Regionalization and its Implications for Price Index Construction: The Case of the International Comparisons Program

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The International Comparisons Program (ICP) compares the purchasing power of currencies through a series of regional comparisons that are then linked together to obtain results at the global level. This process of regionalization complicates the construction of price indexes by essentially forcing the adoption of a two-stage approach. In previous rounds of ICP, data constraints have largely determined how the regions are linked. These data constraints, however, are largely absent in ICP 2005. This raises the question of how best to extend existing price index formulas to cope with the regionalization process that underpins ICP 2005. This paper attempts to answer this question, while at the same time imposing some structure on the existing literature on two-stage methods. (JEL: C43, E31, F01, O47)

Keywords: ICP; Linking Regions; Fixity; Purchasing Power Comparisons; Price Index; Chaining

Word Count: 8706
1. Introduction

Market exchange rates are often unsuitable currency conversion factors for comparisons of per capita income across countries for two reasons. First, exchange rates are typically volatile with short-term movements driven largely by speculative trading rather than changes in relative purchasing power. Second, exchange rate comparisons tend to systematically overestimate differences in per capita income across countries. This systematic bias can be explained either by the fact that nontraded services are more labour intensive in poorer labour abundant countries [see Bhagwati (1984)], or by productivity increases that have been focused predominantly on the tradable goods sector which have driven up wages in both sectors and hence prices in the nontraded service sector in richer countries [see Balassa (1964) and Samuelson (1964)]. Either way the implication is that nontraded services are cheaper in poorer countries, thus driving a wedge between market exchange rates and the actual relative purchasing power of currencies.

The alternative to market exchange rates is to compare the purchasing power of currencies directly by pricing representative baskets of goods and services in each country. These international price indexes are commonly referred to as purchasing power parities (PPPs).\(^1\)

The International Comparisons Program (ICP), which dates back to the late 1960s, was set up to calculate appropriate international price indexes that would allow meaningful comparisons of income levels across countries. The ICP has made benchmark comparisons between groups of countries in 1967, 1970, 1975, 1980, 1985, 1996 and now 2005. These benchmarks underpin the widely used Penn World Table [see Summers and Heston (1991)]. The initial comparison in 1967 involved only six countries. Since

\(^1\)Purchasing power parities should not be confused with the purchasing power parity hypothesis, which in its strong form asserts that (at least in the long run) market exchange rates do fully reflect differences in the purchasing power of currencies.
then the scope of the ICP has expanded significantly. Nevertheless, although affiliated with the United Nations and World Bank, it has traditionally been something of a ‘shoestring’ operation, and the staff involved have been forced to use a certain amount of ingenuity to piece together the results from limited means. The lack of funding has affected the quality of the results [see Heston and Aten (2002)].

This has all changed in the 2005 round. ICP 2005 has received substantial funding from a consortium of international, regional and national development agencies. It covers around 147 countries in all regions of the world. ICP 2005 is being run by the World Bank in collaboration with the UN, OECD and Eurostat.

Price indexes generated by ICP 2005 are based on regional surveys of prices plus a global survey for a selected group of countries. It is a complex operation requiring the cooperation and coordination of a large number of countries with varying abilities and statistical capacity. The resulting price indexes will allow more accurate comparisons of living standards, productivity and inequality across countries.\(^2\)

As a result of the sheer scale and complexity of the project, ICP 2005 has been broken up into two stages. The two stages are illustrated in Figure 1. In stage 1, the countries are divided into five regional blocks. The regions are Africa, Asia, Latin America, Western Asia, and Eurostat/OECD. Originally the intention was to have a sixth region representing the CIS countries. However, this sixth region was eventually abandoned and incorporated into the Eurostat/OECD region. Separate comparisons are then made for each block.

**Insert Figure 1 Here**

A total of 18 so-called ‘ring’ countries are participating in the stage 2 comparison. They are Cameroon, Egypt, Kenya, Senegal, South Africa, Hong Kong, China, Malaysia, The Philippines, Sri Lanka, Chile, Brazil, Jordan, Oman, Estonia, Japan,

\(^2\)See the ICP Handbook (2006) which can be downloaded at \(<\!http://www.worldbank.org/data/icp/\!>\).
Slovenia and the United Kingdom. That is, each of these countries participates both in one of the stage 1 regional comparisons and in the stage 2 ring comparison. By design, there are multiple ring countries from each region. The stage 2 ring comparison generates the links between the five regions.

This process of regionalization in stage 1 matches the UN’s own regional divisions of its Economic Commissions: UNECA (for Africa), ESCAP (for Asia), ECLAC (for Latin America), ECWA (for Western Asia) and ECE (for Europe) – although Eurostat/OECD has replaced ECE in the ICP. Each regional office has assumed complete responsibility for the comparisons in its region. The stage 2 ring comparison is coordinated by the ICP global office based at the World Bank in Washington.

Manageability, however, is not the only reason why ICP 2005 is broken into regional blocks. A second driving force behind regionalization is the desire for regional characteristicity. Most regions prefer to calculate multilateral price indexes between the countries of their own region that are not affected by prices or expenditures in other regions. In particular, they prefer to draw up lists of products for pricing that are representative of countries in their own region. In addition, a region may wish to go further and ensure that its within-region indexes are preserved within wider sets of results including ICP results at a world level. This is known as preserving ‘fixity’. A sub-group of countries within a region may also wish to have their own characteristic results. This is the case for the European Union where the price indexes have financial implications for the EU’s budget. The EU therefore requires that the price indexes between its member countries are based on prices and expenditures in EU countries only and since the 1980’s has insisted that these EU characteristic results remain unchanged at every level of aggregation within wider sets of official results such as those for the OECD, the ECE as a whole and the ICP’s global results.

Regionalization, however, does not fit comfortably with existing methods for constructing multilateral price indexes, since it requires a two stage procedure. The concept
of a two stage method is not completely new. The ICP has been using such methods since 1975 while the OECD has been since 1980. In our opinion, however, neither has achieved best practice.

In the analysis that follows, we will distinguish between two types of two-stage methods. The first type preserves fixity at all levels of aggregation, while the second type does so only at the basic heading level. Methods of the first type in the first stage make separate multilateral comparisons for each region and then in the second stage link the regional results together at an aggregate level (e.g. GDP). Two-stage methods of this type preserve fixity at every level of aggregation from the basic heading level up to GDP.\(^3\) Our focus for methods of the first type is on the way the between-region links at the aggregate level are constructed in stage 2.

For two-stage methods of the first type, the linking at the aggregate level in stage 2 is straightforward when there is only one ring country per region. The between-region links are obtained directly from a multilateral comparison over the ring countries. The methodological complexities arise when there are multiple ring countries per region. There are, however, good reasons for wanting two or more ring countries per region. First, the between-region links are not invariant to the choice of ring countries. The robustness of the results therefore are increased by using multiple ring countries per region. Second, the use of multiple ring countries per region provides insurance against the risk that a ring country’s data might not be usable for some reason. In principle there is no limit to the number of countries that might serve as ring countries per region, but for practical, organizational and financial reasons only a small fraction of the countries within each region can be expected to take on this additional burden.

\(^3\)A basic heading is the lowest aggregate for which expenditure data can be obtained. There are 152 basic headings in ICP 2005. A basic heading corresponds to an elementary aggregate in a temporal index such as a CPI. The price index for a basic heading or elementary aggregate has, by definition, to be an elementary price index that is calculated from price observations only and does not use quantities or expenditures. See Chapter 3 of the ICP Handbook and Chapter 20 of the ILO’s 2004 CPI Manual.
The second type of two-stage method requires that all regions use the same list of basic headings, and that this list is the same as the one used in the ring comparison. This is the case in ICP 2005. In the first stage, price indexes are computed in each region only to the basic heading level. In the second stage, the ring comparison is used to compute separate between-region price indexes for each basic heading. This can be done in a number of ways [see ICP Handbook (2006), chapter 14]. It is then possible to apply a multilateral method directly to the whole set of countries to obtain results at the aggregate level. The stage 2 comparison is universal in the sense that it treats all countries in the same way and ignores the regions to which the countries belong. It should be noted, however, that just because all regions use the same list of basic headings does not imply that they all use the same underlying product lists. For example, in ICP 2005, although the basic headings are common across regions, the underlying product lists for each basic heading differ markedly across regions. It is questionable, therefore, how meaningful will be the between-region results in this case. For methods of the second type, fixity is preserved only at the basic heading level.

The remainder of the paper describes a number of two-stage methods of the first type, considers their strengths and weaknesses, and attempts to reveal the underlying similarities and differences between them. The main objective is to impose some structure on the literature so as to allow users to better appreciate the options available and the resulting trade offs. The strengths and weaknesses of two-stage methods of the second type are also considered. We come down strongly in favour of methods of the first type.

2. Two-Stage Methods that Preserve Fixity at All Levels of Aggregation

(i) Hill’s two-stage EKS method

Two-stage methods of the first type preserve fixity. As noted above, methods of

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4This method was first proposed by Hill (2005) in an unpublished note circulated in August 2005.
this type in stage 1 make separate multilateral comparisons to the highest level of aggregation (e.g. GDP) for each region. The stage 1 comparison itself has two stages. First, price indexes must be computed for each basic heading. This can be done either using the country-representative-product-dummy (CPRD) method or the EKS-Eurostat method. All regions except the OECD are using the CPRD method in ICP 2005, while the OECD is using the EKS-Eurostat method. Second, the basic heading price indexes are used as inputs along with expenditure shares to calculate the price indexes at the aggregate level. A large number of multilateral methods have been proposed in the price index literature for making such calculations [see Hill (1997) for a survey]. The EKS-Fisher method [see Eltető-Kőves (1964) and Szulc (1964)] will be used by all regions in ICP 2005 except Africa, which will probably use the Iklé method [see Iklé (1972)]. The ICP has used Geary-Khamis [see Geary (1958) and Khamis (1972)] in previous rounds.

We begin here from the premise that in stage 1 the EKS method is used to compute the aggregate price indexes within each region. In stage 2 the objective is to link the results for the five regions together.

Price collection is carried out by countries, however, not regions. Between-region price indexes have to be estimated on the basis of samples of prices collected by individual countries. In ICP 2005, 18 countries are participating in the stage 2 ring comparison. The ring countries collect prices for the same list of products established for use at a world level. From a regional perspective, the prices in each ring country can be viewed as providing a sample of prices for the region to which it belongs.

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5The CPRD method is discussed in a later section. We do not consider the Eurostat-EKS method here. Both methods are explained in detail in chapter 11 of the ICP Handbook.

6The EKS transitivization formula [see equation (2)] can be applied to any intransitive price index. The EKS-Eurostat method applies it to Jevons indexes at the basic heading level. The EKS-Fisher method applies it to Fisher indexes above basic heading level. Henceforth, unless otherwise specified, EKS will refer here to the EKS-Fisher method.

7It should be noted that the ring countries are not required to price every product on the list. For those that are priced, the country is able to identify whether or not the product is representative.
The EKS formula is required when a binary approach to international comparisons is adopted. When a binary approach is used, the multilateral price indexes for a group of countries are derived from the binary indexes between every possible pair of countries in the group. The bilateral indexes are usually symmetric superlative indexes, such as Fisher indexes [see Diewert (1976)]. The Fisher index is the geometric mean of a Paasche and Laspeyres index. These indexes are defined as follows:

Paasche:  \[ P_{jk}^P = \frac{\sum_{i=1}^{I} p_{ki} q_{ki}}{\sum_{i=1}^{N} p_{ji} q_{ki}}, \]

Laspeyres:  \[ P_{jk}^L = \frac{\sum_{i=1}^{I} p_{ki} q_{ji}}{\sum_{i=1}^{I} p_{ji} q_{ji}}, \]

Fisher:  \[ P_{jk}^F = \sqrt{P_{jk}^P P_{jk}^L}, \]

where \( j \) and \( k \) denote the two countries being compared, \( p_{ji} \) and \( q_{ji} \) denote the price and quantity indexes for basic heading \( i \) in country \( j \), and \( i = 1, \ldots, I \) indexes the set of basic headings over which the comparison is made.

Bilateral indexes such as Paasche, Laspeyres and Fisher are generally not transitive (i.e., \( P_{jk}^F \times P_{kl}^F \neq P_{jl}^F \)), but transitivity can be imposed on them by using the EKS formula: hence the name ‘EKS method’. The binary approach is preferred by most of the ICP regions for the calculation of the aggregate indexes for the countries of their region.

Similarly, when a binary approach is adapted to regional indexes, bilateral price indexes have to be calculated between each pair of regions. Transitivity can then be imposed on the between-region indexes by means of the EKS formula. The additional complication is that each bilateral index is estimated from the prices of a sample of countries from each region, the ring countries.

Five regions are participating in the stage 2 ring comparison in ICP 2005. This means that a total of 10 bilateral comparisons between pairs of regions can be made. These can then be transitivized using the EKS formula. If there is only one ring country
in each region, in stage 2 we can simply make an EKS comparison between the ring countries. This stage 2 EKS comparison provides the required transitive links between the five regions, which can then be used to generate the global results.

As noted above, however, this is not the situation faced in ICP 2005. To reduce the sensitivity of the results to the choice of ring countries, it is an explicit objective of ICP 2005 to ensure that there are at least two ring countries in each region.

The construction of the 10 between-region bilateral price indexes is more complicated when each region has two or more ring countries. For example, consider the case of a bilateral comparison between the Eurostat/OECD and CIS regions. Eurostat/OECD has four ring countries, while the CIS was supposed to have two.\(^8\) If we are to treat all ring countries in a region symmetrically (another requirement of the ICP) we must use all possible bilateral comparisons between Eurostat/OECD and CIS ring countries. This is also very much in the spirit of EKS which uses bilateral comparisons between all possible pairings of countries. Given that there are four Eurostat/OECD ring countries and two CIS ring countries, there are a total of eight such comparisons, which are listed below. The situation is illustrated in Figure 2.

1. UK-Russia
2. UK-Ukraine
3. Estonia-Russia
4. Estonia-Ukraine
5. Japan-Russia

\(^8\)Originally it was intended that six regions would be represented in the ring comparison. Since constructing this example, the CIS region has dropped out of the ring comparison. In ICP 2005, CIS countries will be linked with countries in the joint Eurostat/OECD program through Russia which has participated in both the OECD and the CIS programs. However, as actual data for OECD and CIS countries are available for 1996, for illustrative purposes we will nevertheless consider here how a two-stage EKS approach can be used to construct a between-region link for the Eurostat/OECD and CIS regions.
6. Japan-Ukraine
7. Slovenia-Russia
8. Slovenia-Ukraine

**Insert Figure 2 Here**

The method proceeds by simply averaging the results obtained from these eight comparisons. Before they can be combined, however, they must all be expressed in units of the same pair of currencies. To do this it is necessary to select a base ring country for each region. If the method is implemented correctly, the choice of base ring countries will have no impact on the results. Suppose we select Russia and the UK as the region bases. The objective then is to convert all eight bilateral comparisons into Pounds versus Roubles. Comparison 1 is already in Pounds-Roubles and so does not require any translation. The conversion to Pounds-Roubles is achieved for comparison 2 by chaining together comparisons between UK-Ukraine and Ukraine-Russia. The method will be base country invariant as long as the formula used to make the between region comparison (in this case UK-Ukraine) satisfies the country reversal test (e.g., Fisher) and the formula used to make the within region comparison (Russia-Ukraine) is transitive (e.g., EKS). For the special case of a region such as CIS with only two ring countries, the within region EKS comparison reduces to a Fisher price index. To convert comparison 3 to Pounds-Roubles we require the following chain: UK-Estonia × Estonia-Russia. The within region Eurostat/OECD comparison (now UK-Estonia) is made using EKS, while the between region comparison is made using Fisher. It should be noted that the EKS comparison here is made over only the ring countries in the Eurostat/OECD region. In a similar manner comparison 4 requires the following chain: UK-Estonia × Estonia-Ukraine × Ukraine-Russia. Comparisons 5 to 8 proceed in an

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9This is also in the spirit of EKS. Hill (1997) shows how an EKS comparison between \( N \) countries can be expressed as the geometric mean of \( N \) star comparisons with each country in turn placed at the centre of the star.
analogous manner.

Using this approach, we obtain eight different estimates of the relative purchasing power of the Pound and Rouble. Taking the geometric mean of these eight estimates we obtain our bilateral parity between the Eurostat/OECD and CIS regions, using UK-Russia as the link. Again it must be emphasized that the choice of base countries will not affect the results. For example, in the Eurostat/OECD region the UK is treated symmetrically with the other three ring countries. If instead Japan was used as the base, this would have no impact on the results except that the bilateral link between Eurostat/OECD and CIS would now be expressed in Yen-Roubles.

Once the bilateral between-region parities have been calculated, they are then transitivized using the EKS formula. If there was only one ring country in each region, then we would jump straight to this step. Finally, these transitivized between-region parities are combined with the within-region parities to generate the overall global parities.

To repeat, the important point to realize is that all within-region comparisons in the chain are made using EKS, while between-region comparisons are made using Fisher. Algebraically, the untransitivized bilateral between-region price indexes are calculated as follows:

\[
P_{a_1,b_1} = \prod_{m=1}^{M} \prod_{n=1}^{N} (P_{EKS}^{a_1,am} \times P_{F}^{am,bn} \times P_{EKS}^{bn,b_1})^{1/(M \times N)},
\]

where \( A \) and \( B \) denote the two regions, \( a_1 \) the base country in region \( A \), \( b_1 \) the base country in region \( B \), \( M \) the total number of ring countries in region \( A \), \( N \) the total number of ring countries in region \( B \), \( P_{EKS}^{a_1,am} \) an EKS price index calculated over the ring countries of region \( A \), \( P_{EKS}^{b_1,bl_1} \) an EKS price index calculated over the ring countries of region \( B \), and \( P_{F}^{am,bn} \) a Fisher price index between ring country \( m \) in region \( A \) and ring country \( n \) in region \( B \). A transitivized EKS price index between regions \( A \) and \( B \), for the case of five regions, is calculated as follows:

\[
P_{EKS}^{a_1,b_1} = \left( \frac{P_{a_1,a_1} \times P_{a_1,b_1} \times P_{a_1,c_1} \times P_{a_1,d_1} \times P_{a_1,e_1}}{P_{b_1,a_1} \times P_{b_1,b_1} \times P_{b_1,c_1} \times P_{b_1,d_1} \times P_{b_1,e_1}} \right)^{1/5},
\]

10
where $A, B, C, D, E$ denote the five regions, and $a1, b1, c1, d1, e1$ the base ring countries for each region.

**An empirical illustration**

Consider a specific pair of regions such as the OECD and the CIS. These two regions are selected because illustrative data are available for them for 1996 from a previous ICP round. In our example, the four ring countries from the OECD are the actual ICP 2005 ring countries – UK, Estonia, Japan and Slovenia – while Russia and the Ukraine are selected here as hypothetical CIS ring countries.

The eight bilateral Fisher price indexes for GDP between the OECD ring countries and CIS ring countries, with the OECD country serving as the base, are as follows:

1. UK-Russia = 3797.13
2. UK-Ukraine = 0.777280
3. Estonia-Russia = 412.428
4. Estonia-Ukraine = 0.0900049
5. Japan-Russia = 24.0126
6. Japan-Ukraine = 0.00508676
7. Slovenia-Russia = 14.7657
8. Slovenia-Ukraine = 0.00313334

Within the OECD group the country price indexes for the OECD ring countries are EKS price indexes. With the UK as the base, they are as follows:

- UK = 1
- Estonia = 8.12102
- Japan = 256.430
- Slovenia = 154.830

Finally, within the CIS region, as there are only two countries, the EKS formula is not needed. The Fisher price index for the Ukraine with Russia as the base is as follows:
Russia-Ukraine = 0.000232245.

The bilateral price index between the two regions is to be expressed in terms of UK pounds and Russian roubles, the currencies of the two selected base countries. The following eight estimates can be made, by pairing the four OECD ring countries with the two CIS ring countries. The indexes between ring countries in different regions are shown in bold.

\begin{align*}
\text{UK-Russia} & \quad = 3797.13 \\
\text{UK-Ukraine} \times \text{Ukraine-Russia} & \quad = 3346.81 \\
\text{UK-Estonia} \times \text{Estonia-Russia} & \quad = 3349.34 \\
\text{UK-Estonia} \times \text{Estonia-Ukraine} \times \text{Ukraine-Russia} & \quad = 3147.25 \\
\text{UK-Japan} \times \text{Japan-Russia} & \quad = 3786.37 \\
\text{UK-Japan} \times \text{Japan-Ukraine} \times \text{Ukraine-Russia} & \quad = 3459.63 \\
\text{UK-Slovenia} \times \text{Slovenia-Russia} & \quad = 3717.87 \\
\text{UK-Slovenia} \times \text{Slovenia-Ukraine} \times \text{Ukraine-Russia} & \quad = 3391.16 \\
\end{align*}

Each country should be viewed not so much as a separate entity but as providing a sample of prices for part of the region to which it belongs. The index between the UK and Russia obviously provides one estimate of the between-region price index. However, the index between, say, Estonia and Russia provides an equally valid estimate once the prices in Estonia have been converted into pounds by multiplying them by the index for Estonia based on the UK. The comparison between Estonia and Ukraine provides a further estimate, but only after the prices in both Estonia and the Ukraine have been converted into pounds and roubles respectively. And so on.

The objective is not to obtain the best possible estimate of the price index between the UK and Russia. If that were the objective, the bilateral index between UK and Russia, being fully characteristic of the two countries, could be regarded as the best estimate on its own. But viewing the UK and Russia as just two parts of the OECD and CIS regions respectively, the bilateral index between them provides an estimate of the
bilateral index between the two regions as a whole that is based on a very limited range of price information. By comparing prices between other parts of the two regions, such as Estonia and the Ukraine, or Japan and Russia, additional information is gained about prices in the two regions which can be used to improve the estimates. Increasing the number of ring countries in each region will obviously improve the estimate of the price index between the two regions and increase its reliability, just as in general increasing the size of a sample reduces errors of estimation.

Our preferred bilateral index between the OECD region and the CIS is obtained by taking the geometric mean of the eight individual estimates, as shown in equation (1). This ensures that the ring countries in each region are treated symmetrically and that no information is wasted. Using the UK and Russia as the base countries for each region, this yields the following between-region price index:

\[ \text{UK-Russia} = 3492.28. \]

The between-region price indexes are not themselves transitive. For example, suppose there are five regions \( A, B, C, D \) and \( E \). In addition to comparing regions \( A \) and \( B \) directly, they can also be compared indirectly via regions \( C, D \) and \( E \). Each path will generate a different answer. The between-region price indexes can be transitivized by taking a geometric mean of the results obtained from each of these paths as shown in equation (2).

The resulting between-region EKS price indexes can then be used to link the various sets of country EKS indexes calculated by the different regions. A complete global set of aggregate price indexes is obtained. At the global level, the EKS price index between country \( a \) in region \( A \) and country \( b \) in region \( B \) becomes a chain index of the following form:

\[ P_{EKS}^{a,b} = P_{EKS}^{a,a_1} \times P_{EKS}^{a_1,b_1} \times P_{EKS}^{b_1,b}, \]

where countries \( a_1 \) and \( b_1 \) are the base countries whose currencies serve as the regional currencies. The first and third terms on the right are the within-region EKS price
indexes between countries a and b and their respective base countries, while the middle term $P_{a1,b1}^{EKS}$ is the aggregate EKS between-region price index denominated in the currencies of the two base countries.

**Base country invariance**

The results are invariant to the selection of the base countries and currencies. For example, for the case of the comparison involving four OECD countries and two CIS considered above, there are eight possible bilateral regional indexes that might be used to link the two regions.

1. UK-Russia = 3492.28
2. UK-Ukraine = 0.811964
3. Estonia-Russia = 430.030
4. Estonia-Ukraine = 0.0998722
5. Japan-Russia = 22.5557
6. Japan-Ukraine = 0.00523844
7. Slovenia-Russia = 13.6188
8. Slovenia-Ukraine = 0.00316291

Each of these indexes is calculated using equation (1), with $a1$ and $b1$ appropriately defined. That is, each index is a between-region index. If UK and Russia are selected as the base countries for each region, respectively, then the following set of indexes are obtained (with UK=1):

UK = 1
Estonia = 8.12102
Japan = 256.430
Slovenia = 154.830
Russia = 3492.28
Ukraine = 0.811064

Alternatively, suppose Japan and Ukraine are selected as the base countries for each
The following indexes are obtained (with Ukraine = 1):

- UK = 1.23295
- Estonia = 10.0128
- Japan = 316.165
- Slovenia = 190.897
- Russia = 4305.80
- Ukraine = 1

The two sets of results differ only by a scalar of proportionality (1.23295). That is, the choice of base countries for the regions is a matter of convenience.

**Numbers of ring countries**

The minimum number of ring countries is one per region. In this case, the between-region indexes would reduce to the set of Fisher indexes between the various ring countries. As the Fishers are not transitive, the EKS formula would still be required in order to obtain a transitive set of between-region indexes. The method of linking is different from, and an improvement on, a situation in which pairs of regions are linked by selecting individual countries to act as bridge countries (as was the case in some earlier version of the ICP). When the ring method is used price indexes are calculated between each ring country and each of the other ring countries whereas a bridge country is used once only to link a specific pair of regions.

As the objective is to estimate between-region indexes, however, samples of one country per region are clearly insufficient. The between-region indexes are too sensitive to the choice of ring countries. Even with as few as three ring countries per region, the situation is vastly improved as each between-region index becomes an average of nine separate price indexes between the ring countries in the two regions. The results must be far more robust than estimates based on only a single index. Given the costs of conducting the ring program, an average of three ring countries per region would seem to be appropriate and adequate.
The issue of weighting in each between-region price index

Sergeev (2006) has criticized the two-stage EKS method as outlined in Hill (2005) on the grounds that it does not treat regions symmetrically in (1) when they have a differing number of ring countries. He argues that the region with the higher number of ring countries has a greater impact on the resulting between-region price index.

We argue below that Sergeev’s concern is unfounded. Nevertheless, he raises an interesting issue that warrants closer examination. Moreover, we show in the next section that his concern is of relevance to the two-stage Geary-Khamis method.

Consider again the example of a comparison between the Eurostat/OECD and CIS regions. Denoting the four ring countries in the Eurostat/OECD region by \(a_1, a_2, a_3, a_4\) and the two ring countries in the CIS region by \(b_1, b_2\), from (1) it can be seen that the untransitivized bilateral price index between regions \(A\) and \(B\) is calculated by taking a geometric mean of the following eight components:

\[
\begin{align*}
    p_{EKS}^{a_1,a_1} \times p_{F}^{a_1,b_1} \times p_{EKS}^{b_1,b_1} \\
    p_{EKS}^{a_1,a_2} \times p_{F}^{a_2,b_1} \times p_{EKS}^{b_1,b_1} \\
    p_{EKS}^{a_1,a_3} \times p_{F}^{a_3,b_1} \times p_{EKS}^{b_1,b_1} \\
    p_{EKS}^{a_1,a_4} \times p_{F}^{a_4,b_1} \times p_{EKS}^{b_1,b_1} \\
    p_{EKS}^{a_2,a_1} \times p_{F}^{a_1,b_2} \times p_{EKS}^{b_2,b_2} \\
    p_{EKS}^{a_2,a_2} \times p_{F}^{a_2,b_2} \times p_{EKS}^{b_2,b_2} \\
    p_{EKS}^{a_2,a_3} \times p_{F}^{a_3,b_2} \times p_{EKS}^{b_2,b_2} \\
    p_{EKS}^{a_2,a_4} \times p_{F}^{a_4,b_2} \times p_{EKS}^{b_2,b_2}
\end{align*}
\]

Suppose now that the base country in region \(A\) is country \(a_1\), while the base country in region \(B\) is \(b_1\). The eight components of the untransitivized price index between regions \(A\) and \(B\) can now be simplified to the following:

\[
\begin{align*}
    p_{F}^{a_1,b_1} \\
    p_{EKS}^{a_1,a_2} \times p_{F}^{a_2,b_1}
\end{align*}
\]
Sergeev’s concern arises from the fact that six within-region-\( A \) EKS indexes will appear in (1) as compared with only four within-region-\( B \) EKS indexes. Does this imply that region \( A \) exerts greater influence than region \( B \) on the resulting between-region price index? The answer is no. Region \( A \) seems to exert more influence in (4) only because more of the region \( B \) EKS indexes in (3) equal one and hence drop out of (4).

\( (ii) \) A Geary-Khamis version of Hill’s two-stage method

Now we begin from the premise that in stage 1 the Geary-Khamis method is used to compute the aggregate price indexes within each region.\(^{10}\) Our problem is to determine how best to link the regions together, given that Geary-Khamis has been used in stage 1. Again, the stage 2 comparison is straightforward if there is only one ring country in each region. In this case, in stage 2 we can simply make a Geary-Khamis comparison between the ring countries. This stage 2 comparison provides the required transitive links between the five regions, which can then be used to generate the global results.

The two-stage Geary-Khamis method also becomes more complicated when each region has two or more ring countries. A Geary-Khamis comparison between all of the ring countries will generate transitive price indexes for the ring countries, which can then be used to link the regional results together. The problem now is that the global results will depend on which ring countries are used to link the regions together. For

\(^{10}\)Iklé can be substituted for Geary-Khamis if desired.
example, suppose $j$ and $k$ denote two ring countries which may belong to the same or differing regions. The Geary-Khamis price index for country $k$ relative to country $j$ in the ring comparison is calculated as follows:

$$P^G_{jk} = \frac{\sum_{i=1}^{I} P_{ki}q_{ki}}{\sum_{i=1}^{I} P_{X_i}q_{ki}} \frac{\sum_{i=1}^{I} P_{X_i}q_{ji}}{\sum_{i=1}^{I} P_{ji}q_{ji}} = \frac{P^P_{X_k}}{P^P_{X_j}},$$

where

$$P_{X_i} = \sum_{n=1}^{N} \left( \frac{q_{ni}}{\sum_{m=1}^{N} q_{mi}} \frac{p_{ni}}{P^P_{X_n}} \right).$$

Here $i = 1, \ldots, I$ indexes the list of basic headings over which the ring country comparison is calculated, $n = 1, \ldots, N$ indexes the set of ring countries, $p_{ni}$ and $q_{ni}$ denote the price and quantity indexes for basic heading $i$ in country $n$, $p_{X_i}$ denotes the Geary-Khamis average price for basic heading $i$, and $P^P_{X_j}$ denotes a Paasche price index between the artificial average country $X$ and country $j$.

Let $a^*$ denote the artificial average country in region $A$ in the stage 1 Geary-Khamis comparison, and $X$ the artificial average country in the stage 2 Geary-Khamis ring comparison. The price index for country $a$ in region $A$ (where $a$ is not a ring country) relative to the artificial average ring country $X$ in the overall world comparison is calculated as follows:

$$P_{aX} = P^G_{aa^*} \times P_{a^*X}. $$

The problem is that the value of $P_{a^*X}$ depends on the choice of ring country in region $A$. Suppose there are two ring countries in region $A$, denoted by $a_1$ and $a_2$. This situation is depicted in Figure 3. We have two star spanning trees, each with an artificial average country in the middle.\(^{11}\) These stars represent the stage 1 Geary-Khamis comparison for region $A$ and the stage 2 ring comparison, respectively. The problem is to decide how the two stars should be linked together. Here they can be linked either using country

\(^{11}\)A spanning tree connects the vertices (here countries) in such a way that there is one and only one path between any pair of vertices.
If $a_1$ is used as the base (i.e., link country), we obtain that
\[ P_{a_1}^{a_1} X = P_{a_1}^{GK} \times P_{a_1}^{GK} = \frac{P_{a_1}^P}{P_{Xa_1}^P}. \]
Conversely, if $a_2$ is used as the base, then
\[ P_{a_2}^{a_2} X = P_{a_2}^{GK} \times P_{a_2}^{GK} = \frac{P_{a_2}^P}{P_{Xa_2}^P}. \]
In general, it will not be the case that $P_{a_1}^{a_1} X = P_{a_2}^{a_2} X$. This problem can be resolved by taking a geometric mean. That is, we can define $P_{a}^{X}$ as follows:
\[ P_{a}^{X} = \left( P_{a_1}^{a_1} X \times P_{a_2}^{a_2} X \right)^{1/2}. \]
This ensures that both ring countries in region $A$ are treated symmetrically when linking the two stars together. The method generalizes in a straightforward manner to regions in which there are more than two ring countries. One problem with this method is that taking geometric means is a technique borrowed from EKS type methods, and sits somewhat uncomfortably in a Geary-Khamis setting.

\textbf{Insert Figure 3 Here}

A second problem with this method is that it falls foul of the criticism raised by Sergeev above. If some regions have more ring countries than others, they will exert more influence on the stage 2 ring comparison Geary-Khamis average price vector $p_X$. At the same time, as is well known, regions with higher incomes will also exert greater influence on the Geary-Khamis average price vector. The latter problem can be dealt with by using the Iklé (1972) method. The Iklé method, however, will still be subject to Sergeev’s criticism. Furthermore, irrespective of how the average price vector is constructed, the results will be distorted by substitution bias [see Dowrick and Quiggin (1997), Hill (2000) and Neary (2004)]. Depending on the amount of heterogeneity in the sample of countries, the magnitude of the distortion can be large.

\textit{(iii) Diewert’s two-stage method}
Even though each region has its own product list at the elementary level, all regions end up with the same list of 152 basic headings [see ICP Handbook (2006), chapter 3]. The basic heading price indexes are constructed using the country-product-representative dummy (CPRD) method (see ICP Handbook, chapter 14).

In the ring comparison, Diewert (2004) suggests calculating price indexes directly at the regional level. To do this, we require a price vector and expenditure vector for each region. Diewert shows how price vectors can be constructed by modifying the CPRD method so that it generates basic heading price indexes at the regional rather than country level. This is achieved by first selecting a base country for each region, and then converting all ring country elementary prices for each region into units of the base country’s currency using the between-region price indexes obtained from stage 1. Now a CPRD regression is run on the ring countries with the modification that the dummies for each country are replaced with dummies for each region. As is pointed out in chapter 14 of the ICP Handbook, this method should more accurately be referred to as the region-product-representative dummy (RPRD) method. The estimated coefficients on the region dummies when exponentiated provide between-region price indexes for each basic heading.

More precisely, the CPRD regression takes the following form:

$$\ln p_{ikr} = \ln \kappa + y_{2i} \ln \beta_2 + y_{3i} \ln \beta_3 + \cdots + y_{Ii} \ln \beta_I + z_{2k} \ln \gamma_2 + v_{2r} \delta_2 + \cdots + v_{5r} \delta_5 + \varepsilon_{ikr},$$  

where $y_{ji}$, $z_{2k}$ and $v_{sr}$ are dummy variables such that $y_{ji} = 1$ if $j = i$ and 0 otherwise, $z_{2k} = 1$ if $k = 2$ and 0 otherwise, $v_{sr} = 1$ if $s = r$ and 0 otherwise, $i$ indexes the elementary products, $k$ whether the product is representative or not for that country, and $r$ the region from which that country is drawn. The regression is run over the 18 ring countries for all the elementary products within a particular basic heading [see Diewert (2004) and chapter 14 of the ICP Handbook (2006)]. The RPRD method converts the prices on the left hand side of (3) into units of the base currency for each region using the within-region price indexes prior to the estimation of the regression equation.
The regional expenditure vectors are more straightforward to compute. First, the expenditure vectors of countries in the same region are converted into units of the base country’s currency, using the stage 1 between-region price indexes. These expenditure vectors are then simply summed to obtain the overall expenditure vector. This procedure can be limited to the ring countries of each region, or it can be applied to the whole set of countries within a region. Diewert (2004) favours the latter approach.

Once the regional price and expenditure vectors have been constructed, any multilateral method can be used to complete the stage 2 comparison, although as a matter of consistency it is recommended that the same multilateral method is used at both stages.

This is the method that will be used in ICP 2005 to link the regions. One potential problem with it is that it may be vulnerable to a modified version of Sergeev’s (2006) critique. Sergeev was concerned that all regions be given equal weight in the stage 2 ring country comparison. In the context of Diewert’s two-stage method, while this concern is clearly relevant when the Geary-Khamis method is used in stage 2 (for reasons discussed in the previous section), a modified version of it even applies when EKS is used in stage 2. The concern here is with the way the expenditure vectors are constructed for each region. Countries with larger expenditure vectors will exert greater influence on the resulting regional expenditure vector. Therefore, even though an EKS comparison in stage 1 treats all countries within a region symmetrically, the stage 2 comparison will not do so.

More generally, this approach of aggregating countries into regions fits naturally with the ethos of the Geary-Khamis and other average price methods, which themselves make use of average artificial countries. The idea of summing expenditures across ring countries in a particular region to construct regional level aggregates, however, sits somewhat uncomfortably with the ethos of the EKS method.

(iv) The OECD method
Regionalization has been forced on the OECD in its internal comparisons by Eurostat’s requirement of fixity in the results for the European Union. This essentially requires the OECD, in the context of its comparisons, to split itself into two regions: the EU, and the OECD excluding the EU. In addition, the EU comparison is made using the EKS method. The OECD has therefore had to devise a way of computing the results for the non-EU region, and then for linking the results of the two regions. This is a special case of the problem posed above. The OECD solution is to make an EKS comparison over the whole OECD block (stage 2), and then scale up or down the results for the whole block so that the total GDP of the EU countries is the same in both the restricted EU comparison (stage 1) and the broader comparison. The results for the EU countries from the broader comparison are then discarded, being replaced by the results from the restricted EU comparison. More precisely, let the EU be denoted by $X$ and the rest of the OECD by $Y$. The scalar adjustment $S_Y$ for region $Y$ is calculated as follows [see Koechlin and Schreyer (2005) and chapter 15 of the ICP Handbook (2006)]:

$$S_Y = \frac{\sum_{j=1}^{K_X} GDP_j / P_{1j}^{X+Y}}{\sum_{j=1}^{K_X} GDP_j / P_{1j}^{X}},$$

where $GDP_j$ denotes the GDP of country $j$ measured in units of domestic currency, $P_{1j}^{X}$ denotes the multilateral price index of country $j$ calculated over the countries in region $X$ with country 1 as the base, $P_{1j}^{X+Y}$ denotes the multilateral price index of country $j$ calculated over the countries in both regions $X$ and $Y$, again with country 1 as the base, $GDP_j / P_{1j}^{X}$ and $GDP_j / P_{1j}^{X+Y}$ denote the GDP of country $j$ measured in units of the currency of country 1, and $K_X$ the number of countries in region $X$. The overall GDP estimate, with fixity imposed on the results for region $X$, for country $j$ in units of the currency of country 1 (where country 1 lies in region $X$) are calculated as follows:

$$\frac{GDP_j}{P_{1j}^{X}}$$ for $j \in X$,  

$$\frac{1}{S_Y} \frac{GDP_j}{P_{1j}^{X+Y}}$$ for $j \in Y$.

This method can be extended to three or more regions. Indeed, in recent years, the OECD has had to do exactly this as a result of its closer links with the CIS countries.
In the new Eurostat/OECD model, the first region is the EU. The second region is the rest of the OECD. The third region is the former CIS countries and other non-OECD countries for which the OECD now gathers data. The method requires a nested approach. First, the results for the EU are computed. Second the results for the rest of the OECD are computed, holding the EU results fixed. Finally, the results for the non-OECD countries are computed holding the results for the OECD countries fixed. Fixity, therefore, is imposed in a nested sequence. This means that fixity can only be imposed for one of the three original regions.

This hierarchical multi-stage methodology, while it solves the problem posed, is itself problematic in that it does not treat regions symmetrically. In particular, this lack of symmetry makes it inappropriate for use in stage 2 of ICP 2005 both because of its arbitrariness, in terms of the ordering of the regions, and because it would contradict the ICP ethos of equal and symmetric treatment of regions.

The two-stage EKS method described above in section 2(i), however, could easily be applied to the Eurostat/OECD context. In this case every country in the data set would effectively be a ring country. Separate EKS comparisons would be made for the three regions. The results for these three regions would then be linked using the between-region price indexes as defined in (1) above. The bilateral between-region price indexes are then transitivized using the EKS formula. This method would impose fixity for all three regions, while treating all countries within each region symmetrically and all regions symmetrically.

3. Two-Stage Methods that Preserve Fixity Only at the Basic Heading Level

It is important to realize that regionalization does not necessarily imply fixity above basic heading level. In fact, this approach has been a feature of previous ICP comparisons. In the context of ICP 2005, since all regions are pricing the same lists of basic
headings (although from different underlying elementary product lists) it is possible in stage 2 to make a universal comparison using say the EKS or Geary-Khamis methods [see chapter 14 of the ICP Handbook (2006)]. To do this, it is first necessary to link the regions at the basic heading level. This can be done by running a CPRD regression over the ring countries for each basic heading. The price index for basic heading \( n \) for country \( a \) in region \( A \) relative to country \( b \) in region \( B \) is obtained as follows:

\[
P_{ab}^n = P_{\hat{a}a}^n \times P_{\hat{b}b}^n \times P_{\hat{a}\hat{b}}^n,
\]

where \( \hat{a} \) and \( \hat{b} \) denote the reference ring countries of regions \( A \) and \( B \), \( P_{\hat{a}a}^n \) and \( P_{\hat{b}b}^n \) are obtained from within-region CPRD regressions for regions \( A \) and \( B \), and \( P_{\hat{a}\hat{b}}^n \) is obtained from the ring country CPRD regression. Using this approach, a complete matrix of comparable basic heading prices at the global level is obtained. Once combined with a corresponding expenditure matrix, standard multilateral methods can then be applied at the global level.

This method is a slight variant on the method used in Phase III (1975) of the ICP [see Kravis, Heston and Summers (1982)]. They began by running a separate country-product-dummy (CPD) regression [see Summers (1973)] for each basic heading in each region. These were used to fill in the gaps in the price matrices. The regions were then combined and CPD regressions estimated again for each basic heading. In the second stage they ran a global Geary-Khamis comparison. The global data set in this case consisted of 34 countries.

The methods used in Phase IV (1980), Phase V (1985), and the 1993-1996 comparisons are not so well documented. In phase IV, it seems that CPD regressions were run on the basic headings for 20 core countries [see United Nations (1987) and Heston and Aten (2002)]. An additional 40 countries were then linked to these 20 core countries. It is not clear exactly how this linking was done. Finally, a global Geary-Khamis comparison was run on the set of 60 countries. Phase V was supposed to make use of selected high-quality bilateral comparisons between pairs of countries in different regions. These
comparisons would then be used to link the regions together. Many of these bilateral comparisons never eventuated. In the end, the OECD and Asia regions were linked through Japan (which participated in both regional comparisons). Hungary, Poland and Yugoslavia were linked to the OECD through Austria, while Africa was linked to the OECD through a bilateral comparison between the United Kingdom and Kenya. Latin America was supposed to be linked through a bilateral comparison between Germany and Argentina. This did not happen, and hence Latin America was excluded. The Caribbean was supposed to be linked through Africa. This also did not happen, but the Caribbean countries were still included via an older bilateral link through Jamaica [see United Nations (1994) and Heston and Aten (2002)]. Overall the comparison included 65 countries. Again, in the second stage a global Geary-Khamis comparison was made. ICP 1993-1996 was also patched together from regional comparisons, in some cases made in different years. Overall it did achieve an impressive participation rate of 116 countries, although at a less fine level of detail than in previous benchmarks. Again, the ICP had to adapt to the realities on the ground [see Heston, Summers and Aten (2001) for further details]. In this sense, ICP 2005 is a major break from the past, in that the comparison has been structured specifically to meet the needs of ICP.

There are three problems with this general approach of linking at the basic heading level and then making universal comparisons at higher levels of aggregation. First, it leads to a violation of fixity – something most of the regions now want – in the results for each region. Second, a global Geary-Khamis or Iklé comparison could be seriously distorted by substitution bias [see Dowrick and Quiggin (1997), Hill (2000) and Neary (2004)]. Third, although each region in ICP 2005 uses the same list of basic headings, the underlying product lists for each basic heading in most instances differ very significantly from one region to the next. Most of the bilateral comparisons that form the building blocks of a universal EKS comparison will be between countries in different regions. These bilateralts will be of dubious quality given that the underlying
product lists priced by these countries could have very little overlap. This mismatch problem is masked but not eradicated by linking regions at the basic heading level using CPRD regressions run over the ring countries.

For these reasons, we recommend applying standard multilateral methods (e.g., Geary-Khamis and EKS) only to countries that are pricing the same elementary product lists, as is the case in within-region comparisons or in a ring comparison.

4. Conclusion

Reliable purchasing power based currency conversion factors are important for international comparisons of income, consumption, productivity and inequality. The ICP was set up in the late 1960s to compute these currency conversion factors. Over time, the scale of ICP activities has increased dramatically. In particular, the sheer scale of ICP 2005 requires it to be broken up into more manageable regional blocks.

This process of regionalization has added a new dimension to the price index problem. We have considered here a few ways in which multilateral price index methods can be modified to take account of regionalization. Invariably, regionalization requires a two-stage approach to price index construction. We have distinguished between two types of two-stage methods. The first type makes separate multilateral comparisons for each region, and then in the second stage links the regions together at an aggregated level (e.g. at the level of GDP). The second type links regions at the basic heading level, and then in the second stage proceeds to make a universal multilateral comparison. This is the approach that has been traditionally favoured by the ICP, although it is not being used in ICP 2005. In the previous section we provide a number of reasons why we do not think this is the best way to proceed. We prefer two-stage methods of the first type.

Examples of two-stage methods of the first type include those proposed by Diewert (2004), Hill (2005) and the method used by the OECD. We believe that Hill’s method
provides the most natural generalization of the EKS method to a two-stage setting, while Diewert’s method provides the most natural generalization of Geary-Khamis (and other average price methods such as Iklé). Both methods warrant consideration in future rounds of ICP. The OECD method, by contrast, is inappropriate for use in an ICP context because of its failure to treat regions symmetrically or impose fixity for all but one of the original regions.

References


Figure 1. — The two-stage structure of ICP 2005
Figure 2. — Linking regions via bilateral comparisons between pairs of countries

Figure 3. — Linking regional and ring country Geary-Khamis comparisons