Welfare-based Approaches to Measuring Real Economic Growth with Application to Thailand

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DISCUSSION PAPER
WELFARE-BASED APPROACHES TO
MEASURING REAL ECONOMIC GROWTH
WITH APPLICATION TO THAILAND

by

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Abstract

This paper is concerned with the measurement of the growth rate of real income. The cost function derived from utility maximization is utilized to define this rate. It is demonstrated that conventionally used measures violate some intuitively natural axioms. A new approach is proposed which satisfies all these axioms. An attempt is also made to make the proposed approach empirically operational.

The basic methodology of this paper based on a one person economy has also been extended to an economy with many consumers. So the measurement of real income growth rate takes into account the changes in income inequality which is an important constituent of people’s welfare. This methodology applied to the Thailand Socio-economic survey data showed that the changes in relative prices in the periods 1988-90 and 1990-92 have adversely affected the poor more than the rich.

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1. Introduction

The measurement of changes in real income plays an important role in applied welfare economics. It is clearly important to know whether and to what extent people are becoming better or worse-off over time. Not surprisingly therefore, magnitudes of growth rates are always the focus of attention not only among economists but also politicians and economic policy makers.

The present paper is concerned with the measurement of growth rates of real income. These growth rates must be related to the change in welfare of an individual or a society between two periods. In practice we observe only the nominal incomes or expenditures of individuals over time.

To compute the growth rate of real income we must separate the effect of changes in prices which influences the growth rate of nominal income.1 Two methods which are conventionally used to tackle this problem are the compensating variation (CV) and the equivalent variation (EV) (Hicks (1946)). In the present paper we demonstrate that these approaches violate some intuitively natural axioms. We propose an alternative approach which always satisfies these axioms.

Our proposed measures of inflation and real income growth rates are defined in terms of a general cost function which is derived from the notion of utility maximization. To make these measures empirically operational, we need to consider specific cost functions. We demonstrate that Fisher’s ideal quantity and price indices are in fact exact (or true) real income growth and inflation rates respectively if the cost function is assumed to be linear in prices. We provide a strong justification for using Tornquist’s

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1 For an excellent discussion of this issue see Usher (1980). Diewert (1980) has provided the most thorough survey of the theoretical results available on this issue.
price index for measuring the inflation rate. It is argued, however, that Tornquist’s quantity index is not appropriate to measure the real income growth rate.

The usual methods of measuring inflation and real income growth rates assume a one person economy. These methods are completely insensitive to changes in income inequality. The methodology of this paper has been extended to a many persons economy. The real income growth then measures the change in social welfare between two periods. This methodology, based on bi-period comparisons, is further extended to the situation of multiperiod comparisons.

The methodology developed in the paper has been applied to the Thailand data to measure the inflation and real income growth rates in the periods 1988-90 and 1990-92.

2. Axioms

Suppose \( q \) is the quantity vector which provides utility to a representative consumer. The conventional assumption in that the consumer maximizes a utility function \( u = u(q) \) subject to a budget constraint \( p'q = x \), where \( p \geq 0 \) is a positive vector of commodity prices and \( x \) is expenditure on the \( n \) commodities. The solution to this maximization problem yields a system of \( n \) Marshallian demand equations \( q = q(x, p) \).

Maximum attainable welfare is then obtained by substituting these demand equations into the utility function \( u(q) \). This gives the indirect utility function

\[
u = u[q(x, p)] = \psi(x, p)
\]

Solving (1) for \( x \) gives the cost function

\[
x = e(u, p)
\]

which is the minimum cost of attaining a level of utility \( u \) at the price vector \( p \).
If the utility function \( u(q) \) is continuous, increasing and concave in \( q \), then the cost function has the following properties

(i) \( e(u, p) \) is increasing in \( u \) for every \( p \).

(ii) \( e(u, p) \) is increasing and concave in \( p \).

(iii) \( e(u, p) \) is (positively) linearly homogeneous in \( p \) for fixed \( u \), i.e.,

\[
e(u, \lambda p) = \lambda e(u, p) \quad \text{for} \quad \lambda > 0.
\]

The cost function will be the basis for measuring the real economic growth and inflation rates.

Suppose we want to measure economic growth between periods 1 (base year) and 2 (terminal year). Let \( p_1 \) and \( p_2 \) be the price vectors and \( u_1 \) and \( u_2 \), the utility levels enjoyed by a representative consumer in periods 1 and 2, respectively, then following Samuelson and Swamy (1974), \( e(u_1, p_1) \) and \( e(u_2, p_2) \) will be the money metric utility levels enjoyed by the consumer in periods 1 and 2, respectively. Suppose \( m_{12} \) is the growth rate of the money metric utility levels from period 1 to period 2, then

\[
\log(1 + m_{12}) = \log e(u_2, p_2) - \log e(u_1, p_1) \quad (3)
\]

Note that \( m_{12} \) is not the real economic growth rate because it includes the effect of price changes as well as the effect of change in the utility level. The major issue then is how to separate the two effects. \( m_{12} \) may be called the nominal growth rate. Suppose \( r_{12} \) is the real income growth rate and \( \varepsilon_{12} \) is the inflation rate (which is the growth rate of nominal income attributed entirely to price changes), then \( m_{12} \) will be some function of \( r_{12} \) and \( \varepsilon_{12} \):

\[
m_{12} = f(r_{12}, \varepsilon_{12}) \quad . \quad (4)
\]

We may now propose the following axioms.
**Axiom 1:** If $\epsilon_{12} = 0$, $m_{12} = r_{12}$ and if $r_{12} = 0$, $m_{12} = \epsilon_{12}$.

This axiom is intuitively natural. It implies that if the inflation rate is zero, the nominal growth rate must be equal to the real growth rate and similarly, if the real growth rate is zero, the growth rate in nominal income must be equal to the inflation rate (because the nominal income has grown only because of changes in prices). This axiom immediately leads to the following weaker axiom.

**Axiom 1A:** If both $r_{12}$ and $\epsilon_{12}$ are equal to zero, then $m_{12}$ must also be equal to zero.

From (3), we note that

$$\log(1 + m_{12}) = -\log(1 + m_{21})$$

where $m_{21}$ is the growth rate of nominal income going from period 2 to period 1, always holds. This leads us to propose the following two axioms.

**Axiom 2:** $\log(1 + \epsilon_{12}) = -\log(1 + \epsilon_{21})$

**Axiom 3:** $\log(1 + r_{12}) = -\log(1 + r_{21})$

where $\epsilon_{21}$ and $r_{21}$ are the inflation and real economic growth rates when going from period 2 to period 1, respectively.

Suppose $y_1$ and $y_2$ are measures of real income in the base and terminal year, respectively, then obviously

$$\log(1 + r_{12}) = \log y_2 - \log y_1$$

which immediately gives

$$\log(1 + r_{21}) = \log y_1 - \log y_2 .$$

These two equations imply Axiom 3. Thus, Axiom 3 will always hold if we assume that there exists a measure of real income in each period. Similarly, Axiom 2 will always hold if we assume that there exists a measure of price level (a price index) in each period. These assumptions are intuitively natural and, therefore, Axioms 2 and 3 are desirable.
Suppose $x_t$ and $P_t$ are the nominal income and aggregate price index in year $t$, respectively, then one should be able to determine the real income $y_t$ in year $t$ by dividing $x_t$ by $P_t$, i.e., $y_t = x_t/P_t$ which immediately gives

$$\log y_2 - \log y_1 = (\log x_2 - \log x_1) - (\log P_2 - \log P_1)$$

This leads to the following axiom:\textsuperscript{2}

\textbf{Axiom 4:} \quad \log(1 + m_{12}) = \log(1 + r_{12}) + \log(1 + \epsilon_{12}) \tag{5}

Note that Axiom 4 will always imply 1 and 1A.

Suppose $q_1$ and $q_2$ are the quantity vectors in periods 1 and 2, respectively, then $u_1 = u(q_1)$ and $u_2 = u(q_2)$, which means that if $q_1 = q_2$, $u_1 = u_2$. It is obvious that if $u_1 = u_2$, $r_{12} = 0$. This gives the following axiom.

\textbf{Axiom 5:} \quad If $q_1 = q_2$, $r_{12} = 0$.

Let us assume that relative prices have not changed between periods 1 and 2, so that $p_2 = (1 + \lambda)p_1$. It is then obvious that $\lambda$ will be the inflation rate. This results in the following Axiom.

\textbf{Axiom 6:} \quad If $p_2 = (1 + \lambda)p_1$, $\epsilon_{12} = \lambda$.

\section*{3. Measures Based on the Equivalent and Compensating Variations}

The equivalent and compensating variations measure the welfare changes in terms of differences in the money value of utility levels in two different periods. We use the same notion of EV and CV to measure the growth rates of real income.

\textsuperscript{2} Axiom 4 corresponds to the weak factor reversal test (Samuelson and Swamy 1974).
The Equivalent Variation

Let \( x_1 = e(u_1, p_1) \) be the initial income of the consumer. The growth rate under the EV may be defined as the percentage of income that must be added to \( x_1 \) in the first period in order to give the consumer a utility level of \( u_2 \) in the second period. Thus, the real growth rate under the EV will be given by

\[
\log(1 + r_{12}) = \log e(u_2, p_1) - \log x_1
\]

where \((1 + r_{12})\) is the Paache-Konüs implicit quantity index (Diewert 1980).

Similarly, the inflation rate under the EV may be defined as

\[
\log(1 + \varepsilon_{12}) = \log x_2 - \log e(u_2, p_1)
\]

where \( x_2 = e(u_2, p_2) \) is the income in period 2 and \((1 + \varepsilon_{12})\) is the Paache-Konüs price index (Diewert 1980).

Combining (6) and (7), we obtain (5) which demonstrates that Axioms 1, 1A and (4) are always satisfied by the real income growth and inflation rates defined in (6) and (7), respectively. It is easy to see that Axioms 5 and 6 are also satisfied. It should be noted, however, that the real income growth rate is derived by keeping the base year prices constant whereas the inflation rate is derived by keeping the terminal year utility constant; and the reverse is the case when we measure growth rates going from terminal year to the base year. Thus, there is some inconsistency with respect to the choice of reference period utility level and prices.

Under the EV, \( r_{21} \) is defined as

\[
\log(1 + r_{21}) = \log e(u_1, p_2) - \log x_2
\]

which, combining (8) with (6), demonstrates that Axiom 3 is violated. Similarly, the inflation rate \( \varepsilon_{21} \) under the EV is given by
which combining with (7) demonstrates that Axiom 2 is violated.

**The Compensating Variation**

The real income growth under the compensating variation is defined as the percentage of income that must be taken away from income $x_2$ in period 2 in order that the consumer is back to the utility level $u_1$ (in period 1). Thus, the real growth rate under the CV is given by

$$\log(1 + \epsilon_{21}) = \log x_1 - \log e(u_1, p_2)$$

which combining with (7) demonstrates that Axiom 2 is violated.

$$\log(1 + e_{21}) = \log x_1 - \log e(u_1, p_2)$$

noting that $(1 + r_{12})$ is the Laspeyres-Konüs implicit quantity index (Diewert 1980).

Similarly, the inflation rate under the CV is defined as

$$\log(1 + \epsilon_{12}) = \log e(u_1, p_2) - \log e(u_1, p_1)$$

Combining (10) and (11) leads to (5), which shows that Axioms 1, 1A, 4 are always satisfied. It is not difficult to see that Axioms 5 and 6 are also satisfied. Further, it is easy to see that $r_{12}$ defined in (10) violates Axiom 3 and $\epsilon_{12}$ defined in (11) violates Axiom 2. This approach also suffers from inconsistency with respect to the reference period’s utility level and prices.

4. A Solution

The EV computes the real growth rate by the percentage change in money metric utility at base year prices, whereas the CV computes it at the terminal year prices. Whether or not one should take base year or terminal year prices is an issue which cannot easily be resolved. We take an agnostic view, that is, the base year prices are not better or worse than the terminal year prices. We define a new measure of the real income growth
rate which is symmetric with respect to the base and terminal years. This measure, denoted by \( r_{12}^* \) is given by

\[
\log(1 + r_{12}^*) = \frac{1}{2} \left[ \log(e(u_2, p_1)) - \log(x_1) + \log(x_2) - \log(e(u_1, p_2)) \right]
\]  
(12)

Following the same logic, we define the inflation rate as

\[
\log(1 + \varepsilon_{12}^*) = \frac{1}{2} \left[ \log(e(u_2, p_1)) - \log(x_1) + \log(x_2) - \log(e(u_2, p_1)) \right]
\]  
(13)

Combining (12) and (13) leads to (5), which ensures that Axioms 1, 1A and 4 are satisfied. It is not difficult to see that \( r_{12}^* \) and \( \varepsilon_{12}^* \) also satisfy axioms 2 and 3 respectively.

Thus, the proposed measures of real income growth and inflation rates satisfy all the axioms proposed in the paper and are consistent with respect to the choice of reference period utility and prices.

**Homothetic Preferences**

When preferences are homothetic, one can write the cost function as

\[ e(u, p) = a(p)c(u) \]

where \( a(p) \) is a homogeneous function of degree 1 and \( c(u) \) is an increasing function of \( u \), then (12) and (13) can be written as

\[
\log(1 + r_{12}^*) = \log(c(u_2)) - \log(c(u_1))
\]  
(14)

and

\[
\log(1 + \varepsilon_{12}^*) = \log(a(p_2)) - \log(a(p_1))
\]  
(15)

respectively. It can easily be seen that exactly the same expressions for \( r_{12}^* \) and \( \varepsilon_{12}^* \) are obtained when we apply the homothetic preferences on the EV and CV measures. Thus, we have demonstrated that the three approaches give identical results when the utility function is assumed to be homothetic.
Constant Relative Prices

Let us now assume that relative prices have not changed between the base and terminal years, then we have \( \mathbf{p}_2 = (1 + \lambda) \mathbf{p}_1 \). It can be easily verified that all three approaches give \( \epsilon_{12} = \lambda \) and \( r_{12} = \frac{x_2}{x_1(1 + \lambda)} - 1 \), which demonstrates that the three approaches give identical results when there is no change in relative prices. Since the relative prices always change, the three approaches will give different results.

5. Specific Cost Functions

To make our measures of real economic growth and inflation rates operational we need to consider specific cost functions.

Linear Cost Function

Let us consider the cost function

\[
e(u, \mathbf{p}) = a' \mathbf{p} + (b' \mathbf{p})c(u)
\]

(16)

where \( a \) and \( b \) are \( n \times 1 \) vectors of constants and \( c(u) \) is an increasing function of \( u \).

Shephard’s (1970) lemma states that

\[
\frac{\partial c(u, \mathbf{p})}{\partial \mathbf{p}} = \mathbf{q}(u, \mathbf{p})
\]

where \( \mathbf{q}(u, \mathbf{p}) \) is the quantity vector. Applying this lemma to (16) gives

\[
\mathbf{q}(u, \mathbf{p}) = a + bc(u)
\]

which immediately gives

\[
e(u_2, \mathbf{p}_1) = \mathbf{p}_1 \mathbf{q}_2
\]

(17)
and
\[ e(u_1, p_2) = p_2 q_1 \]  \hspace{1cm} (18)

Substituting (17) and (18) into our proposed real income growth and inflation rates gives
\[ \log(1 + r_{12}^*) = \frac{1}{2} \left[ \log q_2' p_1 - \log q_1 p_1 + \log q_2 p_2 - \log q_1 p_2 \right] \]  \hspace{1cm} (19)

and
\[ \log(1 + \varepsilon_{12}^*) = \frac{1}{2} \left[ \log p_2' q_1 - \log p_1' q_1 + \log q_2 p_2 - \log q_2 p_1 \right] \]  \hspace{1cm} (20)

Note that \( r_{12}^* \) and \( \varepsilon_{12}^* \) in (19) and (20) respectively are equivalent to Fisher’s ideal quantity and price indices, respectively. Thus, we have demonstrated that real income growth and inflation rates, based on Fisher’s ideal quantity and price indices, respectively are in fact exact (or true) real income growth and inflation rates provided the cost function is assumed to be linear in prices.

**Homothetic Quadratic Cost Function**

A homothetic quadratic cost function can be written
\[ e(u, p) = (p' A p)^{1/2} c(u) \]  \hspace{1cm} (21)

where \( c(u) \) is an increasing function of \( u \) and \( A \) is a symmetric matrix of constants.

Applying Shephard’s lemma to (21) gives
\[ q = \frac{c(u) A p}{\sqrt{p' A p}} \]

which yields
\[ \log x_1 = \log c(u_1) + \frac{1}{2} \log(p_1' A p_1) \]
\[ \log x_2 = \log c(u_2) + \frac{1}{2} \log(p_2' A p_2) \]

\[ \log p'_1 q_2 = \log c(u_2) + \log(p'_1 A p_2) - \frac{1}{2} \log(p'_2 A p_2) \]

\[ \log p'_2 q_1 = \log c(u_1) + \log(p'_2 A p_1) - \frac{1}{2} \log(p'_1 A p_1) . \]

Substituting these four equations into (14) and (15) yields to (19) and (20), respectively. This demonstrates that our proposed measures of real income growth and inflation rates lead to Fisher’s ideal real income growth and inflation rates when the cost function is homothetic quadratic.

**Muellbauer’s PIGLOG Cost Function**

In 1975, Muellbauer proposed the price independent generalized logarithmic model (PIGLOG), the cost function of which can be written as

\[ \log e(u, p) = a(p) + b(p) u \]  \hspace{1cm} (22)

where \( a(p) \) and \( b(p) \) are given by

\[ a(p) = \alpha_0 + \alpha' \log p + \frac{1}{2} (\log p)' A (\log p) \]

\[ b(p) = \beta_0 + \beta' \log p \]

where \( \alpha \) and \( \beta \) are the \( n \times 1 \) column vectors of constants and \( A \) is the \( n \times n \) matrix of constants such that \( \alpha' t = 1, \beta' t = A t = 0 \) and \( A = A' \) (i.e., \( A \) is symmetric); \( t \) being the \( n \times 1 \) vector whose all elements are equal to 1.

Applying the cost function in (22) to our proposed measures gives an inflation rate

\[ \log(1 + \epsilon^*_u) = \alpha' D p + (\log p^*)' A D p + \eta \beta' D p \]  \hspace{1cm} (23)

where \( D p = \log p_2 - \log p_1 \), and \( 2 \log p^* = \log p_1 + \log p_2 \) and \( 2 \eta = u_1 + u_2 \).
Applying Shephard’s lemma on (22) gives the vector of budget shares
\[ \mathbf{w} = \alpha + A(\log \mathbf{p}) + \beta \mathbf{u} . \]

Suppose \( \mathbf{w}_1 \) and \( \mathbf{w}_2 \) are the vectors of budget shares in periods 1 and 2, respectively, then the vector of average budget share defined by
\[ \mathbf{w} = \frac{1}{2}(\mathbf{w}_1 + \mathbf{w}_2) \]
will be given by
\[ \mathbf{w} = \alpha + A(\log \mathbf{p}^*) + \beta \mathbf{u}^*. \]

From (23) it can be seen that
\[ \log(1 + \varepsilon_{12}^*) = \mathbf{w}^T \mathbf{Dp} \] (24)
which immediately gives
\[ \log(1 + r_{12}^*) = \log x_2 - \log x_1 - \mathbf{w}^T \mathbf{Dp} \] (25)

Note that \( 1 + \varepsilon_{12}^* \) in (24) is Tornquist’s (1936) price index. Thus, we have demonstrated that for the PIGLOG cost function, our proposed measure of inflation rate corresponds exactly to Tornquist’s price index.

The Translog Cost Function

A second order approximation to a general cost function \( e(u, \mathbf{p}) \) is written as
\[ \log e(u, \mathbf{p}) = a_0 + a^\top \log \mathbf{p} + \frac{1}{2} (\log \mathbf{p})^\top A(\log \mathbf{p}) + \beta \log \mathbf{u} + \alpha (\log \mathbf{u})^2 + \frac{1}{2} (b^\top \log \mathbf{p}) \log \mathbf{u} \] (26)

where \( a \) and \( b \) are \( n \times 1 \) column vectors of constants and \( A \) is an \( n \times n \) symmetric matrix such that \( a^\top = 1 \) and \( A_1 = 0 \).

With some algebraic manipulations, we could demonstrate that
\[
\log e(u_2, p_1) - \log e(u_1, p_1) = Du \left[ \beta + 2\alpha \log u^* + \frac{1}{2} \beta' \log p_1 \right]
\] (27)

and

\[
\log e(u_2, p_2) - \log e(u_1, p_2) = Du \left[ \beta + 2\alpha \log u^* + \frac{1}{2} \beta' \log p_2 \right]
\] (28)

where \( Du = \log u_2 - \log u_1 \) and \( 2 \log u^* = \log u_1 + \log u_2 \). Combining these two equations, our proposed real income growth rates for the translog cost function give

\[
\log (1 + r^*_i) = Du \left[ \beta + 2\alpha \log u^* + \frac{1}{2} \beta' \log p^* \right]
\] (29)

where \( 2 \log p^* = \log p_1 + \log p_2 \).

Performing similar manipulations, we can demonstrate that the proposed inflation rate \( \varepsilon^*_i \) is given by

\[
\log (1 + \varepsilon^*_i) = a \cdot Dp + (\log p^*) \cdot ADp + \frac{1}{2} (b' \cdot Dp) \log u^*
\] (30)

where \( Dp = \log p_2 - \log p_1 \).

Finally, we demonstrated that the nominal growth rate given in (3) for the translog cost function is given by

\[
\log (1 + m_{12}) = \left( a + A \log p^* + \frac{1}{2} \log u^* b \right) Dp + \left( \beta + 2\alpha \log u^* + \frac{1}{2} \beta' \log p^* \right) Du
\] (31)

in which the first term measures the inflation rate and the second term measures the real income growth rate.

Applying Shephard’s Lemma to (26) gives

\[
\bar{\omega} = a + A \log p^* + \frac{1}{2} \log u^* \cdot b
\] (32)
where $\mathbf{w}$ is the $n \times 1$ vector of average budget shares in periods 1 and 2. Utilizing this equation in (30), we obtain

$$\log(1 + \varepsilon_{12}^\ast) = \mathbf{w}' \mathbf{D} \mathbf{p}$$

(33)

and (31) gives

$$\log(1 + r_{12}^\ast) = \log x_2 - \log x_1 - \mathbf{w}' \mathbf{D} \mathbf{p}$$

(34)

where $\varepsilon_{12}^\ast$ and $r_{12}^\ast$ are the proposed measures of inflation and real income growth rates, respectively.

Thus, we have demonstrated that for the translog cost function, our proposed measure of inflation exactly corresponds to the Tornquist (1936) price index.

The quantity index (as against the price index) is used to measure the change in the standard of living. Tornquist (1936) also proposed a quantity index which in our terminology of real income growth rate can be written as

$$\log(1 + r_{12}) = \mathbf{w}' \mathbf{D} \mathbf{q}$$

(35)

where $\mathbf{D} \mathbf{q} = \log q_2 - \log q_1$, $\mathbf{q}_1$ and $\mathbf{q}_2$ being the quantity vectors in periods 1 and 2, respectively. It should be emphasized that the Tornquist’s quantity index as given in (35) is different from the real income growth rate $r_{12}^\ast$ that we have derived in (34). It can be easily seen that Tornquist’s price and quantity indices will violate axioms 1, 1A and (5).

An implication of this result is that we cannot obtain real income by deflating nominal income by the price index. Whereas our measures of real income growth and inflation rates, viz, $r_{12}^\ast$ and $\varepsilon_{12}^\ast$, (as given in (34) and (33)), respectively, will satisfy all the axioms proposed in the paper.
6. Other Approaches

We may now discuss other approaches which have been proposed in the literature to measure the real income growth and price indices.

The Definition of Samuelson and Swamy

Following Samuelson and Swamy’s (1974) definition of economic price and quantity indices, we obtain the following class of real income growth and inflation rates

\[
\log(1 + r_{12}) = \log e(u_2, p^r) - \log e(u_1, p^r) \tag{36}
\]

and

\[
\log(1 + \epsilon_{12}) = \log e(u^r, p_2) - \log e(u^r, p_1) \tag{37}
\]

respectively, where \(p^r\) and \(u^r\) are the reference price vector and utility level, respectively.

It is easy to see that \(r_{12}\) and \(\epsilon_{12}\) in (36) and (37), respectively, will always satisfy Axioms 2, 3, 5 and 6. However, Axiom 4 is violated, which also leads to a violation of Axioms 1 and 1A. It can be demonstrated that if preferences are homothetic, \(r_{12}\) and \(\epsilon_{12}\), defined in (36) and (37) respectively, will always satisfy Axiom 4. But homotheticity is a highly restrictive assumption. Since our proposed measures \(r^*_{12}\) and \(\epsilon^*_{12}\) do not require the homotheticity assumption and at the same time satisfy all the desirable axioms, we would say that they are more appropriate than the class of measures suggested by Samuelson and Swamy.

The Definition of Pollak

Pollak (1971) derived the quantity index by deflating the ratio of nominal expenditures by the Konüs cost-of-living index. Following this definition, we can write

\[
\log(1 + \epsilon_{12}) = \log e(u, p_2) - \log e(u, p_1) \tag{38}
\]
and

\[ \log(1 + r_{12}) = \log x_2 - \log x_1 - \log(1 + \varepsilon_{12}) \]  \hspace{1cm} (39)

where \( u \) is any arbitrary utility level.

It is easy to see that \( \varepsilon_{12} \) and \( r_{12} \) defined in (38) and (39) respectively will satisfy all our axioms except Axiom 5, i.e. when \( u_1 = u_2 \neq u \). In order to satisfy this axiom, we can substitute either

\[ u = \bar{u} = \frac{u_1 + u_2}{2} \quad \text{or} \quad u = u^* = \sqrt{u_1 u_2} \]

Thus, we obtain the following alternative measures of inflation rates.

\[ \log(1 + \bar{\varepsilon}_{12}) = \log e(\bar{u}, p_2) - \log e(\bar{u}, p_1) \]  \hspace{1cm} (40)

and

\[ \log(1 + \varepsilon^*_{12}) = \log e(u^*, p_2) - \log e(u^*, p_1) \]  \hspace{1cm} (41)

where \( \bar{u} \) and \( u^* \) are the arithmetic and geometric means of the utility levels in periods 1 and 2 respectively. Having determined \( \varepsilon_{12} \) by one of these two methods, \( r_{12} \) is then obtained from (39).

Diewert (1976) has demonstrated that \( \left(1 + \bar{\varepsilon}_{12}\right) \) defined in (41) is exactly equal to Tornquist’s price index when the cost function is a general translog as given in (26). This demonstrates that our proposed inflation rate \( \varepsilon^*_{12} \) is identical to \( \bar{\varepsilon}_{12} \) when the cost function is a general translog. But in general they will differ.

It is not difficult to demonstrate that for the PIGLOG cost function, \( \left(1 + \bar{\varepsilon}_{12}\right) \) is identical to Tornquist’s price index. Thus, we have demonstrated that \( \varepsilon^*_{12} \) and \( \bar{\varepsilon}_{12} \) give exactly the same inflation rates for a general PIGLOG cost function.
Malmquist’s Quantity Index

In the previous sections we defined the real income growth rate in terms of money metric utility which is based on the cost function. An alternative approach to measure the real change in welfare is in terms of changes in quantities which would result in a given level of utility. This approach, suggested by Malmquist (1953), is based on the distance function (which is also called the deflation function).

The distance function $d(u, q)$ defined on $u$ and $q$, is the amount by which $q$ must be divided in order that the consumer enjoys the utility level $u$. $d(u, q)$ is an increasing function of $q$; indeed it is homogeneous of degree one in $q$, i.e. $d(u, \lambda q) = \lambda d(u, q)$. Unlike the cost function, $d(u, q)$ is a decreasing function of $u$.

Malmquist’s quantity index given by $(1 + \hat{r}_{12})$ can be defined by

$$
\log (1 + \hat{r}_{12}) = \log d(u, q_2) - \log d(u, q_1)
$$

(42)

where $u$ is any reference utility level.

When $u = u_1$, $(1 + r_{12})$ in (42) is called the Laspeyres-Malmquist quantity index (Diewert 1980). Similarly, when $u = u_2$, $(1 + r_{12})$ in (42) is called the Paasche-Malmquist quantity index (Diewert 1980).

It can be seen that the Laspeyres (and the Paasche) Malmquist quantity indices will violate Axiom 3. In order to satisfy Axiom 3, we propose a new real income growth rate $\hat{r}_{12}$ which is obtained by averaging the Laspeyres-Paasche Malmquist quantity indices:

$$
\log (1 + \hat{r}_{12}) = \frac{1}{2} \left[ \log d(u_1, q_2) - \log d(u_2, q_1) \right]
$$

(43)

where use has been made of the fact that $d(u_1, q_1) = d(u_2, q_2) = 1$. 


We may now evaluate $\hat{r}_{12}$ using alternative distance functions.

First, we consider a linear distance function:

$$d(u, q) = \frac{b'q}{c(u)} \quad (44)$$

where $c(u)$ is an increasing function of $u$. This distance function is obtained when the direct utility function is linear in quantities (not necessarily homothetic). Utilizing the result that $\frac{\partial d(u, q)}{\partial q} = \frac{p}{x}$, we obtain

$$d(u_1, q_2) = q'_2p_1/x_1$$

and

$$d(u_2, q_1) = q'_1p_2/x_2$$

which on substituting in (43) demonstrates that $(1 + \hat{r}_{12})$ is equal to Fisher’s ideal quantity index.

It is not difficult to prove that for a quadratic distance function

$$d(u, q) = \frac{\sqrt{q'Azq}}{c(u)} \quad (45)$$

where $A$ is symmetric and $c(u)$ is an increasing function of $u$, $(1 + \hat{r}_{12})$ is also exactly equal to Fisher’s ideal quantity index.

If the distance function is translog defined by $\log d(u, q) = \log e(u, p)$ where $e(u, p)$ is defined in (26) such that $d(u_1, p_1) = d(u_2, p_2) = 1$, we could demonstrate that $(1 + \hat{r}_{12})$ is exactly equal to Tornquist’s quantity index defined in (35).

Diewert (1980) (and also Pollak (1971)) has argued that Malmquist’s quantity index is a desirable quantity index because it satisfies a desirable homogeneity property. Thus, when $q_2 = (1 + \lambda)q_1$, $r_{12}$ defined in (36) is equal to $\lambda$; or, in other words, if all
quantities increase by $\lambda$ per cent, welfare must also increase by $\lambda$ per cent. This property may appear intuitively natural but there exists a difficulty with the interpretation of Malmquist’s quantity index as a measure of real income growth rate. Thus $r_{12}$ is defined as the growth rate of real income measured in monetary units, which means that it should be sensitive to the prices used; but Malmquist’s quantity defined in terms of a distance function is completely independent of prices. It follows that the real income growth rate and Malmquist’s quantity indices, both of which are measures of change in the welfare of a consumer, are quite different concepts. There exists no justifiable reason that the real income growth rate $r_{12}$ should satisfy the homogeneity condition mentioned above unless the preference ordering is homothetic to the origin.

If $P_t$ and $Q_t$ are price and quantity indices, then the product of $P_t$ and $Q_t$ should be equal to the actual expenditure ratio for the two periods under consideration. This is called the famous factor reversal test which is equivalent to our Axiom 4. We have argued that Axiom 4 is desirable because it implies that we should be able to determine the real income $y_t$ in year $t$ by dividing the nominal income $x_t$ by the price index $P_t$. Thus, the factor reversal test is indeed important in the measurement of the real income growth rate. But Malmquist’s quantity index does not measure the real income growth rate. It measures the change in quantities to attain a given level of utility. There will exist a one-to-one relationship between quantities and real income only if the preference ordering is homothetic to the origin. In other situations, there is no need to satisfy the factor reversal test when we measure a change in welfare using a quantity index (derived from a distance function). Interestingly, there exists a considerable literature which attempts to relate the price and quantity indices with the ratio of actual expenditures (Diewert 1980, Sato 1976, Theil 1973 and Vartia 1974).
7. **Real Economic Growth in a Many Consumers Economy**

We may now generalize the methodology presented in the previous sections to accommodate a many consumers economy. To accomplish this task we need to define real income growth and inflation rates for the whole society which consists of several individuals. Each individual in the society has different consumption patterns (or preferences). And, therefore, the construction of these growth rates should be based on a social welfare function. In this section, we define the real income growth and inflation rates in terms of a social cost function. The concept of social cost function, as against the individual cost function, is defined in Kakwani (1997).

Suppose there are \( m \) individuals in the society who enjoy the utility levels \( u_1, u_2, \ldots, u_m \), then we may conceptualize \( \tilde{u} \) as the aggregate welfare of the society which is derived from the individual utility levels. Then the social cost function \( E(\tilde{u}, p) \) is defined as the minimum money income which, when given to every individual will allow the society to enjoy \( \tilde{u} \) level of social welfare at a given price vector \( p \). The social cost function should have the following properties.

1. \( E(\tilde{u}, p) \) is an increasing function of \( \tilde{u} \) for all \( p \).
2. \( E(\tilde{u}, p) \) is increasing and concave in \( p \) for every \( \tilde{u} \).
3. \( E(\tilde{u}, p) \) is (positively) linearly homogeneous in \( p \) for every \( \tilde{u} \), i.e.,
   \[ E(\lambda \tilde{u}, p) = \lambda E(\tilde{u}, p). \]

We derive the social cost function from the individual cost functions \( e(u, p) \) such that the three above properties are satisfied. To derive \( E(\tilde{u}, p) \), we utilize the concept of “equally distributed equivalent level of income” (Atkinson 1970). \( E(\tilde{u}, p) \) may be interpreted as the equally distributed equivalent level of income, the level if received by every individual, would result in the same level of social welfare as the present
distribution. Like Atkinson (1970), we assume that the social welfare is utilitarian and every individual has exactly the same utility function. Suppose further that \( x_i \) is the income of the \( i \)th individual who is enjoying a utility level of \( u_i \) at a given price vector \( p \), then obviously \( x_i = e(u_i, p) \). The welfare enjoyed by the \( i \)th individual may then be written as equal to \( g(x_i) = g[e(u_i, p)] \) where \( g(x_i) \) such that \( g'(x_i) > 0 \) and \( g''(x_i) < 0 \) is the function that converts income of the \( i \)th individual into the welfare enjoyed by the \( i \)th individual.

If we assume that the social welfare is equal to the sum of individual welfare levels, we obtain the social welfare as

\[
W = \sum_{i=1}^{m} g[e(u_i, p)]
\]

\( g[E(\tilde{u}, p)] \) will be average welfare enjoyed by the society if every individual receives an income level \( E(\tilde{u}, p) \). This, obviously, should be equal to \( W/m \). Thus, we have

\[
g[E(\tilde{u}, p)] = \frac{1}{m} \sum_{i=1}^{m} g[e(u_i, p)]
\]

which gives the relationship between the cost functions of individuals in the society (given by \( e(u_i, p) \)) and the social cost function \( E(\tilde{u}, p) \).

It is easy to see from (46) that \( E(\tilde{u}, p) \) will be an increasing function of \( \tilde{u} \) and \( p \).

Further since \( e(u, p) \) is concave in \( p \), \( E(\tilde{u}, p) \) will also be concave in \( p \).

The third property, that \( E(\tilde{u}, p) \) is (positively) linearly homogeneous in \( p \) for every \( \tilde{u} \), will be satisfied only if it is assumed that \( g(x) \) is a homothetic function in \( x \). A class of homothetic functions is given by (Atkinson 1970):

\[
g(x) = A + \frac{Bx^{1-\varepsilon}}{1-\varepsilon} , \quad \varepsilon \neq 1
\]

\[
= \log_e (x) , \quad \varepsilon = 1
\]
where $\varepsilon > 0$ is a measure of risk-aversion, which is constant for this utility function. Note that the social cost function is invariant with respect to a (positively) linear transformation of $g(x)$.

Since $\varepsilon$ is a measure of risk aversion it may also be interpreted as a measure of the degree of inequality aversion in the society. As $\varepsilon$ rises, the society’s concern about inequality also increases. If $\varepsilon = 0$, it reflects an inequality neutral attitude in which case the society does not care about inequality at all.

The choice of $\varepsilon$ depends on the society’s value judgement about the degree of inequality aversion. For the purpose of this paper, we choose $\varepsilon = 1$ which gives

$$\log E(\tilde{u}, p) = \frac{1}{m} \sum_{i=1}^{m} \log e(u_i, p)$$

(47)

Note that $E(\tilde{u}, p)$ is a money metric measure of social welfare. So the measure of welfare in period 1 is given by $E(\tilde{u}_1, p_1)$, which denoted by $x^*_1$ is equal to the geometric mean of the nominal incomes of individuals in period 1. Similarly, $x^*_2 = E(\tilde{u}_2, p_2)$ is the money metric measure of welfare in period 2. Then, $\tilde{m}^*_1$, defined by

$$\log\left(1 + \tilde{m}^*_1\right) = \log x^*_2 - \log x^*_1$$

(48)

is the growth rate of nominal money metric measure of welfare. This is not the real change in welfare. It includes the effect of price changes as well as the effect of changes in social welfare.

We may now define the social real income growth and inflation rates as given by

$$\log\left(1 + \tilde{r}^*_1\right) = \frac{1}{2} \left[ \log E(\tilde{u}_2, p_1) - \log E(\tilde{u}_1, p_1) + \log E(\tilde{u}_2, p_2) - \log E(\tilde{u}_1, p_2) \right]$$

(49)

and

$$\log\left(1 + \tilde{\varepsilon}^*_1\right) = \frac{1}{2} \left[ \log E(\tilde{u}_1, p_2) - \log E(\tilde{u}_1, p_1) + \log E(\tilde{u}_2, p_2) - \log E(\tilde{u}_2, p_1) \right]$$

(50)
Combining (49) and (50) with (48) gives

$$\log(1 + \tilde{m}_{12}^{\ast}) = \log(1 + \tilde{e}_{12}^{\ast}) + \log(1 + \tilde{r}_{12}^{\ast})$$

(51)

which is equivalent to Axiom 4.

It is not difficult to see that $\tilde{r}_{12}^{\ast}$ and $\tilde{e}_{12}^{\ast}$ defined in (49) and (50) satisfy all the axioms given in Section 2.

To make the proposed measures of social real income growth and inflation rates empirically operational, we assume that each individual in the society has a general translog cost function. Suppose $\varepsilon_{12}^{i\ast}$ is the inflation rate of the ith person between periods 1 and 2. Then, if the ith person has a general translog cost function, (33) gives

$$\log(1 + \varepsilon_{12}^{i\ast}) = \frac{1}{2}(w_{1}^{i} + w_{2}^{i})' \Delta p$$

(52)

where $w_{1}^{i}$ and $w_{2}^{i}$ are the (n×1) vectors of budget shares of the ith person in the periods 1 and 2, respectively.

Now combining (50) and (47) immediately gives

$$\log(1 + \tilde{e}_{12}^{\ast}) = \frac{1}{m} \sum_{i=1}^{m} \log(1 + \varepsilon_{12}^{i\ast})$$

which in view of (52) yields

$$\log(1 + \tilde{e}_{12}^{\ast}) = \tilde{w}' \Delta p$$

(53)

where

$$\tilde{w} = \tilde{w}_{1} + \tilde{w}_{2}$$

$$\tilde{w}_{1} = \frac{1}{m} \sum_{i=1}^{m} w_{1}^{i}$$

and

$$\tilde{w}_{2} = \frac{1}{m} \sum_{i=1}^{m} w_{2}^{i}$$

are the vectors of average budget shares of the society in periods 1 and 2 respectively.

Having obtained the social inflation rate $\tilde{e}_{12}^{\ast}$ in (53), (48) and (51) immediately yield the social real income growth rate $\tilde{r}_{12}^{\ast}$ as given by
\log(1 + \tilde{r}^*) = \log x_2^* - \log x_1^* - \dot{\hat{w}} D \mathbf{p} \quad (54)

where \( x_1^* \) and \( x_2^* \) are the geometric means of the nominal incomes of all individuals in
the society in periods 1 and 2 respectively.

The usual method of computing growth rates is to assume that all individuals in
the society are identical in the sense that they enjoy the same level of utility. In this
situation, the budget share of a commodity is calculated as the average expenditure on the
commodity divided by the average total expenditure, i.e., \( \bar{w}_j^* = \mu_j / \mu \), where \( \mu_j \) is the
mean expenditure on the \( j \)th commodity and \( \mu \) the mean total expenditure. The inflation
rate for a translog cost function is then obtained as given by

\[
\log(1 + \tilde{\varepsilon}_{12}^*) = \dot{\bar{w}}^* D \mathbf{p} \quad (55)
\]

where

\[
\bar{w} = \frac{1}{2} \left( \bar{w}_1 + \bar{w}_2 \right)
\]

where \( \bar{w}_1 \) and \( \bar{w}_2 \) are the \( n \times 1 \) vectors of aggregate budget shares in periods 1 and 2
respectively. Note that \( \tilde{\varepsilon}_{12}^* \) in (55) is obtained when the inequality aversion parameter \( \varepsilon = 0 \), which is the case in which the society is completely indifferent to income inequality.

Combining (53) and (55) gives

\[
\log(1 + \tilde{\varepsilon}_{12}^*) = \log(1 + \tilde{\varepsilon}_{12}^*) + (\dot{\bar{w}} - \bar{w})^* D \mathbf{p} \quad (56)
\]

In this equation, the first term in the right hand side gives an inflation rate which
is inequality neutral whereas the second term measures the effect of price changes on
income inequality. The second term is equal to \( \sum_{j=1}^{n} \left( \bar{w}_j - \bar{w}_j^* \right) \left( \log p_{2j} - \log p_{1j} \right) \), where
\( p_{1j} \) and \( p_{2j} \) are the prices of the \( j \)th commodity in periods 1 and 2 respectively. Since by
definition, luxury goods are those goods for which budget shares are higher for better off
households and vice versa for necessities; \( \tilde{w}_j \) will be smaller than \( \bar{w}_j \) if good \( j \) is a luxury and larger than \( \bar{w}_j \) if good \( j \) is a necessity. The difference between them can be large if the distribution of income is highly unequal. So equation (56) implies that if the prices of necessities go up more than those of luxuries, \( \tilde{\varepsilon}_{12}^s \) will be larger than \( \bar{\varepsilon}_{12}^s \). Thus, the difference between \( \tilde{\varepsilon}_{12}^s \) and \( \bar{\varepsilon}_{12}^s \) indicates whether the price changes favour richer or poorer individuals in the society. If \( \tilde{\varepsilon}_{12}^s \) is greater (less) than \( \bar{\varepsilon}_{12}^s \), the relative price changes will have an inequality increasing (decreasing) bias.

Having estimated the social inflation rate in (56), the social real income growth rate \( \tilde{r}_{12}^s \) can be estimated from (51). Thus, we have

\[
\log \left(1 + \tilde{r}_{12}^s\right) = \log x_2^s - \log x_1^s - \log \left(1 + \tilde{\varepsilon}_{12}^s\right)
\]

(57)

If all the individuals in the society are identical, the real income growth \( \tilde{r}_{12}^s \) will then be given by

\[
\log \left(1 + \tilde{r}_{12}^s\right) = \log \mu_2 - \log \mu_1 - \log \left(1 + \tilde{\varepsilon}_{12}^s\right)
\]

(58)

where \( \mu_1 \) and \( \mu_2 \) are the mean expenditures of the society in periods 1 and 2, respectively. Note that \( x_1^s = \mu_1 \) and \( x_2^s = \mu_2 \) when we assume that all individuals in the society are identical (in each period) so that the inequality of nominal income in periods \( i \) and \( j \) may be measured by \( A_1 \) and \( A_2 \), respectively as

\[
A_1 = 1 - \frac{x_1^s}{\mu_1}
\]

(59)

and

\[
A_2 = 1 - \frac{x_2^s}{\mu_2}
\]

(60)
Thus, combining (56) with (57) and (58), we obtain

\[
\log(1 + \tilde{r}_{12}^*) = \log(1 + \bar{r}_{12}^*) + \left[\log(1 - A_2) - \log(1 - A_1)\right] - (\tilde{w} - \bar{w})' Dp
\]

which provides an interesting interpretation. The real income growth rates \(\tilde{r}_{12}^*\) and \(\bar{r}_{12}^*\) will differ because of the two reasons. The second term in the righthand side of (61) measures the effect of a change in inequality of nominal incomes from period 1 to period 2 (from \(A_1\) to \(A_2\)). If \(A_2 > A_1\) which means that the inequality has increased between the base and terminal years, its effect on real income growth rate will be negative. This loss of growth will occur even if all prices change uniformly between the two periods. The third term in the righthand side measures the effect changes in relative prices on the growth rate of real income. This term will be negative if the prices of necessities go up more than those of luxuries.

8. When There Are More than Two Periods

So far our methodology of measuring real economic growth has been based on the assumption that there are only two periods of comparison. These may be called the biperiod comparisons. Generally, we wish to make multiperiod comparisons; when there are more than two periods. In this section, we present a generalization of the analysis presented so far.

Suppose there are \(m\) periods for which we wish to measure the real income growth rates. We can always measure these growth rates by making all possible biperiod comparisons. This procedure will violate some intuitively natural axioms which we present below.

Suppose \(m_{ij}\) is the growth rate of nominal income between periods \(i\) and \(j\), where \(i\) and \(j\) vary from 1 to \(m\). It is easy to see that
\[
\log(1 + m_{ij}) = \log(1 + m_{ik}) + \log(1 + m_{kj}) \tag{62}
\]

must always hold for all \(k = 1,2,\ldots,m\).

Suppose \(\hat{\varepsilon}_{ij}\) and \(\hat{r}_{ij}\) are the inflation and real income growth rates between periods \(i\) and \(j\), respectively when we are making multiperiod comparisons, i.e., \(i\) and \(j\) vary from 1 to \(m\). In view of (62), it is intuitively natural to satisfy the following two axioms.

**Axiom 7:** \(\log(1 + \hat{\varepsilon}_{ij}) = \log(1 + \hat{\varepsilon}_{ik}) + \log(1 + \hat{\varepsilon}_{kj})\) for all \(k = 1,2,\ldots,m\).

**Axiom 8:** \(\log(1 + \hat{r}_{ij}) = \log(1 + \hat{r}_{ik}) + \log(1 + \hat{r}_{kj})\) for all \(k = 1,2,\ldots,m\).

Axioms 7 and 8 imply that the inflation and real income growth rates should be transitive. These are the fundamental axioms in the situations of multiperiod comparisons.

Suppose \(P_{ki}\) is the price index measuring the prices in year \(i\) relative to those in year \(k\) and similarly \(P_{kj}\) is the price index measuring the prices in year \(j\) relative to those in year \(k\), then the inflation rate \(\hat{\varepsilon}_{ij}\) between years \(i\) and \(j\) should satisfy Axiom 9:

**Axiom 9:** \(\log(1 + \hat{\varepsilon}_{ij}) = \log(P_{kj}) - \log(P_{ki})\) **must hold for all** \(k=1,2,\ldots,m\)

This axiom implies that the inflation rate must be invariant to what the base year is chosen.

Similarly, suppose \(y_{ki}\) is the income in year \(i\) at the year \(k\) prices and \(y_{kj}\) is the income in year \(j\) at the year \(k\) prices, then the real income growth rate between years \(i\) and \(j\) is given by \(\hat{r}_{ij} = \frac{(y_{kj} - y_{ki})}{y_{ki}}\) where \(k = 1,2,\ldots,m\). It is then natural to expect that the real income growth rate should not be affected by the base year that is chosen for computing the real incomes. This leads to Axiom 10.
**Axiom 10:** \( \log(1 + \hat{r}_{ij}) = \log y_{kj} - \log y_{ki} \) **must hold for all** \( k = 1,2,\ldots,m \).

Note that axioms 7, 8, 9 and 10 are not independent axioms. It can easily be seen that axiom 7 implies axiom 9 and vice versa and, similarly, axiom 8 implies axiom 10 and vice versa. Further, axiom 9 implies axiom 10 and vice versa if we assume that \( \hat{\varepsilon}_{ij} \) and \( \hat{r}_{ij} \) satisfy axiom 4.

We would like to have measures of inflation and real income growth rates which satisfy all the ten axioms presented in the paper. This can be accomplished by making use of the biperiod measures \( \varepsilon_{ij} \) and \( r_{ij} \) which satisfied axioms 1-6. Thus, we propose the new measures of inflation and real income growth rates given by \( \hat{\varepsilon}_{ij} \) and \( \hat{r}_{ij} \), respectively as

\[
\log(1 + \hat{\varepsilon}_{ij}) = \frac{1}{m} \sum_{k=1}^{m} \left[ \log(1 + \varepsilon_{ik}) + \log(1 + \varepsilon_{kj}) \right]
\]  

(63)

and

\[
\log(1 + \hat{r}_{ij}) = \frac{1}{m} \sum_{k=1}^{m} \left[ \log(1 + r_{ik}) + \log(1 + r_{kj}) \right]
\]  

(64)

where \( i, j \) vary from 1 to \( m \). \( \varepsilon_{ij} \) and \( r_{ij} \) are the biperiod measures and may be computed by equations (53) and (54), respectively. It can be easily demonstrated that \( \hat{\varepsilon}_{ij} \) and \( \hat{r}_{ij} \) will satisfy all the ten axioms proposed in the paper.

**9. Inflation and Real Income Growth Rates for Thailand**

In this section, we utilize the methodology developed in the paper to compute inflation and real income growth rates for Thailand. We used the two sources of data. The first source of data is the price indices of various goods and services which were obtained
from the Department of Business Economics, Ministry of Commerce, Bangkok, Thailand.

The second source of data is the socioeconomic survey (SES) data. The National Statistical Office of Thailand conducts these surveys on a regular basis covering all private non-institutional households residing permanently in municipal areas, sanitary districts and villages. We utilized the three surveys 1988, 1990 and 1992.

The NSO Thailand provided us with unit record data giving expenditures on a wide variety of goods and services. Since we could obtain a very detailed disaggregation of goods and services, the matching of the data from the two sources was not a problem.

To estimate the social weights accurately from the sample households, we needed to estimate the weight attached to each sampled household to enable the data provided by these households to be expanded to obtain estimates for the whole population. We estimated these weights using the sample design used in the survey. The population weights were determined by multiplying the population household weight by the household size. Thus, the social weights estimated in the paper relate to individuals rather than households.

The estimates of inflation and real income growth rates are presented in Table 1. Table 2 presents the nominal and real mean incomes along with income inequality.

It can be seen that the nominal mean income of 903.09 baht in 1988 increased to 1274.17 baht in 1990 showing a growth rate of 41.09 percent in the two years. The nominal mean income increased to 1610.24 baht in 1992, showing a growth rate of 26.38 percent in the next two years (between 1990 and 1992).

The nominal mean welfare is measured by the geometric mean-income. This is justified under the assumption that the social welfare function is logarithmic utilitarian. The nominal mean welfare increased from 689.80 baht in 1988 to 915.07 baht in 1990.
and to 1152.40 baht in 1992. The growth rates of nominal welfare was 32.66 percent in the 1988-90 period and 25.94 percent in the 1990-92 period. Thus, the nominal mean welfare grew at a much slower rate than the nominal mean income. This happened because inequality of income has also increased during these periods, thus resulting in a slowing down of growth in the nominal mean income. The income inequality increased from 23.62 percent points in 1988 to 28.18 percent points in 1990 and to 28.43 percent points in 1992. The growth rates estimated on the basis of one person economy are completely insensitive to a change in income inequality in the society. It is, therefore, important to measure growth rates under the assumptions of many persons in the economy.

It is noted that the estimates of inflation rates for a single person economy are lower than those for a many person economy. It implies that the prices of necessities have increased faster than luxuries thus adversely affecting the poor more than the rich. From this, we may conclude that relative price changes in Thailand between 1988 and 1990 and 1990 and 1992 had an inequality increasing bias.

The real income growth rates are substantially lower for a many persons economy than those for a single person economy. The differences between them are attributed to two factors. The first factor is the increase in income inequality which has an effect of lowering the growth rate in the real income. The second factor is the effect of price changes. These factors together have contributed to the lowering of growth rates by 6.188 percent in the 1988-90 period (Table 3). The inequality effect is the major factor which has contributed to lowering of the growth rate by 5.976 percent. The contribution of prices on the growth rate is only -.225 percent. Thus, increasing inequality of income should be the major concern of the Thai government. The growth rates of real income
Table 1: Inflation and real income growth rates in Thailand

<table>
<thead>
<tr>
<th>Periods</th>
<th>Nominal income- Growth rate</th>
<th>Based on biperiod comparisons</th>
<th>Based on multiperiod comparisons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Inflation Rate</td>
<td>Real income- Growth rate</td>
</tr>
<tr>
<td>Economy with one person</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1988-90</td>
<td>41.09</td>
<td>10.36</td>
<td>27.85</td>
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<td>47.09</td>
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<td>1990-92</td>
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<tr>
<td>Economy with many persons</td>
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<td></td>
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<tr>
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<td>1990-92</td>
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</tbody>
</table>

Table 2: Nominal and real mean income and mean welfare and income inequality

<table>
<thead>
<tr>
<th>Year</th>
<th>Nominal Mean income</th>
<th>Nominal Mean welfare</th>
<th>Income Inequality</th>
<th>Real mean income</th>
<th>Real Mean welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988</td>
<td>903.09</td>
<td>689.80</td>
<td>23.62</td>
<td>903.09</td>
<td>689.80</td>
</tr>
<tr>
<td>1990</td>
<td>1274.17</td>
<td>915.07</td>
<td>28.18</td>
<td>1154.49</td>
<td>829.12</td>
</tr>
<tr>
<td>1992</td>
<td>1610.24</td>
<td>1152.40</td>
<td>28.43</td>
<td>1328.45</td>
<td>950.73</td>
</tr>
</tbody>
</table>

Real values are calculated in 1988 prices

Table 3: Explaining growth rates in Thailand

<table>
<thead>
<tr>
<th>Periods</th>
<th>Inequality effect on growth rate</th>
<th>Based on biperiod comparisons</th>
<th>Based on multiperiod comparisons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Price effect on growth rate</td>
<td>Total effect on growth rate</td>
</tr>
<tr>
<td>1988-90</td>
<td>-5.976</td>
<td>-0.225</td>
<td>-6.188</td>
</tr>
<tr>
<td>1988-92</td>
<td>-6.304</td>
<td>-0.359</td>
<td>-6.640</td>
</tr>
<tr>
<td>1990-92</td>
<td>-0.348</td>
<td>-0.217</td>
<td>-0.564</td>
</tr>
</tbody>
</table>
should, therefore, be measured using the methodology of many persons economy which takes into account the effects of changes in income inequality.

We have presented our estimates of inflation and real income growth rates using the two alternative methods. One is based on biperiod comparisons which is an adequate procedure if we are making comparisons between two periods. The more appropriate method is the one based on multiperiod comparisons because we are making comparisons between the three periods. It is interesting to note that two procedures give almost similar results.

It is a well-known fact that Thailand has achieved a remarkable economic growth during the last two decades. Although the rapid economic growth in Thailand has brought about prosperity to many people, the gap between the rich and poor has been increasing. These conclusions are indeed reflected in our results based on household surveys.

10. Concluding Remarks

This paper utilizes the money metric utility approach to measure real economic growth. The paper proposes some intuitively natural axioms which a measure of real income growth should satisfy. The axioms are sufficiently strong to eliminate the measures which have been used in practice. A new approach is proposed which satisfies all the axioms.

To make the approach empirically operational one needs to estimate a demand system derived from the notion of utility maximisation. This may not always be feasible. In practice, statistical measures such as Fisher’s and Tornquist’s are commonly used. In the paper we have attempted to provide an economic justification of these purely statistical measures. Fisher’s measure of growth rate is found to be less attractive because the cost function implied by it is not concave in prices. The paper provides a stronger
justification for using Tornquist’s indices for measuring the inflation rate. However, Tornquist’s quantity index is not considered appropriate to measure the real income growth rate.

The major contribution of this paper has been the measurement of growth rates in a many-persons economy. These growth rates take into account the changes in income inequality which is an important constituent of people’s welfare. The measure of inflation indicates whether or not the price changes have a favourable (or unfavourable) impact on the welfare of the poor. The empirical application to Thailand shows that the relative price changes between 1988 and 1992 have an inequality increasing bias. The increasing inequality in Thailand, however, has a substantial impact the growth of real of income.

Although the rapid economic growth in Thailand has brought about prosperity to many people, the gap between the rich and the poor has been increasing more or less monotonically. This paper has provided a useful link between price changes, economic growth and income inequality. This is accomplished without estimating any consumer demand systems. These demand systems are always very hard to estimate (because not all the parameters are identifiable from a single cross-section data set and also of their non-linear nature) particularly when the number of expenditure items is large.
REFERENCES


