On Measuring Inequality in Taxation: A New Approach

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ON MEASURING INEQUITY IN TAXATION: A NEW APPROACH

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ABSTRACT

In this paper three axioms are given for the equitable income tax, and a measurement system is proposed according to which violation by an income tax system of each one of the axioms would exert a distinct negative influence on the redistributive effect of the tax. The three negative influences associated with axiom violations provide means to characterize the type of inequity in an income tax system and assess its extent. An application to the Australian income tax in 1984 suggests the extent of inequities in the Australian taxation system is very high, severely affecting the people in the bottom decile. The most serious inequity is due to the existence of substantial regressivity in the tax system.
The three axioms

Horizontal equity (HE) and vertical equity (VE) are two of the basic commands of social justice, requiring, respectively, the equal treatment of equals and the appropriately unequal treatment of unequals. These concepts have been applied to the income tax, raising a number of issues. There may be few or no equals in the real world - should we worry about these people? We virtually always deal with the unequals unequally - why is this so important a command? If the applicability of the basic HE and VE commands is open to question, then the measurement of their violations needs to be given clear and understandable significance. Here we define equity in income taxation rather carefully, by means of three axioms.

Let $x_1, x_2, ..., x_n$ be the pre-tax welfare levels of $n$ income units, who are paying $t_1, t_2, ..., t_n$ in tax (denominated in welfare units). For example, $x_i$ can be the per equivalent adult pre-tax income of household $i$, and $t_i$ the per equivalent adult tax payment. We now introduce three axioms:

**Axiom 1**

$$x_i \geq x_j \Rightarrow t_i \geq t_j$$

This says that tax should increase monotonically with respect to people's ability to pay. Moyes (1988) terms this requirement *minimal progression*. Note that because the inequalities are weak, HE is part of this axiom; it also permits the exemption from tax of those with the lowest incomes.

**Axiom 2**

$$x_i \geq x_j \text{ and } t_i \geq t_j \Rightarrow \frac{t_i}{x_i} \geq \frac{t_j}{x_j}$$

According to this axiom, the richer people must pay taxes at higher rates. This is the content of the familiar and much-researched *progressive principle*. See e.g. Blum and Kalven (1953), Blum (1979) and Bös and Felderer (1989) for discussion of a range of politically and economically relevant facets. Again, note the weak inequalities: proportional taxation is permitted by this axiom.

**Axiom 3**

$$x_i \geq x_j, t_i \geq t_j \text{ and } \frac{t_i}{x_i} \geq \frac{t_j}{x_j} \Rightarrow x_i - t_i \geq x_j - t_j$$

This says that a tax satisfying the other two axioms should in addition cause no reranking in people's living standards. Feldstein (1976) formulated the no reranking (NR) criterion. The HE command having been upheld by Axiom 1, and regressivity ruled out by Axioms 1 and 2 taken together, Axiom 3 can be seen as a vertical restriction, ruling out 'too much' progressivity, saying effectively that the marginal tax rate should not exceed 100 per cent (Lambert and Yitzhaki, 1995). This requirement has been termed *incentive preservation* (see e.g. Moyes, 1988). Because of the weak inequalities in Axiom 3, a 100 per cent tax rate is permitted.
If these axioms are applied to an income tax schedule, according to which a person with x pays tax T(x), then HE is automatically satisfied. Let a(x) \equiv T(x)/x be the average tax rate and \(\eta(x) \equiv xT'(x)/T(x)\) the tax elasticity in this case. The content of Axiom 1 becomes the restriction \(\eta(x) \geq 0\) \(\forall x\); Axiom 2 requires that if \(\eta(x) > 0\) then \(\eta(x) \geq 1\), \(\forall x\); and Axiom 3 requires that if \(\eta(x) \geq 1\) then \(\eta(x) < 1/a(x)\), \(\forall x\). An income tax schedule for which \(1 \leq \eta(x) < 1/a(x)\) satisfies all three axioms. Much of the literature on progressive income taxation is based on the assumption that the income tax is like this. See Fellman (1976), Jakobsson (1976) and Lambert (1993, chapters 6-7), for example.

For a general tax system, Axioms 1-3 are lexicographic. By this, we mean that Axiom \(\kappa\) \((\kappa = 2,3)\) is silent for income unit pairs \{i,j\} for which Axiom \(\kappa-1\) is violated. Let \(X, T, A=T/X\) and \(D=X-T\) stand for pre-tax income or living standard, tax, average tax rate and disposable income respectively. Then:

- Axiom 1 is violated iff the rankings of income units by \(X\) and by \(T\) differ.
- Axiom 2 is violated iff the rankings by \(X\) and by \(A = T/X\) of income unit pairs \{i,j\} for which Axiom 1 holds differ.
- Axiom 3 is violated iff the rankings by \(X\) and by \(D = X-T\) of income unit pairs \{i,j\} for which Axiom 2 holds differ.

In order to detect axiom violations, we need to measure the extent of difference between the various rankings (by \(X, T, A, D\)) which are rated as normatively significant in the axioms.

**A proposed measurement system**

We advocate the use of concentration coefficients for the examination of rank differences (Kakwani, 1977a). This provides a straightforward check of Axiom 1: it will be violated if and only if the concentration coefficient for tax payments, lined up by ascending order of pre-tax income, is bigger than the Gini coefficient for tax payments. (We give the formal definitions in a moment).

For Axioms 2 and 3, matters are more complicated, because income unit pairs \{i,j\} for which Axiom 1 fails cannot provide violations of Axioms 2 and 3. If we want to see whether Axiom 2 holds [or is violated], we must look among the subset of pairs \{i,j\} for which the ranking by \(X\) and by \(T\) is the same, and ask whether, for those income units, the ranking by \(A\) concurs, [or not]. Roughly, then, to determine whether Axiom 2 *per se* is upheld [or violated], in the overall population we need to know whether the difference between the rankings by \(X\) and by \(T\) is more [or less] extensive than that between the rankings by \(X\) and by \(A\).

Income unit pairs \{i,j\} for which Axiom 2 fails cannot provide violations of Axiom 3. If we want to see if Axiom 3 holds [or is violated], we must look at pairs \{i,j\} for which Axiom 1 succeeds and check, among them, whether, loosely, the difference between the rankings by \(X\) and by \(A\) is more [or less] extensive than that between the ranking by \(X\) and by \(D\).

These ‘informal’ comparisons can, we suggest, be resolved by the use of concentration coefficients. Let \(L_{zpX}(p)\) be the concentration curve for an attribute \(Z\) of income units, when
ordered by pre-tax income $X$, and denote by $L_Z(p)$ the Lorenz curve for $Z$ (i.e. $L_Z(p) \equiv L_{Z|p}(p)$, $0 \leq p \leq 1$). A fundamental property is that

$$(1) \quad L_Z(p) \geq L_Z(p) \forall p$$

with equality everywhere if and only if the rankings of income units by $X$ and by $Z$ are the same: see Kakwani (1997a) and Lambert (1993, §2.5) on this. Now let $C_{Z|X} = 1 - 2\int_0^1 L_{Z|X}(p) dp$ be the concentration index for $Z$ with respect to $X$, and let $G_Z = 1 - 2\int_0^1 L_Z(p) dp = C_{Z|Z}$ be the Gini coefficient for $Z$.

We propose to measure the difference between the rankings of income units by $X$ and by $Z$ as:

$$(2) \quad R_Z = G_Z - C_{Z|X} \geq 0$$

(with no difference if and only if $R_Z = 0$), and to check the three axioms as follows:

- for Axiom 1, we check $R_T$: if zero [positive], Axiom 1 is upheld [violated];
- for Axiom 2, we check $R_A - R_T$: if zero [positive], this suggests that Axiom 2 is upheld [violated]. On the face of it, $R_A - R_T$ could also go negative. In fact, we have found it always to be non-negative in extensive simulations. Similarly, we have found that the rank correlation between $A$ and $X$ always exceeds that between $T$ and $X$. The reason is not hard to see, intuitively, for:

\[ x_1 > x_2 \text{ and } \frac{t_1}{x_1} > \frac{t_2}{x_2} \Rightarrow t_1 > t_2 \]

i.e. the difference between the $X$- and $T$-rankings cannot be more extensive than that between the $X$- and $A$-rankings: if the $X$- and $A$-rankings agree, so, a fortiori, does the $T$-ranking. Moreover, if the $T$- and $A$-rankings differ, the $X$-ranking agrees with the $T$-ranking:

\[ \frac{t_1}{x_1} < \frac{t_2}{x_2} \text{ and } t_1 > t_2 \Rightarrow x_1 > x_2 \]

adding further strength to our supposition that $R_A \geq R_T$ in general. These observations suggest that $R_A - R_T$ provides the check for Axiom 2, just as $R_T$ does for Axiom 1: if zero [positive], the axiom is upheld [violated];

- for Axiom 3, our check ought, on the face of it, to take into account the relative values of all three measures $R_T$, $R_A$ and $R_D$. After all, we have to determine whether reranking takes place among income unit pairs $\{i,j\}$ for which Axioms 1 and 2 are upheld. $R_D > 0$ if and only if the command $NR$ is violated, but is $R_D > 0$ consistent with Axiom 3? That is, can we have $0 \leq R_A = R_T$ (so that Axiom 2 is satisfied, whether or not Axiom 1 is), $R_D > 0$ and yet no reranking among income unit pairs experiencing minimal progression or progression proper? The answer is no, for:

\[ x_1 > x_2 \text{ and } t_1 < t_2 \Rightarrow x_1 - t_1 > x_2 - t_2 \]

i.e. among income unit pairs $\{i,j\}$ for which Axiom 1 fails, reranking cannot occur, and:

\[ x_1 > x_2 \text{ and } \frac{t_1}{x_1} < \frac{t_2}{x_2} \Rightarrow x_1 - t_1 > x_2 - t_2 \]
i.e. reranking cannot either occur among income unit pairs \( \{i,j\} \) for which Axiom 1 succeeds but Axiom 2 fails. Hence \( R_D \) is the appropriate 'test statistic' for Axiom 3: if zero [positive], Axiom 3 is upheld [violated].

In sum, we have developed here three test statistics, one for each axiom, let us call them (with slight modification for cosmetic purposes) \( S_1, S_2, S_3 \):

\[
S_1 = \tau R_T, \quad S_2 = \tau [R_A - R_T], \quad S_3 = R_D
\]

where \( \tau = \sum T / \sum D \) is total tax divided by total disposable income. Each of these measures is either zero, meaning that the axiom is upheld, or positive, meaning that it is violated.

**Redistributive effect**

Let \( \Delta G = G_X - G_D \) be the redistributive effect of the income tax system. The Kakwani (1984) decomposition can be written:

\[
\Delta G = \tau P - R_D
\]

where \( \tau \) is as above and \( P \) is the Kakwani (1977b) progressivity index:

\[
P = C_{TK} - G_X
\]

measuring the extent of disproportionality in tax payments. In fact, and in light of our proposals, (4) can be further decomposed, as:

\[
\Delta G = \tau [P + R_A] - S_1 - S_2 - S_3
\]

(this is trivial to verify).

Hence not only do \( S_1 - S_3 \) provide easy checks on Axioms 1-3, but also their magnitudes tell us the quantitative significance of these departures from equity, as subtractions from (losses of) redistributive effect. To check for the presence and significance of departures from the \( NR \) criterion, one looks at \( R_D \). If \( R_D = 0 \), \( NR \) holds. If \( R_D > 0 \), \( NR \) fails, providing a subtraction from redistributive effect. This was established in Kakwani (1984), but the present decomposition goes beyond that. All three sources of inequity cause losses of redistributive effect whose significance is told by the value of \( S_1, S_2, S_3 \).

In the decomposition in (6), the first term, \( \tau (P + R_A) \) is non-negative contributing to reduction in income inequality. This term may be called ‘the measure of equity’ in taxation. It is interesting to note that if Axiom 2 holds, so that the progressive principle is upheld and \( R_A = R_T \), then combining with (15) the measure of equity in taxation becomes \( \tau (G_T - G_X) \), which is the product of the tax level and the normatively significant excess inequality in taxes over that in incomes. The inequity in taxation is defined in terms of the violation of the axioms proposed in the paper. So, the sum of three terms, viz. -S1, -S2 and -S3 may be called the ‘measure of inequity’ in taxation. The total redistributive effect of taxation is the net effect of the two measures. The decomposition in (6) also provides methodology to quantify each type of inequity in taxation.

From the following decomposition of the Lorenz difference \( L_D(p) - L_A(p) \) which measures the share of income redistribution to the 100% poorest implicitly by the tax system
relative to a proportional tax with the same yield (see Lambert 1996), the effects of equities and inequities in the tax system on decile shares can be estimated from the following decomposition:

\[
L_D(p) - L_X(p) = \tau[L_A(p) - L_{A'I}(p) + L_{A'I}(p) - L_A(p)]
\]

\[
-\tau[L_{A'I}(p) - L_T(p)] - \tau[L_{A'I}(p) - L_A(p) - L_{A'I}(p) + L_T(p)] - [L_{D'I}(p) - L_D(p)]
\]

In this equation, the first term measures ‘the equity effect’ and the second, third and fourth terms measure the effects of inequities (on decile shares) due to the violation of Axioms 1, 2 and 3, respectively. Multiplying (7) by 2 and integrating from \( p = 0 \) to \( p = 1 \) gives (7).

It should be noted that the inequity terms need not be negative for all \( p \); if any is positive, then we may say that the \( p \)th percentile (or decile group) benefits (gains in income share) from the associated inequity.

**The Australian income tax, 1984**

The methodology developed in the previous sections will now be applied to quantify inequities in the Australian Taxation system. We use the Australian Household Expenditure Survey (HES) for 1984. This is a nationwide survey consisting of 4492 sample households. To measure inequity in taxation we need to measure the economic welfare of each individual in the society. This is done by means of per equivalent adult income. The main components of income are wage and salary, government cash benefits, workers compensation, interest income, investment, property income, alimony, scholarships, regular payments from superannuation, other regular income and income from self employment.

We used an equivalence scale in the calculations as follows: Let \( a \) be the number of adults in the household, \( c_1 \) the number of children aged 5 years or less, \( c_2 \) the number of children between 6 and 14 years and \( c_3 \) the number of children between 15 and 17 years, the number \( N \) of “equivalent adults” in the household is determined as:

\[
N = (a + 2c_1 + 1.4c_2 + 7c_3)^{.8} + .1w
\]

where \( w \) is the number of working adults in the household. The exponent of .8 is used as a power to take account of economies of scale within the households. Since working persons have to incur additional costs such as transport, clothing and baby sitting, we have added 10 percent cost to the household for each working person. Household income as well as the total taxes paid by the household were divided by \( N \) to form per equivalent adult distribution of income and tax.

The various measures of inequity and equity were computed using the individual household data. The results are presented in Table 1.

The before and after tax Gini indices were computed to be 31.13 and 28.73, respectively. It means that the Australian income tax system reduces the inequality of income by 2.4 percentage points. This is the total redistributive effect of taxes. The tax system increases the income share of the bottom 50 percent of the population (first five deciles) and decreases the income share of the top 50 percent of the population. The second decile receives the largest
increase in its share. The inequity due to the violation of Axiom 3 contributes to a reduction of the redistributive effect of taxes by 0.72 percentage points. The inequities due to violation of Axioms 1 and 2 contribute to reduction of redistributive effect by 1.94 and 8.76 percentage points. Thus, the violation of Axiom 2 has the most serious impact on the redistributive effect. This is a striking finding suggesting the existence of substantial regressivity in the tax system which occurs due to non-equivalent-income-based differences in family tax treatment. The total inequity in the Australian tax system reduces the redistributive effect of taxation by 11.42 percentage point. It reduces the share of the bottom decile by 4.29 percentage points and increases the share of top decile by 5.54 percentage points. This suggests that the Australian tax system embodies significant inequities. If these inequities were not present, the Australian tax system would have reduced the inequality of income by 13.82 percentage points (instead of only 2.4 percentage points). Thus, the removal of inequities can substantially improve the redistributive effect of taxation without increasing the marginal tax rates on higher income groups.

Table 1: Equities and Inequities in Australian Taxation 1984

<table>
<thead>
<tr>
<th>Deciles</th>
<th>Decile Shares</th>
<th>Redistributive effect of Taxes</th>
<th>Inequities due to violation of Axiom</th>
<th>Total inequity</th>
<th>Total equity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before tax Income</td>
<td>After tax Income</td>
<td>Axiom 1</td>
<td>Axiom 2</td>
<td>Axiom 3</td>
</tr>
<tr>
<td>1</td>
<td>3.13</td>
<td>3.30</td>
<td>0.17</td>
<td>-0.29</td>
<td>-3.77</td>
</tr>
<tr>
<td>2</td>
<td>4.44</td>
<td>5.07</td>
<td>0.63</td>
<td>-0.22</td>
<td>-0.29</td>
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<tr>
<td>3</td>
<td>5.48</td>
<td>6.00</td>
<td>0.52</td>
<td>-0.40</td>
<td>-0.41</td>
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<tr>
<td>4</td>
<td>6.89</td>
<td>7.13</td>
<td>0.24</td>
<td>-0.20</td>
<td>-0.26</td>
</tr>
<tr>
<td>5</td>
<td>8.22</td>
<td>8.41</td>
<td>0.19</td>
<td>-0.18</td>
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<tr>
<td>6</td>
<td>9.72</td>
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<td>-0.00</td>
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<tr>
<td>7</td>
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<td>11.05</td>
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<td>0.09</td>
</tr>
<tr>
<td>8</td>
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<td>9</td>
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<td>14.96</td>
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<td>0.43</td>
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<td>10</td>
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<td>21.70</td>
<td>-1.31</td>
<td>0.92</td>
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<td>Total</td>
<td>31.13</td>
<td>28.73</td>
<td>2.4</td>
<td>-1.94</td>
<td>-8.76</td>
</tr>
<tr>
<td>%</td>
<td>-</td>
<td>-</td>
<td>7.71</td>
<td>-6.24</td>
<td>-28.14</td>
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</table>
Conclusion

In this paper we have delineated three distinct types of inequity which may be present in an income tax system by means of axioms. Rank orderings lie at the heart of the matter. According to our axioms, for equity the orderings of income units by their tax payments, by their average tax rates and by their final incomes should all be the same as by their pre-tax incomes. Violations of these three requirements provide the three sources of inequity. We have also proposed a measurement system, using which the inequities in a tax system can be identified by means of summary statistics and their importance assessed. Specifically, we have shown that the redistributive effect of the tax system can be decomposed into an equity component and three inequity components, the latter providing subtractions. In Australia in 1984, the most important inequity proves to be regressivity in the income tax system caused by family tax treatment that is not equivalent-income-based. More generally, the summary indicators which are thrown up by our methodology can be used to make informative comparisons of tax systems between countries or regions or demographic groupings, or over time, and can also guide reform: as we have demonstrated, the removal of significant inequities can improve the redistributive effect of the tax system without change to the marginal rate structure which governs incentives.
References


