

A note on portfolio selection, diversification and Fund-of-Funds*

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Abstract

This paper examines the performance and diversification properties of active Australian equity fund-of-funds (FoF). Simulation analysis is employed to examine portfolio performance as a function of the number of funds in the portfolio. The paper finds that as the number of funds in a FoF portfolio increases, performance improves in a mean-variance setting, however measures of skewness and kurtosis behave less favourably given an investor's preferences for the higher moments of the return distribution. The majority of diversification benefits are realised when a portfolio of approximately six active equity funds are included in the FoF portfolio.

JEL Classification: G23

Keywords: Fund-of-Funds, Diversification, Portfolios, Investment Performance

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1. Introduction

This study examines portfolio selection by examining the relationship between investment performance and diversification properties arising from the construction of actively managed equity fund-of-fund portfolios. Fund-of-fund (FoF) investment products are defined as a single managed investment portfolio, where fund assets are allocated across a number of individual managed funds. In essence, FoFs provide investors with the opportunity of accessing a suite of investment products offered by competing investment management institutions through a single investment vehicle. The alleged benefits of investing in actively managed FoFs include enhancement in the execution of an investor's investment strategy as well as improved diversification. However there are a number of trade-offs in the use of FoFs. By increasing the number of unique actively managed funds in the FoF portfolio, such an investment structure might ultimately achieve a diversified portfolio which very closely tracks (or mimics) the underlying benchmark. The diversification benefits may then be significantly eroded, given that

the FoF portfolio charges active management fees for a product that essentially mimics the stock holdings and index weights of the benchmark.

Since Markowitz' (1952) seminal paper on portfolio selection, a number of studies have examined the relationship between risk and return according to the number of stocks held in a portfolio, and therefore the diversification benefits attributable to stock portfolios (Evans and Archer (1968), Elton and Gruber (1977), and Statman (1987)). The literature has since examined the diversification properties of FoF portfolios. Evidence of diversification benefits in a mean-variance setting are documented among U.S. mutual funds by O'Neal (1997) and Potter (2001), and in FoF hedge funds by Park and Staum (1998), Amin and Kat (2002) and Lhabitant and Learned (2002). The additional information content of the higher moments of investment fund return distributions, with respect to performance and risk, is discussed by Bird and Gallagher (2002). However, the diversification properties of FoFs have only been considered beyond a mean-variance framework (i.e. skewness and kurtosis) for hedge funds (Lhabitant and Learned (2002) and Amin and Kat (2002)).

Research examining FoF structures can be motivated in terms of the significant growth in the number and size of fund-of-fund investment vehicles in Australia. Over the last five years FoF products have grown by 119 percent, accounting for \$16.6 billion in assets as at March 2002, over the same period the total funds management industry grew by 93 percent.¹ While FoFs represent a distinct form of investment vehicle available to investors (in that the provider of FoFs selects the underlying funds comprising the portfolio), conceptually a FoF approach can also be extrapolated to include any investment arrangement or product that utilises the services of a diversity of investment providers. Institutional superannuation funds themselves, through their use of external institutional fund managers, exhibit similar features to FoFs.² The bundling of investment products and services can also be extended to include discretionary master trusts and WRAP accounts.

Our research makes a number of important contributions to the literature. Given that FoF research in Australia is non-existent, our study provides attention to FoF portfolio configuration for Australian investors. Furthermore, while the literature has documented the impact on higher return distribution moments for hedge FoFs, the literature has not examined FoF performance and risk properties within traditional asset classes. Using simulations to construct return distributions from active equity fund return data, this study seeks to identify the extent to which FoFs experience diversification benefits from the use of multiple managers and products in equity portfolios. The study documents that in a mean-variance framework, portfolio risk is a decreasing function of the number of funds included in the FoF portfolio, while mean

¹ ASSIRT market share reports March Quarter 1997 and March Quarter 2002.

performance remains constant as the number of funds increases. However, adding funds to the FoF structure leads to deterioration in FoF portfolio skewness and kurtosis.

The remainder of this study is structured as follows. Section 2 describes the data, and this is followed by the research design. Section 4 presents the empirical results, and the final section concludes the study and makes suggestions for future research.

2. Data

Ten years of monthly return data for a sample of active wholesale Australian equities managers was obtained by merging information provided by three asset consultants between the period 1 October 1989 and 30 September 1999, while also ensuring that no fund was double counted in the sample.³ Merging datasets gives rise to a more representative sample and permits cross checking of the returns data. The final sample comprises 134 Australian equity institutional funds offered by 65 investment managers and benchmarked to the ASX All Ordinaries Accumulation Index. The sample represents all open-end Australian wholesale equity funds available to institutional investors over the period. Fund returns are reported before taxes and before expenses (i.e. gross returns).

To date, equity FoF studies (O’Neal (1997), Potter (2001)) employ datasets that include only those funds that have survived the duration of the sample period, which yields results that exhibit survivorship bias. An improvement of this study is the use of a dataset that includes both surviving and non-surviving funds. The average length of fund return data in our sample is 77 months, and there are 66 funds in aggregate which are either non-survivors, or for which the historical returns data ceases (49 percent of the total sample). Amin and Kat (2001) and Lhabitant and Learned (2002) both document the effect of survivorship bias on hedge fund performance, and report that an absence of non-surviving funds in the analysis will overestimate mean return and downwardly bias standard deviation. In addition, skewness can be biased upward and kurtosis downward.

3. Experimental Design

3.1 Method of Portfolio Construction

This section describes the method of FoF portfolio construction employed as a means of understanding the extent to which FoFs experience diversification benefits from including between one and n randomly selected funds in a portfolio. A naïve strategy for portfolio construction involves random selection without replacement from the entire sample and equally

² According to APRA’s Superannuation Trends for the September Quarter 2003 more than \$A185 billion in assets are delegated to investment managers by institutional investors.

³ Asset consultants include Mercer Investment Consulting, The Frank Russell Company and Towers Perrin, and the data are sourced from the internal performance analytics databases of these consultants.

weighting all funds.⁴ Ten thousand portfolios comprising between one through n funds are formed by randomly selecting funds from the sample. Given that our sample includes non-surviving funds, if a fund that is part of a simulated FoF portfolio terminates, it is replaced with another fund in the subsequent period (consistent with Amin and Kat (2002)). The replacement fund is randomly selected from the surviving population of funds, excluding those already included in the portfolio – this ensures no fund is selected more than once in any given portfolio. The performance of the portfolios of funds can then be simulated using data over a given holding period.⁵ The cumulative return and value (given a \$1 initial investment) of each portfolio is calculated for every month over a three-year holding period.⁶ This results in a time-series of monthly FoF returns as well as the terminal wealth level (i.e. the investment value at the end of the last month of the holding period).

3.2 *Attributes of Investment Performance*

The portfolio construction technique described in Section 3.1 requires generation of 10,000 time-series' of returns for the various portfolio sizes (i.e. one through n funds). Consequently, each of the following statistics can be reported as a function of the number of funds in the portfolio. In a mean-variance framework, measures of time-series variance and mean return are appropriate. In order to understand the behaviour of these statistics, the mean monthly return for each of the simulated portfolios (for a given n) is calculated and the average of these means are reported as a function of n (the number of funds in the portfolio). Similarly, the standard deviation of each of the time-series of returns is obtained, and the averages of these standard deviations are reported as a function of n . For each n , the terminal wealth level of the 10,000 simulated portfolios is obtained by assuming an initial \$1 investment in each portfolio and cumulating the portfolio's value over the holding period. Terminal wealth standard deviation (TWSD) is the standard deviation of the terminal wealth levels and is reported as a function of n .

In order to examine risk-adjusted performance, we employ the Sharpe Ratio (or 'reward to variability' ratio). Risk is proxied by the standard deviation of fund returns. Reward is defined as the return in excess of the risk free return (on a 90-day Reserve Bank of Australia (RBA) Treasury note adjusted to a monthly rate).

⁴ Discussions with asset consultants and fund managers indicate that in forming FoF portfolios, it is not standard practice to employ optimisation techniques to determine portfolio weights. As far as the weighting decision is concerned, Elton *et al.* (2004) identify that fund investors tend to allocate assets equally, such that a $1/n$ rule is implemented. Furthermore, Fisher and Statman (1997) indicate that naïve diversification gives rise to portfolios that are remarkably close to mean-variance efficiency. We also examined the sensitivity of the FoF portfolio's configuration according to the investment style adopted by a fund, the aggregate size of the manager offering the fund and the fund's past performance, however the results were largely consistent with those for the naïvely constructed portfolios, and consequently are not directly reported.

⁵ The portfolio return is simply the weighted average of the returns of the component funds. This results from the decisions to equally weight the funds in the portfolio which is equivalent to a monthly portfolio rebalancing strategy.

Beedles (1986) finds evidence of a positive asymmetry in Australian security returns, where an examination of the diversification properties of portfolios of shares shows the measure of skewness increasing, then decreasing with an increase in the number of shares. Alles and Spowart (1995) find significant skewness among 80 percent of securities examined, half of which are negative and the remainder positive. However, the authors do not document statistically significant deviations from zero for the kurtosis measure. In relation to fund managers, Bird and Gallagher (2002) observe that in general, the active return distributions of Australian, Canadian, UK and U.S. investment managers are leptokurtic (more peaked) and positively skewed. Finance theory suggests that a risk-averse investor should prefer distributions that are positively skewed. Investors are also expected to have a preference for lower kurtosis (i.e. less peakedness). Consequently, an investigation of skewness and kurtosis as a function of the number of funds in a portfolio is likely to be of interest as well as of importance to investors.

It is also desirable to assess the statistical significance of a change in the number of funds in the FoF portfolio for each of the performance measures examined. For example, does increasing the number of funds in the portfolio from n to $n+1$ give rise to a significant change in portfolio variance? To overcome problems associated with using t -tests for this purpose, a multiple comparison procedure is employed.⁷ This study employs single factor analysis of variance (ANOVA) and then utilises Tukey's honestly significant difference (HSD) criterion (Tukey (1953)). The latter is a post hoc procedure designed specifically for performing pair-wise comparisons of the means based on the studentized range distribution. The Tukey method is the most powerful multiple comparison test for making pairwise comparisons, and has proven to be robust to violations in its assumptions.⁸

4. Empirical Results

4.1 Mean-Variance Performance

Given the importance of Modern Portfolio Theory in understanding the relationship between risk and return, an examination of the performance properties of FoFs is evaluated. We estimate the change in mean return as a function of the number of funds in the FoF portfolio. For all strategies this value is relatively constant, with a tendency to drift upward slightly. To facilitate comparison across the performance measures and strategies, results are standardised by dividing

⁶ Analysis for a five year holding period was also performed and results were found to be largely consistent with the three-year findings and consequently are not reported.

⁷ The chance of incorrectly finding a significant difference increases with the number of comparisons when t -tests are used. With multiple comparison procedures, the more comparisons that are made, the larger the difference between pairs of means required to find them significant.

⁸ The test assumes that the means come from populations where the observations have equal variances, which is not the case in this analysis, however provided that sample sizes are equal then deviations from this assumption are deemed acceptable. The test does tend to lose relative power with larger sample sizes, which might be of some concern for this analysis.

the performance measure by the result obtained for the one fund case. Table 1 shows that for mean of mean FoF portfolio return, we identify that the maximum change possible is a 0.58 percent difference between the one and thirty fund cases. This drift can be attributed to the absence of a strategy to rebalance the individual gains and losses experienced by the portfolio's component funds, and the cumulative nature of the return calculations.⁹ Importantly, we find that marginal increases in the number of funds tend not to lead to significant changes in the mean of mean value.

<< Insert Table 1 >>

Investors should also be concerned with the risk of FoF portfolios, defined as the time-series variance of returns. Figure 1 demonstrates that average time-series variance decreases as a function of the number of funds in the portfolio, but at a decreasing rate. The relationship observed is comparable for portfolios of directly held stocks (Evans and Archer (1968), Elton and Gruber (1977) and Statman (1987)) as well as in FoFs (O'Neal (1997), Lhabitant and Learned (2002) and Amin and Kat (2002)). Table 1 demonstrates that independent of the portfolio selection strategy, the majority of diversification gains are achieved with a portfolio comprising six funds. After this point, a marginal increase in the number of funds held in the portfolio does not give rise to a significant reduction in average time-series variance. In a relative sense, the diversification benefits (in terms of reducing time-series variance) are slight, and the maximum available reduction in variance for a portfolio of 30 funds is just under seven percent. These results are consistent with the findings reported in O'Neal (1997) and can be explained by a diminishing increase in the number of unique securities added to the FoF as the number of the funds in the FoF portfolio rises, given increasing levels of common stock holdings across funds.

<< Insert Figure 1 >>

Terminal wealth standard deviation (TWSD) is an alternative risk measure which enables investors to understand the ability of their investments to actually meet future monetary obligations. This measure exhibits similar diversification properties to time-series variance, although appears to be more sensitive to increases in the number of funds in the FoF portfolio. There are clear gains from diversification in terms of TWSD; as seen in Table 1 increasing a FoF portfolio from one to thirty funds reduces TWSD to 15.65 percent of the initial level (see Standardised column for TWSD of Table 1).

The Sharpe Ratio provides a means of quantifying the joint effect of the mean-variance observations considered above. According to Figure 2, the Sharpe Ratio increases as funds are added to the FoF portfolio, although the marginal benefit decreases. The total available gain is 3.57 percent of the one fund case (see Standardised column for Sharpe Ratio of Table 1).

⁹ Monthly portfolio rebalancing would result in a portfolio return that is simply a linear combination of the component funds' returns, consequently a portfolio comprised of two funds would be the average of 20,000

Further, Table 1 demonstrates that marginal increases in n are largely insignificant. Regardless of the change in the number of funds in the FoF portfolio, we identify that beyond a portfolio of four funds, it is not possible to significantly improve the portfolio's Sharpe Ratio.¹⁰

<<Insert Figure 2>>

4.2 Higher Moments of the Return Distribution

Bird and Gallagher (2002) highlight the importance of the higher moments of the return distribution in understanding investment performance. Accordingly, this study also examines the effect of FoF portfolio construction on skewness and kurtosis as a function of the number of funds in the portfolio. FoF skewness becomes increasingly negative as a function of the number of funds, albeit at a decreasing rate. This can be explained by the chance of including extremely poor performing funds in the portfolio being positively related to n . Furthermore, the inclusion of non-surviving funds in our sample (which are expected to be the worst performing funds) amplifies this effect. Similar findings are documented in the hedge fund space by Amin and Kat (2002). Lhabitant and Learned (2002) perform a similar analysis for subsets of hedge funds classified according to each of ten investment styles, and they observe the behaviour of skewness to be non-uniform across styles.

Figure 3 illustrates that a FoF portfolio of ten funds realises approximately 90 percent of the total change observed in skewness (i.e. between a FoF portfolio of between one and 30 funds). The total available change in portfolio skewness is approximately nine percent of the one fund case (see Standardised column for skewness of Table 1). According to Table 1, as the FoF portfolio increases beyond six funds, any marginal increase in the number of constituents does not give rise to a significant change in mean skewness.

<< Insert Figures 3 and 4>>

The kurtosis of FoF portfolio returns is consistently positive, increasing with the number of funds in the portfolio as illustrated by Figure 4. The marginal change in kurtosis falls as a function of the total funds included in the FoF portfolio. Lhabitant and Learned (2002) find that changes in kurtosis are unpredictable over time and across styles for U.S. hedge funds, however their results for the latter half of their sample (1998-2001) are consistent with our findings. The total available change in portfolio kurtosis is approximately four percent of the one fund case (see Standardised column for kurtosis of Table 1). This table also shows that a marginal increase in portfolio constituents beyond four funds does not lead to significant increases in kurtosis.

According to the literature, investors prefer positive skewness and are averse to high kurtosis. While FoFs achieve diversification benefits in a mean-variance framework, this research finds evidence of FoF portfolios exhibiting deterioration in the preferred moments of skewness and

individual fund returns, three funds the average of 30,000 etc.

kurtosis as the number of fund constituents increases. However, the deterioration stabilises where FoF portfolios comprise around 10 funds for both measures.

5. Conclusions and Future Research Directions

This study examines the performance and diversification properties of fund-of-fund (FoF) portfolios constructed using a sample of actively managed institutional Australian equity funds. The results show that on average, increasing the number of funds in a FoF portfolio leads to reductions in volatility (time-series return and terminal wealth) while the mean time-series return remains constant. When risk-adjusted performance is examined using the Sharpe Ratio, the results show an improvement in risk-adjusted return as n increases. FoF portfolios provide investors with improvements in diversification, although measures of skewness and kurtosis behave less favourably given an investor's preferences for higher moments of the return distribution. The results presented in this study suggest that although variance declines, FoF returns become more negatively skewed as the number of funds in the portfolio increases. Therefore an investigation into the downside risk properties of these portfolios is warranted and is the subject of future research.¹¹

The majority of diversification benefits are realised when a portfolio of approximately six active equity funds are included. However, the number of funds utilised by investors would also be influenced by the size of assets and the additional administrative, search and review costs when engaging multiple investment managers.¹² Furthermore, portfolios with large numbers of funds may achieve a FoF portfolio that mimics the performance of the underlying index, while also incurring active management fees. Future research is currently underway which examines these issues.

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¹⁰ The analysis was also performed with an alternate risk-adjusted performance measure, the 4-factor alpha (based on Carhart (1997)); the results were equivalent to those for the Sharpe ratio and are therefore not reported.

¹¹ O'Neal (1997) documents a substantial reduction in two downside risk measures (mean shortfall and semivariance) as the number of funds in the portfolio increases, for a sample of U.S. mutual funds. Downside correlations are an additional downside risk measure being considered, these capture the correlation of the FoF portfolio with the aggregate market in periods where markets fall (Ang, Chen and Xing (2002)).

¹² Although this research is primarily from the perspective of superannuation funds whereby total fees are simply a weighted average of all component fund fees, where FoFs are seen for example as master trust products an extra layer of fees coming from the FoF product provider needs to be considered. However, this issue of greater importance for fund-of-hedge funds where in addition to the two layers of fees there also exists an incentive component at the individual fund level, which under certain circumstances can be greater than the realised return of the fund (Brown, Goetzmann and Liang (2002)).

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Table 1

Portfolio Performance as a Function of the Number of Funds in the Portfolio

The mean monthly return is calculated for 10,000 randomly generated FoF portfolios (comprised of one through to thirty funds) and the mean, variance, terminal wealth standard deviation (TWSD), Sharpe Ratio, Skewness and Kurtosis of these means reported as a function of n (the number of funds in the portfolio). Results are also standardised by reporting the ratio of each observation to the one fund case. Statistical significance of the difference between the mean of mean returns following a marginal increase in the number funds in the FoF portfolio is determined by a multiple comparison procedure based on Tukey's honestly significant difference criterion. A significant difference implies that an increase in the number of funds in the portfolio from n to $n+1$ leads to a significant change in the mean of mean portfolio returns and is indicated by ** in the cell relevant to the n fund case. A statistical test is not performed for the TWSD measure.

n	Mean of Mean (%)	Standardised (%)	Mean of Variance (% ²)	Standardised (%)	TWSD (\$)	Standardised (%)	Sharpe Ratio	Standardised (%)	Skewness	Standardised (%)	Kurtosis	Standardised (%)
1	1.158	100	0.1545**	100.00	0.109	100.00	0.1886**	100.00	-1.081**	100.00	4.491**	100.00
2	1.162	100.32	0.1490**	96.44	0.077	71.30	0.1920	101.80	-1.129**	104.40	4.577**	101.91
3	1.165	100.57	0.1473**	95.34	0.062	57.15	0.1936	102.64	-1.146**	105.97	4.610**	102.64
4	1.162**	100.37	0.1465**	94.82	0.054	49.85	0.1933**	102.50	-1.156**	106.93	4.628	103.04
5	1.162	100.36	0.1459**	94.43	0.048	44.56	0.1937	102.68	-1.161**	107.33	4.635	103.21
6	1.164	100.52	0.1455	94.17	0.044	40.07	0.1944	103.05	-1.163	107.58	4.640	103.31
7	1.162	100.38	0.1452	93.98	0.041	37.31	0.1941	102.92	-1.167	107.89	4.646	103.45
8	1.162	100.35	0.1450	93.85	0.037	34.41	0.1941	102.92	-1.168	108.00	4.647	103.47
9	1.163	100.4	0.1449	93.79	0.035	32.52	0.1943	103.00	-1.170	108.24	4.652	103.57
10	1.163	100.47	0.1447	93.66	0.033	30.55	0.1946	103.18	-1.170	108.25	4.653	103.60
11	1.163	100.44	0.1447	93.66	0.031	28.58	0.1945	103.13	-1.171	108.26	4.650	103.53
12	1.163	100.46	0.1446	93.59	0.030	27.54	0.1947	103.20	-1.173	108.47	4.657	103.68
13	1.163	100.45	0.1444	93.46	0.029	26.34	0.1947	103.22	-1.174	108.56	4.661	103.77
14	1.163	100.41	0.1444	93.46	0.028	25.32	0.1946	103.16	-1.174	108.58	4.659	103.73
15	1.164	100.49	0.1444	93.46	0.026	24.01	0.1948	103.30	-1.175	108.65	4.660	103.76
16	1.164	100.5	0.1444	93.46	0.026	23.70	0.1949	103.32	-1.175	108.65	4.660	103.76
17	1.164	100.52	0.1443	93.40	0.024	22.28	0.1950	103.37	-1.175	108.70	4.661	103.77
18	1.164	100.49	0.1443	93.40	0.024	21.72	0.1949	103.33	-1.176	108.79	4.663	103.82
19	1.164	100.5	0.1442	93.33	0.023	21.09	0.1949	103.35	-1.177	108.81	4.663	103.83
20	1.164	100.55	0.1442	93.33	0.022	20.46	0.1951	103.46	-1.176	108.81	4.664	103.84
21	1.164	100.51	0.1441	93.27	0.021	19.64	0.1950	103.41	-1.177	108.82	4.663	103.81
22	1.164	100.52	0.1441	93.27	0.021	19.15	0.1950	103.41	-1.178	108.92	4.667	103.90
23	1.164	100.53	0.1441	93.27	0.020	18.64	0.1951	103.45	-1.177	108.87	4.665	103.87
24	1.164	100.51	0.1441	93.27	0.020	18.48	0.1951	103.42	-1.178	108.94	4.666	103.88
25	1.165	100.57	0.1441	93.27	0.019	17.56	0.1952	103.52	-1.179	109.00	4.669	103.95
26	1.164	100.55	0.1440	93.20	0.019	17.07	0.1952	103.49	-1.178	108.96	4.667	103.91
27	1.165	100.6	0.1440	93.20	0.018	16.66	0.1954	103.59	-1.179	109.01	4.668	103.94
28	1.164	100.54	0.1440	93.20	0.018	16.61	0.1952	103.51	-1.178	108.98	4.667	103.92
29	1.165	100.56	0.1440	93.20	0.017	16.08	0.1952	103.52	-1.179	109.06	4.670	103.97
30	1.165	100.58	0.1439	93.14	0.017	15.65	0.1953	103.57	-1.179	109.05	4.669	103.96

** Significant at 5%

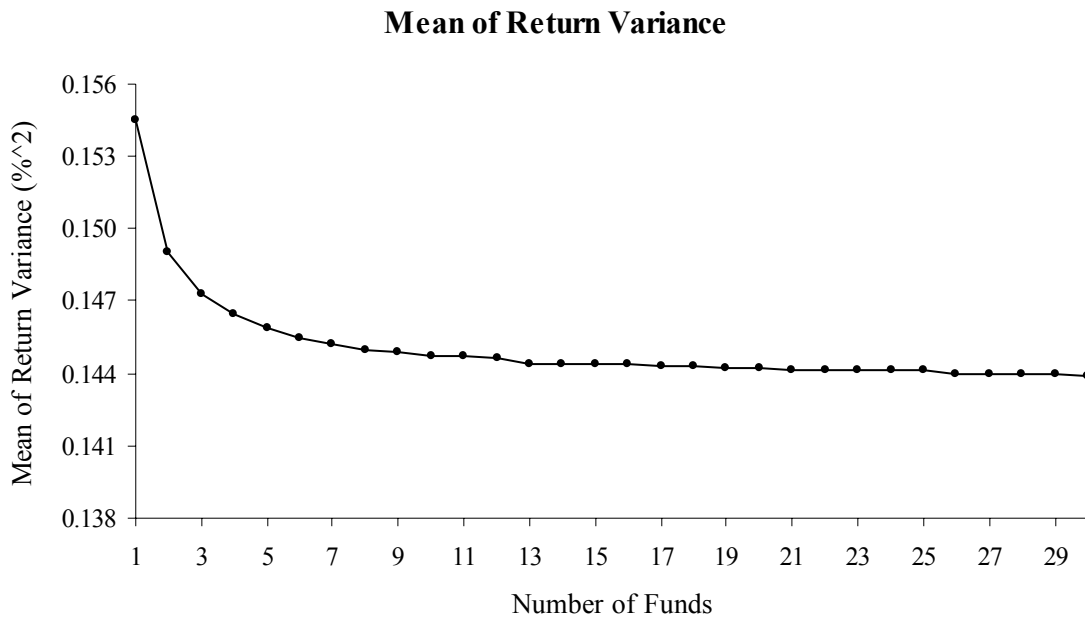


Fig. 1. Plot of the Mean of the Variance of Monthly Portfolio Returns as a Function of the Number of Funds in the Portfolio

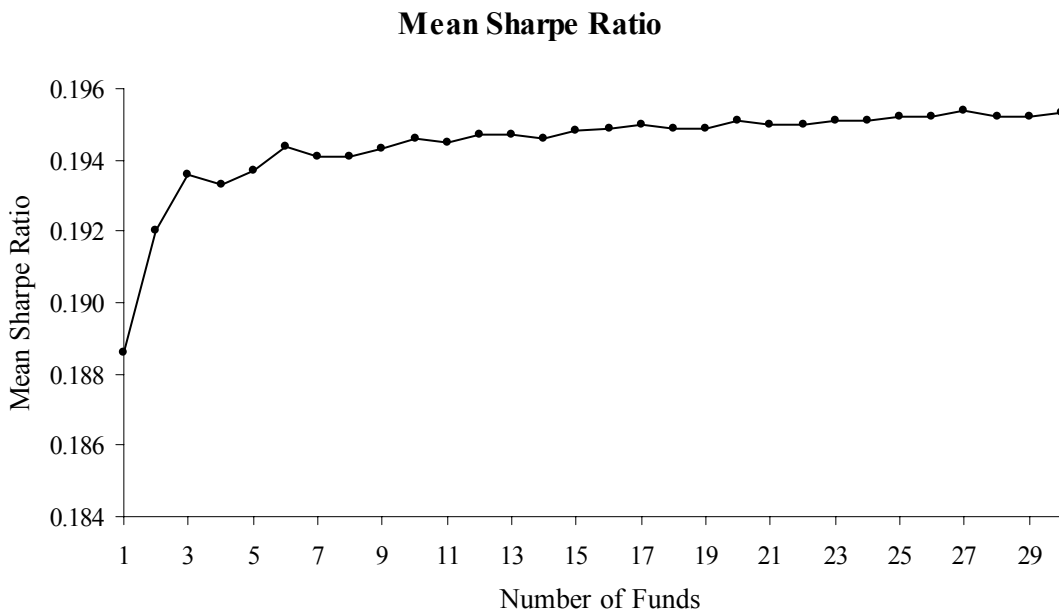


Fig. 2. Plot of the Mean of the Sharpe Ratios of Monthly Portfolio Returns as a Function of the Number of Funds in the Portfolio

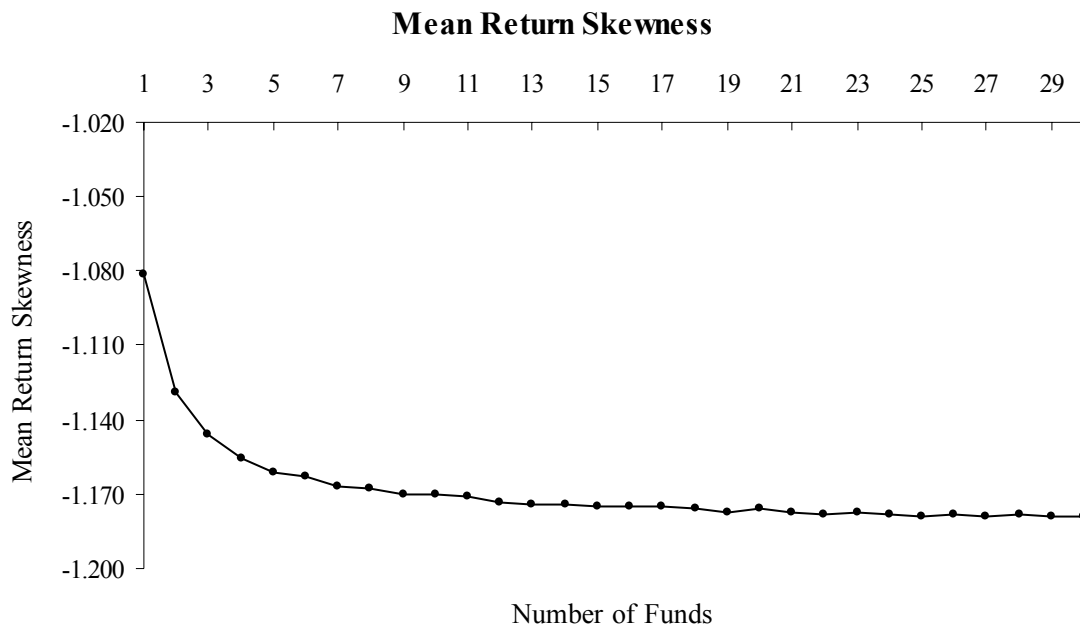


Fig. 3. Plot of the Mean of the Skewness of Monthly Portfolio Returns as a Function of the Number of Funds in the Portfolio

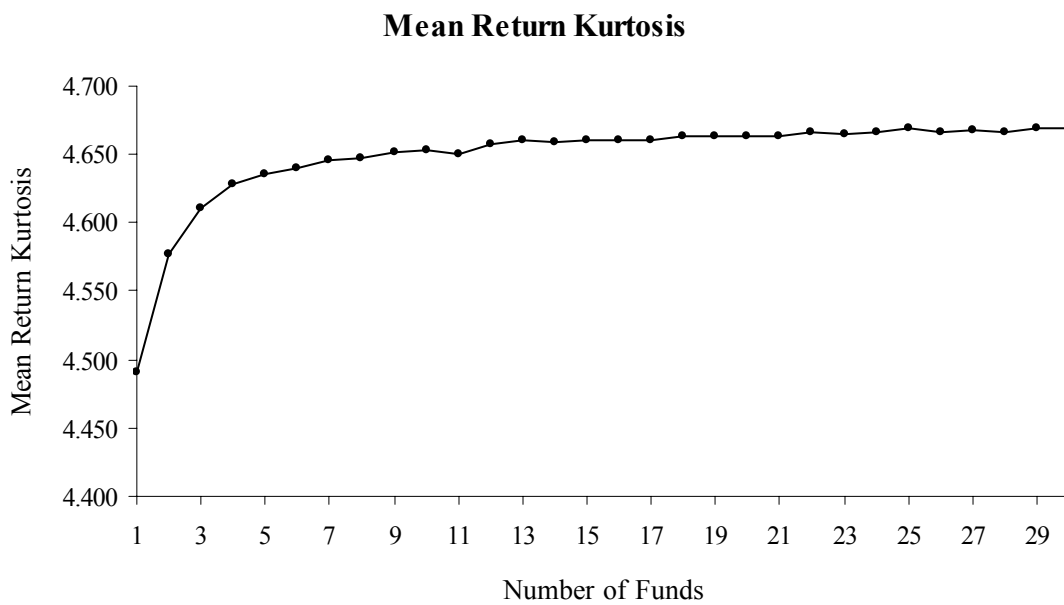


Fig. 4. Plot of the Mean of the Kurtosis of Monthly Portfolio Returns as a Function of the Number of Funds in the Portfolio