Hot Hands and Equilibrium

By

Gil Aharoni •
University of Melbourne

Oded H. Sarig
IDC, Herzeliya
and
The Wharton School

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Hot Hands in Basketball and Equilibrium

Abstract

Behavioral economics casts doubt on the rationality of economic agents using laboratory experiments. A notable exception to the reliance on laboratory data are studies that examine "hot hands" (*HH*) in professional basketball. According to the *HH* belief, players enjoy periods in which they perform better than usual. Gilovich, Vallone, and Tversky (1985) show, however, that actual success rates are independent of past performance, which they interpret as evidence that *HH* is a costly cognitive illusion. We argue that this interpretation ignores concurrent changes in team behavior when a player is *HH*. Accounting for these changes may reverse the predictions and conclusions regarding *HH*.

We model equilibrium changes in player behavior and derive equilibrium characteristics of *HH* periods and tests that differentiate between *HH* as a real phenomenon and as a cognitive illusion. We show that, in both cases, defensive efforts are shifted from non-*HH* players to *HH* players, allowing non-*HH* players easy shots and causing *HH* players to take difficult shots. Therefore, the success rate of an *HH* player nets increased defensive efforts and, if *HH* is a real phenomenon, improved shooting ability. Hence, if *HH* is a real phenomenon, the success rates of *HH* players will remain little changed from normal levels whereas if *HH* is a cognitive illusion the success rate of players erroneously perceived to be *HH* will fall precipitously.

We examine the 2004-2005 data of all NBA players and find that the data largely support our model's predictions. We find a significant reduction in defensive efforts allocated to non-*HH* players and increased efforts allocated to *HH* players. Accordingly, non-*HH* players improve their success rates substantially when other players are identified as *HH*. We also find that *HH* players have the same success rate in the face of increased defensive efforts, implying that their shooting ability does improve when they are identified as *HH*. The overall success rate of teams also materially improves when a player of theirs is identified to be *HH*. We also document improved success rates of *HH* players in free throws, where changes in player behavior do not impact success rates.

Hot Hands in Basketball and Equilibrium

I. Introduction

Traditional economic analysis assumes rational, optimizing behavior of economic agents. Behavioral economics casts doubt on this traditional view by pointing out limitations and biases in the way people behave. In particular, when agents face uncertainty, behavioral economic studies document limited ability of people to properly judge probabilities. For example, it is argued that people expect small samples to look like the properties of the generating distribution, even though probability theory does not imply such a deduction (e.g., Tversky and Kahnemann 1971, Wagenaar 1972). Similarly, people incorrectly generalize from patterns in small samples to the properties of the generating distribution, expecting small samples to "represent" the population. Behavioral economists argue that this bias, called the "Representative Heuristic," illustrates the inability of people to properly infer probabilities from small samples.

Like many other behavioral biases, the Representative Heuristic is documented in laboratory experiments. Some researchers question whether these laboratory results extend to real-world situations. The main reason to suspect the generality of the experimental results is that there are more significant consequences to real-world decisions than to laboratory decisions. Indeed, Slonim and Roth (1998) show that deviations from optimal strategies decline when the financial stakes increase even in laboratory experiments. Hence, behavioral studies attempt to show that such biases impact actual decisions that have significant consequences. For example, several studies attempt to use the

Representative Heuristic to explain patterns in security returns (e.g., DeBondt and Thaler 1985, Barberis, Shleifer, and Vishny 1998). These security return patterns, however, are consistent with alternative, rational explanations of the same phenomena (e.g., Fama and French 1992, Chordia and Shivakumar 2002).

A notable exception to the reliance on laboratory data to study biases is the set of studies that examine beliefs regarding "hot hands" (*HH*) in professional sports, where players have experience and financial stakes are high. According to the *HH* belief, players enjoy periods in which they perform better than their respective averages. Coaches, players, and fans identify these *HH* periods by strings of successes: strings of successful basketball shots, tennis serves, baseball hits etc. Studies of the *HH* belief examine whether *HH* truly exists or is a cognitive illusion that illustrates the Representative Heuristic in that people see patterns in small sports samples where no pattern truly exists. In these real-life settings, misperceptions of the chance of success (i.e., perceiving an *HH* when none exists) affect player behavior and have significant monetary consequences.

One of the first and most cited studies of *HH* is Gilovich, Vallone, and Tversky (1985) (GVT). GVT examine the existence of *HH* in basketball. In particular, GVT test whether a player's chances of scoring a basket ("Field Goal Percentage" or *FG%*) after consecutive successful shots are higher than after consecutive misses. Their main result is that the actual *FG%* of NBA professional players is independent of past performance. GVT argue that the "*data demonstrate the operation of a powerful and widely shared cognitive illusion*." More importantly, GVT argue that this false belief "*has consequences for the conduct of the game"* implying that *"the belief in the "hot hand" is not just* *erroneous, it could also be costly*." Since professional basketball is a multi-billion dollar business involving experienced professionals, these results suggest that the Representative Heuristic operates not just in laboratories but also when decisions have significant monetary consequences. This result, therefore, casts doubt on the universal rationality that classical economic analysis assumes.

In this paper, we reconsider the results of GVT and others and their interpretations. In particular, we examine the potential behavior changes entailed by an *HH* and their implications for the empirical tests of the *HH* phenomenon as a cognitive illusion. Consider, for example, the behavior of the defensive team once it identifies an offensive player as an *HH* player. To minimize the chances of a successful shot, the defensive team should allocate more defensive effort to the player they consider to be *HH*. Hence, if *HH* is a cognitive illusion, the *FG%* of the player erroneously identified as *HH* should decline precipitously because the increased defensive efforts allocated to this player worsen his ability to shoot successfully. Alternatively, if *HH* does exist, the increased defensive effort allocated to the *HH* player should offset the increased shooting ability of the *HH* player, if not completely negate it.

We explicitly account for the changed behavior that follows an identification of a player as *HH*; that is, we analyze and test the *HH* hypothesis in an equilibrium setting. Using a simple model of the game, we show that when a player is identified as an *HH* player, the defense on the *HH* player intensifies, which forces *HH* players to take hard shots while non-*HH* players are allowed easy shots. We consider the equilibrium implications of these strategic changes to the *observed* characteristics of games and compare these characteristics to expected characteristics of the game if *HH* is a cognitive illusion.

We show that if the *HH* hypothesis is valid, the *observable* impact of the strategy changes on the *FG%* of players is on the *FG%* of *non-HH* players. This is because the *FG%* of a truly *HH* player nets the impact of the improved shooting ability and the increased defensive efforts allocated to him. The noticeable difference in the game characteristics of a truly *HH* player is, therefore, not a change in his *FG%* but an increase in the fraction of difficult shots that he is forced to take. The non-*HH* players, however, who are less defended when a member of their team is perceived – correctly or incorrectly – to be *HH*, take easier shots and their *FG%* improves. Importantly, if *HH* is a cognitive illusion, the player who is incorrectly perceived to be *HH* will shoot with *the same offensive abilities* but face a *tougher defense*, which will entail a precipitous *decline* in the *FG%* of a player that is erroneously perceived to have improved shooting ability.

Using an extensive database of more than 1,200 games, we show that the equilibrium predictions regarding player behavior and game characteristics when a player is *HH* are true in the data. Specifically, we examine the game characteristics after the identification of an *HH* player as in GVT and others – when a player has a string of successful shots. We find that after the identification of an *HH* player, there is a significant shift in defensive efforts from *non-HH* players to the *HH* player. This is manifested in the fraction of difficult shots they take, in the number of three-point shots allowed, and in the number of fouls committed. The shift of defensive efforts from non-*HH* players to *HH* players results in a large and significant increase in the *FG%* of *non-HH* players.

Consistent with previous research, we find that the success rate of *HH* players is insignificantly different from their normal shooting ability even though they face more defensive efforts and take more difficult shots than in normal times. The ability of *HH* players to maintain their normal *FG%* in the face of intensified defensive efforts shows that their shooting ability is indeed improved when they are *HH* and that *HH* is not a costly cognitive illusion. The improved shooting ability of *HH* players is also evident in the *overall FG%* of teams when one of their players is *HH*: we find a significant improvement in a team's *FG%* after one of its players is identified as *HH*.

Prior research, including GVT, recognizes that team behavior might change when a player is identified to be *HH.* Thus, prior research attempts to overcome this difficulty by examining the *HH* phenomenon in cases where behavior can little change when ability is perceived to improve temporarily. For example, GVT examine two cases of shots without defense: free throws and practice throws. GVT find no serial correlation in free throws or practice shots, which they interpret as further supporting the conclusion that the belief in *HH* in games is a cognitive illusion. We reexamine the findings of GVT regarding free throws as well.[1] Similar to GVT, we find no simple dependency between the outcomes of first free throws and second free throws. Controlling for player ability (as we do for field goal attempts) and using a much more extensive data set, however, we find that *HH* does exist in free throws as well.

[1] The statistical analysis of GVT with respect to free throws has been criticized (e.g., Wardrop 1995, 1999). Improved statistical analyses show that serial dependency does exist in *FT* in the data analyzed by GVT.

Several studies examine the existence of *HH* in other sports and report mixed results. The evidence suggests that *HH* does not exist or hardly exists in baseball (Albright 1993) and tennis (Klassen and Magnus 2001). Evidence supportive of *HH* is reported in billiard (Adams 1995), bowling (Dorsy-Palmateer and Smith 2004), and horseshoe pitching (Smith 2004).

The remainder of the paper is organized as follows. In Section II, we present our model and derive testable hypotheses. In Section III, we present our data. In Section IV, we present the results regarding *HH* in field goal attempts. Section V tests the existence of *HH* in free throws. Section VI points out some economic implication of the model and results and Section VII concludes.

II. <u>Methodology</u>

While a complete model of basketball games is a daunting task, for our purposes, it is enough to model an individual play within a game.[2] Within an individual play, we analyze offensive shot selections and defensive decisions. Our model is simple enough to be tractable yet rich enough to yield testable hypotheses regarding the recoded variables of the game.

Since the objective of each team is to score more than the opponent team, basketball is a zero-sum game: a point to one team is a minus point to its opponent. Accordingly, in each play, the objective of the offensive team is to maximize the probability of scoring

[2] We ignore 3 point shots, which are a small fraction of all shots. Thus, the analysis of individual shots is justified by Walker and Wooders (2000), who show that in binary Markov games, individual plays are independent of prior results.

while the objective of the defensive team is to minimize this probability.

In our model, all players begin with equal abilities, both defensively and offensively.[3] At some point, both teams identify an offensive player as an *HH* player. Later, we discuss the empirical process by which we identify *HH* in our study. For modeling purposes, however, we abstract from the identification process and assume that the identification is correct in the following sense: the *temporal* offensive ability – O_i – of an *HH* player is perceived to be higher than his *normal* O_i by Δ. Non-*HH* players are called Average Hand – *AH* – and have regular abilities. Thus:

$$O_i^{HH} = O_i^{AH} + \Delta$$

Note that if *HH* is a real phenomenon then Δ will be both perceived and actually positive while if *HH* is a cognitive illusion then the *actual* Δ is zero but both teams *perceive* it to be positive.

Our focus is on the differences between the *HH* player and the four *AH* players. In particular, we do not distinguish between the four *AH* players in our model. Therefore, there is no loss of generality in assuming that the offensive team is comprised of two players: player 1, who can be either *HH* or *AH*, and player 2, who is *AH* and represents four *AH* players. Accordingly, in our model, there are two defensive players, who allocate their defensive efforts between the two offensive players.

[3] While this simplifying assumption may seem untenable to anyone even mildly familiar with the game, as we show in Table 2 below, prudent allocations of defensive efforts counteract natural differences among players. Therefore, this assumption should be viewed as depicting games *after* teams adjust to differential abilities of players.

We consider each shot as a sequence of three events: identification of player 1 as *AH* or *HH*, defensive effort allocation, and shot selection and realization. Accordingly, there are three times in the model:

0		**1**		**2**
HH or AH	*Defense Allocation*			*Shot Selection & Realization*

To model defensive decisions, we simplify the complexity of possible shots by considering two types of shots: difficult shots, which are denoted by *d*, and easy shots, which are denoted by *e*. The *FG%* of an *e* shot is O^e and the *FG%* of a *d* shot is O^d. Defensive efforts affect the mix of shots: the more defensive efforts allocated to a player, the lower the probability that the shot will be an easy one. In our model, when γ_i defensive effort is allocated to player *i*, the probability of a difficult shot is γ_i and the probability of an easy shot is $(1 - \gamma_i)$. Since $O^d < O^e$, the more defensive efforts are allocated to player *i* (i.e., the higher γ_i), the lower the player's *FG%*. The total defensive ability of the defensive team is fixed at Γ:

$$\gamma_1 + \gamma_2 = \Gamma < 2 \tag{3}$$

This means that allocating more defensive efforts to one player lowers the defensive efforts that can be allocated to the other player.

Next, we analyze shot selection and defensive effort allocation in the two possible states of the game: when both player *1* and player *2* are *AH* and when player *1* is *HH* while player *2* is *AH*. Since our analysis focuses on player *1* being either *AH* or *HH*, we call the case where both players are *AH* “the *AH* case”. Similarly, the case where player *1* is *HH* while player *2* is *AH* is called “the *HH* case”.

In both cases, there are four possible realizations of shot difficulties at time 2:

$$\{d,d\},\ \{d,e\},\ \{e,d\},\ \{e,e\}$$

The probabilities of these shot combinations are $\gamma_1\gamma_2$, $\gamma_1(1-\gamma_2)$, $(1-\gamma_1)\gamma_2$, and $(1-\gamma_1)(1-\gamma_2)$, respectively.

<u>The *AH* Case</u>

To maximize the probability of making the shot, at time *2*, the offensive team selects the player with the maximal probability of success to take the shot. Since both players have equal abilities in this case, if one player has an *e* shot and the other a *d* shot the shooting player will be the one with the *e* shot. If both players have equal shots – *e* or *d* – the shooting player will be chosen randomly.

The defensive team allocates its defensive efforts to minimize the *FG%* of the offensive team.

<u>Result 1:</u> In the *AH* case, the defensive team divides Γ equally between the offensive players:

$$\gamma_1 = \gamma_2 = \frac{\Gamma}{2}$$

<u>Proof:</u> See the proof of this result and all other results in the appendix.

Since Result 1 shows that, when both players are *AH*, the optimal allocation of defensive efforts is $(\Gamma/2)$ to each player, in this case, the probability of a *d* shot is $(\Gamma/2)^2$, the probability of an *e* shot is $[1-(\Gamma/2)^2]$, and the resulting *FG%* is:

$$FG\% = \left(\frac{\Gamma}{2}\right)^2 O^d + \left[1-\left(\frac{\Gamma}{2}\right)^2\right] O^e \tag{4}$$

The *HH* Case

While the *HH* case has the same four possible realizations of shot difficulties as in the *AH* case, the choice of the shooting player is less obvious. When both players have the same shot type – *e* or *d* – the *HH* player will shoot as the offensive team attempts to exploit his perceived improved probability. The *HH* player also shoots when he has an *e* shot while the *AH* player has a *d* shot. The potential ambiguity is when the *HH* player has a *d* shot while the *AH* player has an *e* shot. This is because, in this shot combination, the difficulty of the shot the *HH* player faces is offset by his perceived improved offensive ability. We assume that the *FG%* of an *e* shot is higher than the *FG%* of a *d* shot by an *HH* player:

$$O^e > O^d + \Delta \tag{5}$$

Note that if this assumption is not true, the *HH* player will shoot in all plays, which is counter factual. More generally, in our analysis, to mimic actual game conditions, we ignore "corner solutions" in which the *HH* player makes either all shots or no easy shots (i.e., we consider only the cases where $0 < \gamma_i < 1$ $i=1, 2$).

Result 2: In the *HH* case, regardless if *HH* is a cognitive illusion or not, the defensive team allocates more defensive efforts to the *HH* player than to the *AH* player, which improves the *FG%* of *AH* players.

In the Appendix, we prove that the defensive efforts allocated to the *HH* player are:

$$\gamma_1 = \frac{\Gamma}{2} + \frac{\Delta}{2(O^e - O^d - \Delta)} \equiv \frac{\Gamma}{2} + \frac{\varepsilon}{2} \tag{6}$$

Accordingly, the defensive efforts allocated to the *AH* player are:

$$\gamma_2 = \frac{\Gamma}{2} - \frac{\varepsilon}{2} \tag{6'}$$

Result 2 reflects the main thrust of our argument: the *optimal* behavior of the defensive team *changes* when a player is identified to be *HH*. This means that one cannot analyze the *HH* phenomenon by simply comparing the *FG%* of *HH* players to their normal *FG%*. The correct analysis of the problem accounts for the changes in the behavior of all players. As we show below, the resulting characteristics of the game, when all changes are accounted for, differ from the direct impact of the improved shooting ability of the *HH* player, analogous to the difference between a partial equilibrium analysis and a general equilibrium analysis.

Given the increased defensive efforts allocated to a player identified – correctly or incorrectly – as *HH*, the play is characterized as follows:

Results 3a: If *HH* is not a cognitive illusion, *HH* players take a larger proportion of *d* shots and have lower *FG%* than *AH* players

Result 3b: If *HH* is a cognitive illusion, *HH* players take a larger proportion of *d* shots and experience a *decline* in their *FG%* relative to their normal *FG%*

Results 3a and 3b are the basis for our tests of the *HH* hypothesis and, indeed, the reason our results allow us to conclude that *HH* is not a cognitive illusion. This is because Result 3b provides a testable hypothesis regarding the *FG%* of *HH* players if *HH* is a cognitive illusion: in this case, the *FG%* of *HH* players should *decline* once they are erroneously perceived to have temporarily improved shooting ability. As we show, this is not the case: *HH* players do face intensified defensive efforts and are forced to take difficult shots but, despite that, are able to attain virtually their normal *FG%*.

Note that Result 3a, which describes the game if *HH* is valid, does not characterize the *FG%* of *HH* players relative to their *normal* performance; rather, the comparison is to non-*HH* players. This is because the success rate of *HH* players nets offsetting effects: the improved personal ability and the increased defensive efforts allocated to them. Thus, the impact of the perception that a player has a temporarily improved shooting ability on the allocation of defensive efforts is best detected in the shooting characteristics of players who are *not HH – AH* players, whether *HH* is a cognitive illusion or not.

Result 3a is a special case of the Simpson (1951) Paradox: because of the improved shooting ability of the *HH* player, he takes all the difficult shots (while sharing with the *AH* player the easy shots). Hence, the *FG%* of the *HH* player is actually *lower* in equilibrium in that the *FG%* of the *AH* player. Note that, because the fraction of *d* shots in actual games is significantly higher than the fraction of *e* shots, Result 3a implies that the number of shots taken by the *HH* player is higher than his normal number of shots.

Results 2 and 3 illustrate the difference between a *ceteris paribus* analysis and an equilibrium analysis of the *HH* phenomenon. Specifically, the *ceteris paribus* analysis, which underlies the hypotheses tested by GVT, leads us to expect that a temporarily improved shooting ability of *HH* players entails a higher *FG%*. In contrast, when team reactions are accounted for in an equilibrium analysis, it is the *FG%* of the *AH* player that unequivocally improves and the performance of *HH* players can either improve or worsen.

<u>Result 4:</u> The *FG%* of teams with players perceived to be *HH* are better than their normal shooting abilities. The improvement in *FG*% of the team is larger when *HH* is a real phenomenon then when *HH* is a cognitive illusion.

The intuition of Result 4 is straight forward: since defensive efforts are shifted from *AH* players to *HH* players, if *HH* is a cognitive illusion, teams with *HH* players will improve their overall *FG%* because this is a suboptimal allocation of defensive efforts (c.f., Result 1). If *HH* is a real phenomenon, the improved shooting ability of *HH* players will increase the offensive ability of their team and further improve the team's *FG%*.

Note that, in our model, if *HH* is a cognitive illusion, offensive teams that shift throws to players that are erroneously perceived to be *HH* will not bear a cost. This is because, for modeling simplicity, we assume that there are only two types of throws – difficult and easy throws. As offensive teams shift throws to *HH* players only when both players face the same difficulty, the shift will not affect *FG%* if *HH* is a cognitive illusion. In reality, however, shot difficulty is not dichotomous so that players perceived to be *HH* may throw even when their team mates face easier shots. If *HH* is a cognitive illusion, this erroneous shift of throws will be costly. Thus, in reality, if *HH* is a cognitive illusion, the *FG%* of a team with a player erroneously perceived to be *HH* should be little changed from their normal levels since *both* teams err: the defensive team misallocates defensive efforts and the offensive team overly relies on the player erroneously perceived to be *HH*.

The predictions of the above simple model of shot selections and defense allocations are tested using the complete data for all NBA games in the 2004-2005 regular season – 30 teams playing 82 games each.[4] Since players rest for 15 minutes between halves, during which time temporarily improved shooting ability – *HH* – may vanish, our basic unit of analysis is not a game but a half. Like GVT, we define a player as an *HH* player if he successfully shoots two or three times in a row. Since there are no material differences in the results, we report the results of the latter definition only: a player is called

[4] Due to NBA problems, the details of 12 out of 1230 season games (1%) were not recorded.

HH if he makes 3 successful shots in a row in a half. Similarly, a player is called Cold Hand (*CH*) if he misses 3 shots in a row in a half.

By definition, *HH* and *CH* is a transitory phenomenon, affecting the shooting abilities of players temporarily. Therefore, we examine the impact of improved or reduced shooting ability – *HH* or *CH*, respectively – in a short horizon. We do this by focusing on the *Next Shot* following the identification of a player as *HH* or *CH*, which can be made either by the identified player himself or by another team member. This is different from the analysis of GVT. GVT examine the next shot *of the identified player himself*, which may happen quite later than the time the player is identified, even after the player is substituted for a while. Moreover, to ensure that we uniquely identify the status of the game, we consider only time intervals in which there is at most one player identified as either *HH* or as *CH*.

In the model, we consider two types of shots: *e* and *d*. The recording of the games by the NBA does not include all parameters necessary to correctly determine shot difficulty, such as defense intensity, angel and distance of the shot etc. Therefore, we are unable to control fully for the difficulty of shots. Instead, we use the type of shot, which is recoded, to proxy for shot difficulty: we consider jump shots to be *d* shots and all other shots, mainly (75%) lay-up shots, to be *e* shots. As explained where relevant, this partial control for shot difficulty adds some noise to our estimates and we account for it in our interpretations of the results.

In some of our analysis, we separate player data by their abilities. We use two measures of player ability: player salary and player field goal attempts (*FGA*). We report our results using both measures of player ability but note that the two measures are highly correlated (*ρ=0.562*). Moreover, some game participants suggested in private discussions, that superstar players are treated differently by coaches and behave differently than other players. Therefore, we also use these measures to identify the superstar player in each team and examine the *HH* effect on them separately. The results are not different from the results for other players.

In sum, based on the simple theoretical analysis of shot selection and defense allocation, we examine whether the data are consistent with the following conjectures:

- Coaches and players believe in *HH*
- Offensive teams change their shot selection when one of their players is identified as *HH*
- Defensive teams allocate more defensive efforts to *HH* players by reducing the defensive efforts allocated to *AH* players
- The *FG%* of *AH* players and teams improve when one of the players is identified as *HH*
- The FG% of *HH* players reflects locally better shooting ability.

III. Data

We collect all game records for the 2004-2005 NBA regular season from the official web site of the NBA. There are 30 teams, each playing 82 games, for a total of 1230. Due to recording problems of the NBA, we have detailed records for only 1218 – 99.0% of the total number of games in the regular season. A typical game record looks as follows:

Recap | Back to Full Box Score

FULL PLAY-BY-PLAY FOR

76ers 96, Magic 87

11/14/2004 Wachovia Center, Philadelphia, PA

1st Period

(12:00) Jackson Jump Ball Jackson vs Howard
(11:55) [PHL] Iguodala Layup Shot: Missed
(11:54) [ORL] Francis Rebound (Off:0 Def:1)
(11:32) [ORL] Francis Jump Shot: Missed
(11:31) [PHL] Iverson Rebound (Off:0 Def:1)
(11:24) [PHL 2-0] Jackson Jump Shot: Made (2 PTS) Assist: Iverson (1 AST)
(10:55) [ORL] Team Turnover: 24 Second Violation (TO)
(10:35) [PHL] Thomas Layup Shot: Missed
(10:34) [PHL] Thomas Tip Shot: Missed
(10:34) [PHL] Thomas Rebound (Off:1 Def:0)
(10:34) [ORL] Battie Rebound (Off:0 Def:1)
(10:23) [ORL] Battie Layup Shot: Missed
(10:22) [ORL 2-2] Battie Layup Shot: Made (2 PTS)

There are 528 players who were signed in this season and can be potentially included in our sample. We have shooting and salary data for 503 players – 95.2% of the player pool, who account for 98.7% of the all *FGA*. Table 1 reports the main characteristics of players that are included in our analysis. The average NBA player makes about 3.6 million dollars, with the stars making more than 3 times that. Note that the stars earn their pay by playing about 60% more time than the average player and taking more than twice the number of shots. The *FG%* of the star players, however, are similar to the *FG%* of the average players, a point which is amplified in Table 2. This suggests that teams adjust their defensive effort allocation to counteract the differential abilities of players. This point is reflected in the cross-player distribution of *FG%* and the cross-player distribution of successful free throws (*FT%*). The cross-player differences in *FT%*, as measured by the standard deviation and by the inter-quartile spread, are much higher than in *FG%*. This is probably because free throws are not defended while regular shots are defended, allowing teams to strategically allocate more defensive efforts to better shooters than to worse shooters in field goal attempts but not in free throws.

Table 2 further illustrates the strategic defensive effort allocations of teams. In Table 2, we report the average *FG%* and *FT%* of the top 5 players of each team. To make sure that a player that transfers from one team to another is not counted twice, the sample includes only the 150 players who played at least half of the games. (These players account for about 58% of the total game time and shoot about 65% of all *FGA*.) The table shows no ostensible relation between player ability, as measured either by salary or by *FGA*, and *FG%*. For example, the highest *FG%* is of the players ranked #5 by salary and #2 by *FGA* and the difference between the best and the worst *FG%* is less than 2%. This means that, despite the differential abilities of players, defensive efforts are successfully allocated in a strategic way to offset these differences. Thus, when coaches have time to plan their plays, they are able to allocate correctly defensive efforts across players. This, by itself, however, does not rule out cognitive illusion with respect to *temporary* changes in ability – *HH* and *CH*, which is the subject of our research.

The constancy of *FG%* reported in Table 2 is consistent with the simplifying assumption of our model that all players are of equal shooting ability at the outset. This is because, once coaches allocate defensive efforts strategically based on the natural abilities of opponent players, all players do have roughly the same *FG%*. Given this defensive adjustment, star players are noticeable mostly because they play longer and take more shots than average players.

Note that there is no relation between player ability (measured either by salary, or by *FGA*) and success in free throws (*FT%*). This suggests that ability in free throws is a different skill than other play skills (e.g., positioning, jumping, misleading, etc.), which game enthusiasts seem to know. Similar to previous studies, we analyze free throw data to test, using a similar equation to the one used for regular shots and our extensive data set, whether *HH* exists in free throws. As prior research points out, the advantage of examining free throws is that, while they involve different skills than regular throws, players and coaches cannot affect *FT%* by adjusting defensive and offensive efforts to players perceived to be *HH*. Thus, free throws allow for a better controlled experiment to test the *HH* hypothesis than field goal attempts.

IV. *HH* in Regular Throws

Much of the literature on *HH* tests for a serial correlation in success in filed goal attempts. Table 3 presents summary statistics that are similar to those reported by prior studies. Since our sample is much larger than prior samples, we do not break the results to individual players. Rather, we break the results by the rank of the players in their teams based on their season salary. The results are consistent with prior research in that we do not find a positive serial correlation in shot success. Rather, like prior studies, we find a slight negative correlation between past and current success. This negative correlation is evident both in the overall results and in the results broken by rank. Specifically, the overall results show that the *FG%* after three consecutive misses is 45.87% vis-à-vis only 43.00% after three consecutive successes. The similarity of our results to prior results suggests that, even though our sample is much larger than theirs and from a different season, the salient characteristics of shot serial correlations are present in our data too.

As our theoretical model suggests, temporal changes in shooting ability need not manifest themselves in positive serial correlations in shooting records because of changes in team behavior. Therefore, we proceed to examine the impact of *HH* on coach and player behavior. We define a player as an *HH* player after he successfully shoots three consecutive shots within the same half. Similarly, a *CH* player is defined as a player who misses three consecutive shots.[5] We call the shot in which a player is identified as either *HH* or *CH* the *Identifying Shot* and the player is called the *Identified Player*. Most of our analysis focuses on the filed goal attempt that follows the *Identifying Shot* – the *Next Shot*, which can be attempted by either a *HH* player or by a regular player.

While coaches, players, and commentators often talk about the *HH* phenomenon, this may be merely "cheap talk" in that neither coaches nor players actually change their game behavior when there occurs a temporary change in shooting ability (i.e., when a player is identifies as being either *HH* or *CH*). Therefore, we begin our analysis by documenting, in Table 4, the behavior of the identified players and their coaches when we identify a temporary change in shooting ability. Row 2 in Table 4 shows that coaches retain an *HH* player in the game 35 seconds longer than they retain a *CH* player. This difference is both statistically and strategically significant since an *HH* player plays more than 10% longer than a *CH* player. Rows 3 and 4 in Table 4 show that players change their behavior as well. Specifically, *HH* players take statistically significant less time to try another shot and take a larger fraction of *Next Shots* than *CH* players. Moreover, as our model predicts, row 5 shows that players also change their shot selection: *HH* players take significantly more jump shots (which is our proxy for difficult shots) than *CH* players. Therefore, the evidence presented in Table 4 indicates that both coaches and players react to perceived temporary changes in shooting ability.

As we show in Table 4, coaches react to temporary changes in the ability of their players by substituting *CH* players quicker than *HH* players. Coach reactions cause the *CH* sample to be both smaller than the *HH* sample (as is evident in Table 4) and biased. The reason for the bias is that coaches are able to detect temporary changes in shooting ability based on shot characteristics that are not in our data set: number of defenders and their abilities, angle of shot, distance to basket etc. Thus, our *CH* sample includes players that we incorrectly identify as *CH* for lack of better data and excludes data of truly *CH* players who are replaced by coaches. (To illustrate the bias, we note that players who miss consecutive lay-up shots are replaced faster than players who miss jump shots.) Since this problem does not affect our *HH* sample, in the remaining analysis, we focus on comparisons of the *HH* sample to the unidentified-player sample.[6]

In the analysis of the impact of *HH* on team behavior, we focus on the first shot of the team following the Identifying Shot, which we call the *Next Shot*. To make sure that we analyze clearly identified effects, we consider shots only when there is no more than one Identified Player in the offensive team. Throughout, we exclude shots taken within 5

[5] Defining *HH* and CH players by *two* consecutive shots instead of *three* little affects our results.
[6] We note, however, that tests on the *CH* sample, unreported in the paper, yield results that are in line with the predictions of the model. For example, *FG%* of *CH* players are better than their season averages.

seconds of the previous shot of the team (*offensive rebounds*) since these are not necessarily characterized by the same strategies as normal, planned shots. Our final sample consists of 168,959 shots (88% of all attempts) of which 4,993 are Next Shots with an *HH* Identified Player in the offensive team.

In Table 5, we report the characteristics of Next Shots taken by *HH* players, *AH* players, and teams as a whole relative to the same characteristics of all other shots, hereafter called *regular shots*. The first lines of Table 5 report shot characteristics that allow us to examine whether there is reallocation of defensive efforts from non-*HH* players to players perceived to be *HH*. The results show an increase in the difficulty of shots taken by *HH* players – a larger fraction of the shots they take are jump shots. Conversely, *AH* players take fewer difficult shots when there is an *HH* player in their team than usual and than the *HH* players.

Note that the "jump" "non-jump" classification is a noisy proxy for the true difficulty of shots, albeit the only measure of shot difficulty that the NBA records. This is because this measure of shot difficulty does not reflect all information needed to fully determine difficulty, such as: defense intensity, distance, angle, etc. Importantly, since we expect *HH* players to be more defended than *AH* players, we expect both jump and non-jump shots taken by *HH* players to be more difficult than similar shots taken by *AH* players. Due to data limitations, however, we can use only the noisy measure of shot difficulty – "jump" vs. "non-jump" – in the remainder of our analysis.

Our model predicts that if a player is identified as *HH* the mix of shot difficulty will change, which offsets the improved ability of the *HH* player in the *observed* success rate.

To reduce the impact of the mix change, the next two rows of Table 5 show the *FG%* separately in jump shots and in non-jump-shots. Consider *HH* players first. The noisy control for shot difficulty ameliorates the reduction in *FG%* relative to regular shots in jump shots. In the case of non-jump shots, the relative performance of *HH* players is *better* in the *HH* periods than in non-*HH* periods, albeit insignificantly. In the case of *AH* players, even with noisy control for shot difficulty, the *AH* players perform better when there is an *HH* player than in regular shots, which shows that the jump / non-jump classification does not fully capture the reduced defensive efforts allocated to *AH* players in *HH* periods.

The next two lines are additional indications of changes in defensive effort allocation. Consider 3 point shots first. Since 3 point shots are very difficult shots, they are typically taken when players are relatively less guarded. Thus, the significant increase in the number of 3 points made by *AH* players is another indication that they are less defended when another player in the team is *HH*. Similarly, the change in the number of fouls committed shows the differences in defensive effort. Specifically, fouls typically indicate a less careful defense since a foul is a "last resort" defensive effort. Therefore, the reduced number of fouls on *HH* players and the increased number of fouls on *AH* players also indicate a shift of defensive attention from *AH* players to *HH* players.

Tables 4 and 5 demonstrate that defensive efforts are shifted from non-*HH* players to players perceived to be *HH*. Consequently, there is a significant improvement in the *FG%* of the *AH* players – an improvement of nearly four percentage points, as predicted by our model. Most importantly, despite the significant increase in defensive efforts allocated to players perceived to be *HH*, *they are able to attain their regular FG%.* Thus, these players truly have temporarily improved shooting ability. This means that the *HH* belief is *not* a cognitive illusion.

The overall performance of the team is another indication that *HH* is a real phenomenon. If *HH* is a cognitive illusion then there should be little change in the overall *FG%* of the team. This is because the overall *FG%* nets the errors of both teams: defensive teams allocate too much effort to players perceived to be *HH* and offensive teams rely too heavily on the shooting ability of players erroneously perceived to be *HH*. Our results show a significant improvement in the *FG%* of the team of almost 2.5%. This improvement is both statistically significant and practically important, as the difference in season average *FG%* between the *best* and *worst* NBA teams is less than 5%.

It has been suggested to us that a possible explanation for improved team *FG%* is that teams are encouraged by strings of successful throws. Although this possibility, obviously, does not explain the documented significant shifts in defensive efforts following the identification of an *HH* player, it can explain the improvement in *AH* players *FG%*. Hence, we examine this possible explanation in the following way. Since improved team spirits should be observed after *any* successful string of successful shots (i.e., not just by *HH* players), we examine the *FG%* of teams following successful strings of success of the *whole* team – from 3 successful throws to 7 consecutive successful throws. We find virtually identical *FG%* after any length of string of successes. Therefore, our findings suggest that team momentum does not exist in basketball games, suggesting that the improved shooting ability of the team is unique to periods in which a specific player is identify as *HH*.

As our model and results so far indicate, it is difficult to detect the improved ability of *HH* players by a simple comparison of their *FG%*. This is because changes in defensive effort allocations entail changes in the difficulty of the shots *HH* players take. Additionally, the simple comparison of the *FG%* of *HH* (*AH*) players to regular *FG%* implicitly assumes a matching of the players in the sub-samples. To improve our control of player ability, we estimate a PROBIT regression that accounts for several potential determinants of *FG*%. In this regression, we control for the mix of shots – jump or non-jump. Additionally, we improve our control for player ability in three ways. First, we include in the equation a control variable for the normal ability of a player – his season *FG%* relative to the NBA's overall *FG%*. Second, we include two measures of fatigue: the total time that the player played in the half and a dummy variable that takes the value of one if the shot is in the second half of the game and zero otherwise. Finally, we control for the defensive ability of the opponent team. This is because it is possible that the probability of three consecutive successful shots is related to the defensive ability of the opponent team. Hence, we include as a control variable the difference between the *FG% allowed* by the defensive team in the season and the average overall *FG%* in the NBA as a measure of the defensive ability of the defensive team.

Table 6 presents the PROBIT estimates of the following equation:

$$I_{Success} = \beta_0 + \beta_1 \cdot I_{AH} + \beta_2 \cdot I_{HH} + \beta_3 \cdot SFG\% + \beta_4 \cdot I_{Jump} + \beta_5 \cdot I_{Half} + \beta_6 \cdot Time + \beta_7 \cdot Def + \varepsilon$$

where:

$I_{Success}$ is a dummy variable that takes the value of one when the Next Shot is successful and zero otherwise;

I_{AH} is a dummy variable that takes the value of one when the Next Shot is taken by an *AH* player and zero otherwise;

I_{HH} is a dummy variable that takes the value of one when the Next Shot is taken by an *HH* player and zero otherwise;

SFG% is the fraction of successful field-goal attempts of the shooting player in the entire season relative to the NBA's overall *FG%*;

I_{Jump} is a dummy variable that takes the value of one when the field goal attempt is a jump shot, which proxies for difficult shots, and zero otherwise;

I_{Half} is a dummy variable that takes the value of one when the field goal attempt is in the second half of a game and zero otherwise;

Time is the number of minutes the shooting player played in the half prior to the shot;

Def is the difference between the *FG%* allowed by the defensive team in the season and the overall *FG%* in the NBA.

We also report PROBIT estimates of the equation for two sub-samples: one sub-sample is comprised of jump shots only and the other sub-sample of non-jump shots. In these sub-sample regressions we do not include the I_{Jump} control variable.

The results reported in Table 6 confirm the results reported in previous tables. First, even after we control for player ability and shot mix, the improved *FG%* of the *AH* players remains significantly different from zero. The improved *FG%* of *AH* players demonstrates the transfer of defensive efforts away from them to *HH* players. Nonetheless, despite the increased defensive efforts allocated to *HH* players, the estimated coefficient of I_{HH} is small in magnitude and insignificantly different from zero. The fact that *HH* players are able to shoot with essentially their normal *FG%* in spite of the increased defensive efforts allocated to them is an indication of their improved shooting ability in *HH* periods. Other coefficients are of their expected signs. The estimated impact of shot difficulty, proxied by jump shots – I_{Jump} – corresponds to a decrease of roughly 20% in *FG%* when players attempt jump shots. The impact of fatigue is also significantly different from zero: for every minute played, the *FG%* of the player is reduce by roughly 0.1% and *FG%* in second-halves are about 1% lower than in first halves.

In our model, we assume that, except *HH* players, the defensive and offensive abilities of teams are constant throughout a game. It is possible, however, that when a player is perceived to be *HH*, both teams adjust their effort levels, effectively shifting efforts over time within a game. Such changes may spuriously change *FG%* at times when players are perceived to be *HH.* To examine this potential effect on our estimates, we consider periods in which there is little ability to shift efforts over time: when the game gets to the fourth quarter and the score is close. In these periods, it is likely that both offensive and defensive efforts are close to their maximum. Therefore, examining *HH* in such periods is likely to be less affected by effort shifts over time.

We examine shots taken in fourth quarters when the score difference between the teams is less then 10 points.[7] Out of 4,993 Next Shots in our sample, 937 shots are at such times – where effort shifting is not likely. The last row of Table 6 reports the estimate regression with these shots only. Indeed, when efforts cannot be shifted over time (and thus ameliorate the effects we try to estimate), the evidence that *HH* is not a cognitive illusion becomes stronger. Specifically, the coefficient of the *AH* player doubles to 0.156, corresponding to a temporary improvement of 6.2% in his *FG%*, which is very significant given the small cross-sectional differences in player *FG%* (c.f., Table 1). Importantly, despite the shift in defensive efforts, there is no reduction in *FG%* of *HH* players: the coefficient of I_{HH} is *positive*, albeit still insignificantly different from zero. Thus, *HH* players are able to attain their normal *FG%* (or better) even thought they face intensified defense, which reflects their temporarily improved shooting ability. (Recall that if *HH* were a cognitive illusion, the *FG%* of players incorrectly identified to be *HH* should *decline* as they face an increased defense but have no improved ability.) The improved shooting ability of *HH* players also entails a large improvement in team *FG%*: teams' overall *FG%* improves by 5.2% in last quarters of close games with *HH* players. This improvement is statistically significant and larger than the difference between the average *FG%* of the best and worst NBA teams.

In sum, we show that coaches and players react to the identification of a player to be *HH* in regular throws by changing defensive and offensive strategies. We further find that team performance improves during these periods. Importantly, we find that players

[7] We also analyze the data using a 15 point cutoff without any traceable impact on the results.

identified to be *HH* are able to achieve their normal performance despite facing intensified defensive efforts. These effects are stronger in the last quarters of close games than in times where performance matters less. These findings suggest that *HH* in regular throws is a real phenomenon.

V. ***HH* in Free Throws**

While the focus of our analysis is on misperceptions of probabilities when statistical inference is made in a short time span, we use our extensive dataset to examine the *HH* phenomenon in free throws (*FT*) as well. As discussed previously, *FT* are a form of controlled experiment of the *HH* phenomenon since, unlike *HH* in regular throws, in *FT*, players and coaches cannot adjust their behavior once they identify a player as being *HH*. Note, however, that *FT* differ from field goal attempts because they involve different settings and different player skills (e.g., ability to position, jump, mislead etc.). Indeed, the significantly different success rates of these shot types (c.f., Tables 1 and 2) clearly demonstrate the difference in the skills involved. Presumably, these differences explain why basketball fans believe that *HH* has a larger effect in field goals than in *FT* as the survey of GVT shows. Nonetheless, because *FT* are not affected by changes in team strategies, in this section, we use *FT* data to estimate an equation similar to the one estimated for regular shots and test whether *HH* exists in *FT*.

Our raw data are the same data that we use for the analysis of field goal attempts – 1218 games in the regular 2004-2005 NBA season. In these games, there are 60,556 regular *FT* (i.e., no technical fouls or 3-point attempts) from which we construct our sample. We begin by replicating GVT's test of whether success in the second *FT* is correlated with success in the first *FT*. Consistent with their findings, we do not find such correlation. Note, however, that this test is inconsistent with the tests of *HH* in regular throws because it identifies *HH* players in *FT* by a *single shot*. This difference in identification is especially problematic in *FT* because of the high success rate of *FT* and their infrequency (c.f., Table 1). Hence, in what follows, we identify players to be *HH* in *FT* similar to the identification in field throws – by a *series* of successful shots in the same half: we consider a player to be *HH* in *FT* if he succeeds in consecutive shots in a half. Since the probability of success in *FT* (75.6%) is much higher than the probability of success in regular shots (44.3%), we consider a player to be *HH* in *FT* when he succeeds in *four* consecutive *FT* in a half.[8] Because a player cannot succeed in four *FT* unless he attempts at least four such shots, we drop from our sample the first *FT* of a player in a half. This leaves us with a sample of 9,659 *FT* of which 3,229 are attempted when the player is *HH* in *FT*.

Similar to our analysis of regular shots, we estimate a PROBIT regression where the dependent variable is a dummy variable that takes the value of *1* if a player succeeds in a *FT* and *0* otherwise. The explanatory variables are similar to those used in regular throws:

- I_{HH_FT} is a dummy variable that takes the value of *1* when the shooting player is *HH* in *FT* and *0* otherwise
- I_{HH_Reg} is a dummy variable that takes the value of *1* when the shooting player is *HH* in regular shots and *0* otherwise
- *SFT%* is success rate of the shooting player in *FT* in the entire season normalized

[8] The results remain essentially the same when *HH* in *FT* is defined by 2 or 3 successful consecutive *FT*.

by the average *SFT%* in the NBA

- I_{Second} is a dummy variable that takes the value of *1* when this is the second *FT* in the visit to the *FT* line and *0* otherwise
- I_{Half} is a dummy variable that takes the value of one when the field goal attempt is in the second half of a game and zero otherwise
- *Time* is the number of minutes the shooting player played in the half prior to the *FT*

The first two variables indicate a potential temporary improvement in the shooting ability of the player: I_{HH_FT} indicates that the player is identified as *HH* in *FT* and I_{HH_Reg} indicates that the player is identified as *HH* in field goal shots in the previous three minutes before the *FT*. *SFT%* controls for the ability of the player in *FT*. I_{Second} controls for potentially improved shooting ability in the second *FT*. I_{Half} and *Time* control for the potential impact of fatigue on players' shooting ability.

The estimated PROBIT coefficients are:

I_{HH_FT}	I_{HH_Reg}	*SFT%*	I_{Second}	I_{Half}	*Time*
0.090**	-0.068	2.602**	0.112**	-0.007	0.002
(0.033)	(0.059)	(0.148)	(0.029)	(0.031)	(0.003)

** Significant at 1%

Given that no strategic changes in player and coach behavior can be implemented when a player is identified as *HH* in *FT*, it is not surprising that the *HH* phenomenon is clearly evident in these throws. Specifically, the estimated impact of being *HH* in *FT* is significantly positive, implying an improvement of roughly 3% in the success rate in *FT* when a player is *HH* in *FT*. Consistent with the hypothesis that *FT* and regular throws entail different settings and skills, we find no *HH* "spill-over": being *HH* in *regular* throws does not improve the success rate in *FT*.[9]

The controls that have a significant impact on the success rate in *FT* are the overall success rate of the player in *FT* throughout the season (*SFT%*) and the throw being the second one in the same visit to the *FT* line (I_{Second}). (The latter result is consistent with the criticism and findings of Wardrop 1995).

To sum, using a larger sample than GVT and the same methodology as in field goals (i.e., controlling for various determinants of success in *FT*), we find that *HH* is a real phenomenon in *FT*. Given that we document the validity of the *HH* phenomenon in throws where no other player other than the one making it is affected by a belief in *HH*, this is further evidence that *HH* is a real phenomenon.

VI. Economic Implications

While the preceding theoretical and empirical analyses of basketball have natural extensions for other sports, the results and their implications also apply to multiple economic situations. In this section, we briefly point out some of the areas in which the implications of our analysis apply. The main implication we consider are that abnormal ability entails strategy changes, which increases difficulty and reduces the improvement in the *observed* performance of "hot" agents. The strategy changes are more noticeably observed in the results of the *other* agents than in the change in the results of the "hot"

[9] Similarly, we find no "spill-over" effect of being *HH* in *FT* to the success rate in regular throws.

agent. Some of these areas have documented results in the spirit of our analysis and some of the implications call for additional research.

One area where similar results have been modeled and documented is money management. It is well documented (e.g., Jensen 1969, Gruber 1996) that the performance of money managers is not serially correlated: superior investment results in one period are not followed by superior results in subsequent periods. Rather, superior results in a given period are followed by average subsequent investment results. Yet, analysis of money flows into and out of mutual funds suggests that investors tend to invest with money managers who showed superior performance in a preceding period (e.g., Chevalier and Ellison 1997; Sirri and Tufano 1998).

The empirical evidence on investor money flows has been interpreted, similar to the interpretation of the HH phenomenon, as an irrational investor reaction. Specifically, it is argued that investors see patterns in investment results – "hot hands" in investment management – where none exists. Investors transfect investment funds to "hot hand" money managers but obtain no superior results. Berk and Green (2004), however, argue that these results are consistent with superior investment ability of certain managers and with rational investors. Their model, while not quite the same as ours, is based on the fact that managers with superior investment skills receive more money to manage (for an increased fee). Analogous to the drawing of more defensive efforts when a player is *HH*, as the sums managed by the managers with superior performance grow, the ability of these managers to generate superior returns diminishes (for example, because of scale limitations on attractive investment ideas). Thus, managing more money stretches the investment skills of the managers to the point that they can generate just normal returns. The model of Berk and Green (2004) is similar in spirit to our model and, like ours, is consistent rationality and with prior evidence on performance of managers and subsequent money flows. Note that our analysis suggests additional testable hypothesis: as money is withdrawn from managers with *poor* performance their investment returns should improve, akin to the improved performance of *AH* players in our model.

One can transfer the logic of the money management industry to management in any other industry and derive similar implications for industrial organization along the following lines. While the quality of managers is typically little known at the outset of their tenure, as they prove their superior ability they are put in charge of increasingly larger and more problematic assets. For CEO's, increasing firm size is often achieved by acquisitions, which enable superior CEO's to apply their superior skills to large firms and assets. As superior managers attempt to acquire additional assets, however, they are increasingly required to pay larger sums for the acquired assets that they identify as currently being under-managed (or under-priced). This is because sellers realize that the acquiring managers can enhance the value of the assets under their management and bargain for a fraction of the value creation (unless they have no bargaining power at all). Furthermore, as superior managers manage increasingly larger firms, their managerial skills are stretched until they can no longer offer above-normal performance.

Extending the logic of our analysis of hot hands, therefore, implies that, analogous to the observed performance of *HH* basketball players, the *observed* performance of superior managers should be little different from the performance of regular managers. The superior ability of managers should manifest itself in the fact that they are put in charge of larger firms than less able managers, which is analogous to *HH* players playing longer and shooting more than *AH* players. Superior managers should also pay larger premiums in acquisitions than regular managers, which is analogous to the increased defense *HH* players face. Indeed, empirical evidence is consistent with these predictions. For example, while skill and pay are positively correlated in general (e.g., O'Shaughnessy, Levine, and Cappelli 2001), when it comes to top managers, their *observed* performance is uncorrelated with their pay (see, for example, the survey in Bebchuk and Fried 2004). Rather, managerial pay is correlated to assets under management (Bebchuk and Grinstein 2005). While Bebchuk and Fried (2004) interpret these results as "pay without performance", we interpret them as pay *with* performance, albeit when performance is measured by the extent that investors trust superior managers by putting them in control of large sums of money. Indeed, we expect studies in labor economics and organizational behavior to find the same relations between skill, observed performance, and pay of employees.

VII. Conclusions

Behavioral economists use laboratory experiments to document limitations on individual ability to process random data and derive rational probability assessments. The relevance of such limitation to actual decisions that involve substantial sums is often questioned. Studies of "hot hands" in professional basketball are often cited as the prime proof for the relevance of these laboratory results to real-life situations. Specifically, Gilovich, Vallone, and Tversky (1985) and others argue that the chance of scoring is largely independent of past performance, which they interpret to mean that *HH* is a costly cognitive illusion.

We argue that prior empirical evidence regarding *HH* implicitly assumes *ceteris paribus*, which ignores concurrent changes in team behavior when a player is *HH*. In a simple model of basketball, we show that changes in team behavior may reverse the predictions and conclusions of the analysis of the *HH* phenomenon under the *ceteris paribus* assumption. Specifically, when a player is identified as *HH*, defensive efforts are shifted towards him and away from the non-*HH* players. This causes the player with temporarily improved success rate – the *HH* player – to take more difficult shots and ameliorates the temporary improvement in his ability. Conversely, our analysis suggests that the game realignment in *HH* periods is best revealed in the game characteristics of the non-*HH* players. This is because an *HH* player is affected by increased defensive efforts, which negate his improved shooting ability, while a non-*HH* player is affected by a single change – the strategic change in defensive efforts.

We examine an extensive database of more than 1,200 games and show that the equilibrium predictions regarding player behavior and game characteristics when a player is *HH* are true in the data. Specifically, we find that defensive efforts are indeed shifted from non-*HH* players to *HH* players, causing *HH* players to take more difficult shots while less defended, non-*HH* players take more easy shots. Consequently, the performance of non-*HH* players as well as the overall performance of teams improve when one of the players is *HH*. More importantly, we show that, despite increased defensive efforts that are allocated to *HH* players, *HH* players achieve the same success rate that they achieve normally. This means that the shooting abilities of *HH* players are indeed better than their normal abilities. Thus, our empirical findings are consistent with the predictions of our model and with the hot hand phenomenon being real, implying that team behavior and beliefs are consistent with rationality.

To complement our analysis of regular throws, like other studies we also examine *HH* in free throws, which are shot that are not affected by behavior of non-shooting players. We show that there exist *HH* in free throws as well.

An aspect that is highlighted by our model is the importance of distinguishing between easy and difficult tasks in the analysis of performance, in our case – the difference between easy and difficult shots. A simple way to see the importance of this distinction is to think of Kobe Bryant and Allen Iverson, two of the NBA's most talented offensive players in the season we analyze, whose *FG%* is a little *bellow* the NBA's average. This is because their talents cause defensive teams to allocate more defensive efforts to them, which forces them to take difficult shots (yet they are able to attain nearly the average *FG%* because of their talents). Similarly, our finding that *HH* players are able to maintain their normal success rates while taking difficult shots is an indication of their *improved* shooting ability. In a different context, the ability of managers who manage large firms to attain the same operating results that managers of small firms attain is not a proof that they are no better than managers of small firms. Rather, it shows that they are better managers who can get results on par with managers of small firms *despite* the more challenging task they face.

To the best of our knowledge, this is the first time that data are analyzed to show overall equilibrium adjustments to a shock. In the data we analyze, we can observe the reactions of *all* agents to a shock to one of them since our data cover the whole relevant population – the ten players, whose actions are continuously recorded. Thus, we can examine the equilibrium behavior of *all* agents after a shock occurs to one of them and compare the new equilibrium behavior to the pre-shock equilibrium. This analysis illustrates the difference between the analysis of a single agent and outcome and a more complete analysis of multiple effects of a single shock.

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Table 1
Player Summary Statistics

The table presents summary statistic for 503 players (95.3% of all – 528 – NBA players) who attempted at least one field goal (FGA) in the regular season of 2004-2005 and for whom we have salary data. The data reflect 1218 games (99.0% of 1230 regular games) and are taken from the official web site of the NBA. Salary information is taken from HoopsHype and USA Today web sites. We report the means, standard deviations, and quartiles (25%, 50%, and 75%) for each variable. Additionally, we report the characteristics of the star players – the #1 player in each NBA team. Star players are defined as either the player with the highest salary or as the player that has the largest number of field gold attempts (*FGA*) throughout the season. We report the season's salary (*Salary*), average number of minutes played per game (*Minute / game*), fraction of successful field-goal attempts (*FG%*), fraction of FGA that are jump shots (*Jump%*), free throw attempts per game (*FTA / game*), visits to the free throw line per game (*FTA / game*), successful free throws (*FT%*), and Hot-Hand periods a player enjoys per game (*HH / game*).

	Mean	**S.D.**	**25%**	**50%**	**75%**	**Star**	
						Salary	**FGA**
Salary	3.58	3.99	0.81	1.82	5.02	13.95	9.09
Minutes / game	20.37	10.30	11.94	19.19	28.95	33.30	36.98
FGA / game	6.61	4.60	3.00	5.49	9.10	14.19	16.55
FGA / minute	0.31	0.10	0.24	0.30	0.37	0.42	0.44
FG%	44.3	9.60	38.7	42.9	46.9	45.1	45.2
Jump%	60.1	20.4	46.7	61.9	72.9	56.3	61.2
FTA / game	2.09	1.91	0.81	1.42	2.78	4.73	6.25
Visits to line / game	1.10	1.01	0.42	0.77	1.43	2.51	3.27
FT%	75.6	14.4	66.6	75.0	81.2	74.0	78.4
HH / game	0.23	0.24	0.05	0.16	0.37	0.64	0.74

Table 2
Shot Summary Statistics

The table presents average shooting statistics of the top five NBA players in each of the 30 NBA teams who played at least half of their team's game – 41 games – in the regular season of 2004-2005. The sample is comprised of 150 players ranked in the top five in their teams (55% of all NBA players in the 2004-2005 season). These players played 57.9% of the total play time and threw 64.9% of all throws. The data reflect 1218 games (99.0% of 1230 regular games) and are taken from the official web site of the NBA. Salary information is taken from HoopsHype and USA Today web sites. We report the fraction of successful field-goal attempts (*FG%*) and successful free throws (*FT%*) by player rank. Player rank is based on either their season salary or on the number of field goal attempts in the season relative to all team members. The one-way ANOVA test is for the equality of all means. Numbers in parentheses are P values.

Ranked	**By Salary**		**By FGA**	
	FG%	**FT%**	**FG%**	**FT%**
1	45.73	75.88	45.24	78.38
2	44.31	77.40	45.65	78.29
3	44.90	73.82	45.37	78.58
4	45.25	78.17	44.09	79.33
5	46.23	75.04	45.00	74.95
One way ANOVA	1.120 (0.349)	1.978 (0.101)	0.730 (0.573)	0.826 (0.511)

Table 3
Serial Correlation in Shots

The table presents average shooting statistics of the top five NBA players in each of the 30 NBA teams in the regular season of 2004-2005. The data reflect 1218 games (99.0% of 1230 regular games) and are taken from the official web site of the NBA. Ranking of players is by their season salary relative to the salaries of all players in their team. Salary information is taken from HoopsHype and USA Today web sites. We report the fraction of successful field-goal attempts (*FG%*) by player rank and by performance in the preceding 1, 2, or 3 shots. *Unconditional FG%* refers to the overall *FG%*. *FG% 3 (*or *2*, or *1) missed* refers to *FG%* after a player misses 3 (or 2, or 1, respectively) consecutive shots. *FG% 3 (*or *2*, or *1) made* refers to *FG%* after a player succeeds in 3 (or 2, or 1, respectively) consecutive shots. Numbers in parentheses are the total number of shots in each cell.

Player Rank	***FG% 3 missed***	***FG% 2 missed***	***FG% 1 missed***	***Uncon-ditional FG%***	***FG% 1 made***	***FG% 2 Made***	***FG% 3 made***
1	45.87% (2,666)	45.78% (6,144)	46.43% (13,705)	45.78% (29,390)	45.66% (11,741)	44.64% (4,621)	43.78% (1,752)
2	46.42% (1,775)	45.50% (4,275)	45.37% (9,861)	44.31% (21,511)	43.26% (7,995)	42.01% (2, 859)	41.21% (973)
3	47.20% (1,373)	46.23% (3,489)	46.81% (8,520)	45.49% (19,219)	44.78% (7,334)	44.23% (2,667)	42.11% (938)
4	44.07% (1,323)	43.91% (3,352)	44.61% (8,065)	44.63% (18,483)	43.97% (6,853)	42.62% (2, 459)	40.68% (848)
5	48.78% (943)	47.73% (2,531)	47.29% (6,488)	46.62% (15,645)	46.23% (5,888)	46.68% (2,136)	49.33% (750)
Total	**45.87% (12168)**	**45.19% (30,480)**	**45.51% (73,673)**	**44.94% (168,295)**	**44.69% (62,460)**	**43.85% (22,627)**	**43.00% (7845)**

Table 4

Do Coaches and Players Act upon Hot Hand Beliefs?

The table presents couch and player reactions to the identification of an abnormal – Hot Hand (*HH*) – or subnormal – Cold Hand (*CH*) – shooting ability. The data reflect 1218 games (99.0% of 1230 regular games in the 2004-2005 season) and are taken from the official web site of the NBA. We identify players as *HH* if they successfully shoot 3 consecutive shots within a half. Similarly, we identify players as *CH* if they miss 3 consecutive shots within a half. We call the last shot by which a player is identified as either *HH* or *CH* the *Identifying Shot*. *Time Played After* is the time in seconds from the Identifying Shot to the replacement of the identified player by another player. *Time Until Shoots Again* is the time difference in seconds between the Identifying Shot and the next field goal attempt of the identified player in the same half. *Next Shot* is the shot taken by the team of the identified player after the Identifying Shot, independent of which player takes the shot. *% of Next Shots taken* is the fraction of all Next Shots that are taken by the identified players. *Jump Shot Proportion* is the proportion of jump shots in the total number of Next Shots. We exclude Next Shots taken within 5 seconds of the previous shot of the team (*offensive rebounds*). Numbers in parentheses are *t* values.

	HH	***CH***	**Difference**
Number of Identifying Shots	5,009	7,500	
Time Played After	290.95	255.88	35.08** (8.894)
Time Until Shoots Again	196.12	220.84	-24.72** (-5.879)
% of Next Shots taken	23.5%	16.5%	7.0%** (9.247)
Jump Shot Proportion	66.95%	56.46%	10.48%** (5.078)

** Significant at 1%.

Table 5

The Effect of Temporary Changes in Ability on *HH* and *AH* Players

The table presents the effect of having a Hot Hand (*HH*) player in a team on the performance of the *HH* player, the other players in the team – *AH* players, and the whole team. We report average play statistics of all NBA players in the regular season of 2004-2005. The data reflect 1218 games (99.0% of 1230 regular games) and are taken from the official web site of the NBA. We identify players as Hot Hand (*HH*) if they successfully shoot 3 consecutive shots within a half. We call the last shot by which a player is identified as an *HH* player the *Identifying Shot* and focus on the field goal attempt that follows the *Identifying Shot* – the *Next Shot*. . We exclude Next Shots taken within 5 seconds of the previous shot of the team (*offensive rebounds*). *FG%* is the fraction of successful field-goal attempts in the *Next Shot*. *Jump proportion* is the proportion of jump shots in the total number of Next Shots. *FG% - jump shots only* is the fraction of successful field-goal attempts of Next Shots that are jump shots. *FG% - other shots only* is the fraction of successful field-goal attempts of Next Shots that are not jump shots. *3 points made* is the fraction of successful 3 point shots in the total number of Next Shots. *Fouls per shot* is the fraction of shooting fouls called of the total number of Next Shots and free throw attempts that follow identifying shots. Numbers in parentheses are *t* statistics.

	Regular Shots	***HH***	***AH***	**Team**
Number of shots	163,966	1,174	3,819	4,993
Jump proportion	63.62%	3.33%* (2.415)	-2.24%** (-2.816)	-0.93% (-1.343)
FG% - jump shots only	35.27%	-1.68% (-0.983) [N=786]	2.96%** (2.913) [N=2,344]	1.79%* (2.045) [N=3,130]
FG% - non-jump shots only	58.94%	0.33% (0.133) [N=388]	3.56%** (2.791) [N=1,475]	2.89%** (2.528) [N=1,863]
3 points made	6.92%	-0.19% (-0.264)	1.01%* (2.289)	0.73%* (1.909)
Fouls per shot	11.77%	-1.97%* (-2.351)	0.72% (1.414)	-0.08% (-0.193)
FG%	43.88%	-1.80% (-1.247)	3.72%** (4.553)	2.42%** (3.382)

* Significant at 5%
** Significant at 1%

Table 6

PROBIT Estimates of the Impact of Temporal Changes in Shooting Ability

The table presents PROBIT estimates of the impact of having a Hot Hand (*HH*) player in a team on the performance of the *HH* player and other players – *AH* players. We report regression results of all NBA players in the regular season of 2004-2005 who had at least 5 *HH* periods in the season. The data reflect 1218 games (99.0% of 1230 regular games) and are taken from the official web site of the NBA. We identify players as Hot Hand (*HH*) if they successfully shoot 3 consecutive shots within a half. We call the last shot by which a player is identified as an *HH* player the *Identifying Shot* and focus on the field goal attempt that follows the *Identifying Shot* – the *Next Shot*. We exclude Next Shots taken within 5 seconds of the previous shot of the team (*offensive rebounds*). The estimated equation is of the probability of success in each shot controlling for the following variables:

- I_{AH} is a dummy variable that takes the value of one when the Next Shot is taken by the *AH* player and zero otherwise
- I_{HH} is a dummy variable that takes the value of one when the Next Shot is taken by the *HH* player and zero otherwise
- *SFG%* is the *FG%* of each player in the entire season normalized by the overall *FG%* in the NBA in the entire season
- I_{Jump} is a dummy variable that takes the value of one when the field goal attempt is a jump shot, which proxies for difficult shots, and zero otherwise
- I_{Half} is a dummy variable that takes the value of one when the field goal attempt is in the second half of a game and zero otherwise
- *Time* is the number of minutes the shooting player played prior to the shot
- *Def* is the difference between the *FG%* that the defensive team allowed in the season less the average *FG%* in the NBA

The table presents four equation estimates: all shots with a dummy variable for jump shots, jump shots only, non-jump shots only, and shots taken when shifting efforts across time is not likely – in last quarters when the scores of the two teams are close (less than 10 points apart). Numbers in parentheses are asymptotic standard errors.

	I_{AH}	I_{HH}	*SFG%*	I_{Jump}	I_{Half}	*Time*	*Def*
All	0.077** (0.021)	-0.038 (0.038)	1.413** (0.066)	-0.580** (0.007)	-0.029** (0.006)	-0.003** (0.001)	-2.351** (0.227)
Jump Only	0.079** (0.027)	-0.042 (0.046)	0.923** (0.088)		-0.028** (0.008)	-0.006** (0.001)	-0.749** (0.285)
Non-Jump Only	0.073** (0.034)	-0.028 (0.065)	2.069** (0.101)		-0.027* (0.010)	0.003* (0.001)	-5.083** (0.375)
Close games	0.156** (0.050)	0.079 (0.086)	1.486** (0.155)	-0.612** (0.015)		-0.002 (0.001)	-1.807** (0.537)

* Significant at 5%

** Significant at 1%

Appendix: Proofs of Results

Result 1

Since both players are of the same ability and since the offensive team shots are taken by the player with an *e* shot (unless none exists), the defensive team minimizes the probability of a successful shot by minimizing the probability of an *e* shot. The probability that an easy shot exists is $(1-\gamma_1 \cdot \gamma_2)$, which is minimized when the defensive efforts are allocated equally.

Result 2

Given an allocation of defensive efforts $\{\gamma_1, \gamma_1 | \gamma_1 + \gamma_1 = \Gamma\}$, the shot selection of the offensive team is such that the *HH* player shoots unless he has a *d* shot and the *AH* player has an *e* shot. Based on the shot selection, the *FG%* is the weighted sum of the outcomes of the four cases *{d,d}*, *{d,e}*, *{e,d}*, and *{e,e}*:

$$\begin{aligned} FG\% = {} & \gamma_1(\Gamma-\gamma_1)(O^d+\Delta) \\ & +\gamma_1[1-(\Gamma-\gamma_1)]O^e \\ & +(1-\gamma_1)(\Gamma-\gamma_1)(O^e+\Delta) \\ & +(1-\gamma_1)[1-(\Gamma-\gamma_1)](O^e+\Delta) \end{aligned} \tag{A1}$$

Minimizing the *FG%* with respect to γ_1 yields the following first order condition:

$$O^e(2\gamma_1-\Gamma)+\Delta(\Gamma-2\gamma_1-1)+O^d(\Gamma-2\gamma_1)=0 \tag{A2}$$

Solving for γ_1 yields:

$$\gamma_1 = \frac{\Gamma}{2} + \frac{\Delta}{2(O^e - O^d - \Delta)} \equiv \frac{\Gamma}{2} + \frac{\varepsilon}{2} \tag{A3}$$

Since, by assumption, $O^e > O^d + \Delta$, then $\varepsilon > 0$ and $\gamma_1 > \frac{\Gamma}{2}$. Accordingly,

$$\gamma_2 = \frac{\Gamma}{2} - \frac{\varepsilon}{2} < \frac{\Gamma}{2}$$

<u>Result 3a</u>

The first part of the proof follows from Results 2: $\gamma_1 > \gamma_2$ implies that the fraction of difficult shots that an *HH* player takes is higher than the fraction of difficult shots that an *AH* player takes.

Since the *AH* player takes only the shots where he has an *e* shot (while the *HH* player has *d* shots), the *FG%* of the *AH* player ($FG\%^{AH}$) is O^e in the *HH* case.

Since the *HH* player takes all shots where his shot difficulty is no worse than the shot difficulty of the *AH* player, the *FG%* of the *HH* player is:

$$FG\%^{HH} = \frac{\gamma_1(\Gamma - \gamma_1)(O^d + \Delta) + (1 - \gamma_1)(O^e + \Delta)}{\gamma_1(\Gamma - \gamma_1) + (1 - \gamma_1)}$$

Collecting terms we get:

$$\begin{aligned} FG\%^{HH} &= \frac{\Delta[\gamma_1(\Gamma - \gamma_1) + (1 - \gamma_1)] + O^d \gamma_1(\Gamma - \gamma_1) + O^e(1 - \gamma_1)}{\gamma_1(\Gamma - \gamma_1) + (1 - \gamma_1)} \\ &= \Delta + \frac{O^d \gamma_1(\Gamma - \gamma_1)}{\gamma_1(\Gamma - \gamma_1) + (1 - \gamma_1)} + \frac{O^e(1 - \gamma_1)}{\gamma_1(\Gamma - \gamma_1) + (1 - \gamma_1)} \\ &= \Delta + O^e + \frac{\gamma_1(\Gamma - \gamma_1)}{\gamma_1(\Gamma - \gamma_1) + (1 - \gamma_1)}(O^d - O^e) \end{aligned}$$

Therefore,

$$
\begin{aligned}
FG\%^{AH} - FG\%^{HH} &= \frac{\gamma_1(\Gamma-\gamma_1)}{\gamma_1(\Gamma-\gamma_1)+(1-\gamma_1)}(O^d - O^e) - \Delta \\
&> (O^d - O^e) - \Delta \geq 0
\end{aligned}
$$

<u>Results 3b</u>

A player erroneously perceived to be *HH* attracts more defensive efforts than usual:

$$
\gamma_1^{HH} = \frac{\Gamma}{2} + \varepsilon > \frac{\Gamma}{2} = \gamma_1^{AH}
$$

Therefore, the fraction of *d* shots he takes is higher than the normal fraction (i.e., when he is not perceived to be *HH*):

$$
FG\%^{HH} = \left(\Gamma/2 + \varepsilon\right)^2 \left(O^d + \Delta\right) + \left[1 - \left(\Gamma/2 + \varepsilon\right)^2\right]\left(O^e + \Delta\right)
$$

Since the true $\Delta = 0$ and $O^d < O^e$:

$$
\begin{aligned}
FG\%_1^{HH} &= \left(\Gamma/2 + \varepsilon\right)^2 O^d + \left[1 - \left(\Gamma/2 + \varepsilon\right)^2\right] O^e \\
&> \left(\Gamma/2\right)^2 O^d + \left[1 - \left(\Gamma/2\right)^2\right] O^e \\
&= FG\%_1^{AH}
\end{aligned}
$$

<u>Result 4</u>

Since the *FG%* of the *AH* player ($FG\%^{AH}$) is O^e in the *HH* case and it is an average of O^e and O^d in the *AH* case, the *FG%* of the *AH* player is higher in the *HH* case than in the *AH* case.

To see the improvement in the overall *FG%*, note that in the *HH* case:

$$
\begin{aligned}
FG\% &= \gamma_1(\Gamma-\gamma_1)(O^d+\Delta)+\gamma_1[1-(\Gamma-\gamma_1)]O^e \\
&\quad +(1-\gamma_1)(\Gamma-\gamma_1)(O^e+\Delta)+(1-\gamma_1)[1-(\Gamma-\gamma_1)](O^e+\Delta) \\
&\quad > \gamma_1(\Gamma-\gamma_1)O^d+\gamma_1[1-(\Gamma-\gamma_1)]O^e+(1-\gamma_1)(\Gamma-\gamma_1)O^e+(1-\gamma_1)[1-(\Gamma-\gamma_1)]O^e
\end{aligned}
$$

The latter expression is the *FG%* in the *AH* case, which is minimized with $\gamma_i = \Gamma/2$ and not with the γ_i\`s that minimize the *FG%* in the *HH* case. The second part of the result follows from the fact that *FG%* increases in Δ, which means that *FG%* is higher when Δ is positive (i.e., *HH* is a real phenomenon) than when Δ is zero (i.e., *HH* is a cognitive illusion).