Quarterly Beta Forecasting

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Abstract

Recent developments in measuring and modeling asset return covariation and volatility have led to the construction of realized betas, verifying that betas are indeed time-varying. In this paper a variety of proposed models for beta are examined in an out-of-sample forecast evaluation. It is found that an an AR(2) model computed on the previous eighty quarters of realized beta produces the lowest or close to the lowest error for quarterly beta forecasts of Dow Jones stocks. Forecasts from a constant beta model for some stocks produced mean squared forecast errors double that of the AR(2) model.

KEY WORDS. Portfolio management; Realized covariation; Realized volatility.

1 Introduction

Quarterly forecasts of stock betas are a core component of modern portfolio management, asset pricing and financial risk management. A stock beta is the ratio of the stock and market return covariance and the market variance. Often a portfolio is rebalanced based on the individual forecasts of many stock betas. For example, a pension fund manager may optimize subject to their forecast portfolio beta being set to 1, with the forecast portfolio beta being a weighted average of the individual stock beta forecasts. Hedge fund managers often optimize subject to their forecast portfolio beta being set to 0. Accurate individual stock beta forecasts are crucial, see Ghysels and Jacquier (2005). Currently, betas are typically estimated using five to ten years of data and under the assumption that beta is not time-varying, see Campbell, Lo and MacKinlay (1997) and Andersen, Bollerslev, Diebold and Wu (2005a).

Recent advances in financial econometrics have resulted in efficient measures being developed of covariation and volatility. These nonparametric measures, often called realized covariance and realized variance, have been justified by Andersen and Bollerslev (1998), Andersen, Bollerslev, Diebold and Labys (2001, 2003), and Barndorff-Nielsen and Shephard (2002, 2004). A substantial literature has already developed on examining the dynamics of realized variance and traditional time series models have been estimated on realized variance and forecasting performance evaluated, see Andersen, Bollerslev, Diebold and Labys (2003), Andersen, Bollerslev, Diebold and Ebens (2001), Maheu and McCurdy (2002), Martens, van Dijk and Pooter (2004), Ghysels, Santa-Clara and Valkanov (2005), Koopman, Jungbacker and Hol (2005). In these out-of-sample forecast evaluation exercises, an AR(5) model computed on daily realized variance, typically outperforms traditional approaches to forecasting daily volatility, such as GARCH, Engle (1982) and

Bollerslev (1986).

Andersen, Bollerslev, Diebold and Wu (2005a) and Andersen, Bollerslev, Diebold and Wu (2005b) extend the realized covariance and variance literature to the study of realized betas. These authors highlight an emerging consensus that betas are in fact time-varying and verify this by computing quarterly realized betas of Dow Jones stocks with their autocorrelation functions. Based on quarterly realized beta estimates from 1962:3 through 1999:3 they find using an AIC criterion for in-sample fit, that out of their twenty-five Dow Jones stocks, an AR(p) model where p is either 1,2,3,4 or 5 is suitable, with the stocks being relatively evenly distributed between these five models.

This paper continues the work of Andersen, Bollerslev, Diebold and Wu (2005a) by extending their dataset up to 2003:4 and conducting an out-of-sample forecast evaluation. Seven models are evaluated; a constant beta model, AR(1) through AR(5) models and an IV model recently proposed by Ghysels and Jacquier (2005). We find that for one-quarter-ahead forecasts of beta an AR(2) model computed on the previous eighty quarters of realized betas produces the lowest or close to the lowest forecast error. The constant beta model and IV model often produce inaccurate forecasts.

This paper is organized as follows. The next section describes the construction of realized betas and our sample of Dow Jones stocks. The third section evaluates a variety of models for one-quarter-ahead forecasting of beta and the final section section concludes.

2 Measuring beta

To illustrate our approach to beta measurement, we follow Andersen, Bollerslev, Diebold and Wu (2005a) and assume the logarithmic $N\times 1$ vector process $p(t)$ follows a multivariate continuous-time stochastic volatility diffusion,

$$dp(t) = \mu(t)dt + \Omega(t)dW(t) \tag{1}$$

where $W(t)$ is standard N-dimensional Brownian motion, $\Omega(t)$ is the $N \times N$ positive definite diffusion matrix and $\mu(t)$ is the N-dimensional instantaneous drift. Both $\Omega(t)$ and $\mu(t)$ are strictly stationary and jointly independent of $W(t)$.

Let the N^{th} element of $p(t)$ contain the log price of the market and the i^{th} element of $p(t)$ contain the log price of the i^{th} individual stock, and suppose the process is sampled S times per period on an equally spaced grid. Then define the $\delta = 1/S$ period return as $r_{i,t,j} = p(t + j\delta) - p(t + (j-1)\delta)$, $j = 1, 2, \ldots, S$. We denote the realized beta by

$$\beta_{i,t+1} = \frac{\sum_{j=1}^{S} r_{i,t,j} r_{N,t,j}}{\sum_{j=1}^{S} r_{N,t,j}^2} \tag{2}$$

which is a consistent estimator of the true underlying integrated beta

$$\frac{\int_t^{t+1} \Omega_{iN}(\tau)d\tau}{\int_t^{t+1} \Omega_{NN}(\tau)d\tau} \tag{3}$$

almost surely for all t as $S \to \infty$. For additional details see Andersen, Bollerslev, Diebold and Wu (2005a).

We empirically measure realized beta at the quarterly frequency by setting δ equal to a trading day, thus S is set at the number of trading days in the t^{th} quarter. The Dow Jones stocks daily returns and the CRSP Value-weighted (including distributions) market return are obtained from the Center for Research in Security Prices. We construct realized betas for the same set of Dow Jones stocks as Andersen, Bollerslev, Diebold and Wu (2005a), less Union Carbide Corp. These remaining twenty-four stocks have complete data running from 1963:1 through 2003:4 and their quarterly realized betas are displayed in Fig. 1, 2 and 3, alphabetically according to ticker code. (Computations were performed using the Ox programming language, (Doornik (2001)) and the code is available upon request.)

3 Out-of-sample forecast evaluation

We consider the following autoregressive specification for $\beta_{i,t}$

$$\beta_{i,t} = \phi_0 + \sum_{j=1}^{5} \phi_j \beta_{i,t-j} + \epsilon_t, \quad \epsilon_t \sim iid(0, \sigma^2) \;\; t = 1, 2, \ldots, n. \tag{4}$$

All models estimated are nested in this framework. The constant beta model has $\phi_j = 0$ for $j = 1, 2, \ldots, 5$. The AR(1) model has $\phi_j = 0$ for $j = 2, 3, \ldots, 5$, and so on up to AR(5). We also consider the AR(1) model estimated by instrumental variables (IV) with the constant and lag 2 as instruments, as suggested by Ghysels and Jacquier (2005).

One-quarter-ahead forecasting of β_{n+1} is conducted for each of our Dow Jones stocks and the forecast evaluation period is from 1983:1 until 2003:4. All model estimation is based on the previous 80 quarters of realized beta, as smaller sample sizes increased the forecast error. Conditional mean forecasts of beta are evaluated by two alternative measures, mean squared error (MSE) and mean absolute error (MAE) and are reported in Tables 1 and 2.

The results suggest an AR(2) model computed on the previous 80 quarters of realized beta produces the most accurate one-quarter-ahead forecast of beta. For all our Dow Jones stocks, the AR(2) model produces the lowest or close to the lowest, MSE and MAE. For some stocks the AR(2) model MSE is half that of the constant beta model. For Coca-Cola Co. the AR(2) model MSE is 0.106, whereas the constant beta model MSE is 0.293. The respective MAEs are 0.247 and 0.444.

The IV model for some stocks produces very inaccurate forecasts. For United Technologies Corp. the AR(2) model has a MSE of 0.121 whereas the IV model has a MSE of 8.252.

4 Conclusions

Accurate forecasting of stock betas, often at the quarterly frequency, is fundamental to portfolio management, asset pricing and financial risk management. This paper shows for Dow Jones stocks that an AR(2) model computed from the previous eighty quarters of realized betas produces the lowest or close to the lowest error for quarterly forecasts of beta. The constant beta model often produces inaccurate forecasts. For Coca-Cola Co. the constant beta model has a mean squared forecast error almost triple that of the AR(2) model.

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Table 1: MSE of One-Quarter-Ahead Forecast of Beta

Company	Constant	$AR(1)$	$AR(2)$	$AR(3)$	$AR(4)$	$AR(5)$	IV
Alcoa Inc.	0.215	0.197	0.182	0.188	0.194	0.201	0.514
Allied Capital Corp.	0.117	0.123	0.119	0.120	0.122	0.124	2.961
Boeing Co.	0.268	0.163	0.148	0.150	0.154	0.156	0.176
Caterpillar Inc.	0.161	0.107	0.102	0.103	0.105	0.105	0.116
Chevron Corp.	0.218	0.123	0.108	0.103	0.103	0.106	0.137
DuPont Co.	0.172	0.144	0.146	0.149	0.151	0.145	0.155
Walt Disney Co.	0.121	0.097	0.103	0.102	0.103	0.103	0.124
Eastman Kodak Co.	0.158	0.136	0.129	0.115	0.117	0.121	0.162
General Electric Co.	0.102	0.078	0.080	0.083	0.084	0.083	0.081
General Motors Corp.	0.143	0.111	0.108	0.109	0.108	0.110	0.129
Goodyear Tire & Rubber Co.	0.216	0.179	0.163	0.167	0.169	0.175	0.279
Hewlett-Packard Co.	0.157	0.157	0.160	0.161	0.164	0.168	6.223
IBM Corp.	0.108	0.106	0.104	0.107	0.109	0.112	0.408
International Paper Co.	0.190	0.126	0.128	0.132	0.137	0.138	0.153
Johnson & Johnson	0.291	0.167	0.159	0.162	0.171	0.177	0.153
Coca-Cola Co.	0.293	0.106	0.106	0.102	0.103	0.104	0.105
3M Corp.	0.132	0.087	0.086	0.087	0.090	0.090	0.086
Philip Morris Co.	0.248	0.163	0.156	0.157	0.165	0.161	0.145
Merck & Co.	0.197	0.123	0.127	0.128	0.130	0.132	0.155
Procter & Gamble Co.	0.219	0.127	0.108	0.109	0.114	0.115	0.147
Sears, Roebuck and Co.	0.178	0.174	0.153	0.157	0.160	0.160	0.288
AT&T Corp.	0.200	0.128	0.104	0.104	0.105	0.101	0.129
United Technologies Corp.	0.129	0.130	0.121	0.124	0.133	0.135	8.252
Exxon Corp.	0.156	0.074	0.072	0.074	0.075	0.074	0.072

Table 1 reports the mean squared error of the one quarter ahead forecast of beta for the above models. The IV model is an AR(1) with the constant and lag 2 as instruments. Forecasting begins at 1983:1 and ends at 2003:4. Model estimation is based on the previous 80 quarters.

Table 2: MAE of One-Quarter-Ahead Forecast of Beta

Company	Constant	$AR(1)$	$AR(2)$	$AR(3)$	$AR(4)$	$AR(5)$	IV
Alcoa Inc.	0.343	0.323	0.303	0.306	0.312	0.317	0.484
Allied Capital Corp.	0.276	0.276	0.271	0.272	0.278	0.276	0.792
Boeing Co.	0.407	0.307	0.300	0.306	0.307	0.310	0.319
Caterpillar Inc.	0.309	0.241	0.237	0.240	0.241	0.244	0.277
Chevron Corp.	0.372	0.276	0.253	0.249	0.251	0.254	0.278
DuPont Co.	0.307	0.282	0.281	0.284	0.287	0.278	0.294
Walt Disney Co.	0.282	0.239	0.247	0.260	0.259	0.257	0.279
Eastman Kodak Co.	0.307	0.288	0.280	0.279	0.282	0.287	0.318
General Electric Co.	0.248	0.226	0.227	0.233	0.229	0.226	0.235
General Motors Corp.	0.300	0.247	0.244	0.242	0.239	0.240	0.273
Goodyear Tire & Rubber Co.	0.362	0.333	0.314	0.316	0.322	0.327	0.391
Hewlett-Packard Co.	0.294	0.296	0.296	0.295	0.298	0.299	1.239
IBM Corp.	0.265	0.259	0.256	0.259	0.265	0.263	0.408
International Paper Co.	0.339	0.276	0.278	0.281	0.289	0.291	0.301
Johnson & Johnson	0.424	0.313	0.311	0.315	0.315	0.319	0.301
Coca-Cola Co.	0.444	0.250	0.247	0.243	0.243	0.246	0.252
3M Corp.	0.271	0.222	0.218	0.219	0.223	0.224	0.222
Philip Morris Co.	0.388	0.309	0.301	0.301	0.302	0.302	0.291
Merck & Co.	0.313	0.256	0.261	0.263	0.265	0.272	0.282
Procter & Gamble Co.	0.371	0.275	0.251	0.250	0.256	0.257	0.322
Sears, Roebuck and Co.	0.337	0.326	0.300	0.303	0.305	0.308	0.414
AT&T Corp.	0.341	0.279	0.256	0.261	0.264	0.255	0.278
United Technologies Corp.	0.267	0.270	0.262	0.264	0.270	0.271	1.505
Exxon Corp.	0.303	0.224	0.217	0.222	0.222	0.216	0.224

Table 2 reports the mean absolute error of the one quarter ahead forecast of beta for the above models. The IV model is an AR(1) with the constant and lag 2 as instruments. Forecasting begins at 1983:1 and ends at 2003:4. Model estimation is based on the previous 80 quarters.

Figure 1: Quarterly Realized Betas

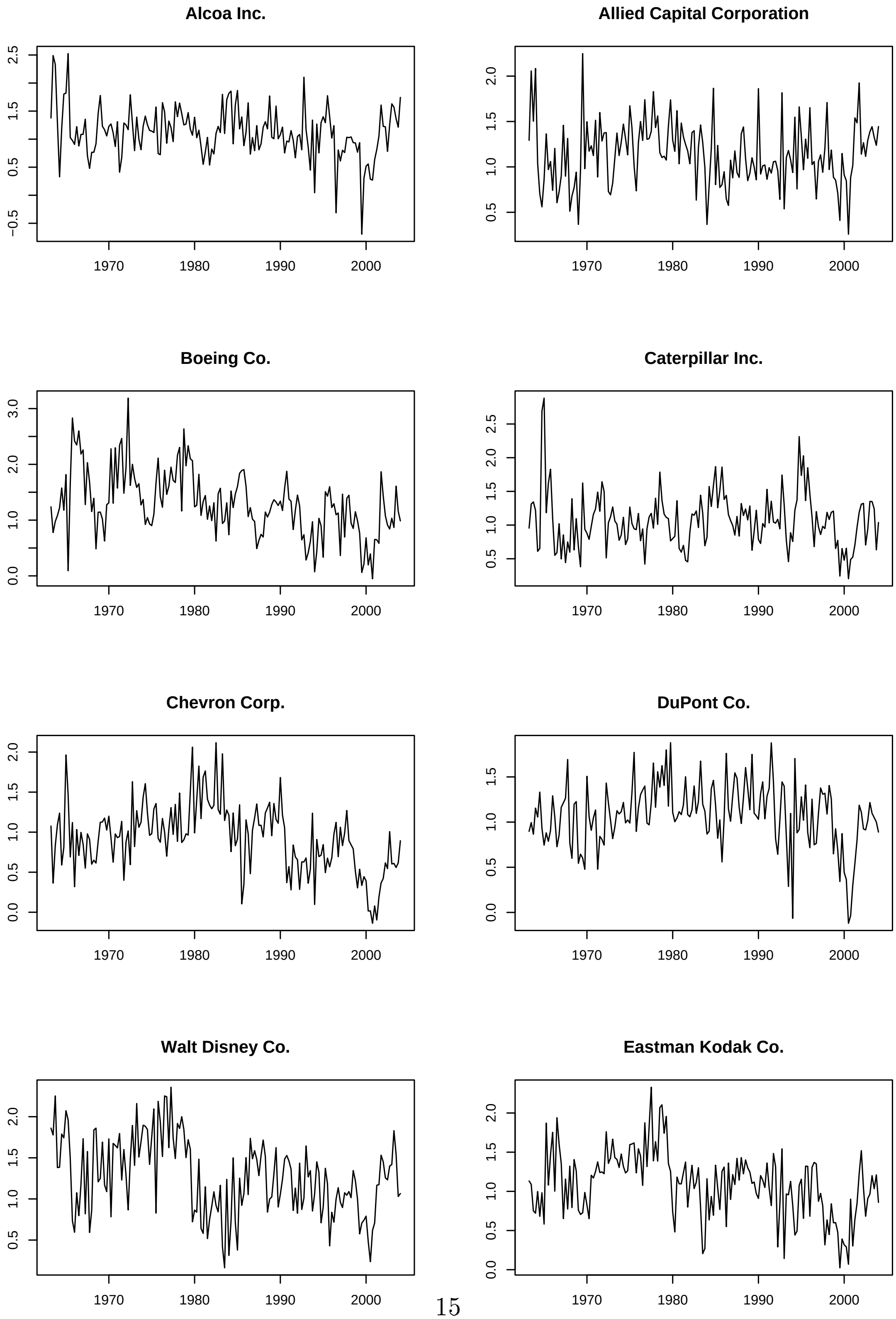

Figure 2: Quarterly Realized Betas

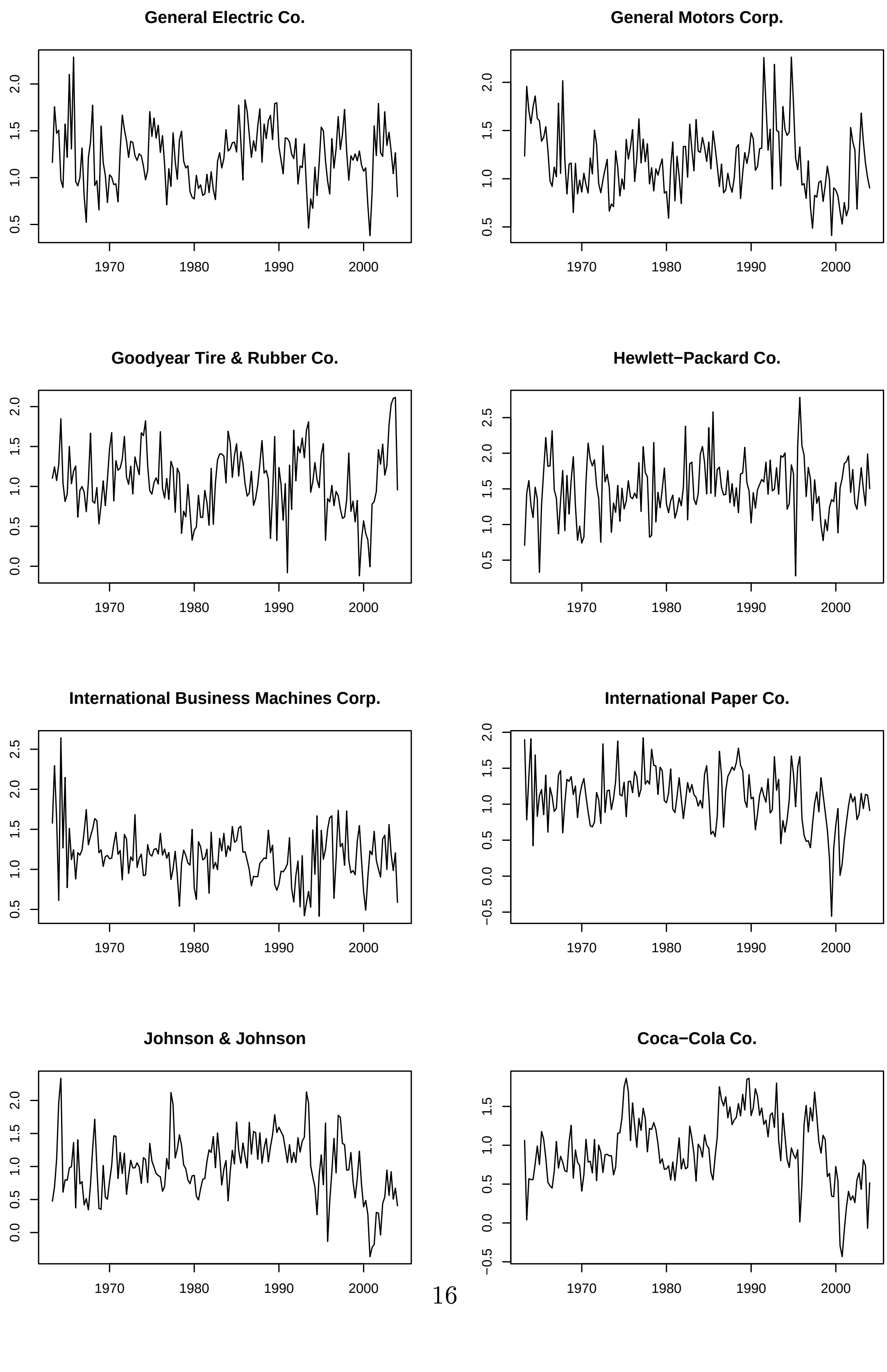

Figure 3: Quarterly Realized Betas