A Multifactor Model of Credit Spreads

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We represent credit spreads across ratings as a function of common unobservable factors of the mean-reverting normal (Vasicek) form. Using a state-space approach we estimate the factors, their process parameters, and the exposure of each observed credit spread series to each factor. We find that most of the systematic variation across credit spreads is captured by three factors. The factors are closely related to the implied volatility index (VIX), the long bond rate, and S&P500 returns, supporting the predictions of structural models of default at an aggregate level. By making no prior assumption about the determinants of yield spread dynamics, our study provides an original and independent test of theory. The results also contribute to the current debate about the role of liquidity in corporate yield spreads. While recent empirical literature shows that the level and time-variation in corporate yield spreads is driven primarily by a systematic liquidity risk factor, we find that the three most important drivers of yield spread levels relate to macroeconomic variables. This suggests that liquidity risk is largely driven by the same factors as default risk.

Key words: Credit Spreads; Macroeconomic Factors; Kalman Filter; State Space Model;

1. INTRODUCTION

The theoretical link between credit spreads and market variables is established by structural models of default. Models such as Merton (1974) and Longstaff and Schwartz (1995) are based on the economic definition of default as the event where a firm's value falls below the face value of its outstanding debt. The unobservable value of the firm is assumed to follow Brownian motion under the assumption of risk-neutrality, allowing the calculation of default probabilities and an endogenous recovery rate. Credit spreads are attributed entirely to the risk-neutral expected default loss, which is positively related to firm leverage and volatility in the firm value. An increase in the firm value through positive equity performance has the effect of reducing leverage and credit spreads. Under the assumption of risk-neutrality the firm value process has a drift rate equal to the risk-free rate. The models predict that an increase in treasury yields increases the drift of the firm value process, leading to lower credit spreads.

In practice, structural models tend to underestimate short-term credit spreads. The use of smooth processes to represent the firm value may exclude the possibility of default by high grade issuers in the short term, which is inconsistent with the observed role of surprise in credit markets. In contrast, reduced-form models are flexible enough to empirically fit the term structure of credit spreads, but they do not provide an economic interpretation of default. Reduced form models such as Jarrow and Turnbull (1995) and Duffie and Singleton (1999), define exogenous stochastic processes for the arrival time of default and exogenous recovery rates. An additional class of models combines the advantages of both structural and reduced-form approaches by incorporating exogenous effects such as jump-diffusions (Zhou, 1997) in the firm value process to allow for surprise default. An empirical overview of structural models by Eom, Helwege and Huang (2004) reveals that existing models cannot simultaneously fit both high-grade and low-grade bond spreads. They conclude that more accurate models would need to correct the common tendency to overstate the credit spreads of firms with high leverage or volatility while at the same time understating the spreads of high-grade bonds.

To the extent that credit spreads reflect expectations on future default and recovery, we would expect aggregate credit spread indices to vary with macroeconomic variables such as interest rates, stock market returns and market volatility. In general, low-grade bond spreads are observed to be closely related to equity market factors (Huang and Kong, 2003) while high-grade bonds are more responsive to treasury yields. Kwan (1996) finds that individual firm yield spread changes are negatively related to both contemporaneous and lagged equity returns of the same firm. On the other hand, lagged yield spread changes do not help explain current equity returns. Campbell and Taksler (2003) show credit spreads to be positively related to the market average of firm-level equity return volatility, and the increase in market and firm volatility documented by Campbell et al. (2001) is consistent with the steady rise in credit spreads throughout the 1990s.

A negative relationship between investment-grade spreads and treasury yields is estimated by Longstaff and Schwartz (1995), while Duffee (1998) finds that the negative relationship is strongest for callable bonds. Collin-Dufresne et al (2001) show that credit spreads have increasingly negative sensitivities to interest rates as ratings decline across both investment and non investment grade bonds. In a study of only high-yield bonds, Fridson and Jonsson (1995) find no significant relationship between credit spreads and treasury rates, which is more consistent with the idea that low-grade bonds are far more responsive to equity variables than interest rate variables. While there is strong empirical evidence of a negative relationship between investment-grade spreads and treasury yields, there is no consensus on its economic causes. Intuitively, lower yields should lead to narrower yield spreads through lower borrowing costs that increase the probability of survival. However, falling treasury yields, particularly in the shorter maturities, also tend to be a feature of recessionary periods when default risk rises and central banks typically lower short-term rates. One recent example of this is the sub-prime crisis, beginning in August 2007, during which short-term treasury yields declined to historical lows while credit spreads widened to historical highs. Duffee (1998) concludes that despite the links of both treasury yields and corporate bond spreads to future variations in aggregate output, it is not obvious that these links explain their observed negative relationship, or that yield spreads are determined by credit quality. To link credit spreads to interest rates and expected aggregate output, the empirical literature has also focused on the slope of the treasury curve, defined as the spread between long-term and short-term yields and often used as a barometer of future economic conditions. Estrella and Hardouvelis (1991) associate a positive slope of the yield curve with future increases in real economic activity, so that an increase (decrease) in the slope of the yield curve should indicate a lower (higher) probability of a recession, in turn reflected in lower (higher) credit spreads. This idea is supported by the findings of Papageorgiou and Skinner (2006) that investment-grade credit spreads are negatively related to changes in both the level and the slope of the treasury curve. In addition they estimate that the negative relationship between credit spreads and the treasury slope is relatively stable over time.

The empirical literature to date supports both the significance and the direction in which structural model variables influence credit spreads, however, recent studies demonstrate that these variables alone are not sufficient to fully explain either the levels or changes in credit spreads. Collin-Dufresne et al. (2001) regress credit spread changes of individual bonds on the changes in treasury yields, the slope of the yield curve, equity index returns, and index volatility, estimating that these variables explain only about 25% of the variation in credit spreads. In addition, the slope of the yield curve is not a significant determinant of credit spread changes when the other variables are taken into account. Using principal components analysis on the residuals they find that the changes in residuals across individual bonds are dominated by a single common systematic component that has no obvious relationship to variables from the interest rate and equity markets. Their conclusion is that yield spread changes are only partly accounted for by the economic determinants of default risk.

To estimate how much of the yield spread levels can be accounted for by default risk, Huang and Huang (2003) calibrate a diverse set of structural models to actual historical default losses then use them to generate theoretical values of credit spreads. In each case the model-based spreads are well below the average observed spreads, suggesting that default risk accounts for only a small fraction of yield spreads. The proportion explained by default risk is highest for low-rated bonds, and decreases for higher-rated bonds that have low historical default losses. The inability of theoretical risk variables to account for most of the levels or changes in yield spreads is sometimes referred to as the 'credit spread puzzle'. Similar to the problem of the equity premium puzzle, the expected returns on corporate bonds, like equities, seem well above those justified by the risks. The explanation of the credit spread premium puzzle has focused on both the presence of additional risks as well as associated risk premiums. Elton, Gruber, et al. (2001) estimate that expected loss accounts for less than 25% of the observed corporate bond spreads, with the remainder due to state taxes and factors commonly associated with the equity premium. Similarly, Delianedis and Geske (2002) attribute credit spreads to taxes, jumps, liquidity and market risk. Factors associated with the equity premium include the Fama and French (1996) ‘High-minus-Low’ (HML) factor, found by Huang and Kong (2003) to account for a significant component of low-grade credit spread changes. The significance of Fama-French factors is also supported by Joutz et al (2001), who conclude that credit spreads are determined by both default risk and systematic market risk.

Structural models contain the assumption that default risk is diversifiable, since yield spreads are assumed to reflect only default loss, with no risk premium for either default risk or the risk of market-wide changes in spreads. Jarrow et al (2001) show that jumps-to-default in credit spreads cannot be priced if defaults across firms are conditionally independent and if there is an infinite number of firms available for diversification. One explanation for the credit spread puzzle is the potential for firms to default on a wide scale not seen historically, a risk that is difficult to eliminate by diversification and is therefore priced by investors. It is also observed that defaults across firms tend to be correlated and concentrated around recessions when investors are most risk averse. Chen (2007) and Chen et al (2008) conclude that in order for structural models to capture observed spreads it is necessary to incorporate a strongly cyclical market price of risk that increases along with default losses during recessions. Another explanation for wide credit spreads is that idiosyncratic risk is priced as well as systematic risk. Amato and Remolona (2003) argue that due to the highly skewed returns in corporate bonds, full diversification requires larger portfolios than typically needed for equities. Given the limited supply of bonds, high transaction costs, and possible constraints on portfolio expected losses, full diversification is difficult to achieve, and it is possible that in practice portfolio managers require a risk premium associated with individual bond value-at-risk. Recent studies confirm the presence of both a firm-specific default risk premium as well as a market risk premium. Drisesen (2005) distinguishes between the market-wide changes in credit spreads and individual credit spread jumps-to-default, finding that both components are priced. Similar results are obtained by Gemmill and Keswani (2008), who show that most of the credit spread puzzle can be accounted for by the sum of a systematic risk premium and a larger idiosyncratic risk premium. While supporting these conclusions, Collin-Dufresne et al (2003) also suggest that it is not surprise default itself that attracts a significant premium, but rather it is the potential for credit events of large individual firms to trigger a flight to quality in the treasury market and cause market-wide increases in credit spreads. So even without directly violating the assumption of conditional independence of defaults across firms, idiosyncratic default risk could matter due to its potential to impact market-wide liquidity, which highlights the difficulty of separating the role of default and liquidity in driving credit spread levels.

A recent stream of literature focuses on the role of liquidity in explaining both the levels and time-variation of credit spreads. The idea that liquidity is an important and priced determinant of yield spreads is not new, with Fisher's (1959) hypothesis being that the risk premium of individual bonds consists of two main components: a default component and a 'marketability' component. The default component is in part a function of a firm's earnings variability and debt ratio, measures that directly correspond to the leverage and asset volatility variables in structural models, while the marketability component is a function of the outstanding issue size. There is no universal proxy for liquidity risk, but measures used in previous studies include the bid-ask spread, trade frequency, the proportion of zeros in the time-series of a bond's returns, a bond's age, amount outstanding and term to maturity. Perraudin and Taylor (2003) estimate that liquidity premiums are at least as large as market risk premiums and far larger than expected default losses. De Jong and Driessen (2006) estimate the liquidity risk premium on US corporate bonds at 0.6% for long-maturity investment-grade bonds and 1.5% for speculative-grade bonds. Recent studies estimate the non-default component of credit spreads directly by subtracting credit default swap (CDS) premiums from corresponding corporate bond yield spreads. Being simply contracts, CDS are regarded as more pure reflections of credit risk[1]. This idea is supported by the findings of Ericsson et al (2005) that

[1] CDS are not subject to the same accessibility issues as physical bonds and difficulties in taking a short position. The absence of coupons also avoids bond-specific taxation considerations.

CDS premiums are driven by the theoretical determinants of credit risk (the risk free rate, leverage and volatility), but that in contrast to the results of Collin-Dufresne et al (2001) on corporate bond yield spreads, there is only weak evidence of a common residual factor affecting CDS premiums. The CDS-based non-default component estimated by Longstaff et al (2005) is strongly time-varying and related to both bond-specific and market-wide measures of illiquidity. There is a strong cross-sectional relationship between the non-default component and individual measures of liquidity such as the bid-ask spread and issue size. The time-series variation of the non-default component is related to macroeconomic or systematic measures of liquidity risk such as i) the spread between on-the-run and off-the-run treasury yields, ii) flows into money market funds, and iii) level of new issuance into the corporate bond market. The cross-sectional results of Longstaff et al (2005) are consistent with equity market evidence of Amihud and Mendelson (1986) that market-average returns are an increasing function of bid-ask spreads, while the time-series results are consistent with the presence of a single common systematic factor found by Collin-Dufresne et al (2001), as well as evidence of systematic liquidity risks in interest rate markets in Duffie and Singleton (1997), Liu et al (2004) and Longstaff (2004). Liquidity risk itself has also been found to be a positive function of the volatility of a firm's assets and its leverage, the same variables that are seen as determinants of credit risk (Ericsson and Renault, 2006).

Our aim is to estimate the factors driving the dynamics of yield spread levels directly from the data, without prior assumption about the specific economic variables that yield spreads could be related to. Based on existing evidence, we take the view that the time-variation in credit spreads is driven by two classes of factors that are non-stationary and mean-reverting, respectively. Our initial guess is that the first group of factors is likely to relate to default risk and have low rates of mean-reversion that reflect relatively persistent macroeconomic conditions. The second group could relate to liquidity premiums that are presumed to change with noisy short-term supply and demand shocks. Given that credit risk explains a lower proportion of high-grade spreads than low-grade spreads, we would then expect high-grade spreads to have stronger mean-reversion that reflects changes in liquidity due to supply/demand. However, from Figure 1 it appears that non investment-grade spreads have far more noise than investment-grade spreads, suggesting that the default risk component may be more highly mean-reverting than the remaining component. One indication this may be true for corporate yield spreads is the study of swap spreads by Liu et al (2006), finding time-varying components relating to both liquidity and default risk, but where the default component is highly mean-reverting and with a flat term structure, while the liquidity component is more persistent and with a steep upward-sloping term structure. It is worth noting that spikes in the lowest-grade spread indices resemble the behavior of the VIX index over the same period. These could be interpreted as short-term increases in default risk under the frictionless market framework of structural models, but in practice sharp increases in the VIX are also closely correlated to declines in liquidity. Without assumption about the source of variations, we observe that while the two bond classes behave in fundamentally different ways during particular sub-periods, they also appear to have different exposures to shared common short-term shocks throughout the sample period.

This study assumes that the time-variation in credit spreads across ratings classes is driven by a common set of unobservable factors to which each observed spread is exposed with some unknown sensitivity. We aim to answer the following questions: 1) how many factors are required to explain the evolution of ratings-based spread indices, 2) what is the exposure of each individual index to each factor, and 3) what economic variables, if any, could be proxies for the factors.

Our choice of the state-space methodology is motivated by its advantage of allowing for both time-series and cross-sectional data simultaneously. It also provides a new and opposite approach to the existing literature on credit spread determinants. Most empirical studies on credit spreads adopt a general-to-specific approach where a range of known potential determinants is tested for statistical significance using OLS regressions. In contrast, state-space models require only an assumption about the structure of the factors that can then be estimated directly from the observed data. Another advantage of state-space models is that they can be applied to both stationary and non-stationary variables. OLS estimation on the other hand requires that both dependent and independent variables are stationary, forcing most studies to focus on explaining the changes in credit spreads as a function of changes in independent variables. In this study we analyze the dynamics of credit spread levels directly.

Given an assumed parametric process form for the latent factors, the Kalman Filter maximum likelihood method can be applied to simultaneously estimate 1) the parameters of each factor process, 2) the sensitivities or loadings of each observed series to the individual factors, 3) the realizations of the factor series, and 4) the covariance matrix of the model errors. The Vasicek (1977) normal mean-reverting process is chosen for the factors since, depending on the size of its mean-reversion coefficient, it is suitable for representing both non-stationary (presumed macroeconomic) as well as stationary (presumed microeconomic) determinants of credit spreads. A multi-factor Vasicek form is also supported by the findings of Pedrossa and Roll (1998) that Gaussian mixtures can capture the fat-tailed distributions of credit spreads.

Early applications of the state-space model in finance literature have focused on the term structure of treasury rates. Babbs and Nowman (1999) find that a three-factor Vasicek model adequately captures variations in the shape of the treasury yield curve, with two factors providing most of the explanatory power. Chen and Scott (1993) and Geyer and Pichler (1999) reach similar conclusions based on a multi-factor CIR (1985) model, and find the factors to be closely related to the short rate and the slope of the curve. Recent studies build upon the two or three-factor term structure of treasury rates and allow for additional factors to explain swap or corporate bond yields. Liu, Longstaff, and Mandell (2006) separate the liquidity and credit risk components of swap spreads through a five-factor model of swap yields. Swap yields consist of three factors driving treasury yields, one influencing credit risk, and the remaining one influencing liquidity risk. Similarly, Feldhutter and Lando (2008) decompose the factors driving the term structure of the swap yield spreads into three factors driving the risk-free rate, two affecting credit risk and one relating to the liquidity premium or ‘convenience yield’ contained in treasury yields over the risk-free rates. They find that while credit risk is important, the strongest determinant of swap spreads is the convenience yield contained in treasury prices. Jacobs and Li (2008) use the state-space approach to estimate a reduced-form model of default, where the probability of default is modeled directly as a stochastic volatility process. They find that the addition of a second, volatility factor to the level factor in the diffusion of default probabilities leads to significant improvements in both in-sample and out-of-sample fits for credit spreads.

Our work is a natural progression in the application of state-space methodology from treasury yield levels to corporate yield spreads. We apply the state space methodology directly to credit spreads to find both the number of factors and compare their behavior to well-known macroeconomic variables. This is the first work to relate the estimated factors driving corporate yield spreads to variables from both equity and interest rate markets.

2. DATA

All data is from Bloomberg with observations taken at the end of each month Apr-96 to Mar-08. We use the 10-year maturity industrial corporate bond yield indices of 14 available ratings: AAA, AA, A1, A2, A3, BBB1, BBB2, BBB3, BB1, BB2, BB3, B1, B2, and B3. Bloomberg ratings are composites of S&P and Moody's ratings, with bonds rated BB1 or lower considered sub-investment grade. The yield indices are converted into credit spreads by subtracting the 10-year benchmark bond yield from each. Other variables sourced are the option-implied volatility index of the S&P500 (VIX) and the S&P500 level.

3. METHOD

A. The Multifactor Vasicek Model in State Space Form

For a given term to maturity, each of *n* observed credit spread indices by rating $R_t = \{R_{1t}, R_{2t}, ..., R_{nt}\}'$ is expressed as a function of *m* independent latent factors or states $X_t = \{X_{1t}, X_{2t}, ..., X_{mt}\}'$ of the Vasicek form. Changes in the *j*-th observed series R_{jt} are a linear combination of the changes in *m* latent factors X_{it} weighted by factor loadings $a_{j1}, a_{j2}, ..., a_{jm}$. Each factor evolves according to its three parameters: the long-term mean θ, the speed of mean-reversion κ[2], and the volatility σ. In continuous time,

$$dR_{jt} = \sum_{i=1}^{m} a_{ji} dX_{it} \qquad j = 1,2,...,n \qquad (1)$$

$$dX_{it} = \kappa_i (\theta_i - X_{it}) dt + \sigma_i dW_{it} \qquad i = 1,2,...,m \qquad (2)$$

The application of the Kalman Filter algorithm to estimate the factor loadings, the process parameters $\psi = \{\kappa_i, \theta_i, \sigma_i\}$ $i = 1,2,...,m$ and the realization of the state vector over time $X = \{X_1, X_2, ..., X_T\}$, requires that the model is expressed in state space form. State space representation consists of the measurement equation and the transition (or state) equation.

$$R_t = D + ZX_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, H) \qquad (3)$$

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[2] The mean-reversion parameter κ is directly related to the time taken for the process to reach its long-run mean θ. In the absence of random shocks the difference between the current level and the mean decays exponentially towards zero. The expected time it takes for the process to decay halfway towards its mean is its 'half-life', equal to $\ln(2)/\kappa$ years.

$$X_t = C(\psi) + \Phi(\psi) X_{t-1} + \eta_t \qquad \eta_t \sim N(0, Q(\psi)) \tag{4}$$

The measurement equation (3) maps the vector of observed credit spreads $R_t(n \times 1)$ to the state vector $X_t(m \times 1)$ via a 'measurement matrix' $Z(n \times m)$ and vector $D(n \times 1)$. Unexpected changes and errors in the sampling of observed series are allowed through n jointly normal error terms $\varepsilon(nx1)$ that have zero conditional means and covariance matrix $H(n \times n)$. Since the computational burden of estimating a full error covariance matrix *H* increases rapidly with additional observed series, most studies assume error independence. In state-space models of the treasury curve, (Chen and Scott (1993), Geyer and Pichler (1996), and Babbs and Nowman (1999)) a diagonal matrix with elements $h_1, h_2, ..., h_n$ was used to capture the effects of differences in bid-ask spreads across *n* maturities. In this study we choose the same form to allow for different bid-ask spreads across *n* bond quality groups. The state equation (4) represents the discrete-time conditional distribution of the states. The terms of the equation follow directly from the discrete form of the Vasicek model for interval size Δt:

$$X_{i,t+\Delta t} = \theta_i \left(1 - e^{-\kappa_i \Delta t}\right) + e^{-\kappa_i \Delta t} X_{i,t} + \eta_{i,t} \tag{5}$$

$$\eta_{i,t} \sim N\left(0, \frac{\sigma_i^2}{2\kappa_i}\left(1 - e^{-2\kappa_i \Delta t}\right)\right) \tag{6}$$

Innovations in the states occur through the normal 'noise' vector η_t, with covariance matrix *Q*. It is assumed that the sources of noise in the state and measurement equations are independent.

In state-space representations of affine models of the term structure, where the observed series correspond to specific maturities, the elements of the measurement matrix *Z* and the intercept vector *D* are usually closed-form functions of the term to maturity, the parameters of each risk factor, factor correlations, and the market risk premium associated with each factor. The difference in this study is that the observed series represent different ratings for a single maturity, without a prescribed formula linking the observed series via factor process parameters. Instead, we estimate the measurement matrix directly by maximum likelihood, along with the process parameters. To reduce the number of parameters in the optimizations we also make the simplifying assumption of a zero intercept vector *D*. Based on numerous experiments we find no observable impact of this assumption on either the estimated factor realizations or the sensitivities of the observed series.

B. The Kalman Filter

At each time step *t*, the filtered estimate $\hat{X}_t$ of the realized state vector consists of a predictive component $\hat{X}_{t|t-1}$, based on information to up to and including time *t – 1* , and an updating component incorporating observations at time *t*. The predictive component of $\hat{X}_t$ is the conditional mean of X_t, $E_{t-1}[X_t]$, which is the optimal estimator of X_t. For $t = 1,2,...,T$

$$\hat{X}_{t|t-1} = E_{t-1}[X_t] = C(\psi) + \Phi(\psi)\hat{X}_{t-1} \tag{7}$$

The covariance $\Sigma_{t|t-1}$ of the predictive component $\hat{X}_{t|t-1}$ is given by

$$\Sigma_{t|t-1} = E_{t-1}\left[(X_t - \hat{X}_{t|t-1})(X_t - \hat{X}_{t|t-1})'\right] = \Phi\Sigma_{t-1}\Phi' \tag{8}$$

$$\Sigma_t = \Sigma_{t|t} = E_t\left[(X_t - \hat{X}_t)(X_t - \hat{X}_t)'\right] = \left(\Sigma_{t|t-1}^{-1} + Z'H^{-1}Z\right)^{-1} \qquad (9)$$

The estimate $\hat{X}_t$ is defined as the sum of $\hat{X}_{t|t-1}$ and an error-correction term v_t weighted by the Kalman Gain matrix K_t. The higher the terms of the Kalman Gain K_t the more responsive $\hat{X}_t$ is to new data.

$$v_t = R_t - \left(C + \Phi X_{t-1}\right) \qquad (10)$$

$$\hat{X}_t = \hat{X}_{t-1} + K_t v_t \qquad (11)$$

$$K_t = \Sigma_{t|t-1} Z'\left[Z\Sigma_{t|t-1} Z' + H\right]^{-1} \qquad (12)$$

The recursive equations are started with guesses for the initial state vector X_0 and covariance matrix Σ_0. In practice, to ensure that the state vector adapts quickly to the first few observations, the initial state noise covariance Σ_0 should be set to an arbitrarily high number so that the Kalman Gain is close to a vector of ones. With further observations it is expected that the covariance terms and the Kalman gain will decrease and stabilize, resulting in a more constant mix of the predictive and error-correcting term in generating state vector estimates. The number of time-steps required for the Kalman Gain to stabilize is usually referred to as the 'burn-in' phase. The part of the estimated state vector coinciding with the burn-in phase is typically excluded in further analysis.

C. Fitting the Model

The state parameters ψ, the elements of the measurement matrix Z, and the measurement error covariance matrix H are estimated by maximizing the log-likelihood function (13) that follows directly from the prediction error decomposition. Given guesses for ψ, Z, and H, and fixed initialization values X_0, and Σ_0, the log-likelihood is

$$\log L(R_1, R_2, ..., R_T; \psi, Z, H, X_0, \Sigma_0) = -\frac{1}{2}\sum_{t=1}^{T}\log|F_t| - \frac{1}{2}\sum_{t=1}^{T} v_t' F_t^{-1} v_t \tag{13}$$

$$F_t^{-1} = H^{-1} - H^{-1}Z\left(\Sigma_{t|t-1} + Z'H^{-1}Z\right)^{-1} Z'H^{-1} \tag{14}$$

$$\left|F_t\right| = \left|H\right| \cdot \left|\Sigma_{t|t-1}\right| \cdot \left|\Sigma_{t|t-1}^{-1} + Z'H^{-1}Z\right| \tag{15}$$

In maximizing the log-likelihood function we force all the factor loadings of the first observed credit spread series (AAA) to equal 1, so that the first observed series is a non-weighted sum of the latent factors. We add this assumption as a way of ensuring that loadings and factor realizations are scaled comparably across factors and across models with different numbers of factors.

4. RESULTS

One, two, and three-factor models are estimated for the period Apr-96 to Mar-03 as well as the full sample period Apr-96 to Mar-08. We are interested in how model estimates are impacted by the changing economic environment. From Apr-96 to Mar-03 lower-grade credit spreads generally increased until reaching their peak in Mar-03 (Figure 1). In the period that followed credit spreads generally narrowed and remained low until 2007. The full sample period includes three major shocks to liquidity: the LTCM crisis of 1998, the bursting of the technology bubble and increase in corporate default rates in 2002, and the sub-prime mortgage crisis starting in 2007.

A. Results for Apr-96 to Mar-03

Table 1 shows the estimates for the mean-reversion speed (κ), mean (μ), and volatility (σ) of each Vasicek factor. The log-likelihood, AIC, and BIC criteria are highest for the three-factor model, under which all parameters (with the exception of one mean) are highly significant. The marginal improvement in the log-likelihood from the addition of a third factor is far smaller than for a second factor, suggesting that a 3-factor model is sufficient in capturing the common sources of variation in credit spreads. For comparison, the log-likelihoods for the one, two, three, and four-factor models are 1246.0, 1840.6, 2040.6, and 2100.10, respectively. The parameter estimates for factor 4 in a four-factor model are largely insignificant[3], supporting the choice of the three-factor model.

Figure 2 shows that the extracted factor under the one-factor model resembles a weighted average of the 14 observed series. Allowing for a second factor reveals two distinct smooth processes as the drivers of the cross-section of credit spreads, while in the three-factor model an additional more noisy process is identified. In the three-factor model the half-life is 2.8 months for factor one, 4.1 years for factor two, and 1.6 years for factor three. The factors under the three-factor model are compared to well-known economic time-series in Figure 3. Under the three-factor model, the noisy first factor resembles the VIX for most of the sample period, the second resembles the (negative of) 10-year bond rate, and the third the S&P500 level[4]. The correlations are 0.08 between factor 1 and the VIX, -0.74 between factor 2 and the long bond rate, and 0.92 between factor 3 and the S&P500 level. If a "burn-in" phase of the first 12 months is excluded under the Kalman Filter approach, the correlation between factor 1 and the VIX increases from

[3] A parameter is significant at the 5% level if the estimated parameter divided by its standard error is greater than 1.96 in absolute value. The standard errors of the estimated parameters are calculated using a finite-difference estimate of the Hessian matrix, as outlined in Hamilton (1994).

[4] S&P500 returns rather than levels are a more appropriate explanatory variable for credit spreads, since S&P500 levels are strongly upward trending over the long term while yields spreads tend to mean-revert. However, for the sample period used the S&P500 level is roughly stationary, and we compare factors to the levels rather than arbitrary measures of rolling returns.

0.08 to 0.47. Given the results of Campbell and Taksler (2003) linking credit spreads to the average of individual firm volatilities, it is possible that factor 1 is more closely related to measures of the average of individual firm implied volatilities than it is to the VIX which measures the volatility of the market average returns.

The estimated loadings of the observed series to each factor under the one, two and three-factor model are shown in Figure 2. Are the sensitivities to the factors consistent with theory? The shape of the loadings on the first factor suggests that equity volatility risk has a positive impact on all credit spreads and that exposure to it increases with declining credit quality. To the extent that equity volatility is a proxy for a firm's asset value volatility, this result is consistent with the prediction of Merton (1974) that the probability of default and credit spreads increase with higher asset value volatility. The sharpest increase occurs in the crossing from investment to sub-investment grade bonds, which is consistent with the observations of Huang and Kong (2003) and others that lower-grade bonds are more sensitive to equity market variables than high-grade bonds.

The positive loadings on factor two and its negative correlation with the level of the 10-year treasury yield are consistent with the strong empirical that increases in treasury yields lower credit spreads. The loadings are also consistent with the finding of Colin-Dufresne et al (2001) that the sensitivity of credit spread changes to interest rates increases monotonically across declining rating groups.

The sensitivities to factor 3, which is closely correlated to the S&P500, change sign from positive to negative as bonds move from investment to sub-investment grade. The estimated positive relationship between equity market performance and investment grade spread indices is at odds with the Merton (1974) model since according to the model, higher equity values increase the value of a firm's assets relative to its fixed level of debt, lowering its probability of default. A possible explanation is that the positive equity performance throughout the 1990s coincided with rising aggregate debt levels during the same period, with highest rated firms raising their leverage the most. The negative effect of higher asset values on spreads may have been more than offset by the positive effect of higher leverage in the case of higher-grade firms. Changing investor risk preferences may also have played a role. It is possible that for all but the lowest credits, prolonged positive equity market performance contributed more to the substitution out of corporate bonds, in favor of equities, than to higher bond values through improved creditworthiness.

B. Results for Apr-96 to Mar-08

We repeat the analysis for the full sample period with results reported in Table 1 and Figure 2. The signs of the correlation coefficients between the factors and macroeconomic variables in the three-factor model remain the same as for the first period: factor 1 and the VIX at 0.71; factor 2 and the long bond rate at -0.54; factor 3 and the S&P500 at 0.76. The general shapes of the loadings and their signs remain unchanged for the 3-factor model, with the exception that loadings on factor 3 are more strongly negative for non-investment grade debt for the full period. This reflects changing market conditions between the first and the second period. Low-grade credit spreads increased throughout Apr-96 to Mar-03, while the S&P500 reached its peak in mid-2000 and declined until Mar-03. The lack of a strong direction in the relationship between low-grade spreads and the equity market is reflected in the estimated loadings of low-grade spreads on factor 3 being close to zero for the first period. For most of the period that followed (Apr-03 to Mar-08) non-investment grade spreads steadily declined, with lowest grade spreads declining the most, while at the same time the S&P500 trended upwards. This feature most likely contributes to the estimated loadings of low-grade spreads being more negative and varied across ratings when based on the full sample period. There is also a change in the shape of the loadings on factor 2 for the full period. The loadings peak for the highest-rated non investment grade index (BB1) but then slowly decline with worse ratings. This is in contrast to the finding of Collin-Dufresne et al (2001), supported by our estimates for Apr-96 to Mar-03, that interest rate sensitivities increase monotonically with declining ratings. We note that for the full period factor 2 is less closely correlated to the long bond yield (coefficient of -0.54), than for Apr-96 to Mar-03 (coefficient is -0.74), and that the factor loadings across the two periods are therefore not entirely comparable. However, the shape of the loadings for the full period raises the question of whether for indices of lower quality than those covered in Collin-Dufresne et al (2001) and this study, the sensitivities to interest rates would decline further across declining ratings. The possibility is also raised by the findings of Fridson and Jonsson (1995), that there is no significant relationship between high-yield spreads and treasury levels.

We find none of the extracted factors in models containing between one to four factors to be correlated to the slope of the treasury curve, either in the spot or forward yields, contemporaneously or with a lag. This is consistent with the findings of Collin-Dufresne et al (2001) that the treasury slope does not help explain credit spread changes, but the result remains surprising given that the treasury curve is commonly used as an indicator of future economic conditions by market participants. One explanation is that the slope of the treasury curve contains no useful information beyond that already contained in the combination of equity returns, volatility and interest rate levels. Another is simply that the period Apr-96 to Mar-08 contains highly contrasting relationships between credit spreads and the treasury slope, due to the sub-prime crisis. The period since August 2007 has been marked by rapidly widening credit spreads while at the same time fears of stagflation, high inflation and low growth, contributed to short-term treasury yields reaching historically low levels relative to long-term yields. Hence the end of the sample period is marked by a strong positive contemporaneous relationship between the slope of the treasury curve and general credit spread levels, which is in contrast to the negative relationship previously documented by Papageorgiou and Skinner (2006).

We examine the estimated measurement error variances, defined as the diagonals of matrix H in the Measurement Equation (3). Figure 4 shows the square-roots of the error variances across rating classes for the one, two, and three-factor models. As expected, for each model the lowest-rated bonds which have the widest credit spreads also have the highest estimated measurement error variance. The variances fall sharply across ratings with the addition of a second factor, particularly for sub-investment grade bonds. The addition of a third factor does not lead to a large reduction in variances, which is similar to the modest impact of a third factor on the maximum likelihood. We note that in the two and three-factor models the measurement error variances also peak around the middle ratings, which is in contrast to the more monotonic shapes of the loadings on each factor, across ratings. The variances increase as indices approach the cross-over point between investment and non-investment grade bonds, with a local maximum for the BB1 index which is the highest-rated non-investment grade index. Both the shape and magnitude of the error variances are comparable to the results of Babbs and Nowman (1999), where a multifactor Vasicek model is used to fit the term structure of 8 observed treasury yields across maturities (0.25, 0.5, 1, 2, 3, 5, 7, and 10-year). In that study, the errors on longer maturities are the highest and decline sharply with the addition of a second factor. The middle maturities around the two-year series have higher error terms than the surrounding maturities.

We provide two possible explanations for the pattern in the error variances. Firstly, the time-series of the first two factors are closely related to averages of the investment-grade bonds and non-investment grade indices, respectively. We would expect that the further an observed index is from the ‘average’ investment-grade or non-investment grade series, the less precisely it will be captured by the two first and most important factors. A second explanation is that the observed BB1 index is a relatively noisy proxy for the yield spreads of BB1 quality. From Figure 1 it can be observed that the BB1 index closely follows investment-grade bonds early in the sample period, but follows non-investment grade bonds more closely for the remainder of the period. This changeover could be related to changes in the composition of the BB1 index, or changes in the pricing of included bonds that does not get reflected by rating changes. The finite sample of bonds within any rating class creates the potential for measurement errors in the relative pricing of various rating indices, and it is possible that these are more pronounced for indices near the cross-over point between investment and non-investment grade. The fact that the estimated measurement error variances peak around the crossing point of rating classes while the factor loadings remain relatively smooth can be interpreted as the effectiveness of state-space models in separating the idiosyncratic and systematic effects. Given a wide enough cross-section of time-series, the features of individual series that are not common to multiple indices can be expected to be absorbed into higher measurement error variances, while leaving factor loadings relatively smooth across the series.

We also analyze the time-series of the fit errors under a three-factor model. Table 2 shows that for each index the average fit errors are close to zero and strongly stationary based on ADF unit root tests. We take this as support for the multifactor Vasicek as an unbiased model of credit spreads across ratings over the sample period.

C. Discussion

The results for both periods suggest that all credit spreads vary in response to three common systematic factors that have proxies in the VIX, the long bond rate, and S&P500 returns. The co-movement between the factors and the variables is particularly evident from the beginning of the sub-prime crisis. Figure 3 shows that from the second half of 2007 factor 1 sharply increased as well as the VIX, factor 2 increased with (the negative of) the long-bond rate, and factor 3 declined with the S&P500 level.

However, the ability of the three factors to explain observed spreads can rapidly decline during financial crises, as shown by the conditional density likelihoods in Figure 5.

Log-likelihoods dropped during the LTCM liquidity crisis of August 1998, the end of the technology bubble in 2002, and since the start of the 2007 sub-prime mortgage crisis. The implication is that credit spreads reached levels that were not accounted for or fully reflected by the macroeconomic conditions at those times. Interestingly, during the LTCM crisis a two or three-factor model does not improve the fit over a one-factor model. One interpretation is that this crisis was of a more exogenous nature and more specifically relating to changes in credit market liquidity than changes in the macroeconomic outlook. While the end of the bubble in 2002 and the sub-prime crisis both had long-lasting impacts real economy, reflected in lower yields, lower equity returns and higher volatility, the LTCM crisis was characterized by a relatively sharper increase in volatility and smaller changes in rate and equity returns. It is likely that almost all of the change in credit spread levels during LTCM is explained by the sharp rise in factor 1 which is representative of the VIX, which is in turn closely related to liquidity risk. The sharp falls in log-likelihood that accompany the largest market moves point either to the presence of additional risk factors and risk premiums that are not captured by the Vasicek form, or the need to allow for time-variation in the factor loadings.

5. CONCLUSION

This study concludes that most of the systematic variation in credit spread indices by rating is explained by three factors. The factors vary broadly with the VIX, the long bond rate, and S&P500 returns, which are the theoretical determinants of credit risk. The sensitivities of credit spread indices to each of the factors suggest that the predictions of the Merton (1974) structural model hold on an aggregate level. While most empirical literature considers liquidity risk, rather than credit risk, to be the major determinant of credit spread levels and changes, we find that the three most important factors driving credit spreads vary with macroeconomic variables. The implication is that the dynamics of a potential liquidity risk premium are not easily separable from those of known macroeconomic variables, a result that is consistent with the findings of Ericsson and Renault (2006) that liquidity risk is determined by the same factors as credit risk. This is the first known study to use state-space representation and the Kalman Filter method to find credit spread factors. By making no prior assumptions about the risk variables driving credit spreads, the approach provides a contrast to existing empirical literature and an independent test of theory.

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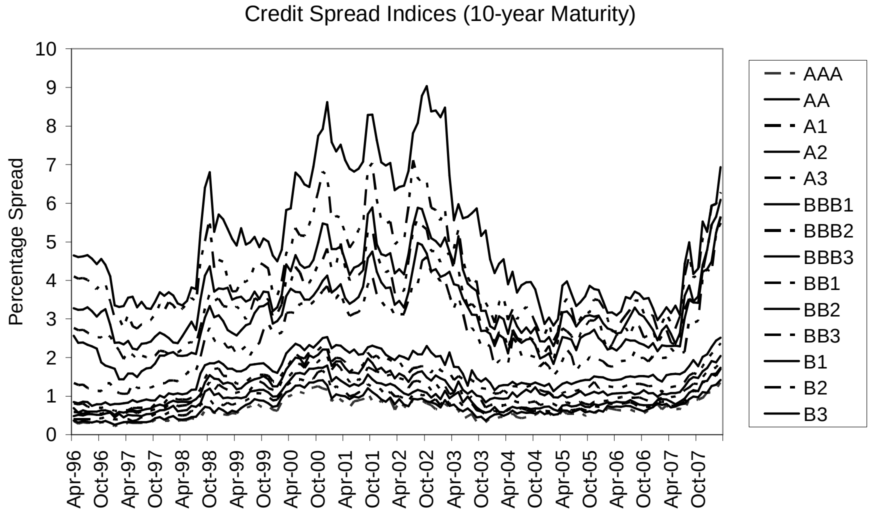
Figure 1 Industrial Corporate Bond Yield Spread Indices by Rating



(source: Bloomberg)

Table 1 Parameter Estimates, for the One, Two and Three-factor Models

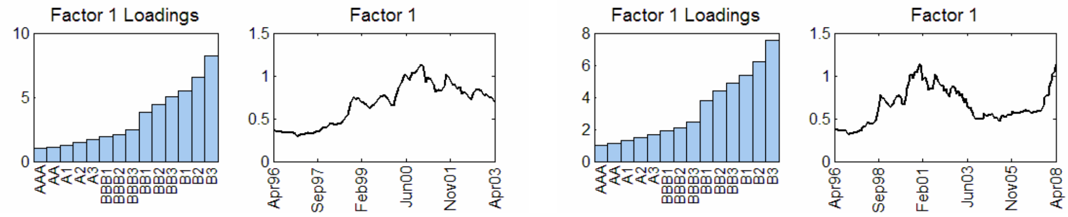
The table shows the maximum-likelihood estimates for each of the three parameters $\{\kappa, \theta, \sigma\}$ of each factor, under the one, two, and three-factor models. The Log-likelihood calculations are based on Equation (12), and used to calculate the Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). Standard errors based on the inverse Hessian matrix are shown below the parameter estimates.

	Period 1: Apr-96 to Mar-03			Period 2: Apr-96 to Mar-08		
	One Factor	**Two Factor**	**Three Factor**	**One Factor**	**Two Factor**	**Three Factor**
LogL	1,246	1,841	2,041	2,316	3,076	3,327
AIC	2,556	3,782	4,217	4,697	6,251	6,790
BIC	2,633	3,903	4,382	4,792	6,400	6,992
κ_1	0.391	0.411	2.981	0.120	0.127	0.486
	(0.281)	(0.401)	(0.231)	(0.040)	(0.145)	(0.197)
κ_2		0.351	0.171		0.757	0.004
		(5.011)	(0.021)		(0.304)	(0.003)
κ_3			0.421			0.786
			(0.041)			(0.357)
θ_1	0.791	0.601	0.251	1.177	1.195	0.329
	(0.211)	(0.201)	(0.021)	(0.299)	(0.653)	(0.043)
θ_2		0.161	0.641		-0.010	17.298
		(0.351)	(0.171)		(0.026)	(2.274)
θ_3			0.161			0.068
			(0.111)			(0.047)
σ_1	0.181	0.141	0.121	0.153	0.175	0.105
	(0.021)	(0.011)	(0.011)	(0.011)	(0.011)	(0.009)
σ_2		0.141	0.081		0.067	0.126
		(0.021)	(0.011)		(0.022)	(0.011)
σ_3			0.131			0.097
			(0.011)			(0.009)

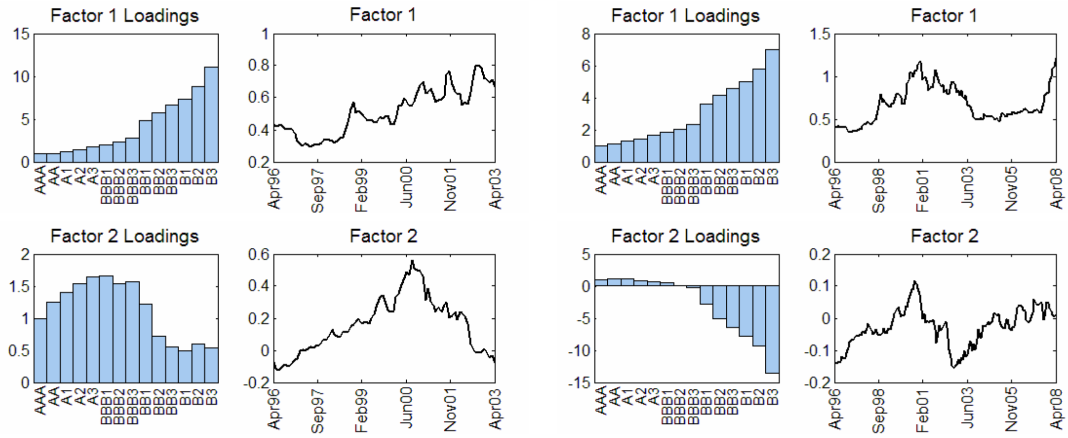
Figure 2 **Estimated Factor Time Series and the Factor Loadings by Rating**

Period 1: Apr-96 to Mar-03 **Period 2: Apr-96 to Mar-08**

One-Factor Model **One-Factor Model**



Two-Factor Model **Two-Factor Model**



Three-Factor Model **Three-Factor Model**

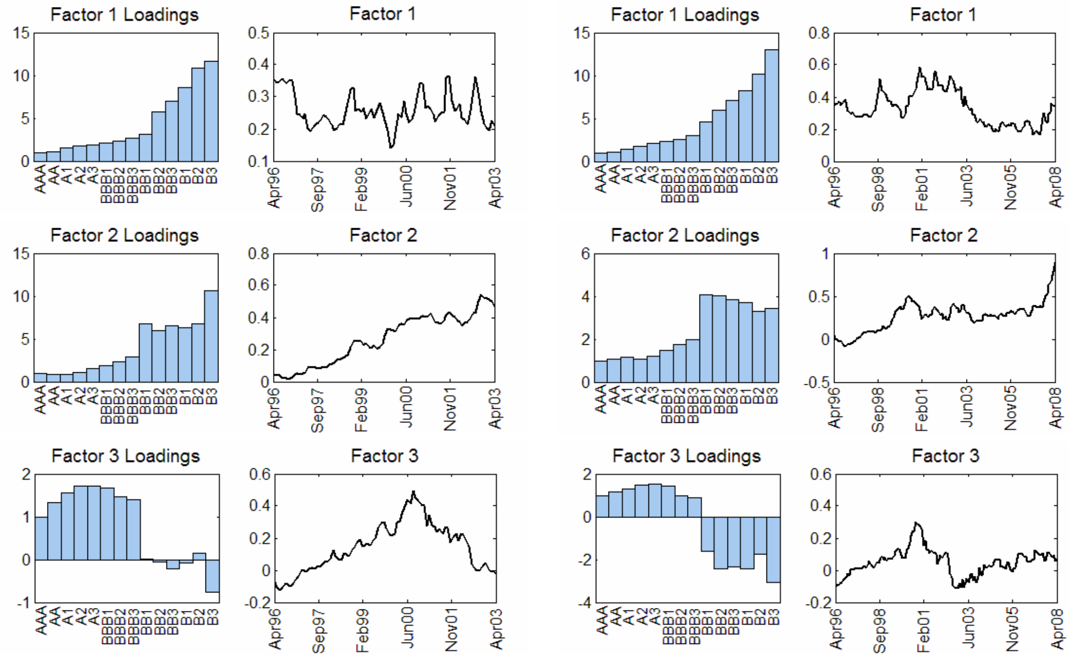


Figure 3 **Estimated Factors of the Three-Factor Model and Macroeconomic Variables**

Period 1: Apr-96 to Mar-03 **Period 2: Apr-96 to Mar-08**

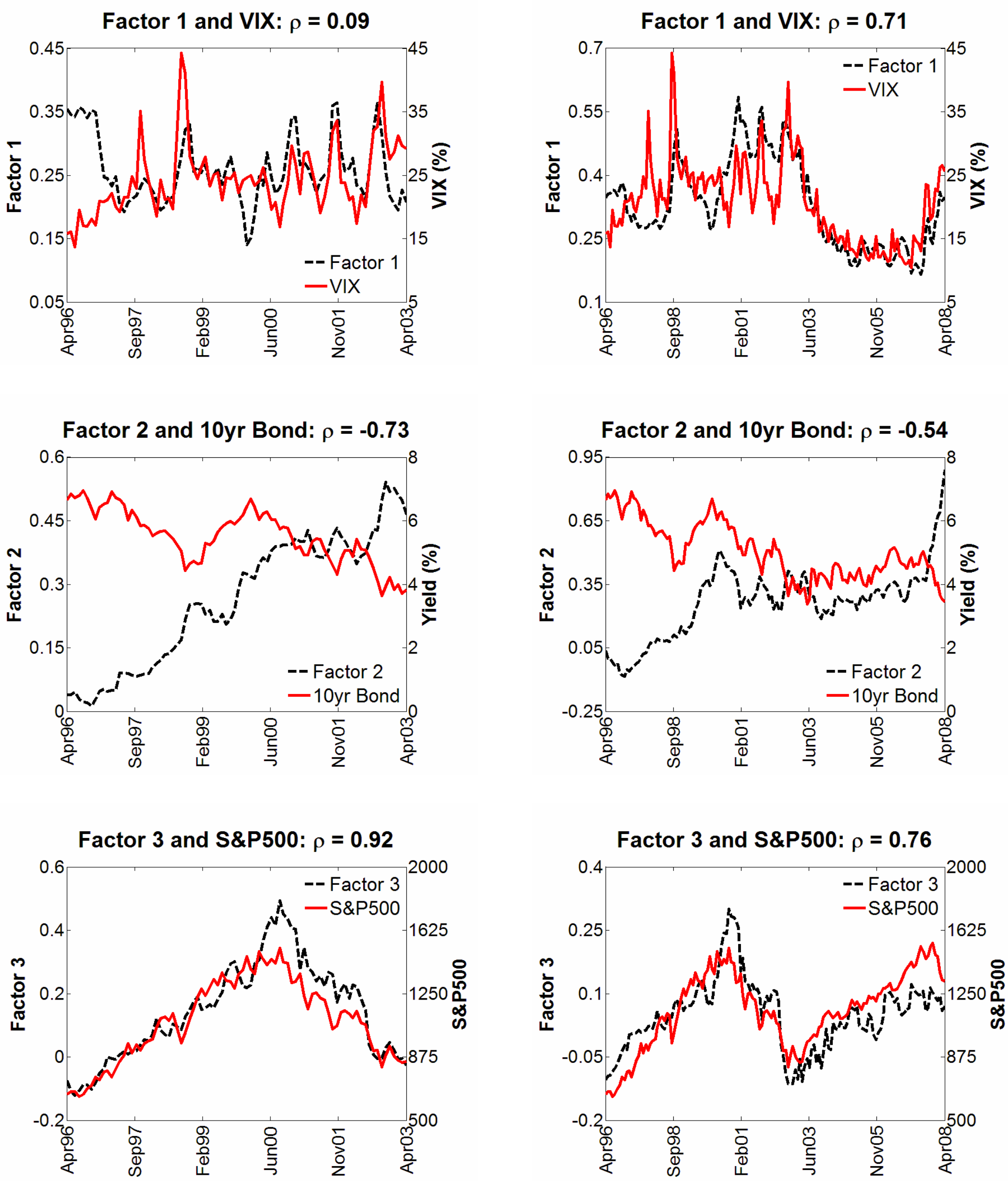


Figure 4 Measurement Error Variances

The figure shows the square roots of the estimated measurement error variances for each rating series under the one, two, and three-factor model. The maximum-likelihood estimates are based on the full period (Apr-96 to Mar-08), where the variances are the diagonal elements of matrix $H(14\times14)$ in the Measurement Equation $R_t = ZX_t + \varepsilon_t$ where $\varepsilon_t \sim N(0,H)$. All variance estimates are significant at the 5% level within each model.

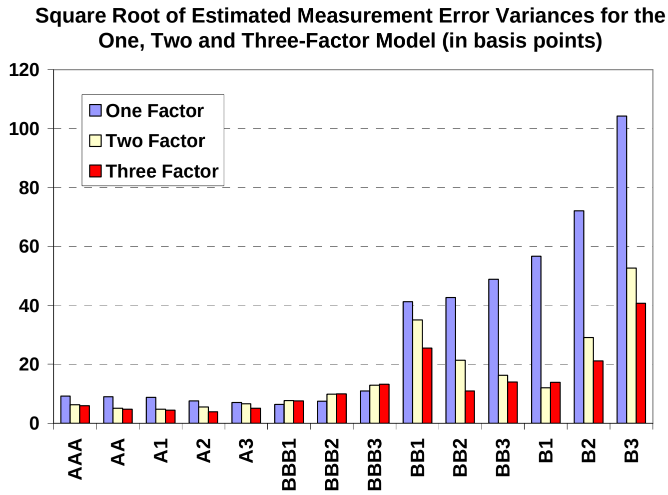


Figure 5 Conditional Density Log-Likelihoods for the One, Two, and Three-Factor Model: Apr-96 to Mar-08

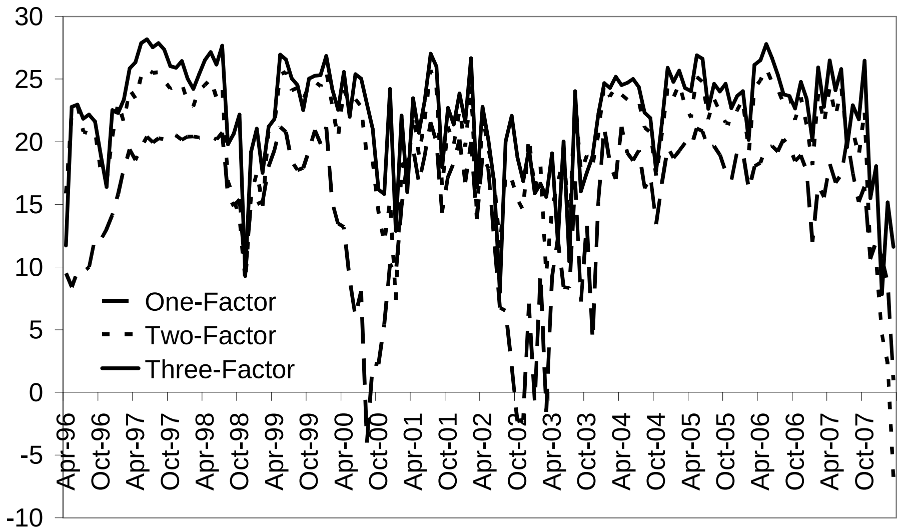


Table 2 Credit Spread Summary Statistics and Model Fit Errors

The following table summarizes observed credit spreads for the 144 months from 30-Apr-96 to 31-Mar-08. For each month we calculate a vector of model fit errors, defined as the difference between the 14 observed credit spreads $R_t(14\times 1)$ and the fitted spreads defined by the Measurement Equation (3) $\hat{R}_t = Z\hat{X}_t$. The estimated state vector $\hat{X}_t(3\times 1)$ and measurement matrix $Z(14\times 3)$ are based on the three-factor full period model. For each of the 14 credit spreads by rating we generate a time-series of 144 fit error terms and calculate the average, standard deviation, and mean absolute percentage error (MAPE). Two Augmented Dickey-Fuller tests (ADF) are performed on each error time-series. The first ADF test is based on an AR model with drift $y_t = c + \phi y_{t-1} + \varsigma \Delta y_{t-1} + \varepsilon_t$ and the second is based on the trend-stationary AR model $y_t = c + \phi y_{t-1} + \delta t + \varsigma \Delta y_{t-1} + \varepsilon_t$. For most of the series the ADF test p-values show that the null hypothesis of a unit root $(\phi = 1)$ can be strongly rejected at the 5% significance level. The stationary error terms with averages close to zero suggest that the three-factor Vasicek model on average provides an unbiased fit for credit spreads across ratings.

	Spread Statistics		Error Statistics			ADF Test p-values	
Index	**Avg (bp)**	**SD (bp)**	**Avg (bp)**	**MAPE**	**SD (bp)**	**No Trend**	**Trend**
AAA	64.5	25.4	0.01	7.8%	5.7	<0.0001	<0.0001
AA	71.9	27.9	-0.02	5.3%	4.3	<0.0001	<0.0001
A1	83.9	31.3	-0.01	4.3%	4.1	<0.0001	0.0032
A2	94.7	33.7	0.14	2.9%	3.4	<0.0001	<0.0001
A3	108.9	38.4	0.30	3.5%	4.4	<0.0001	<0.0001
BBB1	123.3	40.8	-0.41	5.1%	7.3	0.0020	0.0126
BBB2	138.1	43.2	-0.77	6.2%	9.7	0.0075	0.0350
BBB3	159.9	47.9	-1.36	6.9%	13.0	0.0068	0.0522
BB1	247.1	96.5	4.25	9.5%	24.5	0.0045	0.0221
BB2	292.4	88.5	-0.13	2.5%	8.7	<0.0001	<0.0001
BB3	325.5	98.1	0.54	3.3%	12.8	<0.0001	<0.0001
B1	359.1	99.7	-1.19	2.8%	11.9	<0.0001	<0.0001
B2	415.0	119.6	-0.26	3.7%	19.6	<0.0001	<0.0001
B3	502.8	165.3	4.26	6.6%	39.5	0.0025	0.0112