf{B}_{12}^1\boldsymbol{\xi}_{t-1} + \mathbf{B}_{11}^2\Delta\mathbf{z}_{1,t-2} \\ &\quad +\mathbf{B}_{12}^2\boldsymbol{\xi}_{t-2} + \boldsymbol{\varepsilon}_{1t}, \\ \mathbf{B}_{21}^0\Delta\mathbf{z}_{1t} + \mathbf{B}_{22}^0\boldsymbol{\xi}_t &= \mathbf{B}_{21}^1\Delta\mathbf{z}_{1,t-1} + \mathbf{B}_{22}^1\boldsymbol{\xi}_{t-1} + \mathbf{B}_{21}^2\Delta\mathbf{z}_{1,t-2} + \\ &\quad \mathbf{B}_{22}^2\boldsymbol{\xi}_{t-2} + \boldsymbol{\varepsilon}_{2t}.\end{aligned}$$

or

$$\begin{aligned}
\mathbf{B}_{11}^{0}\Delta\mathbf{z}_{1t}+\mathbf{B}_{12}^{0}\Delta\boldsymbol{\xi}_{t} &= \mathbf{B}_{11}^{1}\Delta\mathbf{z}_{1,t-1}+\left(-\mathbf{B}_{12}^{0}+\mathbf{B}_{12}^{1}+\mathbf{B}_{12}^{2}\right)\boldsymbol{\xi}_{t-1} \qquad (22)\\
&\quad +\mathbf{B}_{11}^{2}\Delta\mathbf{z}_{1,t-2}-\mathbf{B}_{12}^{2}\Delta\boldsymbol{\xi}_{t-1}+\boldsymbol{\varepsilon}_{1t},\\
\mathbf{B}_{21}^{0}\Delta\mathbf{z}_{1t}+\mathbf{B}_{22}^{0}\Delta\boldsymbol{\xi}_{t} &= \mathbf{B}_{21}^{1}\Delta\mathbf{z}_{1,t-1}+\left(-\mathbf{B}_{22}^{0}+\mathbf{B}_{22}^{1}+\mathbf{B}_{22}^{2}\right)\boldsymbol{\xi}_{t-1} \qquad (23)\\
&\quad +\mathbf{B}_{21}^{2}\Delta\mathbf{z}_{1,t-2}-\mathbf{B}_{22}^{2}\Delta\boldsymbol{\xi}_{t-1}+\boldsymbol{\varepsilon}_{2t}.
\end{aligned}$$

Matching the coefficients in(20) with (22),and (21) with (23) we now have the following restrictions:

$$\begin{aligned}
\mathbf{B}_{11}^{2} &= \mathbf{0},\ \mathbf{B}_{21}^{2}=\mathbf{0}\\
\boldsymbol{\alpha}_{1}^{*} &= \left(-\mathbf{B}_{12}^{0}+\mathbf{B}_{12}^{1}+\mathbf{B}_{12}^{2}\right),\\
\boldsymbol{\alpha}_{2}^{*} &= \left(-\mathbf{B}_{22}^{0}+\mathbf{B}_{22}^{1}+\mathbf{B}_{22}^{2}\right)
\end{aligned}$$

and the coefficients $\mathbf{B}_{ij}^{k}$ relate to $\mathbf{A}_{ij}^{k}$ and the co-integrating vectors as

$$\begin{aligned}
\mathbf{B}_{11}^{0} &= \left(\mathbf{A}_{11}^{0}-\mathbf{A}_{12}^{0}\boldsymbol{\beta}_{2}^{\prime -1}\boldsymbol{\beta}_{1}^{\prime}\right),\ \mathbf{B}_{12}^{0}=\mathbf{A}_{12}^{0}\boldsymbol{\beta}_{2}^{\prime -1}\\
\mathbf{B}_{21}^{0} &= \left(\mathbf{A}_{21}^{0}-\mathbf{A}_{22}^{0}\boldsymbol{\beta}_{2}^{\prime -1}\boldsymbol{\beta}_{1}^{\prime}\right),\ \mathbf{B}_{22}^{0}=\mathbf{A}_{22}^{0}\boldsymbol{\beta}_{2}^{\prime -1}\\
\mathbf{B}_{11}^{1} &= -\left(\mathbf{A}_{11}^{2}-\mathbf{A}_{12}^{2}\boldsymbol{\beta}_{2}^{\prime -1}\boldsymbol{\beta}_{1}^{\prime}\right),\ \mathbf{B}_{21}^{1}=-\left(\mathbf{A}_{21}^{2}-\mathbf{A}_{22}^{2}\boldsymbol{\beta}_{2}^{\prime -1}\boldsymbol{\beta}_{1}^{\prime}\right)\\
\mathbf{B}_{12}^{2} &= \mathbf{A}_{12}^{2}\boldsymbol{\beta}_{2}^{\prime -1},\ \mathbf{B}_{22}^{2}=\mathbf{A}_{22}^{2}\boldsymbol{\beta}_{2}^{\prime -1}.
\end{aligned}$$

Now we have previously established that $\boldsymbol{\alpha}_{1}^{*}=\mathbf{0}$, and this produces $(-\mathbf{B}_{12}^{0}+\mathbf{B}_{12}^{1}+\mathbf{B}_{12}^{2})=\mathbf{0}$. After substituting this restriction into the first $n-r$ equations of the system they can be written as

$$\mathbf{B}_{11}^{0}\Delta\mathbf{z}_{1t}+\mathbf{B}_{12}^{0}\Delta\boldsymbol{\xi}_{t}=\mathbf{B}_{11}^{1}\Delta\mathbf{z}_{1,t-1}-\mathbf{B}_{12}^{2}\Delta\boldsymbol{\xi}_{t-1}+\boldsymbol{\varepsilon}_{1t},$$

and therefore $\boldsymbol{\xi}_{t-1}$ can be used as instruments for identification and estimation of $\mathbf{B}_{11}^{0}$ and $\mathbf{B}_{12}^{0}$. As noted earlier, since the IV estimators are invariant to non-singular $r\times r$ transformations, it is sufficient also that the co-integrating vectors $\boldsymbol{\beta}$ are known up to a non-singular transformation.

In general the lagged EC terms do not provide sufficient restrictions to estimate all of the structural parameters. Only if there is a single permanent structural shock will there be enough. In other cases the presence of multiple permanent shocks will necessitate extra restrictions, and often these relate to the magnitude of their long-run impact. For example, as we will see later, it is often the case that these extra permanent shocks are assumed to have a zero long-run effect upon some of the $I(1)$ variables in the long-run. Note that the long run effects of any transitory shocks ($\boldsymbol{\varepsilon}_{2t}$) upon these variables will be zero by construction.

To derive the implications of such long-run restrictions we return to (19) and re-write it as

$$(\mathbf{B}_0 - \mathbf{B}_1 L - \mathbf{B}_2 L^2)\mathbf{w}_t = \boldsymbol{\varepsilon}_\mathbf{t},$$

which implies the MA form (note that since $\mathbf{w}_t$ is $I(0)$ then $\mathbf{B}(L)$ is invertible)

$$\begin{aligned} \mathbf{w}_t &= \mathbf{B}(L)^{-1}\boldsymbol{\varepsilon}_t \\ &= \mathbf{C}(L)\boldsymbol{\varepsilon}_t = \mathbf{C}_0\boldsymbol{\varepsilon}_t + \mathbf{C}_1\boldsymbol{\varepsilon}_{t-1} + \mathbf{C}_2\boldsymbol{\varepsilon}_{t-2} + ..., \end{aligned}$$

where the $\mathbf{C}_j$ are the impulse responses to the shocks in $\boldsymbol{\varepsilon}_t$. Hence, recalling that $\mathbf{w}_t = (\boldsymbol{\Delta}\mathbf{z}'_{1t}, \mathbf{z}'_t\boldsymbol{\beta})'$, we have

$$\boldsymbol{\Delta}\mathbf{z}_{1t} = \mathbf{C}_{11}(L)\boldsymbol{\varepsilon}_{1t} + \mathbf{C}_{12}(L)\boldsymbol{\varepsilon}_{2t},$$

and the effects of the permanent shocks on $\mathbf{z}_{1t}$ are given by $\mathbf{C}_{11}(1)$.

Imposing the relation $\mathbf{C}(L) = \mathbf{B}(L)^{-1} \Longrightarrow \mathbf{C}(1)\mathbf{B}(1) = \mathbf{I}_n$ gives

$$\begin{bmatrix} \mathbf{C}_{11}(1) & 0 \\ \mathbf{C}_{21}(1) & \mathbf{C}_{22}(1) \end{bmatrix} \begin{bmatrix} \mathbf{B}_{11}(1) & \mathbf{B}_{12}(1) \\ \mathbf{B}_{21}(1) & \mathbf{B}_{22}(1) \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{n-r} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_r \end{bmatrix},$$

since we know that $\mathbf{C}_{12}(1) = \mathbf{0}$, as these are the long-run effects of transitory shocks upon $I(1)$ variables. Therefore, $\mathbf{C}_{11}(1)\mathbf{B}_{11}(1) = \mathbf{I}_{n-r}$.

Now suppose that interest lies in identifying the parameters of the first equation in the system and this is to be achieved by assuming that the long run effects of all shocks on $\mathbf{z}_{1t}$ are zero. Partitioning $\mathbf{C}_{11}(1)\mathbf{B}_{11}(1) = \mathbf{I}_{n-r}$ according to the first structural equation and the remaining ones, we have

$$\begin{bmatrix} C^1_{11}(1) & \mathbf{C}^1_{12}(1) \\ \overline{\mathbf{C}}_{21}(1) & \overline{\mathbf{C}}_{22}(1) \end{bmatrix} \begin{bmatrix} B^1_{11}(1) & \mathbf{B}^1_{12}(1) \\ \overline{\mathbf{B}}_{21}(1) & \overline{\mathbf{B}}_{22}(1) \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{0}_{1\times n-r-1} \\ \mathbf{0}_{n-r-1\times 1} & \mathbf{I}_{n-r-1} \end{bmatrix},$$

yielding the restriction that $\mathbf{C}^1_{12}(1) = \mathbf{0}$. Now $C^1_{11}(1)\mathbf{B}^1_{12}(1) + \mathbf{C}^1_{12}(1)\overline{\mathbf{B}}_{22}(1) = \mathbf{0}_{1\times n-r-1}$ means that $C^1_{11}(1)\mathbf{B}^1_{12}(1) = \mathbf{0}_{1\times n-r-1}$ i.e. $\mathbf{B}^1_{12}(1) = \mathbf{0}_{1\times n-r-1}$ since $C^1_{11}(1)$ is non-singular. Finally $\mathbf{B}^1_{12}(1) = \mathbf{0}$ implies that the coefficients on the current and lagged values of $\boldsymbol{\Delta} z_{2t}, ..., \boldsymbol{\Delta} z_{n-r,t}$ in the first equation must sum to zero.

Long-run effects of shocks can also be used to restrict the SVECM. It is known that $\boldsymbol{\beta}'\mathbf{C}_{11}(1) = \mathbf{0}$ and $\mathbf{C}_{11}(1)'\boldsymbol{\alpha} = \mathbf{0}$. If $\boldsymbol{\beta}$ is uniquely identified r^2

restrictions need to be placed upon the $n \times r$ elements i.e. there are $(n-r) \times r$ free parameters. In turn this means the same number of free parameters in $\mathbf{C}_{11}(1)$. The constraint $\mathbf{C}_{11}(1)'\boldsymbol{\alpha} = \mathbf{0}$ does not in general impose any restrictions upon $\boldsymbol{\alpha}$. If, however, some of the permanent shocks have zero impact, the number of free parameters in $\mathbf{C}_{11}(1)$ will be smaller than $(n - r) \times r$ and so some restrictions are placed upon $\boldsymbol{\alpha}$.

As far as the second set of r equations are concerned we have

$$\mathbf{B}_{21}^{0}\Delta\mathbf{z}_{1t} + \mathbf{B}_{22}^{0}\Delta\boldsymbol{\xi}_{t} = \boldsymbol{\alpha}_{2}^{*}\boldsymbol{\xi}_{t-1} + \mathbf{B}_{21}^{1}\Delta\mathbf{z}_{1,t-1} - \mathbf{B}_{22}^{2}\Delta\boldsymbol{\xi}_{t-1} + \boldsymbol{\varepsilon}_{2t}.$$

Assuming that $\boldsymbol{\varepsilon}_{1t}$ are identified from the first set of $n-r$ equations, they can then be used as instruments to identify and estimate $\mathbf{B}_{21}^{0}$ and $\mathbf{B}_{22}^{0}$, since by assumption $cov(\boldsymbol{\varepsilon}_{1t}, \boldsymbol{\varepsilon}_{2t}) = \mathbf{0}$. There are also additional moment conditions implied by $cov(\boldsymbol{\varepsilon}_{2t}) = \mathbf{I}_r$ that can be utilized for the identification of $\mathbf{B}_{21}^{0}$ and $\mathbf{B}_{22}^{0}$.

3.4 Modelling with Observable Permanent Shocks

In the analysis above the VECM (5) provides the "reduced form" and (7) is the "structure". To determine identification one examines the relations $\mathbf{A}_0\boldsymbol{\alpha} = \boldsymbol{\alpha}^*$ and the fact that $cov(\boldsymbol{\varepsilon}_t) = \mathbf{I}_n$. There are n^2 structural parameters in $\mathbf{A}_0$. Since the reduced form VECM will provide identified values for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, once $\mathbf{A}_0$ is known $\boldsymbol{\alpha}^*$ can be recovered. Now there are two types of restrictions available to identify the structural parameters - those that come from "dynamics", namely $\mathbf{A}_0\boldsymbol{\alpha} = \boldsymbol{\alpha}^*$, and those from "orthogonality" assumption, $cov(\boldsymbol{\varepsilon}_t) = \mathbf{I}_n$. The first set delivers $r(n-r)$ restrictions from $\boldsymbol{\alpha}_1^* = \mathbf{0}$ and the second $n(n+1)/2$. Hence the system is exactly identified when $n^2 = r(n-r) + n(n+1)/2$. Clearly when $n = 2$ and $r = 1$ we have exact identification.

In the above analysis we have not distinguished whether the permanent shocks are observable or not. Wickens and Motto (2001) suggest that there is identifying information when the shocks are observable and when the number of endogenous variables equals r. To examine this case let $\mathbf{z}_t' = (\mathbf{x}_t', \mathbf{y}_t')$, where $\mathbf{x}_t$ is an $q \times 1$ vector of exogenous $I(1)$ variables and $\mathbf{y}_t$ is a $p \times 1$ vector of endogenous variables $n = p + q$. The system of structural equations can be regarded as having the form (where we have normalized the coefficients on the contemporary exogenous variables in the first q equations)

$$\begin{aligned}\mathbf{\Delta x}_t &= -\boldsymbol{\alpha}_x\boldsymbol{\beta}'\mathbf{z}_{t-1} + \mathbf{\Psi}_{1x}\mathbf{\Delta z}_{t-1}+\boldsymbol{\varepsilon}_{xt}, &(24)\\ \mathbf{A}^0_{yy}\mathbf{\Delta y}_t+\mathbf{A}^0_{yx}\mathbf{\Delta x}_t &= -\boldsymbol{\alpha}^*_y\boldsymbol{\beta}'\mathbf{z}_{t-1}+\mathbf{\Psi}_{1y}\mathbf{\Delta z}_{t-1}+\boldsymbol{\varepsilon}_{yt}. &(25)\end{aligned}$$

In the present case where $\mathbf{x}_t$ is assumed to be weakly exogenous we have $\boldsymbol{\alpha}_x = \mathbf{0}$. However, this restriction does not help in identification of the structural parameters of (25).

The relations between the SVECM and the VECM remain the same as before but, since

$$\mathbf{A}_0 = \begin{pmatrix} \mathbf{I}_{q\times q} & \mathbf{0}_{q\times p} \\ \mathbf{A}^0_{yx} & \mathbf{A}^0_{yy} \end{pmatrix}, \text{ and } \boldsymbol{\alpha} = \begin{pmatrix} \mathbf{0}_{q\times r} \\ \boldsymbol{\alpha}_y \end{pmatrix},$$

we now have $\mathbf{A}^0_{yy}\boldsymbol{\alpha}_y = \boldsymbol{\alpha}^*_y$ as the dynamic restrictions on the equations for $\mathbf{y}_t$. There are p^2 unknown structural parameters in $\mathbf{A}^0_{yy}$ and pr in α^*_y. To estimate these we only have $r(p-r)$ "dynamic" restrictions from $\mathbf{A}^0_{yy}\boldsymbol{\alpha}_y = \boldsymbol{\alpha}^*_y$ and $p(p+1)/2$ $+pq$ from orthogonality of the shocks. The latter come from $E(\boldsymbol{\varepsilon}_{yt}\boldsymbol{\varepsilon}'_{yt}) = \mathbf{I}_p$ and $E(\boldsymbol{\varepsilon}_{yt}\boldsymbol{\varepsilon}'_{xt}) = \mathbf{0}_{p\times q}$, reflecting the fact that imposing a zero correlation between the shocks driving the $\Delta\mathbf{x}_t$ equations would be of no use for identifying the equations corresponding to $\Delta\mathbf{y}_t$. Note that the dynamic restrictions do not depend upon the number of exogenous shocks, so that exogenous permanent shocks have an impact only through the orthogonality restrictions, $E(\boldsymbol{\varepsilon}_{yt}\boldsymbol{\varepsilon}'_{xt}) = \mathbf{0}_{p\times q}$. Consequently a special treatment of the implications of co-integration is not needed.

4 Some Applications of the Framework

4.1 Blanchard and Quah (1989)

Blanchard and Quah have a two equation system in GNP (y_t) and the unemployment rate (un_t). The variables y_t and un_t are assumed to be $I(1)$ and $I(0)$, respectively. They assume that there is one permanent (supply) and one transitory (demand) shock. These are denoted by ε_{1t} and ε_{2t}, respectively. Although there is no co-integration in this case, our methodological approach can be applied by treating the $I(0)$ variable as if it "co-integrates" with itself.

Let us set up a pseudo co-integrating vector of the form $\boldsymbol{\beta} = (0, \beta_2)'$ which produces the lagged "EC term" given by $\beta_2 un_{t-1}$. According to our results (see section 3.1) the equation with the permanent shock will have the form (normalizing on Δy_t)

$$\Delta y_t = \alpha_{12}^0(\beta_2 un_t) + \alpha_{11}^1 \Delta y_{t-1} + \alpha_{12}^1 \Delta(\beta_2 un_{t-1}) + \varepsilon_{1t},$$

and the second equation (normalizing on un_t) will be

$$un_t = \alpha_{21}^0 \Delta y_t + \alpha_{22}^0 \beta_2 un_{t-1} + \alpha_{21}^1 \Delta y_{t-1} + \alpha_{22}^1 (\beta_2 \Delta un_{t-1}) + \varepsilon_{2t},$$

It is clear that in this set up $\beta_2 un_{t-1}$ does not enter the first equation and can therefore be used as instrument for un_t in it. So long as $\beta_2 \neq 0$, the value of β_2 does not matter as the instrumental variable estimator is invariant to it. However, unlike the co-integration case where β_2 could be estimated super-consistently, this is not possible when un_t is $I(0)$, so that we would need to treat un_{t-1} as a regressor in the second equation. That means un_{t-1} is not available as an instrument for Δy_t. But the residuals from the first equation form a suitable instrument. This instrumental variable interpretation of Blanchard and Quah is due to Shapiro and Watson (1988). The problem with this procedure is that un_{t-1} is often a very poor instrument for Δun_t and this can lead to highly non-normal densities for the instrumental variables estimator. Using the same data as Blanchard and Quah this is shown in Fry and Pagan (2005). There are many applications of this idea e.g. Peersman (2005).

4.2 Identification of Capital Share in a DSGE Model

In many DSGE models, one of the structural equations is a Cobb-Douglas production function of the form

$$y_t - l_t = \alpha(k_t - l_t) + u_t,$$

where y_t is the log of output, k_t the log of the capital stock, l_t the log of labour input and the technology shock u_t is generally assumed to be an $I(1)$ variable i.e. $\Delta u_t = \varepsilon_t$. Hence the structural equation to be estimated can be transformed to

$$\Delta(y_t - l_t) = \alpha(\Delta k_t - \Delta l_t) + \varepsilon_t$$

and instruments are needed for $\Delta k_t - \Delta l_t$. In most DSGE models l_t is an $I(0)$ variable, but there is co-integration between y_t and k_t, so that the capital-output ratio $y_t - k_t$ is an $I(0)$ variable. Therefore, using the result established above, $y_{t-1} - k_{t-1}$ could be used as an instrument for $(\Delta k_t - \Delta l_t)$. Of course there are also lags of Δk_t and Δl_t excluded from this equation and these might provide other instruments. To assess how useful the lagged ECM term might be as an instrument we simulate data from the RBC model set out in Ireland (2004), but change the technology process to one having a unit root. Then one finds that the correlation between $y_{t-1} - k_{t-1}$ and $(\Delta k_t - \Delta l_t)$ would be 0.729, making it an excellent instrument, and therefore α should be quite accurately estimated.

4.3 Wickens and Motto's Four Equation Monetary Model

Wickens and Motto (2001) give an example which has four equations

$$\begin{aligned}
\Delta i_t &= a_{12}^0 \Delta p_t + a_{12}^1 i_{t-1} + \varepsilon_{it} && (26)\\
\Delta p_t &= -\Delta y_t + \Delta m_t + \lambda i_t + \alpha_2^*(m_{t-1} - p_{t-1} - y_{t-1}) + \varepsilon_{pt} && (27)\\
\Delta y_t &= \gamma + \alpha \Delta y_{t-1} + \varepsilon_{yt} && (28)\\
\Delta m_t &= \mu + \theta \Delta m_{t-1} + \varepsilon_{mt}, && (29)
\end{aligned}$$

where i_t is an interest rate, p_t is the log of the price level, m_t is the log of the money supply and y_t is the log of real output. All the four structural shocks, ε_{it}, $\varepsilon_{pt}, \varepsilon_{yt}$, and ε_{mt}, are assumed to be uncorrelated. It is clear that y_t and m_t are $I(1)$ and exogenous. This leaves two potentially $I(1)$ variables. If both variables were $I(1)$ there would be two co-integrating vectors. In their paper they state "there is only one long-run structural relation among the $I(1)$ variables, namely $m_t - y_t - p_t$, and hence only one co-integrating vector" (p. 380), which renders i_t as an $I(0)$ variable.[5]

The presence of i_t creates some issues in using the identification conditions of section 3, since these relate to $I(1)$ variables. Initially then suppose that i_t is not in the system and that the model consists of (27)-(29) with the term λi_t excluded. Then $p = 1, r = 1$, and $n = p+q = 3$ which gives $pn = 3$ structural parameters to be estimated based on the $p(p-r)+p(p+1)/2+pq = 3$ available restrictions; namely the system composed of (27)-(29) without λi_t would be exactly identified. However, Wickens and Motto's argument was that when $p = r$ then $\alpha_2^* = 1$ and it is this restriction which is used for identification. But the conditions above show that if i_t is not in the system the restriction

[5] The fact that i_t is to be taken as $I(0)$ is repeated in their footnote 14.

$\alpha_2^* = 1$ would be unnecessary, since (27) can be estimated with OLS noting that all regressors are predetermined or exogenous. So it is not the condition $r = p$ which leads to identification, but rather the restrictions imposed by the exogeneity of y_t and m_t and the orthogonality of the shocks.

Now introduce the $I(0)$ variable i_t into the system. An instrument is needed for it. If the coefficients on Δy_t and Δm_t in (27) were known then these could be instruments, but Wickens and Motto decide to treat these as having unknown coefficients. Consequently, the assumption that if $\alpha_2^* = 1$ would allow the lagged EC term to be used as an instrument, and this is what Wickens and Motto do. As observed in section 3.4 the fact that $r = p$ does not guarantee that $\alpha_2^* = 1$. An assignment of such a value to α_2^* is simply an assumption which may or may not be correct and the truth of which is not testable.

4.4 Gali's IS-LM Model (Gali, 1992)

Gali presents a four equation model that is meant to be an analogue of the IS-LM system. It consists of four $I(1)$ variables, the log of GNP (y_t), the inflation rate (π_t), the growth rate of the money supply (Δm_t) and the nominal interest rate (i_t). He assumes that there are two co-integrating vectors among these four variables - $\xi_{1t} = \Delta m_t - \pi_t$ and $\xi_{2t} = i_t - \pi_t$ so that

$$\boldsymbol{\beta}' = (\boldsymbol{\beta}_1', \boldsymbol{\beta}_2') = \begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \text{ with } \boldsymbol{\beta}_2' = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}.$$

Gali works with an SVAR in $\Delta y_t, \Delta i_t, \xi_{1t}$ and ξ_{2t} rather than the SVECM that is implied by the assumptions that there are $I(1)$ variables and co-integration. It is clear that $\boldsymbol{\beta}_2$ is non-singular and we have shown how to move between the SVECM and the SVAR under this condition in section 3.3. It emerged there that the implied SVAR would have the form (for the first equation)

$$\begin{aligned} \Delta y_t \;=\; & \alpha_{12}^0 \Delta i_t + \alpha_{13}^0 \Delta \xi_{1t} + \alpha_{14}^0 \Delta \xi_{2t} + \alpha_{11}^1 \Delta y_{t-1} + \alpha_{12}^1 \Delta i_{t-1} + \\ & \alpha_{13}^1 \Delta \xi_{1,t-1} + \alpha_{14}^1 \Delta \xi_{2,t-1} + \varepsilon_{1t}. \end{aligned}$$

We can clearly use $\xi_{j,t-1}$ $(j = 1, 2)$ as instruments for $\Delta \xi_{jt}$ in this equation but still need another one for Δi_t. To get this Gali assumes that the long-run effect of the second permanent shock upon y_t is zero. This shock must be in the Δi_t equation in his SVAR. As seen in section 3.3 the restriction means that $\alpha_{12}^0 = -\alpha_{12}^1$, and so the equation can be re-expressed in terms of $\Delta^2 i_t$, allowing Δi_{t-1} to be used as an instrument.

The second equation will have the form

$$\begin{aligned}\Delta i_t &= \alpha_{21}^0 \Delta y_t + \alpha_{23}^0 \Delta\xi_{1t} + \alpha_{24}^0 \Delta\xi_{2t} + \alpha_{21}^1 \Delta y_{t-1} + \alpha_{22}^1 \Delta i_{t-1} + \\ &\quad \alpha_{23}^1 \Delta\xi_{1,t-1} + \alpha_{24}^1 \Delta\xi_{2,t-1} + \varepsilon_{2t}.\end{aligned}$$

Now to estimate this equation we can still use the lagged ECM terms as instruments but we also have available the residuals of the first equation, assuming that the shocks are uncorrelated. Gali adopts the latter instruments but not the former, as he does not recognize that the lagged ECM terms are available as instruments. Instead he imposes short-run restrictions. Pagan and Robertson (1998) found that the instruments coming from these short run restrictions were extremely poor and so the distributions of Gali's estimators of the structural parameters were highly unlikely to be normal. In contrast the lagged EC terms are excellent instruments. Using the same data as in Pagan and Robertson $\xi_{2,t-1}$ is found to have a correlation with Δy_t of .36 and with ξ_{2t} of .49, while $\xi_{1,t-1}$ has a correlation of .1 with Δy_t and .58 with ξ_{1t}. Thus, since Gali did not fully work out the implications of his twin assumptions that all variables are $I(1)$ and that there are two known co-integrating vectors, he was forced to search for alternative short-run restrictions.

The third and fourth equations in Gali's model need some extra short-run information as we need three instruments for each equation but only have the two residuals that correspond to the two permanent shocks available by an assumption that permanent and transitory shocks are uncorrelated. We can make the transitory shocks uncorrelated but this means we are still one restriction short. One of Gali's short-run restrictions can therefore be used. In summary, once one recognizes the structural restrictions imposed by co-integration only one rather than three of the short-run restrictions used by Gali are needed to completely estimate the impacts of transitory and permanent shocks.

4.5 Shapiro and Watson's (1988) Business Cycle Decomposition Paper

Shapiro and Watson considered a five variable system comprised of hours worked (h_t), the real price of oil (po_t), the level of output (y_t), the inflation rate (π_t), and the nominal interest rate (i_t). All variables are taken to be $I(1)$ and $\xi_{1t} = i_t - \pi_t$ is assumed to be the only co-integrating relationship in the model. The real price of oil is taken to be exogenous. Discussion in the paper identifies three permanent shocks - technology, real oil prices, and "labour supply" shocks. Given these assumptions it must be the case that there are two co-integrating relations, but Shapiro and Watson do not specify a second co-integrating relation. This creates a problem for their analysis. To see why note that they work with an SVAR in $\Delta y_t, \Delta h_t, \Delta\pi_t, \Delta po_t$ and ξ_{1t}. However the assumptions about the nature of the variables and the number of permanent shocks means that the evolution of the $I(1)$ variables is as an SVECM in the original five variables. This SVECM can be converted to an SVAR in $\Delta y_t, \Delta\pi_t, \Delta po_t$, ξ_{1t} and ξ_{2t}. Thus the SVAR they work with is mis-specified as it should contain two EC terms and not a single.

4.6 Gali and Fisher's Technology Shock Identification Papers

Gali (1999) has a five variable model consisting of labour productivity (x_t),the log of per capita hours (n_t), the inflation rate (π_t), the nominal interest (i_t) and the growth rate of the money supply (Δm_t). He assumes all variables are $I(1)$ and that there are two co-integrating relations, $\xi_{1t} = i_t - \pi_t$ and $\xi_{2t} = \Delta m_t - \pi_t$. The system therefore has three permanent shocks. Gali uses the SVAR form of system which contains the five variables $\Delta x_t, \Delta n_t, \Delta\pi_t, \xi_{1t}$ and ξ_{2t}, and the first equation will be

$$\begin{aligned}\Delta x_t &= \alpha_{12}^0 \Delta n_t + \alpha_{13}^0 \Delta\pi_t + \alpha_{14}^0 \Delta\xi_{1t} + \alpha_{15}^0 \Delta\xi_{2t} + \\ &\quad \alpha_{12}^1 \Delta n_{t-1} + \alpha_{13}^1 \Delta\pi_{t-1} + \alpha_{14}^1 \Delta\xi_{1,t-1} + \alpha_{15}^1 \Delta\xi_{2,t-1} + \varepsilon_{1t}.\end{aligned}$$

Now we need instruments for $\Delta n_t, \Delta\pi_t, \Delta\xi_{1t}$ and $\Delta\xi_{2t}$. Two are provided by the two lagged EC terms and so two more instruments are needed. Gali assumes that the long-run effects of the non-technology permanent shocks upon labour productivity are zero so that $\alpha_{12}^0 = -\alpha_{12}^1$ and $\alpha_{13}^0 = -\alpha_{13}^0$, and, as a result, Δn_{t-1} and $\Delta\pi_{t-1}$ can be used as instruments for $\Delta^2 n_t$ and $\Delta\pi_t^2$. (see the derivations at the end of section 3.3).

In a recent paper Fisher (2006) augments Gali's five equation model with log real investment price (denoted here by q_t), drops the money supply growth variable and the co-integrating relations, ending up with an alternative five equation model in $q_t, x_t, n_t, \pi_t,$ and i_t. Denote the structural shocks in the system $\{q_t, x_t, n_t, \pi_t,\ i_t\}$ by $u_{qt},\ u_{xt},\ \varepsilon_{nt},\ \varepsilon_{\pi t}$ and ε_{rt} respectively. Fisher's analysis certainly assumes that q_t and x_t are non-cointegrated $I(1)$ processes, but it is not entirely clear what assumptions he means to make about the other variables. It seems that the preferred assumption for n_t is that it be $I(0)$, as in his comments on this variable (p430) he says that "the discussion focusses on the levels specification". Later he presents results that have Δn_t replacing n_t in the system. In what follows we shall assume that n_t is $I(0)$, although some mention will be made of how the analysis changes if this is not the case. To determine the order of integration assumed for other variables we examine the equations presented. Thus, his equation (14) combines together $\Delta q_t, \Delta x_t, n_t, \pi_t$ and i_t and has an $I(0)$ error term, so that either π_t or i_t are $I(0)$ or they must be co-integrating $I(1)$ variables. But this would mean that the system would involve an EC term, and thus is not present in the formulations, so we are led to the conclusion that they are taken to be $I(0)$.

The first structural equation in his system therefore would have the form (ignoring lagged values except for x_t)

$$q_t = \alpha_{12}^0 x_t + \alpha_{13}^0 n_t + \alpha_{14}^0 \pi_t + \alpha_{15}^0 i_t + \alpha_{12}^1 x_{t-1} + ... + u_{qt}$$

and, when the shock in this equation is $I(1)$, we have

$$\Delta q_t = \alpha_{12}^0 \Delta x_t + \alpha_{13}^0 \Delta n_t + \alpha_{14}^0 \Delta \pi_t + \alpha_{15}^0 \Delta i_t + \alpha_{12}^1 \Delta x_{t-1} + ... + \varepsilon_{qt}$$

where $\Delta u_{qt} = \varepsilon_{qt}$. He then makes the identifying assumption that only investment-specific shocks (ε_{qt}) have a long run impact on q_t. Of course, the transitory shocks $\varepsilon_{nt}, \varepsilon_{pt}$, and ε_{rt} have a zero long run effect by definition, so this assumption only imposes the restriction $\alpha_{12}^0 = -\alpha_{12}^1$ leading to the estimable equation

$$\Delta q_t = \alpha_{12}^0 \Delta^2 x_t + \alpha_{13}^0 \Delta n_t + \alpha_{14}^0 \Delta \pi_t + \alpha_{15}^0 \Delta i_t + ... + \varepsilon_{qt}, \qquad (30)$$

which is his equation (15). Note that his argument is somewhat confused as he implicitly differences the $I(0)$ variables at the second step rather than the first one. Only if these variables were $I(1)$ would his argument be correct that the lag polynomials attached to them have a unit root, but if that was the case then all these variables should be expressed as second differences. **(30)** can then be estimated using Δx_{t-1} as an instrument for $\Delta^2 x_t$ and n_{t-1}, π_{t-1}, i_t as instruments for $\Delta n_t, \Delta \pi_t$ and Δi_t.[6]

[6] If n_t were taken to be $I(1)$ then there is a third permanent shock and it would be $\Delta^2 n_t$ rather than Δn_t that appears in the first and second equations and the choice of the instruments would change accordingly.

The next structural equation is for x_t and it also has an $I(1)$ shock attached to it. After differencing to remove the $I(1)$ shock the equation becomes

$$\Delta x_t = \alpha^0_{21}\Delta q_t + \alpha^0_{23}\Delta n_t + \alpha^0_{24}\Delta \pi_t + \alpha^0_{25}\Delta i_t + ... + \varepsilon_{xt}.$$

Here the same instruments can be used for $\Delta n_t, \Delta \pi_t$ and Δi_t as before while the residuals from (30) are a suitable instrument for Δq_t.

Once one has found the investment and technology shocks one can compute impulse responses and estimate the proportion of the variance of Δx_t they explain, provided it is assumed that the structural shocks are uncorrelated. In fact it is not possible to identify the remaining shocks without additional *a priori* information. Fisher suggests that it can be done by using the residuals from the structural equations with permanent shocks and "N lags of $\mathbf{y}_t$", where $\mathbf{y}_t$ contains the five variables $\Delta x_t, \Delta q_t, n_t, \pi_t$ and i_t, while N is said to be the order of the VAR in $\mathbf{y}_t$. But if this is true then the lagged values of $\mathbf{y}_t$ do not provide any instruments. If the VAR is of order less than N, then "N lags of $\mathbf{y}_t$" would not yield useful instruments.

4.7 King et al. (1991) Macroeconomic Model and Related Studies

King, Plosser, Stock, Watson (KPSW, 1991) dealt with a six equation model containing six $I(1)$ variables output (y_t), consumption (c_t), investment (inv_t), real money ($m_t - p_t$), the nominal interest rate (i_t) and inflation (Δp_t). They assumed that there were three permanent shocks - a balanced growth shock, a real interest rate and an inflation shock. It will prove to be convenient to re-write the variables in the structural system as $\mathbf{z}_t = (y_t, i_t - \Delta p_t, m_t - p_t, c_t, inv_t, i_t)'$.

KPSW assume that there are three co-integrating relations of the form

$$\begin{aligned} &c_t - y_t - \phi_1(i_t - \Delta p_t) \\ &inv_t - y_t - \phi_2(i_t - \Delta p_t) \\ &(m_t - p_t) - \beta_y y_t + \beta_r i_t. \end{aligned}$$

Since at least three restrictions have been applied to each of the three relations the co-integrating vectors are uniquely identified, and their unknown coefficients, ϕ_1, ϕ_2, β_y, and β_r can be super consistently estimated. In what follows we assume that the values of these long run parameters are given. In terms of $\mathbf{z}_t$ the co-integrating vectors are

$$\boldsymbol{\beta}' = \begin{pmatrix} -1 & -\phi_1 & 0 & 1 & 0 & 0 \\ -1 & -\phi_2 & 0 & 0 & 1 & 0 \\ -\beta_y & 0 & 1 & 0 & 0 & \beta_r \end{pmatrix},$$

so that

$$\boldsymbol{\beta}_2' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \beta_r \end{pmatrix},$$

which will be non-singular provided $\beta_r \neq 0$. Thus we can re-formulate the implied SVECM system as an SVAR in $\Delta y_t, \Delta(i_t - \Delta p_t), \Delta(m_t - p_t)$ and the three EC terms. The first three structural equations have the three permanent shocks. We will look at each of the six equations in turn.

Based on their equation (8) and attendant discussion, we can determine the long-run impact of the permanent shocks upon our selected set of variables (notice that we have ordered the shocks to be balanced growth, real interest rate and inflation). The long-run effect of permanent shocks upon the six variables will be

$$\mathbf{C}_{11}(1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta_y & -\beta_r & -\beta_r \\ 1 & \phi_1 & 0 \\ 1 & \phi_2 & 0 \\ 0 & 1 & 1 \end{pmatrix}.$$

From the first row of $\mathbf{C}_{11}(1)$ there are two long-run zero restrictions on output of the real interest rate and inflation shocks. Hence the equation can be estimated in exactly the same manner as in Gali's models. In turn estimation of the output equation produces a residual that can be adopted as an instrument in any equation whose shock is uncorrelated with the technology shock. Since KPSW assume that all permanent shocks are uncorrelated this means the equations for $\Delta(i_t - \Delta p_t)$ and $\Delta(m_t - p_t)$.

The second row of $\mathbf{C}_{11}(1)$ shows that balanced growth and inflation shocks have no impact upon the real interest rate in the long-run. Hence this equation can be estimated in the same way as the output equation. However this would result in an excess of instruments since there are three lagged EC terms and the residuals from the first equation so one only one long-run effect of the balanced growth and inflation shock is needed. It would seem to make sense that this be the inflation shock. Having estimated these two equations the residuals from them combine with the three lagged EC terms to estimate the real money equation.

It is not possible to estimate the remaining three equations without some additional restrictions. If the transitory shocks are uncorrelated with the permanent shocks then the permanent shocks can be used as instruments. But this still leaves us with a lack of two instruments in each equation. The constraint that the transitory shocks are uncorrelated allows one to estimate three of these six unknown coefficients but the remaining ones need to be determined with short-run restrictions. However, because KPSW did not seek to estimate the transitory shocks the only restrictions that were needed were those coming from co-integration and long-run restrictions.

5 Concluding Remarks

This paper considers the implications of the permanent/transitory decomposition of shocks for the identification of structural models when one or more of the structural shocks are permanent. It provides a simple and intuitive generalization of the work of Blanchard and Quah (1989), and shows that structural equations for which there are known permanent shocks must have no error correction terms present in them. This insight can be used to construct suitable instruments for the estimation of the structural parameters. The usefulness of the approach is illustrated by re-examinations of the structural identification of the monetary model of Wickens and Motto (2001), the business cycle decomposition of Shapiro and Watson (1988), the macroeconometric model of King et al. (1991), the influential IS-LM model of Gali (1992), and its extensions in Gali (1999) and Fisher (2006) that consider the identification of alternative technology shocks. It is shown that often empirical work has not fully exploited the identification provided by the structural co-integration. This information acts to restrict the SVECM and so affects model design. Fort this reason the general results provided in this paper are also likely to be relevant to a number of DSGE models with more than one permanent shock that have been recently advanced in the literature.

References

Blanchard, O.J. and D. Quah (1989), "The Dynamic Effects of Aggregate Demand and Supply Disturbances", *The American Economic Review*, 79, 655-673.

Edge, R., M. Kiley and J.P. Laforte (2005), " An Estimated DSGE Model of the US Economy with an Application to Natural Rate Measures", paper presented to the *IRFMP/IMP Conference on DSGE Modeling at Policy making Institutions: Progress and Prospects*, Washington, December.

Fisher, Jonas D. M. (2006), "The Dynamic Effects of Neutral and Investment-Specific Technology Shocks," Journal of Political Economy, 114, 413-451.

Gali, J. (1992), "How Well does the ISLM Model Fit Postwar U.S. Data?", *Quarterly Journal of Economics*, 107, 709-735.

Gali, J. (1999), "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?", American Economic Review, 89, 249-271.

Garratt, A., Lee, K., M.H. Pesaran and Y. Shin (2006), *Global and National Macroeconometric Modelling: A long-run structural approach*, Oxford University Press, Oxford.

Gonzalo, J. and S. Ng, (2001), "A Systematic Framework for Analyzing the Dynamic Effects of Permanent and Transitory Shocks", *Journal of Economic Dynamics and Control*, 25, 1527-1546.

Fisher, J.D.M. (2006), "The Dynamic Effects of Neutral and Investment-Specific Technology Shocks", Journal of Political Economy, 114, 413-451.

Fry,R. and A. R. Pagan (2005), "Some Issues in Using VARs for Macroeconometric Research", CAMA Working Paper #19.

Ireland, P. (2004), "A Method for Taking Models to the Data", *Journal of Economic Dynamics and Control,* 28, 1205-1226.

Johansen, S., (1995), *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*, Oxford University Press, Oxford.

Juselius, K. (2006), *The Cointegrated VAR Model* (Oxford University Press, Oxford)

Justiniano, A. and B. Preston (2006), "Can Structural Small Open Economy Models Account For the Influence of Foreign Disturbances", CAMA Working Paper 12/2006, *Centre for Applied Macroeconomic Analysis*,The Australian National University (http://cama.anu.edu.au).

King, R.G., C.I. Plosser, S.H. Stock, and M.W. Watson (1991), "Stochastic Trends and Economic Fluctuations", American Economic Review, 81, pp.

819-40.

Luktepohl, H. (2005), *New Introduction to Multiple Time Series*, Springer-Verlag, Berlin.

Pagan, A.R. and J. Robertson (1998), “Structural Models of the Liquidity Effect”, *Review of Economics and Statistics*, 80, 202-217.

Peersman, G. (2005), "What Caused the Early Millenium Slowdown? Evidence Based on Autoregressions", *Journal of Applied Econometrics*, 20, 185-207.

Pesaran, M.H. and Y. Shin (2002), "Long-Run Structural Modelling", *Econometric Reviews*, 21, 49-87.

Shapiro and Watson (1988), "Sources of Business Cycle Fluctuations", NBER Macroeconomics Annual, Vol. 3, 111-148.

Wickens, M.R. and R. Motto (2001), "Estimating Shocks and Impulse Response Functions", *Journal of Applied Econometrics*, 16, 371-387.