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# Is public capital provision efficient?

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**Abstract**

We examine whether public investment undertaken in Australia over the last three decades satisfies conditions for intertemporal efficiency. We find that the conditions are satisfied over the sample period but only after allowance for changes in the relative price of public and private capital. In contrast to previous research, we do not find any evidence of excessive returns to public investment; rather, the average real investment return for both private and public capital is estimated at about 9% per annum. © 1998 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

A stylized fact for many industrialized economies is the marked decline in the ratio of public investment to gross domestic product that has occurred over the last three decades. This decline has raised concerns that public capital stocks have fallen to sub-optimal levels, possibly reducing private sector productivity and imposing a constraint on economic growth (see Aschauer, 1989). These concerns have motivated a substantial literature which seeks to quantify the contribution of public capital to private production and, at least indirectly, assess the implications of the declining share of public investment. Gramlich

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(1994) provides a survey of the existing literature and a critical review of many of the arguments that have been raised.[1]

While previous empirical studies are able to provide some information about public capital within the aggregate economy, they are unable to provide direct evidence as to whether or not the observed declines in public investment are appropriate given the state of private production. In contrast to these studies, we seek to explicitly evaluate public capital provision. We characterize the efficient allocation of resources in an economy with both a private and public capital sector and then examine whether the political and market processes in Australia deliver allocations that satisfy these conditions.

The approach we pursue is a natural extension to the existing literature which is based upon relatively simple neoclassical models of aggregate production dependent upon private and public capital. However, the added dimension we pursue, the specification and testing of efficient allocation conditions, has our analysis most closely related with the part of the asset pricing literature which is concerned with testing Euler equations (e.g. Hansen and Singleton, 1982). In particular, our model can be viewed as a sectoral version of Cochrane's production-based asset pricing model (Cochrane, 1991), where investment returns are modelled using production variables, which we estimate and test using Hansen's generalized method of moments (Hansen, 1982).

Our approach has some advantages worth noting. Previous studies approach the provision of public capital indirectly using either growth accounting techniques or a simple comparison of implied sectoral investment returns based upon estimated production functions.[2] Neither of these is fully satisfactory. Growth accounting exercises, while able to identify declining growth in public capital as a contributing factor to declining productivity, cannot determine whether this decline is sub-optimal; this requires a model of intertemporal behaviour such as the Euler equations we employ. The comparisons of investment returns, which often find very high returns to public investment, is unsatisfactory as it is generally done without any statistical basis. In contrast, our approach has efficient provision of both public and private capital as the maintained hypothesis and examines whether this hypothesis is rejected by the data. Furthermore, Gramlich (1994) and others view the very high returns to public investment with scepticism: if these investment returns are true, it raises

[1] Gramlich discusses a variety of different approaches used to assess the provision of infrastructure and their shortcomings. The discussion here concentrates on what he identifies as macroeconometric productivity studies.

[2] The adequacy of public capital provision is an extension of the more basic question, first pursued by Ratner (1983) and Aschauer (1989), as to whether public capital is a productive input into private production. The latter question is itself unresolved, see for example Holtz-Eakin (1994).

the question as to why they remain unexploited. One advantage of our estimation is the entirely reasonable levels of investment returns we estimate for both the public and private sector.

The second advantage of our approach is that we treat public investment as endogenous, both theoretically and for purposes of estimation. Many of the previous studies have treated public investment as exogenous, either explicitly or implicitly, and have been criticized on this basis (see Gramlich, 1994).

Finally, and perhaps most importantly, our approach highlights an aspect of public investment which has not received much, if any, attention to date. This is the role of the relative prices of public and private investment goods. To foreshadow a principal result of the paper, the declining growth in public capital in Australia can be viewed as one component of a more general trend – the growing private capital intensity of aggregate production; this change in capital intensity can be explained by the rising price of public investment goods relative to private investment goods.[3]

The paper proceeds as follows. Section 2 characterizes testable conditions of allocative efficiency across the private and public sectors for a given set of preferences and two specifications of production. Section 3 presents the empirical analysis of these conditions for the Australian economy using a quarterly data set for the period 1959–1992. Section 4 concludes.

## 2. Resource allocation

We consider an economy which produces a single output with two capital sectors – private and public. The sole defining feature of public capital in this economy is public ownership; otherwise, its accumulation and depreciation are similar to privately-owned capital.[4] The underlying presumption is that public ownership of capital is warranted on the basis of either the public good aspects of certain investment projects (i.e. all the returns are not privately appropriable) or the presence of increasing returns (or large fixed costs) in the supply of public services. For a more complete discussion of this framework, see Arrow and Kurz (1970).

[3] The measures of public and private capital which we employ in this paper are designed to avoid issues of privatization so that this cannot be the explanation for the increasing private capital intensity.

[4] This is the structure employed in Arrow and Kurz (1970) and implicit in the existing literature. For an alternative specification of public investment with quite different implications for sectoral efficiency, see Weitzman (1970).

### 2.1. Efficiency conditions

With two capital sectors in the economy, there is a return on investment associated with each sector. We can define $R_{it+1}$ as the gross investment return in sector $i$. Further, let $m_{t+1}$ be defined as a stochastic discount factor. Then, assuming a common stochastic discount factor for the aggregate economy, the conditions for efficient resource allocation are:

$$\mathrm{E}_t(m_{t+1}R_{it+1}) = 1, \qquad i = 1,2,$$

where $\mathrm{E}_t$ is the expectations operator conditional on the time $t$ information set of the agents in the economy. To estimate and test these conditions we need to specify functional forms for the stochastic discount factor and the returns in both sectors. The former is specified using a parameterization of consumer preferences while the latter comes from a specification of aggregate production.

### 2.2. Preferences

Consistent with intertemporal theories of consumption and asset pricing, the stochastic discount factor used here is the intertemporal marginal rate of substitution for a representative agent with a time separable life-time utility function:

$$m_{t+1} \equiv \beta u'(c_{t+1})/'u(c_t),$$

where $c_t$ is per capita consumption and $\beta$ is the subjective discount factor. For purposes of estimation, we use constant elasticity of substitution preferences specified as

$$u(c) = c^{\sigma}/\sigma$$

with $\sigma \leq 1$. Here $1/(1-\sigma)$ is the intertemporal elasticity of substitution (and $1-\sigma$ is the coefficient of relative risk aversion).

### 2.3. Production

Two functional forms are considered for aggregate production. The first is a simple Cobb–Douglas model which is common throughout much of the macroeconometric literature concerning public capital for both time series and cross-section studies. This model is

$$Y = K_1^{\rho_1}K_2^{\rho_2}N^{1-\rho_1-\rho_2},$$

where $Y$ is total output, $K_1$ is the private sector capital stock, $K_2$ is the public sector capital stock, and $N$ is labour for both sectors. Ordinarily one would also include some measure of technology. However, our focus is only on sectoral returns and these may often be expressed independently of the specification of

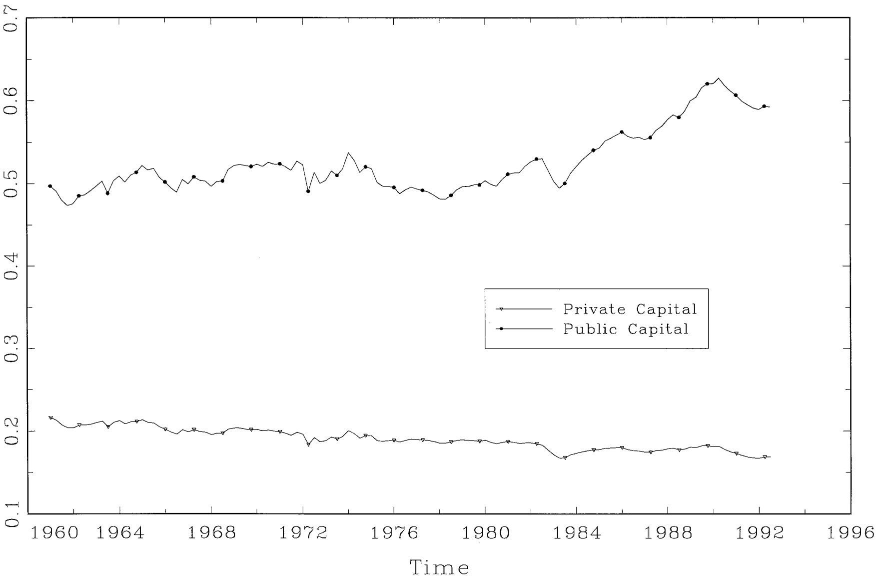


Fig. 1. Output–capital ratios.

technology. For example, specifying Hicks neutral technology will not alter our parameterization of sectoral investment returns. One point of difference between this model and the previous literature is that we are specifying production for the entire economy and not just private production. Since measured public capital may contribute to both private production (for example, transportation infrastructure) and public production (for example, the public service) its return should be measured relative to aggregate production of the economy as a whole.

The Cobb–Douglas representation provides a simple linear model which can be easily estimated. However, in the current context we can anticipate why it may prove inadequate. The returns $R_{it+1}$ for the Cobb–Douglas model depend linearly on the output–capital ratio for each sector (see Eq. (1)). For the efficiency conditions to hold, these sectoral ratios should be highly correlated. In fact, however, for our data these ratios trend in opposite directions (see Fig. 1). It seems likely then that the efficiency conditions for the Cobb–Douglas model will be rejected.

The rejection of the efficiency conditions, however, may reflect an incorrect representation of production rather than evidence of inefficiency.[5] We may therefore want to consider a more flexible representation of production. A

[5] We are grateful to a referee for suggesting we pursue this issue.

natural generalization is suggested by the data. For the observed patterns of output capital ratios to be consistent with the efficiency conditions, we require technology which permits a high degree of substitutability between private and public capital: the fall in $Y/K_1$ is offset by the rise in $Y/K_2$. To allow for greater substitution between the two sectors, the second model we consider has an effective capital stock which is a CES aggregate of the two capital sectors:

$$Y = [\mu K_1^{\phi} + (1-\mu)K_2^{\phi}]^{\rho/\phi} N^{1-\rho}.$$

The above nests the Cobb–Douglas production function as $\phi \to 0$, while the two capital stocks are perfect substitutes when $\phi = 1$. This is, *ex ante*, a more plausible representation for production in terms of the efficiency conditions since the sectoral returns are linearly dependent upon ratios of output to weighted averages of the capital stocks (see Eq. (3)). Consequently, the returns themselves need not inherit the same trend behaviour of the output to capital ratios.

Both models are written assuming constant returns to scale across all inputs. This implies that the services from public capital are not pure public goods but suffer from congestion: public capital must expand proportionally with private imports. While other possibilities have been considered in the literature (e.g. Aschauer, 1989) for our purposes it is not necessary as the conditions we test are not affected by relaxing the CRS assumption. However, to back out estimates of the output elasticity of labour, which provide an informal evaluation of our empirical results, we use the constant returns to scale representation.

### 2.4. Parameterized models

The conditions for efficient resource allocation require a measure of the real return to investing one unit of the production good. In an economy with a single good that may be consumed or invested in either capital sector with a unit relative price in each case, the real return for investment in sector $i$ is

$$R_{it+1} \equiv f_{it+1} + (1-\delta_i),$$

where $f_{it+1} = \partial Y_{t+1}/\partial K_{it+1}$ and $\delta_i$ is the depreciation rate for sector $i$ capital. The interpretation is straightforward: one unit of foregone consumption at time $t$ invested in sector $i$ provides an increase in production at time $t+1$ of $f_{it+1}$ as well as an increase in the $t+1$ capital stock of $(1-\delta_i)$.

This model, although very simple, is implicit in much of the existing discussion concerning the provision of public capital. The returns as defined above together with the efficiency conditions imply, roughly, that the marginal product of capital in each sector should be the same (weighted for consumption risk and allowing for differences in depreciation rates). Many of the studies which estimate models of private production have parameter estimates which imply substantial discrepancies between the marginal products of private and public capital with the latter being larger. These discrepancies have been interpreted as supernormal returns to public investment, suggesting under provision in recent years. (See for example Finn 1993; Gramlich provides a discussion of this aspect of the literature).

One aspect of the above model which may cause it to be rejected by the data is the assumption of a constant unit relative price for private and public investment goods. To see why this might be the case, consider the output to capital ratios for each sector presented in Fig. 1. These ratios clearly indicate that aggregate production in Australia has become more intensive in private capital and less intensive in public capital. Taken by itself, the latter is just another means of describing the fall in public investment which is the basis for the concerns raised by Aschauer and others. Taken in conjuction with the increase in private sector capital intensity, a possible explanation for the behaviour of these series may be a change in the relative prices of public and private investment goods.[6] Fig. 2 presents the gross fixed capital expenditure deflators for each sector relative to the GDP deflator. It is clear that public investment goods have become relatively more expensive over time while private capital investment goods have become relatively cheaper. This is certainly consistent with the changing capital intensities observed in Fig. 1.

Since the assumption of constant unit relative prices of sectoral investment goods appears to be at odds with our data, we amend our model to allow for a variable relative price. Recall that the model outlined above assumes a single output which may be consumed or invested one for one in either capital sector. Now, we imagine an economy where output can be transformed into investment goods according to some technology specific to each sector. Although we could introduce such technology explicitly into the model, it is simpler to do so by introducing the concept of a relative price $p_{it}$ which measures the cost of investment good in sector $i$ in terms of aggregate output.

The framework we are employing is closely related to models of investment with adjustment costs. In those models, adjustment costs arise from the transformation of output (or equivalently the consumption good) into physical capital. These costs are modelled explicitly as functions of investment

[6] An alternative explanation for the increasing private capital intensity of aggregate production is that sectors of the economy which are relatively intensive in private capital have become increasingly dominant in the economy. While a complete examination of this is beyond the scope of this paper, we have made a preliminary examination of private capital intensity on a sectoral basis. We examine, using an index number approach, the behaviour of the output to private capital ratio holding the share of each sector (in terms of output) constant at 1960s values. The resulting output–capital ratio has the same time series properties as the measured series so we conclude against sectoral change as an explanation for the growing private capital intensity of aggregate production. Details are available in an appendix available at http://www.economics.unsw.edu.au/staff/voss.html or directly from the authors.

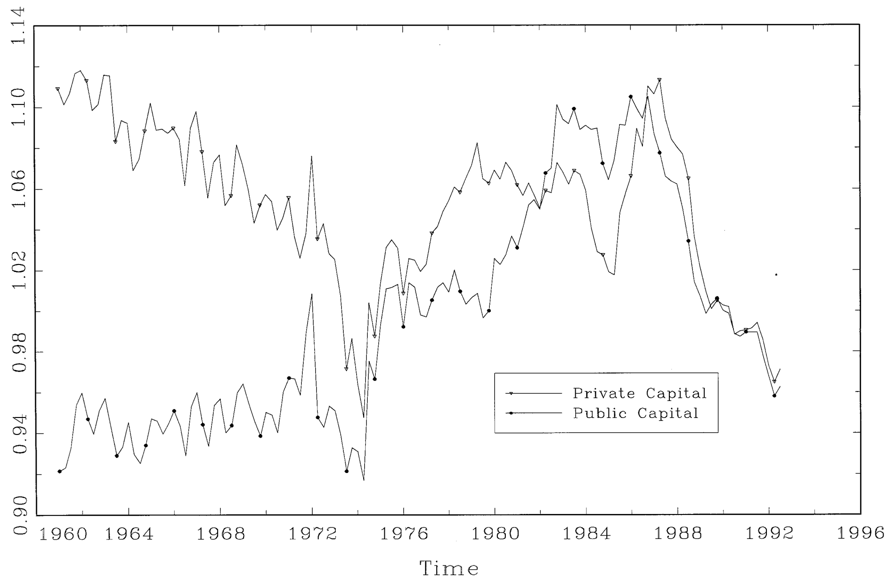


Fig. 2. Relative prices of investment goods.

expenditure and capital stocks.[7] However, for our purposes, it is useful to represent the transformation of output into capital expenditure using relative prices since these are readily available from standard data sources. The procedure here also has the advantage of not relying on an unknown functional form for modeling adjustment costs.

For sector $i$, the real return is

$$R_{it+1} \equiv \frac{1}{p_{it}}[f_{it+1} + (1-\delta_i)p_{it+1}]$$

and has the following motivation. One unit of foregone production at time $t$ provides $1/p_{it}$ units of the investment good for sector $i$. This increases future output by $f_{it+1}$ and the future capital stock by $(1-\delta_i)$ valued at $p_{it+1}$.

In the light of these arguments, we estimate a series of models. The first is the Cobb–Douglas model which assumes a unit relative price for both investment sectors. This model is most closely related to the existing literature and serves as a benchmark to compare our methods and results. The efficiency conditions for

[7] For example, Cochrane (1991) models and estimates investment returns with adjustment costs for investment. The Euler conditions from his paper are very similar in interpretation to those we present here.

the basic Cobb–Douglas model are

$$\mathrm{E}_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{\sigma-1}[\rho_i Y_{t+1}/K_{it+1}+(1-\delta_i)]\right]=1,\qquad i=1,2. \tag{1}$$

This model is then generalized in two ways. First, to include relative prices for investment goods. The conditions become

$$\mathrm{E}\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{\sigma-1}\left(\frac{1}{p_{it}}\right)[\rho_i Y_{t+1}/K_{it+1}+(1-\delta_i)p_{it+1}]\right]=1,\qquad i=1,2. \tag{2}$$

Then to allow a non-unit elasticity of substitution (the CES model),

$$\mathrm{E}_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{\sigma-1}\left(\frac{1}{p_{1t}}\right)\left(\rho\mu\frac{Y_{t+1}K_{1t+1}^{\phi-1}}{\mu K_{1t+1}^{\phi}+(1-\mu)K_{2t+1}^{\phi}}+(1-\delta_1)p_{1t+1}\right)\right]=1, \tag{3}$$

$$\mathrm{E}_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{\sigma-1}\left(\frac{1}{p_{2t}}\right)\left(\rho(1-\mu)\frac{Y_{t+1}K_{2t+1}^{\phi-1}}{\mu K_{1t+1}^{\phi}+(1-\mu)K_{2t+1}^{\phi}}+(1-\delta_2)p_{2t+1}\right)\right]$$
$$=1$$

For each model, the efficient resource allocation conditions define a $2\times 1$ error vector, denoted $u_{t+1}$, which has conditional mean zero. Let $z_t$ be a $q\times 1$ vector of instruments which are elements of the time $t$ information set. Then the conditional moment conditions for each model may be written as a $2q$ unconditional moment conditions suitable for estimation by Hansen (1982) generalized method of moments:

$$\mathrm{E}(u_{t+1}\otimes z_t)=0.$$

Of the parameters involved in these moment conditions, only the production parameters are estimated. The subjective discount factor, the intertemporal rate of substitution and the depreciation rates are set at particular values. The subjective discount factor is set at 0.99 which is consistent with a discount rate of approximately 4% per annum. The parameter $\sigma$, related to the intertemporal elasticity of substitution, is set at a value of $-1.0$. Setting this parameter rather than estimating it is not ideal, however, it serves to focus the study on the parameters of interest.[8] The depreciation rates are set at $\delta_1=0.017$ and $\delta_2=0.011$; these numbers match observed depreciation of the respective capital stocks.

[8] Attempts to estimate this parameter have not yielded economically sensible results. To obtain an accurate estimate of the intertemporal elasticity of substitution is likely to require a more careful model of consumers' preferences including consideration of leisure, non-separability of preferences, and possibly consideration of durables and non-durables consumption (see e.g. Hansen and Singleton, 1982; Eichenbaum et al., 1988). In practice, our results are not particularly sensitive to the choice of this parameter.

## 3. Empirical results

### 3.1. Data

We use Australian data which are quarterly and in constant 1989/90 prices for the sample period 1959:3–1992:2. Consumption is private final consumption expenditure, seasonally adjusted and in per capita terms. Output is the production-based seasonally adjusted measure of gross domestic product. The measures of private and public capital are derived from annual estimates published by the Australian Bureau of Statistics. Our quarterly estimates of the capital stocks are obtained by interpolation using investment data.[9] The private capital stock consists of both private sector capital and the capital stock of all public enterprises. The latter are institutions which provide marketed services and are not considered, in this study, as public capital. This has the additional advantage of avoiding issues of privatisation since this has mostly affected public enterprises. Public capital is the capital stock reported for general government, including federal, state and local governments, and corresponds most closely to the concepts of public capital discussed above (public goods in production). Price deflators for gross domestic product and for gross fixed capital expenditure of each sector are used to measure output and investment prices.

Conditions for consistency and asymptotic normality of the estimators are given in Hansen (1982). These conditions include the assumption that the series used in the estimation are covariance stationary. For the models we estimate, the assumption of covariance stationarity is reasonable for the growth rate of consumption as well as the growth rates of investment prices. Concern arises, however, over the remaining series which enter the moment conditions in such a manner so as to prevent transformation to covariance stationary series. For the benchmark Cobb–Douglas model, the series of concern are the output capital ratios, denoted $\hat{y}_{it+1} \equiv Y_{t+1}/K_{it+1}$, $i = 1,2$. These series, presented in Fig. 1, clearly exhibit some form of trend behaviour over the sample period. Indeed, it is this trend behaviour which is the basis for concern about public investment: relative to total production, public capital stocks have fallen in recent years while private capital stocks have risen. For the Cobb–Douglas model, the problem is mitigated by the investment price series $p_{it}$ which weight the output capital ratios; the compound series are $(1/p_{it})\hat{y}_{it+1}$, $i = 1,2$. As noted previously, for each sector the price series moves in the same direction as the output capital ratio so that the compound series exhibit markedly less trend

[9] Exact details on our data sources and method of construction are provided in a data appendix available at http://www.economics.unsw.edu.au/staff/voss.html or directly from the authors.

behaviour. Finally, the greatest difficulties arise for the CES model. Here the return component is a non-linear function of output to capital ratios as well as ratios of sectoral capital stocks, neither of which satisfy stationarity assumptions nor is there any obvious transformation to the series to recast the problem with stationary compound series.

The nonstationary series which form important components of our models mean that the GMM estimator may not have its standard asymptotic properties. Our approach is to proceed with the estimation of the models and consider what information is available from these results despite the failure of our series to formally satisfy the assumptions of GMM. As we argue in detail below, we have reasonable confidence in our parameter estimates but are less confident concerning formal inference. Despite these concerns over inference, we also argue that the results from the model are supportive of the maintained hypothesis of an efficient allocation of capital.

The choice of instruments is important in instrumental variables estimation. Three sets of instruments are chosen. The first set contains a constant and a measure of the real interest rate and is used to estimate all three models.[10] The second set of instruments is used for the benchmark Cobb–Douglas model and contains a constant and the two output capital ratios. The third set, which is used for all models that include investment prices, contains the price-weighted output–capital ratios and the gross growth rates of the investment prices. The reason for presenting results for each model using two sets of instruments is that while a larger instrument set provides asymptotically more efficient estimates there is evidence that smaller sets result in less finite sample bias (see the discussion and references cited in Davidson and MacKinnon, 1994).

In addition to choosing the instrument variables, it is necessary to decide on the appropriate dating of these instruments. As developed, the moment conditions of the model are orthogonal to $z_t$. However, elements of $z_t$ may not be appropriate instruments and we may wish to consider instruments dated further back in time. A number of authors, e.g. Christiano et al. (1991), argue that time-averaging (consumption decisions are made at finer intervals than measured data) of consumption data is sufficient to introduce a first-order moving average structure into the error terms of consumption-based Euler equation models. To address this, as well as the possibility of serial correlation in the Euler conditions, we consider instruments dated $t-1$ or earlier.

[10] We follow Mishkin (1981) in computing the real interest rate. The real interest rate used is the predicted values from a regression of the *ex post* real rate $r_t - \pi_{t+1}$ regressed on a constant, a time trend and four lags of: inflation, the nominal interest rate and output growth. Details of this regression are presented in the data appendix.

Table 1
Cobb–Douglas model

Instruments
Set 1: $\{c, r_{t-1}\}$
Set 2: $\{c, \hat{y}_{1,t-1}, \hat{y}_{2,t-1}\}$
Set 3: $\{c, \hat{y}_{1,t-1}/p_{1,t-2}, \hat{y}_{2,t-1}/p_{2,t-2}, p_{1,t-1}/p_{1,t-2}, p_{2,t-1}/p_{2,t-2}\}$

| | $\rho_1$ | $\rho_2$ | $J$ |
|---|---|---|---|
| *Cobb–Douglas without prices* | | | |
| Instrument Set 1 | 0.2010 | 0.0654 | 10.1042 |
| | (0.0067) | (0.0026) | (0.0064) |
| Instrument Set 2 | 0.1908 | 0.0583 | 17.0807 |
| | (0.0066) | (0.0028) | (0.0019) |
| *Cobb–Douglas* | | | |
| Instrument Set 1 | 0.2125 | 0.0587 | 1.1965 |
| | (0.0117) | (0.0037) | (0.5498) |
| Instrument Set 3 | 0.2123 | 0.0589 | 9.2021 |
| | (0.0090) | (0.0027) | (0.3255) |

Notes: Sample is 1960:04–1992:02. $J$ is Hansen's J-statistic distributed $\chi^2(r-k)$, where $r$ is the total number of moment conditions estimated and $k$ is the number of estimated parameters. Numbers in parentheses are standard errors except for the reported statistics; these numbers are marginal significance levels. The covariance matrix is estimated following Newey and West (1987) using a lag truncation parameter of four and three iterations of the estimation procedure.

### *3.2. Estimation results*

The estimation results for the benchmark Cobb–Douglas model (without investment prices) are presented in Table 1. For this model, both sets of instruments provide qualitatively similar output elasticity estimates for public and private capital and the estimates seem reasonable. For example, for Instrument Set 1, private capital output elasticity is 0.20, public capital output elasticity is 0.07 and, if we assume constant returns to scale for the aggregate economy, the implied output elasticity of labour is 0.73. This latter figure is very close to average income share for labour in Australia over this period of 0.69. Notably, the public capital elasticity estimates obtained from the Euler equation are significantly lower than those obtained in previous studies such as Finn (1993), Lynde and Richmond (1993) and Aschauer (1989) as well as our own study for Australia, Otto and Voss (1996), using the same data set employed here. The principal reason for this is the maintained hypothesis of efficient provision which acts as a restriction on the parameter estimation. Previous studies do not impose this restriction.

While the estimated elasticities seem reasonable, Hansen's test of the over-identifying restrictions imposed by the model are rejected for both sets of instruments at the 1% level. This indicates that the orthogonality conditions

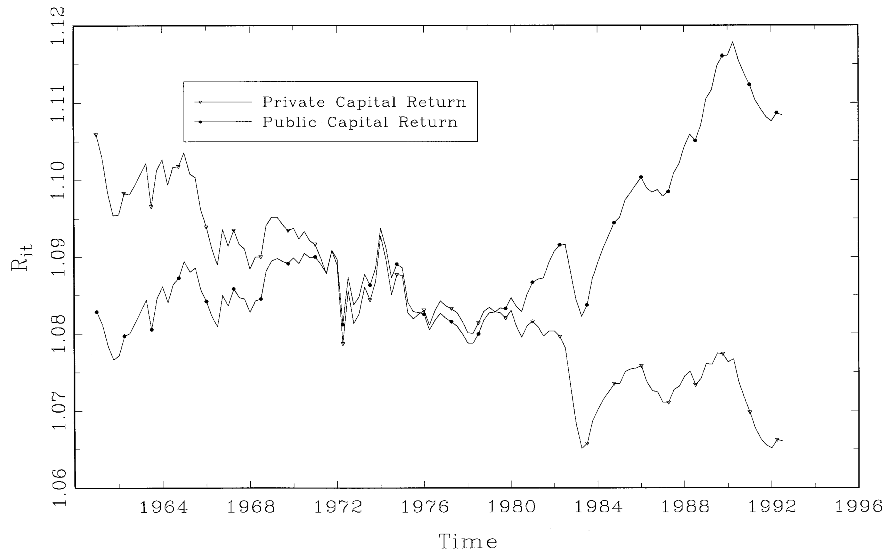


Fig. 3. Cobb–Douglas model without prices (Instrument Set 2).

imposed by the two Euler conditions are rejected by the Australian data. To identify the source of this rejection, consider the investment returns, $R_{it+1}$, $i=1,2$ implied by the point estimates associated with Instrument Set 2. These returns are presented in annual terms in Fig. 3 (Instrument Set 1 provides similar conclusions). Initially, the return on private sector investment exceeds that of public investment while, in contrast, during the 1980s the return to public sector investment exceeds that of private investment. This pattern of returns is not unexpected given the Cobb–Douglas production function, which has the return to each sector as a function of the output to capital ratio in that sector, and the pattern of output capital ratios presented in Fig. 1.

With Fig. 3 in mind, it is relatively straightforward to identify the source of rejection of the over-identifying restrictions. Based upon the two efficiency conditions, the estimation procedure is producing parameter estimates which provide average returns in each sector as close as possible to each other over the sample period. The test for over-identifying restrictions then examines whether there has been a statistically significant divergence from these average returns. The pattern of returns in Fig. 3 suggests there has and the test for over-identifying restrictions supports this conclusion.

Put somewhat differently, the question of whether there is currently a shortage of public investment spending requires a benchmark – some measure of the optimal level of public investment spending. For example, one might consider

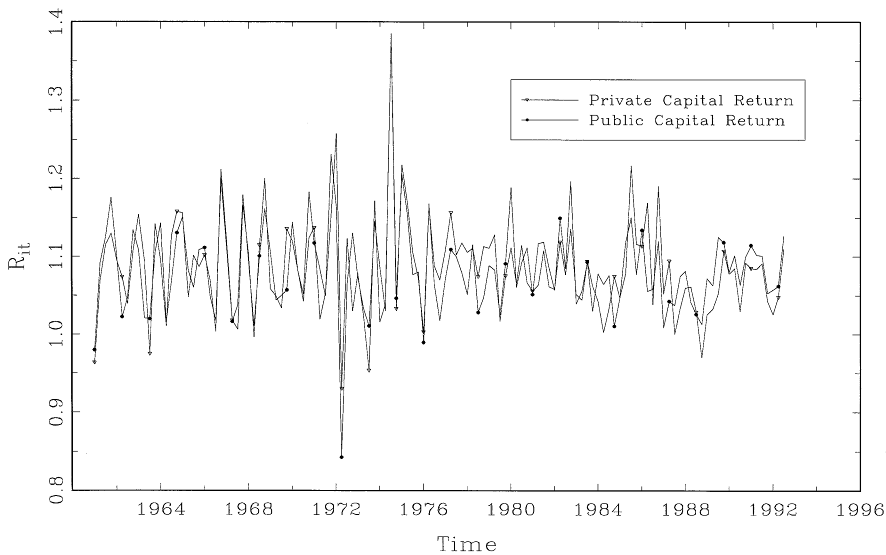


Fig. 4. Cobb–Douglas model (Instrument Set 3).

the pattern of spending in the 1960s as optimal in which case the reduced spending in more recent decades is sub-optimal; this seems to be implicit in much of the discussion in the public capital literature. Our estimation strategy implicitly uses the average over the sample period as a benchmark for the optimal level of public capital. Given this benchmark, the pattern of returns suggests over-investment in the public sector during the early part of the sample and under-investment in the later part of the sample.

We now consider the same model but amended to include the price of investment goods in each sector. These results are also reported in Table 1. For both sets of instruments, the parameter estimates are very similar to the previous model. Now, however, the over-identifying restrictions are not rejected by the J-test. Further indication of the the importance of including investment prices in the model can be found by inspecting the investment returns for each sector, presented in Fig. 4 (based on Instrument Set 3). Clearly, the inclusion of investment prices greatly alters the nature of these returns and consistent over- or under-investment in either sector is not evident. The return series are stationary around a mean real return of approximately 9% per annum. This contrasts markedly with the behaviour of the investment returns to the benchmark model, a point we return to below.

Although the results for the Cobb–Douglas model with prices are quite good, it seems worthwhile to examine whether a more general representation for production is required. Table 2 reports the results for the unrestricted CES

Table 2
CES model

| | $\rho$ | $\mu$ | $\phi$ | $J$ | LR($\phi = 0$) | LR($\phi = 1$) |
|---|---|---|---|---|---|---|
| CES | | | | | | |
| l Instrument Set 1 | 0.2716 | 0.7276 | 0.2982 | 0.5466 | 0.6499 | 3.8464 |
| | (0.0152) | (0.0712) | (0.3513) | (0.4597) | (0.4201) | (0.0500) |
| Instrument Set 3 | 0.2792 | 0.6599 | 0.6253 | 5.8270 | 3.3751 | 2.2780 |
| | (0.0120) | (0.0382) | (0.1622) | (0.5601) | (0.0619) | (0.1312) |
| CES ($\phi = 1$) | | | | | | |
| Instrument Set 1 | 0.2754 | 0.5677 | 1.0 | 4.3930 | | |
| | (0.0151) | (0.0068) | – | (0.1112) | | |
| Instrument Set 3 | 0.2730 | 0.5692 | 1.0 | 8.1050 | | |
| | (0.0119) | (0.0055) | – | (0.4233) | | |

Notes: See Table 1. LR is a $\chi^2(1)$ test statistic based on the likelihood ratio principle.

model as well as for the CES model with the restriction that the capital stocks are perfect substitutes ($\phi = 1$).

For the unrestricted CES model, both sets of instruments provide reasonably similar parameter estimates with the exception of the elasticity of substitution parameter $\phi$ which varies to some extent between the two estimated models. Neither value is, however, unreasonable; both suggest that the parameter lies between zero and one or, equivalently, between the Cobb–Douglas version of the model and the perfect substitutes version of the model. More importantly, the test for over-identifying restrictions is not rejected for either set of instruments so, as before, we do not find any evidence of consistent over- or under-investment in either sector. (As before, the investment returns are stationary and average approximately nine percent.)

Since the elasticity of substitution parameter $\phi$ is not very precisely estimated, we test two restrictions for this parameter. The first is the Cobb–Douglas model which sets $\phi = 0$; the second is the perfect substitutes model which sets $\phi = 1$. In both cases, tests based on the likelihood ratio principle (see Davidson and MacKinnon, 1994) do not provide any conclusive evidence. Consider the hypothesis $\phi = 0$. For the first instrument set, the hypothesis cannot be rejected; for the second, there is weak evidence against the hypothesis. At best then, there appears to be some weak evidence in favour of the Cobb–Douglas model. Now consider the hypothesis $\phi = 1$. There is weak evidence, again depending upon the instrument set, against the hypothesis. While these results are insufficient to make any firm conclusions they are suggestive at least that the Cobb–Douglas specification may be a reasonable representation of aggregate production. Essentially it is not the change from a Cobb–Douglas to a CES production function that improves the ability of the Euler equation to fit the data, rather it is the inclusion of the relative prices of investment goods.

So far, discussion of the estimation results has proceeded as if we are confident about the distributions of the estimators we employ. However, two potential problems with the estimation results merit attention. The first concerns instrument quality, the second the covariance stationary assumption underlying GMM.[11]

The quality of instruments is well recognized as an important determinant of the performance of instrumental variables estimation in general and within the GMM framework. Poor quality instruments have implications for finite sample bias and the finite sample distributions of the estimators (see the discussion and references cited in Shea, 1996). One obvious and important concern for the exercise here is that the failure to reject the efficiency hypothesis arises from poor instruments.

To examine the quality of our instruments, we follow Shea (1996) and consider a sample partial correlation statistic which measures instrument quality in multivariate models. Shea suggests that for each endogenous variable $X_i \in X$ in a linear model, the sample-squared correlation between two series: (i) the component of $X_i$ orthogonal to $X_{-i}$ and (ii) the component of the $X_i$ predicted by the instruments which is orthogonal to the component of $X_{-i}$ predicted by the instruments. For our models, which are nonlinear, we perform this procedure for each derivative of the moment condition with respect to each parameter (this follows arguments in Pagan and Jung, 1993). The sample-squared correlations are reported in Table 3. Generally speaking, Instrument Set 1 performs poorly relative to the other two instruments sets with the results for the CES model especially poor. The other two instrument sets seem to perform well and we can be reasonably confident that poor instrument quality is not responsible for any bias or distributional problems of our estimators. On this basis, we do not believe instrument quality to be responsible for the failure to reject the efficiency conditions.

The second potential problem with the estimation results is that the moment conditions comprise series that do not satisfy the assumption of covariance

[11] There is an additional problem. In a previous version of this paper we test and reject parameter stability for models similar to those considered here. Using the tests suggested in Andrews (1993), parameter stability is also formally rejected for the models estimated here. However, the concerns we raise about the distribution of the GMM estimators and test statistics also apply to the stability test statistics so it is difficult to be confident about these results. For example, examination of the estimated results reveals that the standard errors for the parameter estimates are very small which is going to favour finding evidence of parameter instability. In favour of our models is the fact that split sample estimation of these models provides parameter estimates and conclusions similar to the full sample estimation (these results are available from the authors). The exception is the CES model which is very difficult to estimate in the smaller split samples, possibly due to the nonlinearities of the model.

Table 3
Instrument quality

| | | | | |
|---|---|---|---|---|
| Cobb–Douglas without prices | | | | |
| | | $\partial u_{i,t+1}/\partial \rho_1$ | $\partial u_{i,t+1}/\partial \rho_2$ | |
| Instrument Set 1 | $i=1$ | 0.191 | – | |
| | $i=2$ | – | 0.358 | |
| Instrument Set 2 | $i=1$ | 0.887 | – | |
| | $i=2$ | – | 0.916 | |
| Cobb–Douglas | | | | |
| Instrument Set 1 | $i=1$ | 0.321 | – | |
| | $i=2$ | – | 0.096 | |
| Instrument Set 3 | $i=1$ | 0.803 | – | |
| | $i=2$ | – | 0.894 | |
| CES | | | | |
| | | $\partial u_{i,t+1}/\partial \rho$ | $\partial u_{i,t+1}/\partial \mu$ | $\partial u_{i,t+1}/\partial \phi$ |
| Instrument Set 1 | $i=1$ | 0.003 | 0.013 | 0.000 |
| | $i=2$ | 0.000 | 0.004 | 0.094 |
| Instrument Set 3 | $i=1$ | 0.263 | 0.269 | 0.258 |
| | $i=2$ | 0.255 | 0.255 | 0.259 |
| CES ($\phi=1$) | | | | |
| | | $\partial u_{i,t+1}/\partial \rho$ | $\partial u_{i,t+1}/\partial \mu$ | |
| Instrument Set 1 | $i=1$ | 0.357 | 0.355 | |
| | $i=2$ | 0.320 | 0.321 | |
| Instrument Set 3 | $i=1$ | 0.839 | 0.838 | |
| | $i=2$ | 0.817 | 0.817 | |

Notes: Numbers are sample-squared correlations from the artifical regressions suggested in Shea (1996).

stationarity. As a consequence, it is unlikely that the parameter estimates and test statistics will have the distribution assumed. Despite this, there are a number of reasons to believe that our estimation results are informative and that our conclusion of efficient provision is robust. First, the estimated parameters yield economically sensible results. The output elasticities for public and private capital are consistent with prior expectations as is the implied output elasticity of aggregate labour. Second, with the exception of the benchmark model, the implied sectoral returns are stationary and very highly correlated. Regardless of statistical measures, the close mapping of these returns is evidence in favour of the maintained hypothesis of efficient resource allocation between the two sectors.

This latter point could be extended by noting that for all but the benchmark model, the estimated residuals $\hat{u}_{it+1}$ are stationary. (The error terms are dominated by the return component $R_{it+1}$ so if the latter are stationary for a given set of parameters so are the estimated residuals.) Consequently, the estimated parameters combine a set of nonstationary (and stationary) series into a stationary residual in much the same manner as a cointegrating vector does in a linear environment. While little is known concerning estimation of nonlinear models involving non stationary series, it seems reasonable to argue that the stationarity of the residuals here provides some support for our estimated models.[12]

Finally, we could consider the following experiment. Suppose we knew the true model was CES with perfect substitution between private and public capital and further the value of $\mu$ was known and was such that $Y_{t+1}/(\mu K_{1,t+1} + (1-\mu)K_{2,t+1})$ was stationary. In this case, we could estimate the model and be quite confident about inference since all our series would be stationary. There is in fact a range of $\mu$ that satisfy this criteria and estimating the model conditional on such $\mu$ provides similar parameter estimates and conclusions. While this does not provide direct support for our estimation results, it does indicate that the pattern of capital stocks which have been described as sub-optimal are consistent with at least one reasonable (and simple) model and that it takes more than declining growth in public capital to identify a problem.

## 4. Conclusion

This paper investigates the provision of public capital in Australia for the last three decades. We identify conditions of efficient resource allocation for an economy with two capital sectors, private and public, and then examine whether or not the Australian data satisfies these conditions. Our estimation results support the hypothesis that resources have not been misallocated; specifically, there is no evidence of systematic over- or under-investment in either the public or private sector.

An important aspect of assessing public investment which our analysis highlights is the role of investment goods prices. For the Australian economy, the price of public investment goods in aggregate has risen relative to that of aggregate production (both measured by the relevant price deflators). In contrast, private investment goods have become relatively less expensive. This behaviour of investment goods prices seems to account, as one would expect, for the growing private capital intensity (relative to public capital) of aggregate

[12] This informal extension of the principles of cointegration to method of moments estimation has also been applied in Fuhrer et al. (1995).

production within Australia. To put this more simply, the declining growth in public capital can be explained by the rising cost of public investment.

A natural question which our analysis raises is the cause of the rise in the relative price of public investment goods. While this may reflect circumstances beyond the control of policymakers, it may also be driven by their behaviour. One possibility is that the rising costs might reflect strengthening regulations on the capital provided by the public sector; for example, new roads might have to satisfy higher quality specifications which raises the cost of new road capital. This is, of course, the standard problem with price indices and quality changes and suggests, if valid, that the decline in public capital growth may be in part a measurement problem. Whatever the cause, the behaviour of the price deflators for public and private investment seem to merit further investigation. An additional direction for further research is whether similar price movements have occurred in other countries which have also experienced a decline in the growth rate of public capital.

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