Transitional Growth Paths
in Developing Economies

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SCHOOL OF ECONOMICS

DISCUSSION PAPER
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7 May, 1997

This paper develops model of growth in an economy where the capital stock is rationed across labour inputs, as in the dual, or segmented, labour market literature on developing economies. In this economy, the increased use of labour in the formal sectors can sustain high marginal and average products of capital and high growth rates for periods of 15-30 years. This provides an interesting insight into the current growth and convergence debate. The model is shown to overcome the empirical problems of the standard Ramsey growth model and also avoids some recent criticisms of endogenous growth models.

Key words: growth, development, endogenous growth, dual labour markets, convergence.

JEL Classification: OO, O1, O3, D9

* I would like to thank Kevin Fox, Geoff Kingston, Murray Kemp, Jim Melvin, Anu Rammohan, Allan Würtz and Minxian Yang and seminar participants at the Third Australian Macroeconomics Workshop, and the 23rd Australian Conference of Economists. Special debt is owed to Rick Harris. All errors and omissions are my own.

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1. Introduction

In the last half of this century we have witnessed the dramatic economic success of Japan, South Korea and other newly industrialising economies (NIEs) and the equally dramatic economic failure of many countries, especially in Africa and South Asia. These differences are well known and illustrated in Figures 1. Japan and South Korea, for example, achieved average growth rates of above five percent over twenty year periods, compared to average growth rates below two percent in much of the developing world. In order to design policies which can assist growth and development in low income economies, it is necessary to understand the causes of the rapid growth in the successful developing economies.

(Fig. 1 here)

Mankiw, Romer and Weil (1992), Barro and Sala-i-Martin (1992, 1995), Sala-i-Martin, (1994) and Mankiw (1995) argue that the standard Ramsey growth model, with diminishing returns to capital, explains why growth rates should be high in low income economies.1 A relatively small stock of capital results in high average and marginal products of capital. This causes a high, but monotonically falling, growth rate which, the authors suggest, accounts for the performance of the NIEs. King and Rebelo (1993) show, however, that the transition paths in this model, are too short, display too little persistence, and imply an initial growth rate and marginal product of capital that is too high, (King and Rebelo, 1993 p. 929).2

1 The standard Ramsey growth model is discussed, for example, in Blanchard and Fisher (1989) or Barro and Sala-i-Martin (1995). See also Solow (1956) and Swan (1956), who pioneered the neoclassical growth model.

2 For example, they argue that to explain the Japanese economic miracle in the traditional growth model requires a marginal product of capital of 500% in 1950. King and Rebelo’s criticisms apply even if human capital is included in the aggregate capital stock, which is the remedy suggested by Mankiw, Romer and Weil (1992), Mankiw (1995) pp. 290-291, and Barro and Sala-i-Martin (1995) pp. 80-87. In this case the higher aggregate capital share implies transitional implies unrealistically high long run investment rates. Further doubt has been shed on the empirical reliance of the Mankiw et al (1992) model by Caselli
Along with Romer (1986, 1990) and Lucas (1988, 1993), King and Rebelo (1993) therefore contend that an accurate understanding of growth miracles depends primarily on an understanding of the process of productivity growth, or on conditions where the production function does not exhibit a diminishing marginal product of capital. There are however, several difficulties with these endogenous growth models. First many predict increasing long run growth rates, whereas there is no strong evidence of this in time series data, (Jones 1995, Ben–David and Papell, 1995). Second, many also predict high levels of productivity growth in rapid growth economies. Recent, growth accounting studies of the East Asian NIEs, however, do not support this prediction, (Young 1994, 1995, Kim and Lau 1993, and Pilat 1994).3

The purpose of this paper is to describe an important aspect of growth in developing economies, which has hitherto been overlooked in this debate. If the economy has a dual or segmented, labour market structure then a significant part of the growth process may be due to high rates of capital accumulation, without unusually high rates of technology change. This is because diminishing returns to capital may be offset by the increased employment of labour from low wage informal sectors of the economy, which does not otherwise have access to modern sector capital. In this case the marginal product of capital, the savings rate and the growth rate are relatively constant over the early stages of development. It is shown that the modified model generates high and persistent growth rates and realistic levels of investment and the marginal product of capital. Thus rapid accumulation can occur in the medium term without high rates of technology change.

3 et al (1996). They argue that after correcting for mis specification bias the human capital augmented growth model also implies unrealistically fast transitions.

3 This results are supported by Havrylyshyn, (1992), who shows that developing countries have typically been found to have lower productivity residuals than the developed countries.
The paper is organised as follows. Section 2 briefly discusses the nature of labour markets in developing countries. It then incorporates the dual, or segmented, labour market hypothesis into the standard Ramsey growth model. In section 3 the transition properties of the dual labour market model are discussed and section 4 assesses the model using numerical simulations, calibrated to Japanese and South Korean post war data. Section 5 concludes.

2. Growth, convergence and development with dual labour markets.
Dual labour market theory was pioneered by Lewis (1954) and developed into formal models of development by Sen (1966), Ranis and Fei (1961), Marglin (1984), Dixit (1968), Stern (1971), Minami (1973) among others. In these models, growth in the modern sector of a developing economy induced labour migration from the less developed rural regions. Harris and Todaro (1970), Fields (1975), Mazumdar (1985) and Kannapan (1985) extended this idea in the theory of segmented labour markets – the existence of different real wage rates for labour of similar skills. The common element in all of these theories is that entry to the high productivity, or formal, sector is rationed. Thus an expansion of the formal sector labour demand results in an increase in labour supply. Informal sector output declines, but by less than the increase in formal sector output.

The theory of segmented labour markets in developing countries is not uncontroversial. Nevertheless empirical evidence shows the presence of urban-rural wage gaps for low skilled labour, Squire (1981), wage gaps

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4 For discussion of microeconomic models of labour market conditions in LDC’s see Gregory (1980), Basu (1984), Mazumdar (1983), Stiglitz (1988), Rosensweig (1988), Williamson (1988), Dasgupta (1993), and Freeman (1993). The segmented labour market theory should also be distinguished from the “surplus labour” hypothesis which maintains that output in the informal sector does not fall as labour is withdrawn, Dasgupta (1993). The claim of the segmented labour market hypothesis is that labour’s marginal contribution to output in the formal sector is larger than the decline in the informal sector.
according to factory size, Mazumdar (1979, 1985), and rationed access to education and capital markets resulting in high rents to those with skills, Sundrum (1990), Williamson (1993). There are also significant migrant labour supply responses to these wage gaps, (Ghatak, Levine and Wheatley-Price, 1996). The dual nature of labour markets has featured heavily in many descriptive accounts of the development in Japan (Fei, Ohkawa and Ranis, 1985, Minami, 1973, 1986), and other NIEs, (Amsden, 1989 Bai, 1982, Fields, 1994, Ranis, 1995). Moreover, the role of improved resource allocation in general for the East Asian NIEs has been stressed by Olson (1996) and The World Bank (1993). Thus, the segmented or dual labour market has received considerable attention in the development literature. In view of this the remainder of this paper is concerned only with investigating the implications of the dual labour market hypothesis for the pattern of growth.

Assume that the economy has two sectors, informal and formal, and labour in the formal receives a constant wage, \( w \) which is greater than the informal wage rate. This follows earlier growth models with dual labour markets and generates job rationing in the formal sector for sufficiently low capital stocks. Also following earlier growth models, we simplify matters by considering the formal sector only. The equations for capital accumulation and output are;

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5 While wage gaps exists, they are not ubiquitous and may vary substantially. Recent country studies supporting the segmented labour market hypothesis for developing countries include: Marcouiller, Ruiz de Castilla and Woodruff (1995), who report formal to informal wage ratios of 1.5 - 2 in El Salvador and Peru, but find no segmentation in Mexico; Beckar, Hamer and Morrison (1994) who site urban to rural wage ratios of 5-10 in some African economies and Ranis (1995) who reports ratios of approximately 1.5 - 2.5 in Taiwan and South Korea. See also Basch and Paredes–Molina (1996), and Magnac (1991). See Rosensweig (1988) for a critique of the measurement problems involved.

6 The most commonly suggested cause of segmented labour markets is efficiency wages, see Leibenstein, (1963), Bliss and Stern (1978), and Mazumdar (1983) and Dasgupta (1993). Fields and Wan (1989) present evidence that the institutions have been an important part of wage setting in Latin American. The difference between this model and models by Dixit (1968) and Stern (1972) and Marglin (1984) is that they considered the intertemporal optimal allocation of labour in the context of a normative planning problem, and restricted savings to be from profits, and not wages. See also Solow’s (1956) discussion of real wage rigidity.
\[
\dot{k}(t) = y(t) - c(t) - nk(t), \quad (1)
\]
\[
y(t) = A(t) f(k(t), l(t)) f_i > 0, f_{ii} < 0, i = 1, 2, \quad (2)
\]
\[
N(t) = N_0 e^{nt}, \quad (3)
\]

where \(N(t)\) is the labour force and \(l(t) \equiv L(t)/N(t)\) is the ratio workers employed in the formal sector to the total labour force. The remaining \(1-l(t)\) workers are in the informal sector. \(A(t)\) represents the level of technology, \(k(t)\) is capital per worker, \(y(t)\) is output of the formal sector per worker and \(f(\cdot)\) is assumed to be homogeneous of degree one. It will useful to define all \(t\) such that \(l(t) < 1\), as phase 1 and all \(t\) such that \(l(t) = 1\), as phase 2.

Firms in the formal sector are assumed to maximise current profits, \(\pi(t)\), taking prices as given. Thus the representative firm maximises,

\[
\pi(t) = A(t) f(k(t), l(t)) - w(t) l(t) - r(t) k(t), \quad (4a)
\]

subject to the constraint that,

\[
w(t) \geq \bar{w}. \quad (4b)
\]

Dropping time subscripts, the necessary first order conditions for profits maximisation are:

\[
Af_2(k, l) = w, \quad (5a)
\]
\[
Af_1(k, l) = r. \quad (5b)
\]

If the constraint (4b) holds with equality, then firms adjust their labour inputs, \(l\). That is, if \(Af_2(k, 1) < \bar{w}\), then,
Because the production function is homogeneous of degree one, \( f_2(k, l) \) is homogeneous of degree zero and can be written as a function of the ratio \( k/l \). Thus the value of \( l \) which solves equation is (6), \( \tilde{l} \), is linear in \( k \), 
\[ \tilde{l} = \tilde{l}(A, \bar{w})k. \]

The optimal choice of labour per worker can be expressed as, \( \hat{l} = \min(\tilde{l}, 1) \).

In phase 1, \( \hat{l} = \tilde{l} \) and \( y = f(k, \hat{l}) \equiv g(A, \bar{w})k \). Output must also be a linear function of the capital stock in phase 1. In general, output may be expressed as,

\[ \hat{y} = A f(k, \hat{l}) = \min \{ g(A, \bar{w})k, f(k, 1) \}. \]

The representative consumer in the formal sector is assumed to maximise utility subject to (1), (3) and (7). The intertemporal utility function depends on consumption per person and the number of people in the formal sector, 
\[ U(c, l) = \int_{t=0}^{\infty} u(c, l) e^{-\rho t} dt. \]
To solve the consumer’s optimisation problem the current value Hamiltonian is formed,

\[ H(k, c, l, \lambda) = Nu(c, l) + \lambda[Af(k, \hat{l}) - nk - c]. \]

Assume \( u(c, l) \) takes the isoelastic form, 

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7 This result generalises in a natural way to the isoelastic case with \( n \) capital inputs. Output will be a linearly homogenous function of all the capital inputs. For more general functions it can be shown that the capital inputs in the reduced form production function are weighted by a number greater than 1.

8 Output is homogeneous of degree one in \( k \), and \( l \), where \( l \) is itself a linear function of \( k \). Thus output is homogeneous of degree one in \( k \). See also Dixit (1968, 1973).
where $\overline{c}$ is a subsistence consumption level. The necessary conditions for a maximum are:

\[
(c - \overline{c})^{-\sigma} f^{\sigma-1} - \lambda = 0, \\
\dot{\lambda} = \rho \lambda - \lambda(f_1 + f_2 \hat{l}_k - n), \\
\dot{k} = A f(k, \hat{l}) - c - \overline{c} - nk. 
\]

A unique solution to this problem is obtained by requiring that the solution approaches a steady state as $t$ tends to infinity, such that consumption and capital grow at constant rates. As $t \to \infty$, $f_2(k, 1) \to \infty$, and so the economy must enter phase 2. In phase 2, $\hat{l}_k = 0$ and $l = 1$. Making these substitutions the system (8a)-(8c) is identical to the standard Ramsey model. In phase 1 however, the transitional dynamics will be different from the standard model. The next section discusses these transitional dynamic paths.

3. Transitional growth paths in a developing economy

Equation (8c) describes the evolution of the capital stock. Dividing by $k$, using equation (7) and assuming that $\hat{l} = \bar{l}$, gives

\[
\frac{\dot{k}}{k} = g(A, \bar{w}) - (c + \overline{c}) / k - n. 
\]
subsistence level, or is constant, then the growth rate of the capital stock will also be constant or increasing. Thus in phase 1 the economy behaves like the “AK” linear endogenous growth models as discussed, for example, by King and Rebelo (1990). This result contrasts with the standard model where growth rates decline monotonically. In contrast to endogenous growth models, however, the economy still approaches a long run steady state growth rate.

Next, consider the path of consumption which is given by equations (8a) and (8b). Combining these gives the “Keynes-Ramsey” rule for this model.

\[
\frac{\dot{c}}{c} = \frac{(c - \bar{c})}{\sigma c} \left[ A(f_1 + f_2 \cdot \hat{l}_k) - \rho + (\sigma - 1) \frac{\hat{l}}{\bar{l}} \right],
\]

In phase 1, the first term in the square brackets, \(A(f_1 + f_2 \cdot \hat{l}_k)\) is equal to the average product of capital, \(g(A, \bar{w})\) and is therefore constant or increasing. The last term in the square brackets reflects the effect of induced changes in the formal sector population on the rate of consumption. It is small when \(\sigma\) is close to unity, and zero if \(\sigma = 1\). In this case, which is empirically valid, the growth rate of consumption will also be constant or increasing.

The transitional dynamics are illustrated in the phase diagrams, Figures 2a and 2b. These are drawn in \(\{c, k\}\) space for the case where the level of technology is constant. The equation \(\hat{l} (A, \bar{w}) = 1\) defines the level of \(k\), say \(\bar{k}\), which defines the phase 1-phase 2 boundary. Setting \(\dot{k}\) equal to zero in equation (8c), gives an expression for the locus of stationary points for capital accumulation.

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9 A similar result is obtained in a descriptive growth model by Dixit (1973).
10 The class of endogenous growth models based on Lucas’ (1988) model also have this feature when no externalities are assumed. This class of models, however, have transitional dynamics more akin to the standard Ramsey model than to models based on strictly non-convex production possibilities, as in Romer (1986).
11 The elasticity of substitution, \(\sigma\), is generally thought to be around 1 or lower, Blanchard and Fisher (1989).
per worker. Recalling that \( f(k, \dot{l}) = g(A, \bar{w}) k \) in phase 1, then \( \dot{k} = 0 \) is a ray with slope \( g(A, \bar{w}) - n \), which intersects the vertical axis at \( -\bar{c} \). This is shown as line segment \(-\bar{c} \ a\). This intersects the curve \( a \ b \ d \) which is the \( \dot{k} = 0 \) line in phase 2. Everywhere below \(-\bar{c} \ a \ b \ d\), the per worker capital stock is increasing.

(Fig. 2a and 2b here)

Setting the growth rate of consumption per worker equal to zero in equation (10), gives the locus of stationary points for consumption. In phase 1, stationary consumption occurs if \( g(A, \bar{w}) = \rho + (1 - \sigma) \hat{l} / \lambda \). This is independent of the level of \( k \) when \( \sigma = 1 \), and the stationary consumption paths are drawn for this case. The growth rate of consumption will be positive if \( g(A, \bar{w}) > \rho \) and this is drawn in Figure 2a. Otherwise \( g(A, \bar{w}) < \rho \) and this is shown in Figure 2b. In phase 2, equation (8b) defines a unique value of \( k \), \( k^* \) where \( \dot{c} / c = \dot{\lambda} / \lambda = 0 \). This is a vertical line in Figures 2a and 2b, \( k^*-\ b \). To the right of this line consumption per worker is falling and to the left it is rising. Possible optimal trajectories are drawn from \( k(0) \) to \( k^* \).

Consider the economy as it crosses the phase 1-phase 2 boundary at \( \bar{k} \). At this boundary it is approximately true that \( y = f(\bar{k}, 1) = g(A, \bar{w}) \bar{k} \). But for \( k < \bar{k} \), \( \partial y / \partial k = A (f_1(\cdot) + f_2(\cdot) \hat{l}_k) \) where \( f_2(\cdot) \hat{l}_k \) is positive. For values of \( k > \bar{k} \), \( \partial y / \partial k = A f_1(\cdot) \), so that \( \partial y / \partial k \) falls across this boundary. From equations (9) and (10) it follows that the growth rate of consumption and capital also fall as the economy enters phase 2.

These results show that linear approximations of the growth rates in the neighborhood of the steady state may be poor approximations to the growth rate of the economy when \( k < \bar{k} \). The use of steady state conditions has been
widespread, in particular in the cross country econometric growth literature, following Mankiw, et al. (1992). This result therefore indicates that if developing economies are characterised by dual labour markets, then regression equations based on the growth equations in the neighborhood of a steady state may be misspecified.

Thus we have shown that the dual economy model behaves like the linear endogenous growth model in phase 1, and that the growth pattern may change in response to structural changes occurring during the course of development. The next section considers these results in the light of the experience of the Japan and South Korea.

4. Numerical Solutions

King and Rebelo’s (1993) critique of the Ramsey growth model is based on the relative magnitudes of important variables in numerical solutions. This section therefore presents numerical solutions in order to compare the two models further. First, following King and Rebelo (1993), the results of the transitional growth paths of a Ramsey growth model are plotted against the actual growth path of Korea and Japan in Figure 3. The solutions use a Cobb-Douglas production function, as in Barro and Sala-i-Martin (1995) and King and Rebelo (1993) and are parameterised using data from Summers and Heston (1991) *Penn World Table Mk. 5.6*. For the numerical solutions the assumption that technology is constant is relaxed so that, $A(t) = A(0)e^{\gamma t}$. In

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12 The procedure of explaining the growth of countries in an aggregate closed economy framework is also adopted by Mankiw et al. (1992), Mankiw (1995) and Barro and Sala-i-Martin (1995). The omission of the role of foreign capital may be particularly important in this context. Nevertheless, in South Korea for example, national savings still accounted for 75 percent of domestic investment during the rapid growth in the 1970’s, and up to 100 percent in the 1980’s, Dornbusch and Park (1987). A full account of the Japanese and South Korean growth should also account for the effects of trade, technology transfer, post-war reconstruction and other government policies. The aggregate closed economy framework may, nevertheless, be viewed as a useful starting point.

13 The most recent version of this data is the *Penn World Table Mk 5.6* and was obtained from the NBER internet site http://nber.harvard.edu/pwt56.html.
Figure 3 results are presented for different values of $\gamma$. In all solutions, however, the steady state marginal product of capital is held constant at 0.065, by adjusting the discount rate, $\rho$, for different values of $\gamma$.

(Fig. 3 here)

The results confirm King and Rebelo’s (1993) findings. The growth rate falls rapidly from its maximum value displaying very little persistence. This contradicts the actual pattern of growth in Japan and South Korea. Similarly the marginal product of capital is unrealistically high when $\gamma$ is assumed to be low. If higher values of $\gamma$ are assumed the model performs better. Nevertheless King and Rebelo’s objections are still valid as one is then explaining the miracle by the exogenous growth rate of technology and not by endogenously determined returns to capital.

To implement numerical solutions for the dual labour market model, we require approximate dates at which the economies made the transition from phase 1 to phase 2. In this context, Minami (1973, 1994) presents evidence of “tightening” formal sector labour markets in Japan, such as reduced sectoral and skill based wages gaps and accelerating wage increases, and concludes that informal sector labour was exhausted around the early to mid 1960’s. Similar evidence is presented for Korea by Bai (1982), which suggests that the Korean labour markets tightened during the mid 1970’s.

Taking these dates as approximate timing for the phase 1 phase 2 transition then one can determine the implied level of $\bar{w}$. In addition simulations

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14 Let $y$ be the GDP in the formal sector per worker in both sectors, as above. Then in phase 1, the share of formal sector labour in formal sector output is $s_{f_1} \equiv \bar{w} l_1 y$. On the phase 1 phase 2 boundary, $l_1 = 1$, so $\bar{w} = s_{f_1} y$. Japan’s GDP per worker was $\text{Sm} 7.3$ in 1965 and Korea’s was $\text{Sm} 6.2$ in 1975, measured in constant 1985 dollars (PPP). Thus assuming that the labor share is 60 percent, gives values of $\bar{w}$ equal to $\$4500$ for Japan and $\$3700$ for Korea.
require an estimate of the size of the informal sector relative to the formal sector. For the purpose of this exercise it is assumed that the economy-wide GDP was initially equally divided between formal and informal activities in the initial year.\textsuperscript{15}

Figure 4 shows numerical solutions derived using these assumptions. Results are shown for different levels of $\gamma$, with the initial level of technology, $A(0)$, chosen so that phase 2 is reached in 1965 for Japan and 1975 for Korea. The numerical results confirm the analysis so far. The growth rate and marginal product of capital are relatively persistent and increasing in some cases. In addition investment rates are high initially and then fall in phase 2. The relative magnitudes of prices, investment rates and growth rates are quite plausible, and quite different to the pattern of growth in the standard Ramsey model described by King and Rebelo (1993).

(Fig. 4 here)

Figure 5 shows the results of comparative dynamic analysis of the growth path in response to changes in the models parameters. Cells 5.1 and 5.2 of show the effect of different levels of $\overline{w}$, holding constant the steady state growth marginal product of capital, $r^*$, the initial average product of capital $g(A(0), \overline{w})$, and initial level of GDP per worker. Different exogenous wages do not radically alter the nature of the transition, but require a higher level of $y$ be obtained before phase 2 is reached. Thus a higher level of $\overline{w}$ could potentially account for Korea’s growth through to the 1990’s.

(Fig. 5 here)

\textsuperscript{15} There may also be large measurement problems. Informal sector activity is likely to be understated in national accounts data.
The effects of altering the initial average product of capital (for given \( \bar{w} \), \( r^* \), \( g(A(0), \bar{w}) \)) are shown in cells 5.3 and 5.4. Higher initial levels of technology induce a higher growth rate over phase 1 so that the growth rate increases in response to a constant rate of technology change. This property is desirable in that “growth miracles”, by definition, begin with an acceleration of growth from an initially low rate.\(^\dagger\) Cells 5.5 and 5.8 show that introducing a minimum consumption constraint, reduces the growth rate and delays the arrival of phase 2.

Finally cells 5.7 and 5.8 show solutions where the share of capital is varied from 0.4 to 0.5 and 0.8, holding the initial average product of capital and \( \bar{w} \) fixed. This is of interest since Mankiw et al (1992), Barro and Sala-i-Martin (1995) and Mankiw (1995) argue that the capital share should be about 0.6-0.8, to take account of human capital. As the capital share increases the production function approaches the linear case so that so there is less difference in the growth paths between the two phases.\(^\ddagger\) The higher capital share also results in a lower labour demand elasticity with respect to changes in technology. Thus with a higher capital share, the marginal and average products of capital rise more slowly as capital accumulates in phase 1.

The simulations thus indicate several desirable properties of the dual, or segmented, labour market model in accounting for stylised facts of growth. Specifically growth rates may be low when the capital stock is low, and a modest rate of exogenous technology change can produce the observed pattern of slow but accelerating growth as the average product of capital rises. This high growth phase, however, comes to an end eventually as demand for labour in the formal sector reaches sufficiently high levels. Further, during the high

\(^\dagger\) Similarly large scale technology investments, such as the U.S. investment in of Japan and Korea, may be seen has technology shifts which induce higher growth rate over phase 1.

\(^\ddagger\) Also when the capital share is higher, the opportunity cost of consumption is also higher, so that consumption tends to grow slowly and the economy has high savings and investment rates.
growth phase the marginal product of capital remains relatively constant and never exceeds 25 percent. Thus rapid growth is achieved through the persistence of moderate marginal product of capital, rather than extremely high but rapidly diminishing marginal product of capital.

The model is, however, highly stylised. The dualistic structure is an oversimplification of the many segmented labour markets formal sector as is the assumption of a constant reservation wage. A possible consequence of this is that while the model generates a growth slowdown after the informal sector labour supply is absorbed, Japan experienced a slowdown in growth in the 1970’s, about 10 years after Minami’s date for the tightening of labour markets. The period of rapid growth could be extended however if \( \bar{w} \) were endogenous. The construction of multi-sector general equilibrium growth models which endogenise the wage in the informal sector, is therefore an interesting area for further research.\(^{18}\)

5. Conclusion
The accelerated transition to a modern industrial economy eludes many LDC’s. In recent literature, beginning with seminal papers by Romer (1986, 1990), Lucas (1988, 1993) and Grossman and Helpman (1992), there has been considerable emphasis on the productivity changes such as knowledge diffusion and positive externalities. An important insight of the standard growth model, however, is that relatively capital scarce economies have high average and marginal products of capital which induce rapid capital accumulation. While the importance of this mechanism has recently been questioned, this paper shows that a high marginal products of capital does produce an empirically plausible growth pattern, if the economy has a dual

\(^{18}\) An alternative interpretation is that endogenous technology change may be relatively more important in the latter stages of a growth miracle, for example from 1960-1970 in Japan, 1980-1990 in Korea, whereas the early stages can be explained without appealing to large technology shifts.
labour market. High returns to capital are sustained by increased labour employed in the formal sector, resulting in persistently high growth rates for period of 15-30 years. This is in accordance with the early stages of growth in the economies that have recently achieved successful industrialisation and development such as South Korea and Japan.
Appendix: Parameter choices for numerical solutions

Using the Cobb-Douglas form of the production function (as in King and Rebelo (1993) and Barro and Sala-i-Martin(1995)) the formal sector output is,

$$\hat{y}(t) = A(t)k(t)^{\alpha}\hat{l}(t)^{(1-\alpha)}, \quad (A1)$$

and the labour demand equation is,

$$\tilde{l} = \left(\frac{(1-\alpha)A(t)}{\tilde{w}}\right)^\frac{1}{\alpha} k(t). \quad (A2)$$

Further, differentiating equation (10), the balanced path value of the capital stock at any time, $t$, is given by,

$$k(t)^* = \left(\frac{\alpha A(t)}{r^*}\right)^\frac{1}{(1-\alpha)}. \quad (A3)$$

where $r^*$ is the balanced path marginal product of capital and equal to $\rho + \sigma \kappa$, where $\kappa$ is the constant balanced path growth rate of consumption, and is equal to $\gamma(1-\alpha)$. In all simulations $r^*$ is set at 0.065, as in King and Rebelo (1993), and $\sigma$ was set at unity. The model was solved using actual labour force growth rates with an assumed long run growth rate of 0.05 percent.\(^\text{19}\)

For the standard solutions in Figure 3 it was assumed that the economy had reached a steady state in 1990. Using Summers and Heston’s (1991) estimate of GDP in that year and the relationship, $y^{*1-\alpha} = A(a/r^*)^{\alpha}$, from equations

\(^{19}\) The FORTRAN routines used for solving these problems is available from the author upon request.
(A1) and (A3), determines the value of $A(t)$, for $t = 1990$. The remaining values can be calculated as $A(t) = A(1990)e^{-\gamma(1990-t)}$. Given $A(0)$, $k(0)$, and $k^*$, the differential equation system (8b) and (8c) was integrated using finite difference method for the solution a the two point boundary value problem – the “relaxation method”, as described in Press et al (1990).

For the numerical solutions of dual labour market model require values of $\bar{w}$, as described above, and the initial marginal product of capital, given by $(y/k)^\alpha = A((1-\alpha) / \bar{w})^{1-\alpha}$. Given, $y$, then this expression determines $A$. Identical numerical procedures were used to solve the model in this case and the parameters of the production function and utility function were as for the standard model, with the same long run steady state growth path.

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Fig. 1. Post war economic growth in the developing regions, natural logarithm of GDP per worker, $1995 PPP. Source: Summers and Heston, (1991), Penn World Table, Mk 5.6. Region definitions: “North East Asia” is Taiwan, Hong Kong and China; “South Asia” is Bangladesh, India, Pakistan and Sri Lanka; “South East Asia” is Indonesia, Malaysia, Singapore, Philippines and Thailand. “Latin America” consists of countries 52, 55, 57, 58, 60-67, 71, 73-84 in the Summers and Heston (1991) country code. “Africa” consists of countries 1-50 in the Summers and Heston (1991) country code, excluding 13, Djibouti, and 41, South Africa.
Figure 2a: Phase Diagram when $g(A, \bar{w}) > \rho$.
Note: Diagram drawn for $\sigma = 1$ and $g = 0$.

Fig. 2b: Phase Diagram when $g(A, \bar{w}) < \rho$.
Note: Diagram drawn for $\sigma = 1$ and $g = 0$. 
Fig. 3. Japanese and Korean post war growth in the Ramsey model. Key: bold line, actual GDP, solid line, $\gamma = 0$; dot-dash line, $\gamma = 0.01$; dashed line, $\gamma = 0.02$; dotted line, $\gamma = 0.03$, where $\gamma$ is the exogenous rate of technology growth. Notes: Estimates based on an assumed steady state growth path with the steady state marginal product of capital, $r^*=0.065$, and actual labour force values from Summers and Heston (1991).
Fig. 4. Japanese and Korean post war growth in the dual, or segmented, labour market model. Key: bold line, actual GDP, solid line, $\gamma = 0$; dot-dash line, $\gamma = 0.01$; dashed line, $\gamma = 0.02$; dotted line, $\gamma = 0.03$, where $\gamma$ is the exogenous rate of technology growth. Notes: Estimates based on an assumed steady state growth path with the steady state marginal product of capital, $r^*=0.065$, and actual labour force values from Summers and Heston (1991).

Figures 4
Fig. 5. Sensitivity analysis of dual labour market model. Note: In all solutions unless, otherwise specified, the initial APK is 0.06, the capital share, $\alpha$, is 0.4, the rate of exogenous technology change, $\gamma$, is 0.02, and the reservation wage, $\bar{w}$, is $4000. In all solutions the steady state marginal product of capital is 0.065.