ECONOMIC GROWTH AND THE RETURN TO CAPITAL IN DEVELOPING ECONOMIES.

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An important stylized fact of economic growth is that the rate of return to capital is relatively constant across countries and time. This paper gives an explanation for this using a model of growth in a developing economy that has dualistic structure. Three conditions are derived, each of which may be responsible for the observed stability of the return to capital. The results address Lucas’ (1990) criticism of conventional growth models and supports Young’s (1995) hypothesis of growth in the East Asian economies.

Keywords: growth, development, endogenous growth, dual economy, convergence.

JEL Classification; O0, O1, O3, D9
1. Introduction

A major debate exists in the economic growth literature over how to account for the mutual existence of large differences in per capita incomes and small differences in returns to capital. In particular, a neoclassical production function, with conventional factor shares, implies unrealistically high differences in returns to capital across countries, (Lucas 1990). There have been two main responses to this finding. Lucas (1988, 1990) suggests that the focus of inquiry should shift from theories of accumulation to theories of growth based on economies of scale and productivity improvements as in endogenous growth models, (see also King and Rebelo 1993, and Romer 1986).\textsuperscript{1} An alternative response has been to argue that the conventional treatment of capital ignores the role of human capital, and thus overstates differences in returns to capital for different income levels (Mankiw 1995, Barro and Sala-i-Martin 1995).

In this paper I propose an alternative explanation for the constancy of the return to capital. Changes in the return to capital may be small because of a decline in the use of traditional methods of production in a dual economy. To show this I derive a model of a dual economy, where the sectoral division reflects different methods of production – traditional and modern. It is shown that changes in the return to capital are relatively small if either: (i) traditional production is relatively labour intensive or; (ii) traditional assets and labour are relative substitutes. Further I also consider the effect of a productivity gap between the traditional and modern sectors. It is show that this also reduces the extent to which large changes in the return to capital is required to account for changes in income levels.

\textsuperscript{1} For example, King and Rebelo (1993) argue that to explain the Japanese economic miracle in the traditional growth model requires a marginal product of capital of 500% in 1950.
The results provide theoretical support for the growth accounting studies of the East Asian economies by Young (1995, 1994), Kim and Lau (1993), and Pilat (1994). These studies have emphasized the importance of accumulation and structural change and the relative unimportance of technological change. The dual economy also demonstrates an important link between standard and endogenous growth theory in developing economies. In the early stages of development, growth in the modern sector behaves like a ‘AK’ endogenous growth model. In the long run however, the economy reverts to the standard Ramsey model.

The paper is organized as follows. The dual economy model is described in Section 2. In Section 3 I show how labour migration affects the growth of the modern sector and the return to capital. Numerical simulations are presented in Section 4, using South Korea’s post war growth path as a reference. Imperfect labour markets are introduced in Section 5 and it is shown how this enhances the model’s ability to account for the stylized facts of growth. Section 6 concludes.

2. The dual economy growth model

Variations on Lewis’ (1954) dual economy model have been used for analyzing many aspects of developing economies, especially normative planning problems and economic policy questions. Planning models were developed by Marglin (1967) and Dixit (1968). Recent applications are: Becinvenga and Smith (1997) who study of the nature of unemployment in LDCs; Gangopadhyay (1998) who studies the persistence of poverty; and Drazen and Eckstein (1988), who discuss the impact of traditional sector organization on the steady state

2 By ‘the standard theory’ I refer to models following Solow (1956), Swan (1956), and Ramsey (1928).
levels of capital.\(^3\) The model in this paper is similar to the model of Drazen and Eckstein (1988). Whereas they are concerned with welfare problems in alternative steady states of an OLG model, in this paper I focus on positive issues concerning the behaviour of the transition path of a continuous time model.

First assume there exists a modern and a traditional sector. Each sector employs labour and a specific factor. Denote the specific factors as capital, \(K(t)\), for the modern sector and traditional assets, \(A(t)\), in the traditional sector. Examples of traditional assets are livestock and simple tools. Second, assume that all the traditional sector output is consumed. Hence only modern sector goods can be used for factor accumulation. Third, I assume that labour, \(N(t)\), is mobile between the two sectors.

Let \(Y(t) = F(K(t), L(t))\) be the production function in the modern sector and let \(X(t) = G(A, N(t) - L(t))\) describe the output of the traditional sector, where \(L(t) \leq N(t)\) is the labour employed in the modern sector. Let \(n\) be the exogenous growth of labour so that \(N(t) = N(0) e^{nt}\). Assume that \(F(K(t), L(t))\) and \(G(A, N(t) - L(t))\) are homogeneous of degree one and exhibit positive but diminishing marginal products for all positive values of each input. I assume that \(F(K(t), L(t))\) satisfies the Inada conditions.\(^4\) For the traditional sector I assume that the demand for labour is not infinitely inelastic. This requires that the second derivative of \(G(A, N(t) - L(t))\) with respect to labour inputs is always bounded above negative infinity, that is \(\frac{\partial^2 G}{\partial (N - L)^2} > -\infty\). Finally, I also assume that traditional output approaches zero as labour inputs become small and \(G(A, 0) = 0\). Expressing the functions and variables in per-worker terms and suppressing

\(^3\) For a comparison of the different types of dual economy models in the early literature see Jorgenson (1967) and Dixit (1973).
time indices, gives \( y = f(k, l) \) and \( x = g(a, 1 - l) \) respectively where \( l \equiv L/N, a \equiv A/N, k \equiv K/N, \)
y \( \equiv Y/N, \) and \( x \equiv X/N. \)

Consider the decisions faced by an infinitely lived extended family whose members may be
located in the modern and the traditional sector. The family chooses the location of different
family members at each moment in time and the family’s total consumption over time to
maximize utility. Let \( c^x \) and \( c^y \) denote consumption in each sector divided by the total labour
force, \( N. \) Following Drazen and Eckstein (1988), I assume that the two consumption goods are
perfect substitutes. The objective function for the family is

\[
U(c^x + c^y) = \int_{t=0}^{\infty} u(c^x + c^y) e^{-\rho t} \, dt
\]

Assuming that the economy consists of many such identical families, the solution to a
representative family’s problem will correspond to that of a social planner.\(^5\) I therefore
consider the optimization problem for a social planner who maximizes (1) subject to

\[
\dot{k} = y - c^y - nk \tag{2}
\]

\[
c^y = x \tag{3}
\]

The current value Hamiltonian associated with this problem is

\[
H(k, c^y, l, \lambda) = u(g(a, 1-l) + c^y) + \lambda(f(k, l) - c^y - nk) \tag{4}
\]

\(^4\) That is \( \lim_{k \to 0} (F_K) = \infty, \lim_{l \to 0} (F_L) = \infty, \lim_{k \to \infty} (F_K) = 0, \lim_{l \to \infty} (F_L) = 0. \) These ensure that a steady state exists
when the economy consists only of the modern sector.

\(^5\) From the first welfare theorem, this holds because there are no economies of scale or externalities.
where \( \lambda \) is the co-state variable. The necessary conditions for an optimum solution to this problem are that there exists \( \lambda > 0 \) such that \( k, c^y, \) and \( \lambda \) simultaneously satisfy eqs (5)–(7).\(^6\)

\[
u' - \lambda = 0 \tag{5}
\]

\[
u'g_i + \lambda f_i = 0 \tag{6}
\]

\[
\dot{\lambda} = \rho\lambda - \lambda(f_k - \rho - n) \tag{7}
\]

where a prime denotes a partial derivative of the function, and a subscript denotes a partial derivative with respect to the subscripted variable. Combining (5) and (7) gives the Keynes-Ramsey condition

\[
\dot{c} / c = \sigma(c)(f_k - \rho - n) \tag{8}
\]

where \( \sigma(c) \equiv -u'(c) / u''(c)c \) is the inverse of the elasticity of marginal utility with respect to consumption, and \( c \equiv c^y + c^x \). The solution to the social planner’s problem is thus described by eq. (6), the pair of ordinary differential eqs (2) and (8), and a pair of boundary conditions which are yet to be defined.

First, consider eq. (6). This describes the optimum allocation of labour between the two sectors. If we denote the equilibrium wage rate as \( w \), then substituting (5) into (6) gives

\[
g_{1-i} = f_i = w \tag{9}
\]

where \( g_{1-i} = -g_i \). Optimality requires that the marginal physical product of labour is equated across the two sectors through labour migration. Because capital accumulation occurs in the

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modern sector, \( f_k \) increases with \( k \), for a given value of \( l \). Hence the equilibrium wage, \( w \), also increases. Given the assumptions above, traditional output becomes infinitesimally small or zero.\(^7\)

**Proposition 1**

The steady state of the Ramsey model is the unique steady state for the dual economy. It can only be obtained when traditional output is zero or, equivalently, when \( l = 1 \).

**Proof:**

First, inspection of eqs (5), (6), and (7) shows they are identical to the Ramsey model when \( l = 1 \). Thus the Ramsey model steady state is also a steady state of this model. Second, suppose another steady state exists such that \( l < 1 \) and \( \dot{c} / c = \dot{k} / k = 0 \). Given \( A \) is constant, eq. (9) requires that \( \dot{l} / l > 0 \). From eq. (8), however \( f_k \) is constant and this requires that \( \dot{l} / l = \dot{k} / k > 0 \), which is a contradiction.

Given proposition 1, a boundary condition is therefore obtained from the standard Ramsey model steady state. From eq. (8) this requires that \( k(t) = k^* \) such that \( k^* \) satisfies \( f_k = \rho + n \).

The second boundary condition is the initial condition \( k(0) = \bar{k} \). The equilibrium growth path of the economy thus follows the verbal description given by Lewis (1954). The modern sector absorbs labour from the traditional sector until the latter vanishes. At this point the growth path is described by the standard Ramsey model

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\(^7\) Note that the sectors here refer to factor usage rather than types of output or geographic location. Also note from eq. (8) that if the marginal product of capital is initially low, consumption may fall. This could result in a poverty trap. In this paper however, I focus only on successful growth paths and therefore assume the initial marginal product of capital exceeds \( \rho + n \).
3. Labour migration and the pattern of growth in the benchmark model

It remains to show how the labour supply conditions affect the rate of accumulation and this affects the return to capital during the course of its development. First I consider the pattern of growth in the modern sector. This proves some preliminary results and also provides intuition for later results.

3.1 Migration and convergence in the modern sector

To determine the behaviour of growth in the modern sector I consider the magnitude of the elasticity of modern sector output with respect to capital. This determines the rate at which diminishing returns occur as the economy accumulates capital. If this is large, then growth rates will be relatively persistent.

Let $\eta_{ij}$ and $\varepsilon_{ij}$ denote total and partial elasticities of $i$ with respect to $j$ respectively, where $i$ and $j$ are elements of the set of outputs, inputs, and prices, $(x, y; a, 1-l, k, l; w, r)$. Using the implicit labour demand eq. (9), modern sector output can be expressed as $y=f(k, l(k, a))$.

Differentiating the production function gives $\eta_{yk} = \varepsilon_{yk} + \varepsilon_{yl} \varepsilon_{lk}$. If each factor is paid its marginal product then $\varepsilon_{yk}$ and $\varepsilon_{yl}$ are also the factor income shares in the modern sector. Because $f(k, l)$ is homogeneous of degree one, the shares sum to one. Following convention, let $\alpha^y$ denote the share of capital in the modern sector, then $\varepsilon_{yk} = \alpha^y$ and $\varepsilon_{yl} = 1-\alpha^y$.

Similarly let $\alpha^x = \varepsilon_{xa}$ be the asset share for the traditional sector. Differentiating eq. (9) then gives an expression for $\varepsilon_{lk}$. As shown in the appendix, this is

$$\varepsilon_{lk} = \frac{1}{1 + s\alpha^x \sigma^y / \alpha^y \sigma^y}$$  \hspace{1cm} (10)
where $\sigma^y \equiv f_k f_l / f_{y, l l}$ is the elasticity of factor substitution in the modern sector, $\sigma^x$ is similarly defined for the traditional sector and $s \equiv l/(1-l)$. It can be confirmed that (10) reverts to the standard Ramsey case as the traditional sector becomes small, $(\lim_{l \to 1} (\varepsilon_{\alpha}) = 0)$.

**Proposition 2**

The presence of labour migration from the traditional to the modern sector raises the total elasticity of output with respect to capital, $\eta_{yk}$, above the partial response, $\varepsilon_{yk}$. The difference is greater when there are relatively high substitution possibilities in the traditional sector, $(\sigma^y / \sigma^x < 1)$, and when the modern sector is capital intensive relative to the traditional sector, $(\alpha^x / \alpha^y < 1)$.

**Proof:** This follows from inspection of eq. (10).

**Corollary**

Consider the classical dual economy model where traditional sector output is a linear function of labour, (see for example Stern, 1972). In this economy, $w$ is constant when $l < 1$. From eq. (14) therefore, $\varepsilon_{wa} = 0$ and $\eta_{yk} = \varepsilon_{yk} + \varepsilon_{yl} = 1$. The pattern of accumulation in the modern sector is therefore given by a linear $AK$ endogenous growth model up until the point where $l = 1$.

Proposition 2 and the corollary show that the accumulation of capital in the modern sector of a dual economy model exhibits greater persistence than the one sector economy. This is because the supply of labour from the traditional sector offsets diminishing returns.
The results can be used to illustrate the phase diagram for growth in the modern sector in \( \{c^y, k\} \) space. Figure 1 shows an example where the traditional sector becomes small asymptotically, as the wage rate increases. Setting \( \dot{k} = 0 \) gives the locus of stationary points for capital accumulation per worker. This is shown as curve (b). Curve (a) shows the higher consumption possibilities associated with \( \dot{k} = 0 \), if all labour was employed in the modern sector. Curve (a) is therefore the stationary locus for a Ramsey economy. Because \( l < 1 \) in the dual economy, (b) is below (a). Curve (b) is initially more linear than (a) but as \( k \) becomes large, \( l \) approaches unity and the curves converge. Setting \( \dot{c} = 0 \) and \( l = 1 \), gives the locus of stationary consumption points when the Ramsey phase is reached. The intersection of the \( \dot{c} = 0 \) and \( \dot{k} = 0 \) curves then determines the steady state level of \( c^y \) and \( k \).

Figure 2 shows the classical case where traditional output is a linear function of labour, so that \( \varepsilon_{sk} = 0 \) and \( \eta_{yk} = 1 \). In this case \( f_l \) is constant. The \( \dot{k} = 0 \) locus is a ray with slope equal to the average product of capital minus the population growth rate \( n \). In this example there exists a level of \( k, \bar{k} \), where \( l = 1 \). For all \( k > \bar{k} \), the transition is identical to the Ramsey model.

(Figs. 1 and 2 here)

Thus the dual economy model shows how the pattern of accumulation may differ in developing economies relative to industrialized ones. In particular growth in the modern sector will tend to be relatively persistent, like an AK endogenous growth model. This effect is larger when \( \sigma^x \) is relatively large or \( \alpha^x \) is relatively small.
3.2 Accumulation and the return to capital in general equilibrium

As discussed above, the standard Ramsey growth model predicts unrealistic differences in marginal product of capital, \( r \), during the course of development. In this section I derive the behaviour of \( r \) in the dual economy model.

From the Inada conditions, as the stock of capital becomes small with given quantity of labour, \( r \) tends to infinity. That is, \( \lim_{k \to 0} F_k = \infty \). In the dual economy model, however, the value of \( f_k \) is bounded because as \( k \) becomes small, \( l \) also becomes small. To see this note that constant returns to scale implies that \( f_k \) depends on the ratio, \( l/k \). Thus \( r = f_k (l(k,a)/k) \). Taking logarithms and differentiating with respect to \( k \) gives

\[
\eta_{rk} = \varepsilon_{rk} (1 - \varepsilon_{rk})
\]

where \( \varepsilon_{rk} \equiv f_{rk} k / f_k \). Denote the value of total output, GDP, as \( z \). Because traditional and modern sector outputs are perfect substitutes in consumption, GDP is simply the sum of the two outputs, \( z = x + y \). We wish to derive an expression for \( \eta_{rz} = \eta_{rk} / \eta_{zk} \). It is straightforward to show that \( \eta_{zk} = \alpha^y z / y \). Finally given \( \varepsilon_{rk} = \varepsilon_{yk} / \sigma^y = (1 - \alpha^y) / \sigma^y \) then substitution in (11) gives

\[
\eta_{rz} = \frac{(1 - \alpha^y) (1 - \varepsilon_{rk}) y}{\alpha^y \sigma^y z}
\]

In the standard one sector case \( \varepsilon_{rk} = 0 \) and \( y = z \), so the second term is equal to unity and (12) reduces to

\[
\eta_{rz} = \eta_{ry} = \frac{(1 - \alpha^y)}{\alpha^y \sigma^y}
\]
which is the expression derived by Mankiw (1995). Thus \( \varepsilon_{rk} > 0 \) reduces the responsiveness of the return to capital, relative to the one sector model, for given ratio of \( y/z \). From the appendix however the ratio \( z/y = 1 + x/y \), can be expressed terms of the factor shares and the distribution of labour between sectors, \( x/y = (1 - \alpha^y) / (s(1 - \alpha^x)) \). Using this and substituting eq. (10) into (12) gives

\[
\eta_{rc} = \frac{(1 - \alpha^y) (1 - \alpha^r) / (1 - \alpha^r) + s}{\alpha^r \sigma^y \left( \frac{\alpha^x \sigma^r}{\alpha^r \sigma^y} \right) + s}
\]  

(14)

As in eq. (12), the second term in this product reflects the effects of labour migration on the return to capital. Note that if the factor shares and the elasticity of substitution are the same across sectors, then the second term is equal to unity and (14) reduces to (13).

**Proposition 3**

In a dual economy the return to capital is relatively unresponsive to changes in income if:

(i) The traditional sector is more labour intensive than the modern sector, \( \alpha^y > \alpha^x \); and,

(ii) The elasticity of factor substitution in the traditional sector is larger than the modern sector, \( \sigma^y > \sigma^x \).

**Proof:** This follows from inspection of eqs (14) and (13).

Hence, in order to demonstrate that the return to capital responds relatively inelastically to changes in income levels, it is sufficient to show that the traditional sector of the economy has a high elasticity of substitution, or is relatively labour intensive.
3.3 Discussion

As shown by Mankiw (1995), the ability of the standard growth model to account for the stylized facts of growth, depends on the plausibility of the parameters in eq. (14). Lucas (1990) and King and Rebelo (1993) use conventional values, which give $\eta_{rz} > 1$. The constancy of the return to capital, however, suggests that the elasticity is close to zero. Mankiw (1995) argues that the elasticity of substitution may be around three, and that the aggregate capital share is $2/3$. This gives $\eta_{rz} = 1/6$. This is consistent with, for example, South Korea’s marginal product of capital falling from 14% to 7% since the early 1950’s.\(^8\)

These values are unconventional however. Mankiw (1995) justifies them with arguments relating to factor price equalization and the production function for human capital.\(^9\) His view of human capital is criticized by Phelps (1995). Similarly the assumption of high elasticities of substitution is apparently contradicted by the evidence of large differences in returns to similarly skilled labour across countries, Romer (1995).

The explanation offered here however, is based on highly plausible conditions. LDCs do have labour intensive technologies. Similarly, traditional capital, such as oxen and carts, are likely to be more easily substituted with labour than modern sector machinery. For example, assume that the modern sector is characterized by conventional values, $\alpha^v = 0.4$, $\sigma^v = 1$. For the traditional sector assume $\alpha^s = 0.2$ and $\sigma^s = 4$. Then eq. (14) gives $\eta_{rz} = 0.16$, if $l = 0.5$,

\(^8\) South Korea has experienced a 600% increase in GDP since 1953. The percentage fall in $r$ would be $600%/6 = 100\%$. What merits an unrealistically high level however seems to be unresolved in the literature. King and Rebelo (1993) give evidence suggesting that real rates of return rarely exceed 10%. The opposing view is given by Christiano (1989) who reports stock market real rates of return that have a mean of approximately 20%.

\(^9\) Ventura (1997), shows that high elasticities of substitution may be a consequence of international trade. Most LDCs are relatively closed economies however. Even the high export economies such as Taiwan and South Korea, achieved high growth rates in the 1960s with import substitution policies.
\[ \eta_{rz} = 0.52 \text{ if } l = 0.99 \text{ and } \eta_{rz} = 1.5 \text{ if } l = 1. \text{ These values give changes in the return to capital similar to those hypothesized by Mankiw (1995).}^{10} \]

4. Numerical simulations with competitive labour markets

An unresolved issue is how long the period of dualistic growth lasts. Although the return to capital is inelastic when \( l < 1 \), its level must still be sufficiently high to induce any accumulation after the traditional sector vanishes. For example, Bai (1982) argues that South Korea reached the Lewisian turning point in the mid 1970’s, yet high growth rates have continued into the 1990’s. This section addresses this by obtaining numerical solutions to the growth path of the dual economy described above.

To implement numerical solutions I follow Barro and Sala-i-Martin (1995) and King and Rebelo (1993), and assume that aggregate output is described by the Cobb-Douglas function,

\[ y = \phi k^{\alpha} l^{1-\alpha}. \]

In this case \( \varepsilon_{yk} = \varepsilon_{wk} = \alpha y \) and \( \sigma y = 1 \). Similarly, I assume that the instantaneous utility function is of the Stone-Geary form

\[ u(c^x + c^y) = \ln(c^x + c^y - \sigma) \]

where \( \sigma \) is a parameter that defines the minimum consumption level.\(^{11}\) Further assume that there is Hicks neutral technological change of 2\% per year, \( \dot{\phi} / \phi \equiv g = 0.02.\(^{12}\) Labour force growth rates for 1953-90 are taken from Summers and Heston (1991). It is assumed that the

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10 These arguments are not mutually exclusive to those of Mankiw (1995) and Barro and Sala-i-Martin (1995). If one accepts their views then \( \eta_{rz} \) will be even lower.

11 Christiano (1989) shows that the Stone-Geary utility function is required in standard models to prevent accumulation occurring too rapidly in the initial years. It is used by Barro and Sala-i-Martin (1995) and King and Rebelo (1993). See also Cochrane (1997).

12 The average annual growth rate of real (SPPP) GDP per worker in Korea from 1953-1990 was 5.3\%. Thus in the simulations, 5.3-2 = 3.3 percentage points of this was due to capital accumulation. A reasonable
growth rate of labour beyond that date is \( n = 0.005 \). I assume \( \alpha_y = 0.4 \), which is in accordance with data presented by Pilat (1994) for South Korea. The steady state marginal product of capital is given by the expression, \( r^* = \rho + n + (g/(1 - \alpha_y)) \). This is set equal to 0.065, which gives \( \rho = 0.027 \). The parameterized differential equation system consists of a two point boundary value problem. The solution is obtained using a finite difference method known as the ‘relaxation method’, (Press et al 1990). Using these functional forms and parameters I consider two cases utilizing different production functions for traditional sector output.


Assume that the traditional sector production function is, \( x = g(a, 1 - l) = a^\alpha (1 - l)^{1 - \alpha} \). We know from Proposition 3 that in this case the changes in the return to capital will be the same as the one sector model. This case therefore provides a useful benchmark. Equation (9) becomes, \( a^\alpha (1 - l)^{-\alpha} = \phi k^\alpha l^{-\alpha} \) and rearranging gives, \( l = (1 + \phi^{-1/\alpha} (a / k))^{-1} \). The traditional sector becomes arbitrarily small as the marginal product of labour rises and hence the long run behaviour of the economy asymptotically approaches the Ramsey model. For the initial conditions, income per person in South Korea in 1950 was ($1985 PPP) 2,200. Thus I choose \( y(0) + x(0) = 2.2 \). The remaining parameters are the level of \( \bar{c} \) and initial average product of capital, \( y(0)/k(0) \). These last two variables are used to calibrate the model to actual data. The higher the initial average product of capital, \( y(0)/k(0) \), the greater will be the initial rate of accumulation.

decomposition would be to attribute 3.3/5.3 = 62% of growth to capital accumulation. All the data cited are from Summers and Heston (1991), Penn World Table (Mark 5.6).
The results are shown in Figs 3 and 4. Figure 3 shows an example where $y(0) = 0.5$ and $x(0) = 1.7$. The dashed line shows modern sector output. This grows rapidly so that the traditional sector has all but disappeared by 1970. The maximum value of $r$ is about 30%. Figure 4 shows solutions for alternative initial values of $y(0)$, holding $z(0)$ constant. As $y(0)$ becomes large and $x(0)$ becomes small, the transition path becomes closer to the transition path of the one sector model. It can be seen from the Figs that the results are not very sensitive to these changes. This is because both sectors share the same production function.

It may be noted that in these solutions the value of $r$ is rising initially. This is because the rate of investment is low initially, due to the minimum consumption requirement, and so technological change raised the marginal product of capital.

(Figs. 3 and 4 here)

4.2. Case 2: Traditional output is a linear function of labour

The assumption that the traditional sector production is a linear function of only labour inputs has been common in the development literature. Moreover, the dynamics of the modern sector in this case are the same as in the AK endogenous growth model, with $\eta_{rc}$ equal to zero. I assume that the modern sector’s output is as described above, but the traditional sector’s output and consumption is $x(t) = \bar{w}(1 - l(t))$. The modern sector labour demand equation for $l < 1$ is found by using eq. (9). Noting that $\partial x / \partial w = -\bar{w}$, this gives $l = ((1 - \alpha)\phi(t) / \bar{w})^{1/\alpha} k(t)$. Hence labour demand is linear in $k$. Substituting this back into the

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13 Again this follows King and Rebelo (1993). Note that with this practice of choosing parameters so that $r^* = 0.065$, it is not important whether we assume that the representative family’s utility function is invariant to family size, as assumed here, or is sensitive to population size.
production function gives output as a linear function of capital. This dual economy reverts to the standard Ramsey model when $\phi f_i > \overline{w}$. The numerical simulations use the same parameters as above, for $\alpha$, $\sigma$ and $\rho$.

The results are shown in Figs. 5 and 6 using the same initial values of $y(0)$ and $z(0)$ as those in Case 1. The Figs show that marginal product of capital in 1953 is relatively low at 14%. However, due to technological change, $r$ increases over time to a maximum of approximately 30%. The impact of using a different production function in the informal sector is that $r$ is now much lower in the initial years. The difference in $r$ associated with a six-fold increase in GDP, $z$, and a 28 fold increase in modern sector income, $y$, is just four percentage points. This example indicates that the largest international differences in rates of return may not be between the richest and poorest economies. Rather gaps exist between mature industrialized countries and the rapidly growing economies such as South Korea in the mid 1960’s. This holds even when the economies share the same steady state growth paths.

(Figs. 5 and 6 here)

5. Migration and efficiency when labour markets are imperfect

The discussion so far has ignored the existence of urban-rural wage gaps in LDCs. Quantitative studies, however, show that a significant fraction of growth in these countries is due to improvements in efficiency of resource allocation. This section therefore considers

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14 This is lower than the values that were reported by Barro and Sala-i-Martin (1995) and King and Rebelo (1993) for similar experiments. The main reason is that I assume that technological progress is Hicks neutral rather than Harrod neutral.

15 This is the rate required to induce sufficient capital accumulation to account South Korea’s growth for a conventional Ramsey model, after the time when the dual structure disappears.

16 For example, Pilat (1994) attributes 13.6% of South Korea’s growth to improved resource allocation, assuming that the average traditional sector wage is twice the marginal product of labour in the traditional sector.
the affects of gains in allocative efficiency due to labour migration between the traditional and modern sectors. It is shown that this further reduces the maximum level of $r$ to less than 20% during the course of development. This is because the endogenous efficiency gains reduce the size of the initial average and marginal product of capital required to produce a given change in GDP.

5.1. Case 3: Differences in family organization by sector.

In the extended family model discussed above, all family members are assumed to have claims on all factor returns in both sectors. Ranis and Fei (1961) argue that although traditional sector labour has claims on asset returns by virtue of family membership, this is not the case in the modern sector. The opportunity cost of migration is therefore the average product of labour in the traditional sector, rather than the marginal product.\(^{17}\) This results in too little employment in the modern sector and a wedge between the marginal product of labour in each sector.

Assume a labourer in the traditional sector receives the average product of labour. Equation (9) is replaced by

$$g(a,1-l)/(1-l) = \phi f_i.$$ \hspace{1cm} (16)

Following Drazen and Eckstein (1988), I refer to this as the share economy ($SE$). The dynamic growth path of the $SE$ is thus described by eq. (16), and the pair of ordinary differential equations (2) and (8). The identical Cobb-Douglas production model, Case 1, is re-calibrated with the new labour allocation condition (16).
The results are shown in Fig. 7. It can be seen that the same period of growth is consistent with a maximum value of $r$ less than 20%. This is because every migrant raises the marginal product by an amount equal to the gap between their marginal product and their average product.\footnote{Given identical Cobb-Douglas production functions, the ratio of the marginal product to the average product is just $\alpha = 0.4$.} The additional growth means that the economy achieves rapid growth with lower initial values of the average and marginal products of capital.

(Figs. 7 and 8 here)

5.2. Case 4: Segmented labour markets in the modern sector.

Alternative explanations for the existence of urban rural wage gaps have focused on the role of institutions and incomplete markets (see for example Harris and Todaro 1970, Stiglitz 1974, and Bliss and Stern 1978). I consider the effect of an exogenously given markup between the return to labour in each sector, $\tau$. Equation (9) is replaced by (17).

$$g_{t-1} = f_t - \tau.$$  \hspace{1cm} (17)

Formally $\tau$ is equivalent to a tax on employment in the modern sector, financed by a lump-sum subsidy. This markup economy ($ME$) is described by eqs (2), (8), and (17). For simplicity I restrict attention to the linear traditional technology example. I assume that $\tau = 0.5$, which is consistent with evidence of urban/rural wage gaps discussed by Williamson (1988), and values used by Pilat (1994). The results are shown in Fig. 8.\footnote{Much larger gaps are often reported, for example see Becker, et al (1994).} Again $r$ is significantly lower through the transition beginning at 7% and reaching a maximum of 17%. The initial value of $r$

\footnote{In the some dual economy models the marginal product is also assumed to be zero due to overpopulation, thus giving rise to surplus labour. The existence of productivity gaps and surplus labour are however distinct concepts, and this paper is concerned only with the former.}
is only 0.5 of a percentage point above the assumed steady state level. The intuition for this result in the \textit{ME} is the same as for the \textit{SE} example.

6. Conclusion

An important stylized fact of economic growth is that the rate of return to capital is relatively constant in cross sectional or time series data. This is despite the existence very large differences in income levels and capital per worker ratios. Significantly this cannot be explained using growth models based on conventional aggregate production functions.

In this paper I have shown that structural changes are crucial determinants of the return to capital. This is demonstrated using a dual economy model that incorporates constant returns to scale and conventional values of the labour and capital cost shares. The argument therefore does not rely on economies of scale, or externalities associated with factor accumulation. The model is also consistent with recent growth accounting studies of the East Asian growth, that emphasize the importance of capital investment and efficient resource allocation in the development process, (Young 1995, 1994, Kim and Lau 1993, and Pilat 1994).

It is show that in a dual economy, the stability of the return to capital can be explained by a relatively elastic labour supply from traditional to modern sectors. This occurs when traditional sector labour is easily substituted for assets, or when traditional sector production is labour intensive, relative to the modern sector. It is also shown that imperfect labour markets between these sectors reduces the magnitude of the difference in the return to capital during the development process.
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Appendix

1. Derivation of eq. (10).

As in the text I let \( \eta_{ij} \) denote a total elasticity and \( \varepsilon_{ij} \) represents the partial elasticities. Thus \( \varepsilon_{yk} = f(k/l) \) and \( \varepsilon_{yl} = f(y/l) \). Similarly denote the cost share of capital in the modern sector as \( \alpha_y \), so \( \varepsilon_{yk} = \alpha_y \) and \( \varepsilon_{yl} = 1 - \alpha_y \) and for the traditional sector \( \varepsilon_{xa} = \alpha_x \) and \( \varepsilon_{xl} = 1 - \alpha_x \). We wish to derive an expression for \( \varepsilon_{ik} \), from the properties of the \( l(k, a) \) function, which is implicitly defined by eq. (9).

Because \( f(k, l) \) and \( g(a, 1-l) \) are homogeneous of degree one with respect to two inputs, their derivatives are homogeneous of degree zero, and can be expressed as a ratio of the inputs. Thus I restate eq. (9)

\[
f_i(k/l) = g_{1-l}(a / (1-l)) = w \tag{9}
\]

Logarithmic differentiation of (9), assuming \( dA = dN = 0 \) gives

\[
\varepsilon_{uk} d\ln k + \varepsilon_{wl} d\ln l = -s \varepsilon_{v_{1-l}} d\ln l \tag{18}
\]
where \( s = l/(1-l) \), and the partial elasticities are \( \varepsilon_{wk} \equiv f_{ik}k / f_i > 0 \), \( \varepsilon_{wl} \equiv f_{il}l / f_i < 0 \) and
\[ \varepsilon_{w(1-l)} \equiv g_{(1-l)k}(1-l) / g_{1-l} < 0. \]
Because of homogeneity of degree one \( \varepsilon_{wk} = -\varepsilon_{wl} \), and
\( \varepsilon_{w(1-l)} = -\varepsilon_{wa} \). Simplifying gives

\[
\varepsilon_{wk} d\ln k = (\varepsilon_{wk} + s\varepsilon_{wa})d\ln l \quad (19)
\]

and therefore

\[
\frac{d\ln l}{d\ln k} = \frac{\varepsilon_{wk}}{\varepsilon_{wk} + s\varepsilon_{wa}} \equiv \varepsilon_{ik} \quad (20)
\]

This expression is denoted as a partial elasticity because to derive it I assumed \( dA = dN = 0 \).

Note that this can be expressed in terms of the factor cost shares \( \varepsilon_{wk} = \alpha^y / \sigma^y \), and
\( \varepsilon_{wa} = \alpha^x / \sigma^x \) where \( \sigma^y \equiv f_k f_i / f_{ki} \), is the elasticity of factor substitution in the modern sector and \( \sigma^x \) is similarly defined for the traditional sector. Making these substitutions gives

\[
\varepsilon_{ik} = \frac{1}{1 + s\alpha^x \sigma^y / \alpha^y \sigma^x} \quad (10)
\]
as required.

2. Derivation of eq. (14).

The elasticity of GDP, denoted \( z = x+y \), with respect to capital is

\[
\eta_{zk} = (f_k + f_{i}l_k - g_{1-l}l_k) \frac{k}{z} \quad (21)
\]

In equilibrium \( w = f_i = g_{1-l} \), and so
\[ \eta_{zk} = \frac{f_j k}{z} \]  

(22)

Dividing by \( y \) gives,

\[ \eta_{xk} = \frac{\alpha^y}{x/y+1} \]  

(23)

This can rearranged in terms of the labour income shares in each sector. Note labour markets equilibrium implies \((1-\alpha^y) = w_l/y \) and \((1-\alpha^x) = w_t(1-l)/x \) Then,

\[(1-\alpha^y)/(1-\alpha^x) = s\frac{x}{y} \]  

(24)

so that

\[ \eta_{zk} = \frac{\alpha^y}{s^{-1}(1-\alpha^y)/(1-\alpha^x)+1} \]  

(25)

Now consider the return to capital \( r = f_k (l(k,a)/k) \). Taking logarithms and differentiating with respect to \( k \) gives

\[ \eta_{rk} = \epsilon_{rk} \]  

(26)

However \( \epsilon_{rk} = -\epsilon_{rk} \), so

\[ \eta_{rk} = \epsilon_{rk} (1-\epsilon_{rk}) \]  

(11)

Given \( \eta_{rz} = \eta_{rk}/\eta_{zk} \) and \( \epsilon_{rk} = (1-\alpha^y)/\sigma^y \), then

\[ \eta_{rz} = \frac{(1-\alpha^y)(1-\epsilon_{rk})}{\sigma^y \eta_{zk}} \]  

(27)
From above

\[ 1 - \varepsilon_{ik} = \frac{s}{\alpha^x \sigma^x / \alpha^y \sigma^y + s} \]

Then substituting for \( \eta_{zk} \) and \( 1 - \varepsilon_{ik} \), gives the required expression

\[ \eta_{cz} = \frac{(1 - \alpha^x)(1 - \alpha^y) / (1 - \alpha^z) + s}{\alpha^y \sigma^y / (\alpha^x \sigma^x / \alpha^y \sigma^y) + s} \]  \hspace{1cm} (14)
References


Fig. 1. Phase diagram for the modern sector of dual economy when the traditional sector employs labour and assets.

Fig. 2. Phase diagram for the modern sector of dual economy when traditional sector output is a linear function of labour.
**Fig. 3.** Case 1: identical Cobb-Douglas production functions. *Key*, Bold Line, Korean GDP, Solid Line, \( z = y + x \); Dotted line, \( y \). The marginal product of capital is a solid line in the second figure. This assumes \( y(0) = \text{PPP}$500 \). See text for other parameter values.

**Fig. 4.** Alternative values of \( y(0) \) for case 1: identical Cobb-Douglas production functions. *Key*, Solid Lines, \( z = y + x \); Dotted lines, \( y \). The marginal product of capital are solid lines in the second figure. Example (a), \( y(0) = \text{PPP}$100 \), example (b), \( y(0) = \text{PPP}$1000 \). See text for other parameter values.
\textit{Output per worker, }z, \ y, \textit{ Korea,} \\
($1000s$)

\textit{Marginal product of capital, }r, \textit{ Korea.} \\

\begin{figure}
\centering
\includegraphics[width=\textwidth]{graph1}
\end{figure}

Fig 5. Case 2: linear traditional production function. Key, Bold Line, Korean GDP, Solid Line, $z = y + \alpha$; Dotted line, $y$. The marginal product of capital is a solid line in the second figure. This assumes $y(0) = \text{PPP}$\$500. See text for other parameter values.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{graph2}
\end{figure}

\textit{GDP per worker, }z, \ y, \textit{ Korea,} \\
($1000s$)

\textit{Marginal product of capital, }r, \textit{ Korea.} \\

\begin{figure}
\centering
\includegraphics[width=\textwidth]{graph3}
\end{figure}
FIG. 6. Alternative values of $y(0)$ for case 2: linear traditional production function. *Key*, Solid Lines, $z=y+x$; Dotted lines, $y$. The marginal product of capital is are solid line in the second figure. Case (a) $y(0) = $100, case (b) $y(0) = $1000. See text for other parameter values.
Fig 7. Case 3: Share Economy with identical Cobb-Douglas production functions. Key, Bold Line, Korean GDP, Solid Line, Predicted GDP=$y+x; Dotted line, modern sector output; Broken Line, marginal product of capital in the modern sector. This assumes $y(0) = PPP$500. See text for other parameter values.
Fig 8. Case 4: Productivity Gap Economy with linear traditional production function.

Key: Bold Line, Korean GDP; Solid Line, Predicted GDP = y + x; Dotted line, Predicted modern sector output; Broken Line, marginal product of capital in the modern sector.

This assumes y(0) = PPP$500. See text for other parameter values.